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AN ALTERNATIVE APPROACH FOR CHOICE MODELS IN TRANSPORTATION: USE OF POSSIBILITY THEORY FOR COMPARISON OF UTILITIES

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Abstract: Modeling of human choice mechanism has been a topic of intense discussion in the transportation community for many years. The framework of modeling has been rooted in probability theory in which the analyst's uncertainty about the integrity of the model is expressed in probability. In most choice situations, the decision-maker (traveler) also experiences uncertainty because of the lack of complete information on the choices. In the traditional modeling framework, the uncertainty of the analyst and that of the decision-maker are both embedded in the same random term and not clearly separated. While the analyst's uncertainty may be represented by probability due to the statistical nature of events, that of the decision maker, however, is not always subjected to randomness; rather, it is the perceptive uncertainty. This paper proposes a modeling framework that attempts to account for the decision maker's uncertainty by possibility theory and then the analyst's uncertainty by probability theory. The possibility to probability transformation is performed using the principle of uncertainty invariance. The proposed approach accounts for the quality of information on the changes in choice probability. The paper discusses the thought process, mathematics of possibility theory and probability transformation, and examples.

Keywords: Choice model, random utility theory, uncertainty treatment, fuzzy sets, possibility theory.

1. INTRODUCTION

Understanding of the choice process of humans is an essential component of the transportation planning process. Given a set of stimuli in the form of transportation service alternatives, humans, individually or collectively, make certain responses or choices. To predict these responses and choices, models of human stimulus-response pattern are used. Traditionally, such models estimate human choice based on the stochastic modeling framework.

Questions have been raised regarding the interpretation and structure of such choice models - in particular, handling of uncertainty in the model. Over the years, various formulations with different degrees of sophistication have been devised to handle the uncertainty embedded in human choice process. The basic mathematical framework of the models, however, has always been probability theory with the probability distribution used to characterize the uncertainty of the analyst. One of the problems of the traditional approach has been the lack of an appropriate representation of confidence in the result that reflects the analyst's uncertainty.

This paper examines the nature of uncertainty considered in the traditional choice model and questions the appropriateness of using probability theory as the sole mathematical framework for the model of choice. It then proposes an alternative approach based on possibility theory, and shows that this approach can incorporate various levels of uncertainty. The paper then shows that possibility measure can be transformed to the probability measure through an appropriate mathematical treatment, so that the choice can still be predicted in terms of probability. This is useful for practical application of the model. The issues discussed in the paper are generic with regard to the treatment of uncertainty, and, hence, the underlying logical bases should be valid for any human choice modeling.

2. MOTIVATION AND BACKGROUND

2.1. Motivation

The traditional stochastic choice model uses utility as the agent to measure the decision-maker's preference of one alternative over the others. Utility is generally expressed by a linear combination of the performances of the attributes of the alternatives with each attribute weighted by a coefficient. Added to these terms is a random variable that is assumed to follow a certain probability density function, and it is called the random term.

While there are variations with respect to the assumptions of the probability distributions for the random terms and other details, the basic mathematical framework to deal with uncertainty is the same for all the models. It is based on probability theory. The question posed at the outset of this paper is whether the uncertainty of the analyst is reflected truthfully in the result. As seen later, in stochastic models only the fixed terms of the attributes of the alternatives affect the probability of choice. Consequently, the degree of the analyst's uncertainty does not appear to matter in the end.

2.2. Background

In choice processes, uncertainty affects a person's decision since the knowledge of alternatives is rarely complete and precise. Traditionally, randomness is the characteristics associated with uncertainty, and therefore, it is the potential for manifestation of different choices. First, in early 1970's, relevant theoretical works have been carried out about random utility models and, in particular, about the multinomial logit model. These works were formalized by Domencich and McFadden [9], while theoretical fundamentals of random utility models have been analyzed by Stopher and Meyburg [23], Williams [24], Ben Akiva and Lerman [2] Nested logit models have been investigated by Ortuzar [17] and Sobel [22], and the probit model has been deeply analyzed by Daganzo [7].

Uncertainty imbedded in different situations of choice has been studied by these stochastic approaches: De Palma, Ben Akiva, Lefevre, and Litinas [8] developed a model for stochastic equilibrium for departure time choice. Afterwards, Ben Akiva, De Palma and Kanaroglou [3] extended this model including route choice and the option of not making the trip. More recently, Cascetta [4] has analyzed day-to-day dynamics and Cascetta and Cantarella [5] within-day dynamics in a transportation network. Influence of Information - or its dual uncertainty - on users' behavior has also been studied by means of simulation frameworks proposed by Kaysi [11] for ATIS services and by Hu and Mahamassani [10].

During the same period, a set of new paradigms of uncertainty was being developed. This development started with fuzzy set theory in the late 1960's and evidence theory in the 1970's. Different measures of uncertainty emerged in the 1980's, and in the 1990's, treatment of uncertainty has been systematized by Klir [16]. These new paradigms are developed in the context of the evidence–proposition connection. While the field is not yet fully matured in terms of real world applications, a unified theory of uncertainty have provided a better insight into the understanding of uncertainty and redefined the place of probability theory when dealing with uncertainty.

Among the new theories, possibility theory is said to be amenable to the framework for representation of human perceptive uncertainty. This point has been suggested by prominent systems scientists such as Shackle [19, 20] and Cohen [6]. They argue that the traditional approaches for choice modeling using probability theory do not completely represent the true level of uncertainty in people's behavior. Possibility theory deals with uncertainty when the evidence points to a nested set of propositions; and hence, it can deal with propositions that refer to an interval as well as a single value. Under this framework, the measures of possibility and necessity are used to capture the optimistic and conservative views of the truth of a proposition. Furthermore, the new paradigm of uncertainty invariance allows for conversion of possibility measure to probability, and vice versa. These developments in theories of uncertainty provide an opportunity for us to examine handling of uncertainty in the choice model critically and to improve its logical integrity.

2.3. Review of the Terms of Uncertainty in the Traditional Choice Model

The stochastic choice models express utility of alternative i, U_i, as sum of two terms: a fixed term V_i and a random term ε. The random term represents all the uncertainties in the model. The fixed term on its turn is expressed, for a given person, as a linear function of the attributes of the alternative:

$$V_i = a_0^i + a_1^i x_1^i + a_2^i x_2^i + \cdots + a_m^i x_m^i$$

Therefore, utility of alternative i can be written as:

$$U_i = V_i + \varepsilon = a_0^i + a_1^i x_1^i + a_2^i x_2^i + \cdots + a_m^i x_m^i + \varepsilon \qquad (1)$$

The IID Assumption for the Uncertainty Term ε

This section reviews the nature of ε in terms of its mathematical property. We will use logit model as the platform, but this discussion is valid for other probability based choice models as well. Given the utility function as shown above and a set of alternatives, the probability that the alternative i is chosen over the others by an individual is found by:

$$P_{iq} = \text{Prob}(U_{iq} > U_{jq};\ i \neq j;\ i, j \in A_q) \quad \text{or}$$

$$P_{iq} = \text{Prob}(V_{iq} + \varepsilon_{iq} > V_{jq} + \varepsilon_{jq};\ i \neq j;\ i, j \in A_q) \qquad (2)$$

where, for the *q*-th decision-maker:

- *i* and *j* are alternatives;
- A_q is the set of alternatives;
- U_{iq} and U_{jq} are the utilities of alternatives:
- V_{iq} and V_{jq}, are the fixed terms;
- ε_{iq} and ε_{jq} are the random terms.

ε_{iq} and ε_{jq} are assumed to be independent and identically distributed (IID). The IID assumption suggests that ε_{iq} and ε_{jq} are not correlated and the analyst's degree of uncertainty about the representation of the choice situation is the same for all the alternatives.

The outcome, the probability that one chooses an alternative, is given by:

$$P_{iq} = \frac{e^{V_{iq}}}{\sum_j e^{V_{jq}}} \qquad (3)$$

The above approach is based on the assumption that only the expected values are known for the attributes associated with each alternative. In such a case, it is perhaps reasonable to assume that the uncertainty associated with the utility is the same among alternatives, and hence, the assumption of the IID property may be upheld.

However, different attributes and different alternatives harbor different patterns of variations in most cases. In these cases, the decision-maker's imprecise perception of the attributes is not compatible to the concept of IID. Consequently, from a theoretical point of view, logit model in the form of eq. (3) cannot adequately deal with situations in which the variances in the attributes are different.

Consider a situation in which a bus route shares the freeway lane with private cars. In this case, a conventional logit model application would assume the same variance in travel time for both buses and cars. Suppose now that a proposal to construct an exclusive bus lane is to be analyzed. The exclusive lane would bring about better reliability of travel resulting in reduced variance in travel time without really changing the mean travel time. The conventional logit model would show no effect on the share of bus rider ship, since it should use the same variance for all alternatives. This aspect has been pointed out in the past by several authors, among them Abdel-Aty, Kitamura, and Jovanis [1].

In summary, use of ε with the IID property is valid under the following conditions: given everything the same, what the analyst cannot capture in the model is identical across the alternatives. Therefore, to be consistent with the IID assumption, any differences in the uncertainty of the characteristics of the alternatives should be represented in the fixed term. The fixed term captures all the differences in the decision-maker's behavior and what is left is the same for all the alternatives.

3. THE PROPOSED MODEL: APPROACH

Given the observations above, we develop a choice model with the following objectives in mind. Terms used in this discussion, such as possibility theory and uncertainty invariance will be explained in later sections.

3.1. The Objective

Our aim is to develop a mathematical framework that captures uncertainty more faithfully than the existing models. The proposed model has the same basic axioms as the traditional choice model that the decision-makers are rational and choose the alternative with the highest value of utility. However, it will differ in the following aspects:

1. Approximate values (or interval of values), which represent decision maker's perception of the attributes, are introduced. Therefore, utility is expressed as an approximate number (or an interval) not a random number;
2. Choice of an alternative is then performed by comparing utilities expressed in approximate numbers (or intervals). The difficulty in comparing two approximate numbers signifies the difficulty of making choices when two alternatives have very close values of utility;
3. The choice probability is computed along with the confidence that the analyst can place to the conclusion.

3.2. Use of intervals and approximate numbers for comparing utilities

Intervals and approximate numbers are used to represent the imprecision in the information about the attributes as perceived by the decision-maker and by the analyst. Consequently, an approximate number, not the random value, characterizes the value of the utility. We then propose possibility theory, not probability theory, as the mathematical framework to compare the utilities expressed in approximate numbers.

Possibility theory provides a means to preserve uncertainty when comparing numbers (in our case, the values of utility) that are expressed in an interval or an approximate value. It introduces two measures, the possibility measure and the necessity measure, to capture the optimistic and conservative views when comparing two approximate values whose intervals overlap. These measures, when combined, yield a measure of confidence of the decision-maker when choosing one alternative over the others. Further, these measures are converted to the probability of choice using the principle of uncertainty invariance.

4. THE PROPOSED MODEL: MATHEMATICAL FRAMEWORK

Consider that n alternatives $A_i\ (i=1,...,n)$ exist, and each is characterized by a vector of m attributes $x_1^i, x_2^i, ..., x_m^i$. Thus $A_1 = \{x_1^1, x_2^1, ..., x_m^1\}$, $A_2 = \{x_1^2, x_2^2, ..., x_m^2\}$ etc. Utility of alternative i is expressed by: $U_i = a_0^i + a_1^i x_1^i + a_2^i x_2^i + \cdots + a_m^i x_m^i$. This section explains mathematical operations of the model.

4.1. Use of a Fuzzy Number for Representation of an Approximate Value

The values assigned to $x_j^i\ (i=1,...,n;\ j=1,...,m)$ are given either as an approximate number (interval) or as an exact number. The fuzzy number is introduced to represent the imprecise feeling for the values for some of the attributes. A fuzzy number is characterized by the membership function that defines the range and the compatibility of a specific number with the linguistic notion of the approximate value.

While the details of fuzzy set theory must be referred to many references on fuzzy sets, it must be pointed out that the membership function and the probability density function are fundamentally different. The former is the characteristic function of a fuzzy set, while the latter indicates the distribution of available evidence pointing to clearly define random events; it is a measure function. The shape of the membership function needs to be defined either subjectively or by one of several methods using data, such as the use of the neural networks. The triangular shaped membership function, defined by the center value and the spread, may be a simple and practical assumption and it is often used in the application of fuzzy logic. Fundamentals of fuzzy set theory are found in Klir and Yuan [15], Zadeh [26], and Klir [13].

4.2. Utility Expressed as an Approximate Number

Utility is now computed as a sum of fuzzy numbers according to the linear utility formulation. The arithmetic operations of fuzzy numbers representing combination of weight and attributes are performed by the extension principle of fuzzy theory. The operation is as follows:

$$h_{U_i}(x) = \max_{(x_1, x_2, ...x_m) \in f_i^{-1}(x)} \min \{h_1^i(x_1),\ h_2^i(x_2), ...h_m^i(x_m)\} \tag{4}$$

where

$f_i(x_1, x_2, ..., x_m) = U_i = a_0^i + a_1^i x_1^i + a_2^i x_2^i + \cdots + a_m^i x_m^i$;

$h_1(x_1), h_2(x_2), ..., h_m(x_m)$ are the membership functions of $x_1, x_2, ..., x_m$, respectively;

$h_{U_i(x)}$ is the membership function of utility of the alternative *i.*

A fuzzy number now expresses the value of the utility of an alternative; this means that the value of utility is an interval characterized by the membership function. In the case of logit model, the value of utility is defined as a random number distributed according to a distribution function.

4.3. Comparison of Utility: Use of Possibility Theory

To compare the values of utility expressed in fuzzy number is not simple, because each value is not a single number but an interval (or an approximate number). This may be the reason why one experiences anxiety when faced by a choice situation, particularly true when the intervals overlap. We introduce possibility theory to measure the truth that one approximate value is greater than the other.

The truth of a proposition is measured based on the available evidence. The patterns that a body of evidence points to different sets (or alternative outcomes) can be grouped into three as shown in Figure 1. They are called the conflicting pattern, the nested pattern, and the mix of these two. Each is associated with different theories of uncertainty.

When all pieces of evidence are independent of one another and each point to one and only one set as shown in Figure 1a), the body of evidence is said to be in conflict. The truth of a proposition (referring to one of the sets) is measured by the total amount of evidence pointing to the set. The mathematical framework that deals with uncertainty in this pattern of evidence is the frequency based probability theory. This situation allows us to add probabilities to obtain the probability of the union of sets (additive principle). Klir and Yuan [15] note that "probability theory is the ideal tool for formalizing uncertainty in a situation where class frequencies are known or where evidence is based on outcomes of a sufficiently long series of independent random experiments".

If the pieces of evidence point to the sets in a nested manner as shown in Figure 1b, then the evidence is generally in agreement and consistent, so that a piece of evidence supporting one set also supports its supersets and subsets. The mathematical framework that deals with the truth of a proposition in this type is called possibility theory. The important difference between probability and possibility theory is that the latter can treat uncertainty when evidence supports an interval instead of a single value or event. Klir and Yuan [15] also state "Possibility theory is ideal for formalizing incomplete information expressed in terms of fuzzy proposition…" This means that it can deal with uncertainty associated with a vague proposition, for example a proposition such as, the travel time is "less than approximately 60 min." This is the framework we propose in this paper.

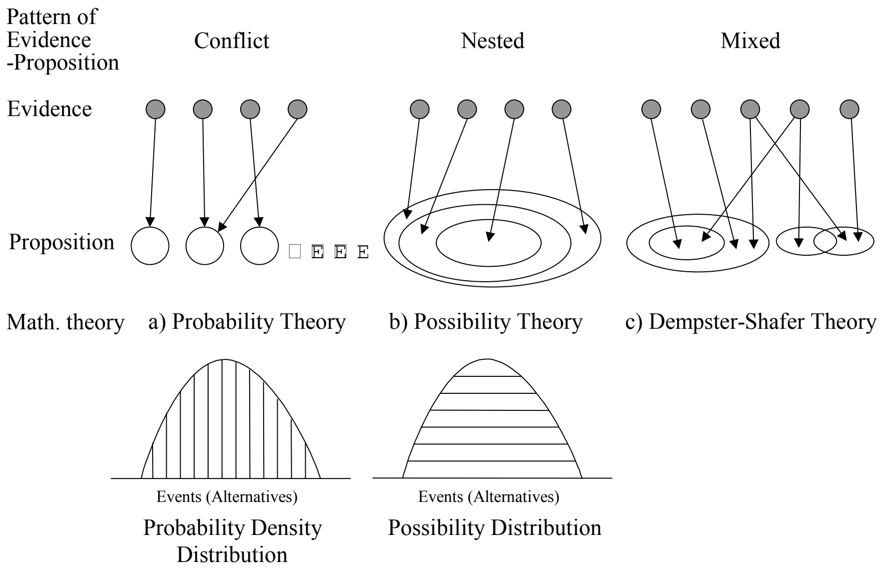

Figure 1: Three Patterns of Evidence-Proposition Connection.

When the pieces of evidence are both in conflict and nested, then the appropriate theory of uncertainty is Dempster-Shafer theory. Hence, this theory subsumes probability theory and possibility theory. For the Dempster-Shafer theory a number of references is available, among them are Shafer [21] and Yager, Fedrizzi and Kacprzyk [25].

4.4. Possibility and necessity measures

Given n alternatives, A_1 thorough A_n, suppose that the analyst's uncertainty regarding the value of utility of each alternative is represented by possibility; then, as the degree of uncertainty increases, the possibility measure (possibility that A_i is chosen) should approach one: this is the case of "anything is possible". The possibility and necessity are expressed by:

$$\text{Poss}(A_1) = 1 \text{ and } \text{Nec}(A_i) = 0 \quad \text{for all } i = \{A_1, A_2, ..., A_n\}$$

In this case, the principle of uncertainty invariance (explained later) provides:

$$\text{Poss}(A_1) = 1 \text{ for all } i = 1 \text{ to } n \;\Rightarrow\; \text{Prob}(A_i) = 1/n \text{ for all } i = 1 \text{ to } n.$$

This indicates that as uncertainty increases to the point that the analyst is totally uncertain, the choice probability approaches uniform among all alternatives. This seems reasonable.

Under possibility theory, the truth of a proposition can be stated in two ways depending on how the evidence is weighted. One way is to weigh all pieces of evidence that at least point to the proposition including ones that point to the superset as well as the ones pointing to the subsets. The other way is to weigh only the evidence that exclusively points to the proposition (the subsets). The former is called the possibility measure and the latter is called the necessity measure with the value of the former being always equal or greater than the latter. The following are basic characteristics of possibility and necessity measures:

$$\text{Poss}\,(A \cup B) = \max\ \{\text{Poss}\,(A)\,,\ \text{Poss}\,(B)\,\}\ \text{and}$$

$$\text{Nec}\,(A \cap B) = \min\ \{\text{Nec}\,(A)\,,\ \text{Nec}\,(B)\,\}$$

where $\text{Poss}\,(A)$ and $\text{Poss}\,(B)$ are possibility measures and $\text{Nec}\,(A)$ and $\text{Nec}\,(B)$ are necessity measures for *A* and *B*, respectively.

These two measures have the following dual relations:

$$\text{Possibility of } A = 1 - \text{Necessity of “not } A\text{”} \quad \text{or} \quad \text{Poss}\,(A) = 1 - \text{Nec(“not } A\text{”)}$$

$$\text{Necessity of } A = 1 - \text{Possibility of “not } A\text{“} \quad \text{or} \quad \text{Nec}\,(A) = 1 - \text{Poss(“not } A\text{”)}$$

This can be interpreted that necessity of *A* is impossibility of “not *A*”. It should be noted that the fuzzy set and possibility theory have a close link. Zadeh [26] states that a membership function of fuzzy set *A*, $h_A(x)$, induces a possibility distribution, $\pi(x)$; hence, numerically, $h_A(x) = \pi(x)$. Given the evidence in possibility distribution, $\pi(x)$, the truth of a proposition *Z* is computed by:

$$\text{Poss}\,(Z) = \max \pi(x),\ x \in Z \quad \text{if } Z \text{ is a crisp set} \tag{5}$$

or, if *Z* is a fuzzy set expressed by $h_Z(x)$

$$\text{Poss}\,(Z) = \max\,(h_Z(x), \pi(x)),\ x \in Z\ . \tag{6}$$

4.5. Application to comparison of two values

Explanations of these operations are found in many references of fuzzy theory. Comparing two fuzzy numbers is in effect the same as determining the truth of the event that one number is greater than the other. The possibility that fuzzy number B is greater than *A* is found by the truth that *B* is included in a fuzzy set of “greater than *A*”. Applying Eq (6) above:

$$\text{Poss}\,(B \geq A) = \max \min(h_B(x), \pi(x)) \ \text{ for } \ x \in X \tag{7}$$

where *X* is the range in which *x* is considered, $h_B(x)$ is the fuzzy set of *B* and $\pi(x)$ is the possibility distribution derived from fuzzy set “greater than *A*”: $\pi(x) = h_{>A}(x)$.

Because of the dual relationship, the necessity that B is greater than A is:

$$\begin{aligned}\text{Nec}\,(B \geq A) &= \{1 - \text{Poss}\,(\text{Not}\, B \geq A)\} \ \text{ or } \ \{1 - \text{Poss}\,(B < A)\} \\ &= 1 - \max \min(h_B(x), \pi'(x)) \quad \text{for } \ x \in X\end{aligned} \tag{8}$$

where $\pi'(x)$ is the possibility distribution derived from a fuzzy set “less than *A*”, $\pi'(x) = h_{<A}(x)$. This indicates that the necessity of $(B \geq A)$ is the impossibility that *B* belongs to

the set of less than *A*. Using the principles shown in Eq. (7) and (8), Figure 2 illustrates the way to calculate $\text{Poss}\,(B \geq A)$ and $\text{Nec}\,(B \geq A)$ for various shapes of membership functions for *A* and *B*.

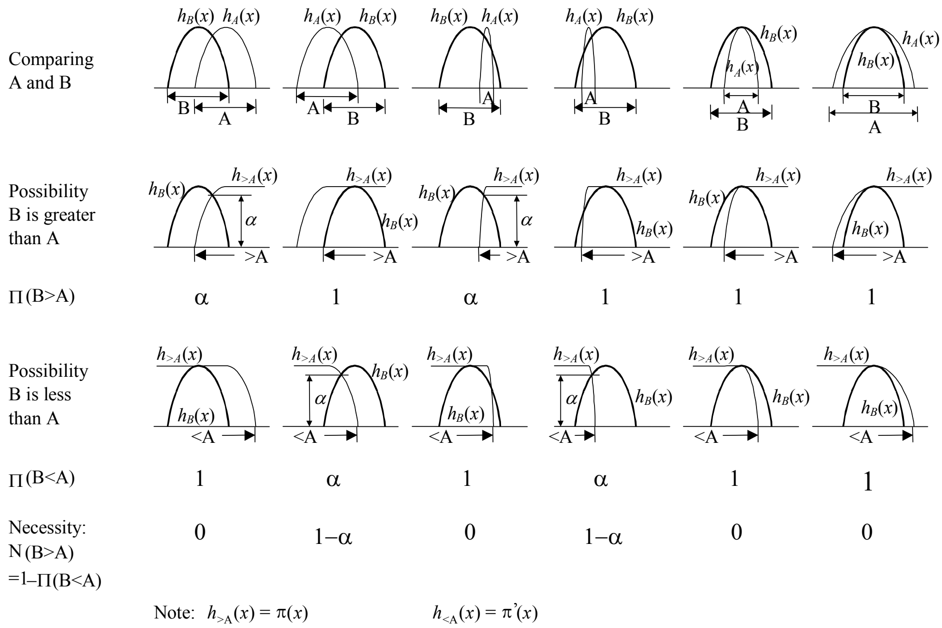

Figure 2: Possibility and Necessity Measures of "*B* is greater than *A*": $\Pi(B \geq A)$ and $\text{N}(B \geq A)$.

Based on the derivation by Perincherry [18], the confidence level associated with the proposition *Z* is computed by the amount of the evidence that strictly pointing to the proposition:

$$C(Z) = \text{Poss}(Z) - \text{Poss}(\text{Not}\, Z) = \text{Poss}(Z) - \{1 - \text{Nec}(Z)\} = \text{Poss}(Z) + \text{Nec}(Z) - 1 \qquad (9)$$

Applying this formula, the confidence of the proposition that *B* is greater than *A* is found:

$$C(B \geq A) = \text{Poss}(B \geq A) + \text{Nec}(B \geq A) - 1 \qquad (10)$$

The principle of uncertainty invariance links different modalities of mathematical representations of uncertainty. The principle, systematized by Klir and Wang [14], specifies that the uncertainty (or demand for additional information) in a given situation should be the same irrespective of what mathematical formulation is used to describe the situation. In the traditional probabilistic choice modeling situations, the amount of uncertainty associated with the probabilities can be assessed in terms of entropy of the probability distribution:

$$H = \sum_i (p_i \log_2 (p_i)) \tag{11}$$

where H is the entropy measure and p_i is the probability of choosing alternative A_i.

The same choice situation can be represented in terms of possibilities of choosing each of the alternatives. The principle of uncertainty invariance states that the U-uncertainty measure associated with the possibility distribution should be equivalent to the entropy measure of the corresponding probability distribution. The U-uncertainty measure of a possibility distribution is the counterpart of entropy in a probability distribution and it is given as:

$$U = \sum_i [\{\mathrm{Poss}(A_i) - \mathrm{Poss}(A_{i+1})\} \log_2 i] \tag{12}$$

where U is the U-Uncertainty associated with the possibility distribution of choice, and $\mathrm{Poss}(A_i)$ and $\mathrm{Poss}(A_{i+1})$ are possibilities that alternatives i and $i+1$ are the best alternatives when the alternatives are ranked in descending order from 1 to n.

The Principle of Uncertainty Invariance suggests that for the two distributions (Probability and Possibility) to describe the same situation, entropy and U-Uncertainty are equal; in other words, the following condition must be satisfied.

$$H = U \tag{13}$$

Basic ideas regarding uncertainty-preserving transformation between probability and possibility have been developed by Klir [12]. He found that interval and log-interval scales are potentially unique under the requirement of uncertainty equivalence. Given the n-tuples $p = (p_1, p_2, ..., p_n)$ and $\pi = (\pi_1, \pi_2, ..., \pi_n)$, denoting respectively ordered probability and possibility distributions that do not contain zero components, log-interval scale transformations have the form:

$$\pi_i = \beta \cdot p_i^{\alpha} \quad i = 1, 2, ..., n$$

where α and β are positive components. Hence:

$$p_i = (\pi_i / \beta)^{1/\alpha}$$

From probabilistic normalization, we obtain $\beta^{1/\alpha} = \Sigma_k \pi_k^{1/\alpha}$ and then, putting $\gamma = 1/\alpha$:

$$p_i = \frac{\pi_i^{\gamma}}{\sum_{k=1}^{n} \pi_k^{\gamma}} \tag{14}$$

This relation yields that when $\mathrm{Poss}(A_1)$, $\mathrm{Poss}(A_2)$,…$\mathrm{Poss}(A_n)$ are given, the corresponding probability that A_1 will be chosen is:

$$p_1 = \frac{\mathrm{Poss}^{\gamma}(A_1)}{\sum_{k=1}^{n} \mathrm{Poss}^{\gamma}(A_k)} \tag{15}$$

The value of parameter γ is calculated by solving numerically Eq.(13), in which the right-hand side is a constant and each p_i on the left-hand side is replaced by the corresponding expression of Eq.(15). Further, the values of Possibility and Necessity measures provide the upper and the lower bounds of the probabilities. Possibility theory may be interpreted in terms of interval-valued probabilities if the normalization requirement is applied.

$$\mathrm{Nec}(A_i) \le \mathrm{Prob}(A_i) \le \mathrm{Poss}(A_i)\,. \tag{16}$$

Thus, once Possibility and Necessity measures are derived, then the range for the value of probability can be derived.

Treatment when two alternatives have similar utilities

When the utilities of two alternatives are near equal, in the classical probabilistic choice model, the probability of choice between the two alternatives approaches 0.5. It is interesting to see that when the decision-maker has no information about the attributes of the alternatives, then his/her choice probability should become 0.5 also. Therefore, in the probability based modeling the distinction between the complete ignorance (lack of information) and the case of equal attributes (under the full information) cannot be made clearly.

In the proposed modeling framework, the confidence that one utility is greater than the other can be used to indicate the preference between two alternatives. Suppose that the fuzzy sets of utility for two alternatives are *A* and *B*, and they share the same center value. According to Eq. (9), as $\mathrm{Poss}(A < B)$ approaches 1, and at the same time, $\mathrm{Nec}(A < B)$ approaches 0, and then $C(A < B) = \{\mathrm{Poss}(A < B) + \mathrm{Nec}(A < B) - 1\}$ approaches 0. This indicates that the utility values are more or less equal, and in this case, the confidence of the proposition that one alternative is better than the other is very low (unknown). But, if the two utilities are in fact near equal (with full information), then the confidence that one is better than the other is –1, or highest confidence of negation (because $\mathrm{Poss}(A < B) = 0,\ \mathrm{Nec}(A < B) = 0$ in Eq(10)). This conclusion makes sense.

In this situation, if we assume the decision-maker's preference as "the larger utility is more or less acceptable", then the ambiguity of selecting one alternative over the others is represented better by the most possible framework than the case of probabilistic models, in which "one alternative must be absolutely better than the others". If such a preference is represented by a fuzzy set with a membership function that looks as a downward slope or one side of a triangle on the axis of utility (this fuzzy set may be called "small utility"), then this membership function can be used as the reference for comparing the membership functions of two nearly equal utilities.

Let us assume two nearly equal utility values of two alternatives, *A* and *B*, and the vague preference of the decision-maker, "the larger value is more or less acceptable", *C*. Let the membership function of fuzzy set *C* be a slope or one side of a triangle. Then the degree of compatibility between *A* and *C*, and *B* and *C* is measured separately. It is measured by

$$\mathrm{Poss}(A \subseteq C) = \max\min\{h_A(x), h_C(x)\}$$
$$\mathrm{Poss}(B \subseteq C) = \max\min\{h_B(x), h_C(x)\}$$

Similarly the necessity measures are computed for the compatibility between *A* and *C*, and *B* and *C* respectively.

$$\text{Nec}(A \subseteq C) = 1 - \text{Poss}(A \not\subset C) = 1 - \max\min\{h_A(x), h_{\text{Not}C}(x)\}$$
$$\text{Nec}(B \subseteq C) = 1 - \text{Poss}(B \not\subset C) = 1 - \max\min\{h_B(x), h_{\text{Not}C}(x)\}$$

After calculating the confidence of $A \subseteq C$ and $B \subseteq C$, the alternative that has the higher confidence is considered the acceptable alternative, because this alternative is compatible with set *C*. Then, the possibility and necessity measures for $A \subseteq B$ are used to define a probability interval.

The concept of choosing an alternative with maximum utility is reasonable when the decision-maker clearly has the objective of "only the maximum utility is acceptable" (the winner takes all) regardless how close the utilities of two alternatives is. If, on the other hand, the decision-maker does not have such a clear conviction, and rather, he has an idea of "the larger utility is more or less acceptable", then the alternatives can be compared by the method we propose above. The latter may indeed be the case of travel path choice process of most daily trips, in which the trip maker does not have the absolute preference, and the choice criterion itself is vague.

5. EXAMPLE: USE OF THE PROPOSED FRAMEWORK

Assume an urban area road network as shown in Figure 3. A fuzzy number with a triangular shaped membership function indicates the estimated travel time $T(A_i)$ on the generic link i. The travel times in minutes are represented by two-tuples with mid-values and spread. For example, (10,2) is a fuzzy number with a mid-value of 10 and a spread of 2 on either side. Thus, the lower value is 8 and the upper value is 12.

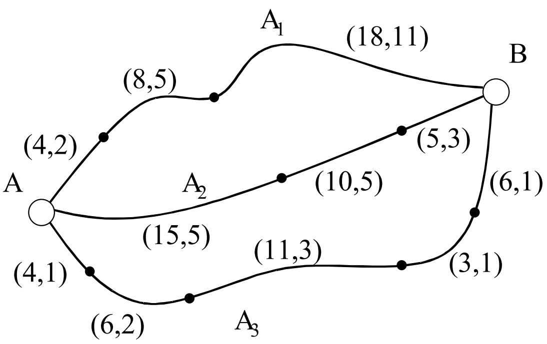

Figure 3: Hypothetical Travel Times and Three Paths.

Let us consider the route choice from node *A* to node *B*. Three alternative paths are indicated as A_1, A_2, A_3. The driver is to select the shortest path.

5.1. Expressing the travel times and comparing travel times

Travel time on each path is given as a fuzzy number with mid-value and spread. The membership functions of these values are shown in Figure 4.

$$\begin{aligned} &\text{Path 1} \quad T(A_1) = (4,2) + (8,5) + (18,11) = (30,18) \\ &\text{Path 2} \quad T(A_2) = (15,5) + (10,5) + (5,3) = (30,13) \\ &\text{Path 3} \quad T(A_3) = (4,1) + (6,2)+(11,3)+(3,1) +(6,1) = (30,8) \end{aligned} \tag{17}$$

Since travel times are the same, in comparing the travel times among the paths we introduce the vague preference of the decision-maker as a downward slope *C*. The travel time of each path (given in fuzzy values) is compared to *C* and then the possibility and necessity measure that the travel time on path i belongs to the preference set, $\text{Poss}(T(A_i) \subseteq C)$ and $\text{Nec}(T(A_i) \subseteq C)$, are computed referencing Figure 4.

$$\begin{aligned} &\Pi(T(A_1) \subseteq C) = 0.743; \\ &\Pi(T(A_2) \subseteq C) = 0.725; \\ &\Pi(T(A_3) \subseteq C) = 0.705; \\ &\text{N}(T(A_1) \subseteq C) = 0; \\ &\text{N}(T(A_2) \subseteq C) = 0; \\ &\text{N}(T(A_3) \subseteq C) = 0; \end{aligned} \tag{18}$$

where $\Pi(T(A_1) \subseteq C)$ is identical to $\text{Poss}(T(A_1) \subseteq C)$, and $\text{N}(T(A_1) \subseteq C)$ is identical to $\text{Nec}(T(A_1) \subseteq C)$.

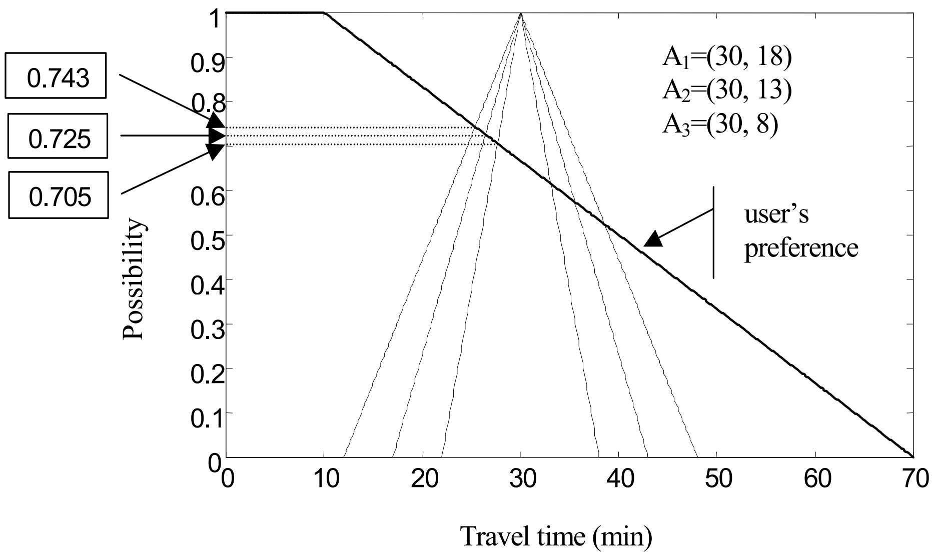

Figure 4. Comparison between travel time and decision-maker's preference

What we see above is the possibilistic and necessity views of preference of each route. For example, the truth that path A_1 is the preferred one is given by the two measures, the possibilistic view 0.743 ($\Pi(T(A_1)\subseteq C)=0.743$), and necessity view 0 ($\mathrm{N}(T(A_1)\subseteq C)=0$).

The confidence that each path is the preferred one is found using Eq (10):

$$C(A_i \subseteq C)=\Pi(A_i \subseteq C)+\mathrm{N}(A_i \subseteq C)-1$$

accordingly,

$$C(A_1 \subseteq C)=-0.257;\quad C(A_2 \subseteq C)=-0.275;\quad C(A_3 \subseteq C)=-0.295 \tag{19}$$

The negative values of confidence indicate that the truth that each of these routes is the preferred is negated, and the degree of negation is the absolute value of the confidence. Based on the computation above, the alternative A_1 has the higher (although negative) value of confidence and therefore would be the route preferred by a traveler who has no other biases in choosing the route.

5.2. Transformation to the Probability Measure

The possibility values for the choice of individual routes, can now be computed according to the uncertainty invariance as described before.

The probabilities that Paths A_1, A_2 and A_3 are chosen are given by Eq. (14):

$$\begin{aligned}
p(A_1)&=\Pi(A_1 \subseteq C)^{\gamma}/[\Pi(A_1 \subseteq C)^{\gamma}+\Pi(A_2 \subseteq C)^{\gamma}+(A_3 \subseteq C)^{\gamma}]\\
p(A_2)&=\Pi(A_2 \subseteq C)^{\gamma}/[\Pi(A_1 \subseteq C)^{\gamma}+\Pi(A_2 \subseteq C)^{\gamma}+(A_3 \subseteq C)^{\gamma}]\\
p(A_3)&=\Pi(A_3 \subseteq C)^{\gamma}/[\Pi(A_1 \subseteq C)^{\gamma}+\Pi(A_2 \subseteq C)^{\gamma}+(A_3 \subseteq C)^{\gamma}]
\end{aligned} \tag{20}$$

and the value of parameter γ is calculated by:

$$H(p(A_i))=U(\Pi(A_i \subseteq C)) \tag{21}$$

where $H(p(A_i))$ represents the entropy of the probability distribution about A_i, and $U(\Pi(A_i \subseteq C))$ represents the U-Uncertainty of the possibility distribution about $A_i \subseteq C$.

The entropy corresponding to this probability distribution is, according to Eq. (11):

$$H=p(A_1)\log_2 p(A_1)+p(A_2)\log_2 p(A_2)+p(A_3)\log_2 p(A_3) \tag{22}$$

The U-Uncertainty of the possibility distribution is obtained according to Eq. (12), given the values of possibilities from Eq (18), $\mathrm{Poss}(A_1)=0.743$, $\mathrm{Poss}(A_2)=0.725$, and $\mathrm{Poss}(A_3)=0.705$:

$$U=(0.743-0.725)*\log_2(1)+(0.725-0.705)*\log_2(2)+(0.705-0)\log_2(3)$$

or

$$U=0.02*\log_2(2)+0.705*\log_2(3)=1.14 \tag{23}$$

Equating H and U, entropy and U-Uncertainty we obtain, $\gamma = 42.6$ and

$$p(A_1) = 0.69, \quad p(A_2) = 0.24, \quad p(A_3) = 0.07 \tag{24}$$

In terms of the range of probability:

$$\begin{aligned} \text{Nec}(A_1) < p(A_1) < \text{Poss}(A_1) \Rightarrow 0 < p(A_1) < 0.743 \\ \text{Nec}(A_2) < p(A_2) < \text{Poss}(A_2) \Rightarrow 0 < p(A_2) < 0.725 \\ \text{Nec}(A_3) < p(A_3) < \text{Poss}(A_3) \Rightarrow 0 < p(A_3) < 0.705 \end{aligned} \tag{25}$$

What is shown here is a simple example that involves three choices (routes) and the travel time as the only attribute or decision criterion. Each value of the travel times, however, is considered an interval incorporating the notion of variation. Possibility and necessity that each route will be chosen by the decision-maker are computed. The confidence associated with the decision is also computed. Finally, the corresponding probability of choice is derived. We believe that this presentation provides more information as to the nature of uncertainty of the estimate.

6. SUMMARY

The proposed approach differs from the traditional approach to model choice in three aspects. First, the decision-maker's perceptive uncertainty about the information on the attributes is accounted for by the use of the approximate number (fuzzy number). Second, the values of utility of different alternatives (expressed in fuzzy numbers) are compared in terms of the possibility and necessity measures. Third, the possibility and necessity-based comparison of the utilities are converted to the probability that utility of an alternative is greater than the other, using the technique of uncertainty invariance. The obtained value of probability has a range. This range is derived as a result of the possibilistic and necessity views. Further, confidence associated with the probability is provided.

The proposed framework will be useful in dealing with choice situation when the information about the value of attributes is incomplete, e.g., in approximate number, interval, or linguistic expression. The approach can respond to the degree of accuracy of the data, and yields choice probabilities faithfully to the quality of information. It is the authors' belief that different mathematical frameworks are possible depending on the nature of uncertainty embedded in the problem situation. The authors do not undermine the classical stochastic choice model approach if the nature of uncertainty in the problem is purely probabilistic. This research presents what possibility theory and uncertainty invariance can offer in dealing with situation in which the quality of information about the attributes and the decision-maker's perception of the information is in question.

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