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# BUILD TO ORDER AND ENTRY/EXIT STRATEGIES UNDER EXCHANGE RATE UNCERTAINTY

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**Abstract:** Under uncertainty of exchange rate, we extend the build to order production model of Lin *et al*. (2002) by considering the export-oriented manufacturer to make decisions to switch production location freely between domestic and foreign ones. The export-oriented manufacturer is risk neutral and has rational expectations. When we transfer the production location from domestic (foreign) to foreign (domestic), and the production location transferring cost and the drift of real exchange rate are both equal to zero, then the optimal entry and exit threshold value of Cobb-Douglas production function are equal, no matter whether we use real options or net present value method. Thus export-oriented manufacturer can make decisions at the optimal transfer threshold value for transferable locations wherever the production locations are. It provides the export-oriented manufacturer with another way of thinking.

**Keywords:** Build to order, entry and exit, exchange rate uncertainty, real options.

## 1. INTRODUCTION

Following the collapse of the Bretton Woods System in 1973, industrialized uncertainty significantly influences the cash flows of exporting manufacturers and influences export-oriented manufacturer' choices concerning the production location. Most industries use real exchange rates to determine expected remuneration. Blonigen (1997) and Tomlin (1998) have claimed that the threshold value of the real exchange rate is an important decision-making index. Management's flexibility to respond to alter future market conditions increases an investment opportunity's value by improving its upside potential while limiting the downside losses, in relation to initial expectations under the assumption of passive management. The omit asymmetry due to managerial adaptability requires an "expanded NPV" rule that reflects both value components: the traditional (static or passive) NPV of direct cash flows, and the option value of operating and strategic adaptability. This claim doesn't mean that traditional NPV should be ignored, but rather that it should be seen as a crucial and necessary input to an options-based, expanded NPV analysis, like that of Trigeorgis (1995). Accordingly, the complete NPV method, correctly stated, is as follows:

*Expanded* (*Strategic*) *NPV = Static* (*Passive*) *NPV of expected cash flows*

*+ value of options from active management*

Dixit (1989a) applies the real options method to model multiple industries entry to or exit from the U. S. market. Dixit (1989b) applies the real options method to model single industry entry to or exit market. When the output price follows a random walk is examined. An idle firm and an active firm are viewed as assets that call options on each other. The solution is a pair of the optimal threshold prices for entry and exit. Weeds (1999) considers optimal investment behavior for a firm facing both technological and economic uncertainty and gets the optimal investment strategy. If the lump-sum exit and entry cost are equal to zero, then the optimal entry and exit threshold value will converge to the same value. Furthermore, Abel (1983) determined the value of a competitive firm, using the Cobb-Douglas production function for essentially planned production. Abel's model, assumes that the firm is risk-neutral and maximizes the expected present value of its cash flow subject to the capital accumulation equation. The revenue is a function of the two decision-variables capital and undertakes gross investment; the revenue function is the sale function. Thus, the sale function depends on the capital and undertakes gross investment, such that the production is basically planned. The value of the firm is then becomes the maximized expected present value of cash flows in an uncertain market price.

Industrial structural changes and build to order production have slowly replaced planned production, making the method of determining the value to industry of adopting the build to order process, extremely important. For example, TSMC (Taiwan Semiconductor Manufacturing Company) and UMC (United Microelectronics Corporation) produce the more expensive chips and adopt the BTO (Built to Order) to proceed with the production. In Taiwan, the investment is more uncertainty and the real wages are too high, so they will transfer the productive base from Taiwan to China or other countries. Therefore, few investigations have discussed investment decisions relating to the build to order process and referring mainly to Lin *et al.* (2002). Lin *et al.* (2002) establish a decision valuation model for selecting the optimal location, determines the optimal entry threshold value, and then explains its economic meaning. The model is established using one export-oriented manufacturer who produces domestically, and one that produces at the overseas point of sale (foreign). Either the domestic and foreign production locations can be selected, or export-oriented manufacturer can then transfer local production overseas and find an optimal production location where production can be achieved for less than the market value of the existing production location. To calculate the optimal entry and exit threshold value of the export-oriented manufacturer decides to transferable location. This investigation yields the optimal entry and exit threshold value of the export-oriented manufacturer's decisions to transfer production location, and performs a sensitivity analysis between the internal and external factor with the optimal entry and exit threshold value of the export-oriented manufacturer who decide to transfer their production location. They examine the manufacturing export-oriented manufacturer's production in the build to order process given exchange rate uncertainty. This investigation will extend Lin *et al.* (2002) to transfer freely between domestic and foreign and we will establish the transferable model freely in the Cobb-Douglas build to order process.

## 2. ENTRY/EXIT IN COBB-DOUGLAS BUILD TO ORDER PROCESS MODEL

This section states the assumptions and notation of the build to order process using the Cobb-Douglas production function. An export-oriented manufacturer with a constant capacity is considered to produce a fixed quantity of goods, $\psi$ , which exactly meets market demand. Only the labor, $L$ , the raw materials, $\xi$ , and the fixed technology parameter, $A$ , influence the production function. The export-oriented manufacturer sells overseas and the net profit is measured in local currency. Tariff, $\tau$ are levied on the export-oriented manufacturer overseas, and the price of the goods is $P_s^*$ , in foreign currency, at time, $s$ . Let $R$ be the real exchange rate, namely the real price of the foreign currency in terms of the local currency. The real exchange rate is assumed to be following geometric Brownian motion:

$$\frac{dR_t}{R_t} = \mu dt + \sigma dZ(t) . \tag{1}$$

Here $dz$ denotes an increment in the standard wiener process; $\mu$ represents the drift of the real exchange rate, and $\sigma$ is the volatility of the real exchange rate. As a first approximation, this approach has considerable empirical support from Frankel and Meese (1987). The build to order process model produces output according to a constantly elastic substitution production function in domestic (or foreign) market. The factors in the function include domestic (foreign) labor, $L_s$ ( $L_s^*$ ), at time, $s$ , domestic (foreign) raw materials, $K_s$ ( $K_s^*$ ), at time, $s$ , and the technological parameters, domestic (foreign), $A_s$ ( $A_s^*$ ), at time, $s$ . Other external factors include as real wages, $\omega_s$ ( $\omega_s^*$ ), in domestic (foreign) market, at time, $s$ , the prices of real raw materials, $\xi_s$ ( $\xi_s^*$ ), in domestic (foreign) market, at time, $s$ , and the quantity of sales, $\psi_s$ ( $\psi_s^*$ ), in domestic (foreign) market, at time, $s$ . $r$ is the risk-free interest rate. Assuming the coefficient correlation between the real exchange rate and the market portfolio is equal to zero. Thus the risk-adjusted rate is equal to risk-free interest rate. If a local export-oriented manufacturer decides to transfer production from the domestic (foreign) market to foreign (domestic) market, then the cost of transferring the production location, $E^*$ ( $l^*$ ), should be paid in foreign currency.

This section investigates the build to order process, using the Cobb-Douglas production function, satisfying the maximum value of the export-oriented manufacturer's product both domestically and overseas. This study is to optimize Labor, $L_s$, and raw materials, $\xi_s$, for which special solutions are then obtained. This solution depends on the value matching and smooth pasting conditions from the real options to determine the approach solution and optimize the threshold value for export-oriented manufacturer who decide to transfer their production location. The optimal entry (exit) threshold value for export-oriented manufacturer deciding to transfer their locations from domestic (foreign) to foreign (domestic) using real options is the $R^L_{ROA}$ ( $R^H_{ROA}$ ) method, while that for the export-oriented manufacturer deciding to transfer their location using the NPV method is $R^L_{NPV}$ ( $R^H_{NPV}$ ).

**Definition 1.** *The value of export-oriented manufacturer's domestic production is defined as*

$$V^d\left(R_t\right) = \underset{\{L_t, K_t\}}{Max}\, E_t\left[\int_t^\infty e^{-r(s-t)}[\left(1-\tau\right)R_s\psi_s P_s^* - \omega_s L_s - \xi_s K_s]ds\right] \tag{2}$$

$$s.t.\ \ \psi_s = A_s L_s^\alpha K_s^{1-\alpha}.$$

The value of the export-oriented manufacturer's domestic production is the maximized expected present value of the cash flows. In the Cobb-Douglas Build to order Process Model, the cash flow at time $s$ is discounted to time $t$ using the risk-free interest rate $r$, and the decision-variables are labor, $L_s$, and raw materials, $K_s$. The export-oriented manufacturer considers customer demand and production output according to the Cobb-Douglas production function. In Eq. (2), the revenue is $(1-\tau)R_s\psi_s P_s^*$ , where $\psi_s$ represents sales volume and is constant. The revenue function equals to sale function and the sale function doesn't depend on Labor and raw materials. Herein, the sales volume is defined in build to order terms, such that export-oriented manufacturer produce output according to orders for goods. Export-oriented manufacturer thus consider customer demand and production output according to order levels, using optimal labor, $L_s$, and raw materials, $K_s$.

By considering the problem of finding the optimal control $\{L_t, K_t\}$ in a small time interval from t to t + dt, one can write the maximum productive value of export-oriented manufacturer produces in domestic. According to *Stochastic Dynamic Programming* (Dixit and Pindyck, 1994), you can find the answer for this question (See Page 93-132):

$$V^d\left(R_t\right) = \underset{\{L_t, K_t\}}{Max}\, E_t\left[\left[\left(1-\tau_0\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right]dt + e^{-rdt}V^d\left(R_t + dR_t\right)\right]$$

$$= \underset{\{L_t, K_t\}}{Max}\left[\left[\left(1-\tau\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right]dt + E_t\left(1-rdt\right)\left[V^d\left(R_t\right) + dV^d\left(R_t\right)\right]\right]$$

$$s.t.\ \ \psi_t = A_t L_t^\alpha K_t^{1-\alpha}.$$

Implying

$$rV^d\left(R_t\right) = \underset{\{L_t, K_t\}}{Max}\left\{\left[\left(1-\tau\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right] + \frac{1}{dt}E_t dV^d\left(R_t\right)\right\}$$

$$s.t.\ \ \psi_t = A_t L_t^{\alpha} K_t^{1-\alpha}\ .$$

The export-oriented manufacturer's maximum domestic production must satisfy the following optimality condition, Eq. (3)

$$rV^d(R_t)dt = \underset{\{L_t, K_t\}}{Max}\left[\left(1-\tau\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right]dt + E_t\left[dV^d(R_t)\right] \tag{3}$$

$$s.t.\ \ \psi_s = A_s L_s^{\alpha} K_s^{1-\alpha}\ .$$

The optimality condition in Eq. (3) has a straightforward economic interpretation. If the owners of the firm require the risk-free interest rate $r$, then the left-hand side of Eq. (3) is the total mean return required by the owners of the firm over the time interval, $dt$. The right-hand side of Eq. (3) specifies the total return expected by the owners of the firm, and includes cash flows and the expected capital gain. Optimality requires that the expected return equals the required mean return. The capital gain, $dV^d$ is calculated by recognizing that the value of the firm is a function of a single state variable, $R_t$, and then applying the It'o Lemma to obtain,

$$dV^d\left(R_t\right) = V_R^d dR_t + \frac{1}{2}V_{RR}^d\left(dR_t\right)^2. \tag{4}$$

Substituting Eq. (1) into Eq. (4), and recognizing that $E_t(dz) = (dt)^2 = (dt)(dz) = 0$. We obtain the expected change in the value of the firm over the time interval $dt$:

$$E_t\left[dV^d\left(R_t\right)\right] = \left[\mu R_t V_R^d\left(R_t\right) + \frac{1}{2}\sigma^2 R_t^2 V_{RR}^d\left(R_t\right)\right]dt. \tag{5}$$

Substituting Eq. (5) into Eq. (3) yields

$$rV^d(R_t) = \underset{\{L_t, K_t\}}{Max}\left[\left(1-\tau\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t + \left(\mu R_t V_R^d\left(R_t\right) + \frac{1}{2}\sigma^2 R_t^2 V_{RR}^d\left(R_t\right)\right)\right]. \tag{6}$$

Applying the Lagrange multiplier, it is easily shown that (see appendix A)

$$\underset{\{L_t, K_t\}}{Max}\left[\left(1-\tau\right)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right] = \left[(1-\tau)R_t\psi_t P_t^* - \frac{\psi_t\omega_t^{\alpha}\xi_t^{1-\alpha}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right]\right]$$

$$s.t.\ \ \psi_t = A_t L_t^{\alpha} K_t^{1-\alpha}\ . \tag{7}$$

Substituting Eq. (7) into Eq. (6) yields

$$\frac{1}{2}\sigma^2 R_t^2 V_{RR}^d + \mu R_t V_R^d - rV^d + \left[(1-\tau)R_t\psi_t P_t^* - \frac{\psi_t \omega_t^\alpha \xi_t^{1-\alpha}}{A_t} \times \left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right]\right] = 0 . \tag{8}$$

Where the value of export-oriented manufacturer's domestic production is $V^d\left(R_t\right)$

$$V^d\left(R_t\right) = \frac{R_t}{(r-\mu)}(1-\tau)\psi_t P_t^* - \frac{\psi_t \omega_t^\alpha \xi_t^{1-\alpha}}{rA_t}\left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right]. \tag{9}$$

The above observation resembles that for a static economy. From a financial perspective, we can get $r - \mu > 0$. Otherwise, investing in the asset depends on risk aversion, since money could be borrowed at $\mu$, and then invested without risk at $r$, producing unlimited profits. Furthermore, from a traditional economic perspective, the value of an export-oriented manufacturer's domestic production equals the market value of the export-oriented manufacturer's domestic production. However, if the real exchange rate continues to rise, then export-oriented manufacturer will transfer the location of their domestic production to reduce the cost of production. Otherwise, they will come into being opportunity costs. Given uncertain exchange rates, meaning that, export-oriented manufacturer's domestic production is likely to hold an American call. Thus, the value of the option to transfer the location of production (i.e., the productive value of the export-oriented manufacturer produces in domestic prior to acceptance), $F^d(R_t)$, satisfies an ordinary differential equation of the form specified in Dixit and Pindyck (1994, pp.140-144). (See appendix B)

$$\frac{1}{2}\sigma^2 F_{RR}^d R_t^2 + \mu F_R^d R_t - rF^d = 0 . \tag{10}$$

The solution is given by

$$F^d(R_t) = A_1 R_t^{\beta_1} + A_2 R_t^{\beta_2} , \tag{11}$$

where the two roots are defined as

$$\beta_1 = \frac{1}{\sigma^2}\left\{-(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}\right\} > 1,$$

$$\beta_2 = \frac{1}{\sigma^2}\left\{-(\mu - \frac{1}{2}\sigma^2) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2}\right\} < 0 .$$

$A_1 R_t^{\beta_1}$ diverges if the real exchange rate, $R_t$, approaches infinity. The export-oriented manufacturer won't transfer production location perpetually and the waiting–time value approaches zero. Therefore, $A_1 = 0$ must be set and Eq. (11) must be corrected to

$F^d\left(R_t\right)=A_2 R_t^{\beta_2}$. **The market value of the export-oriented manufacturer's domestic production is then defined as**

$$V^D\left(R_t\right)=V^d\left(R_t\right)+A_2 R_t^{\beta_2}$$

$$=\frac{R_t}{(r-\mu)}(1-\tau)\psi_t P_t^* - \frac{\psi_t \omega_t^{\alpha} \xi_t^{1-\alpha}}{rA_t}\left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}+\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right]+A_2 R_t^{\beta_2}\ . \quad (12)$$

Notably, the market value of the export-oriented manufacturer's domestic production is a nonlinear function of the real exchange rate.

**Definition 2.** *The value of export-oriented manufacturer's foreign production is defined as*

$$V^f\left(R_t\right)=\underset{\left\{L_t^*, K_t^*\right\}}{Max}\, E_t\left[\int_t^{\infty} e^{-r(s-t)} R_s\left[\psi_s^* P_s^*-\omega_s^* L_s^*-\xi_s^* K_s^*\right] ds\right] \quad (13)$$

$$s.t.\ \ \psi_s^*=A_s^* L_s^{\alpha^*} K_s^{1-\alpha^*}$$

$$dR_s=\mu R_s ds+\sigma R_s dZ(s)\ .$$

The production value of the export-oriented manufacturer's overseas production is the maximized expected present value of cash flows, and the economic mean resembles Definition 1: By the same definition, the value of the export-oriented manufacturer's foreign production should satisfy Eq. (14).

$$\begin{aligned}&\frac{1}{2}\sigma^2 R_t^2 V_{RR}^f+\mu R_t V_R^f-rV^f\\&+R_t\left[\psi_t^* P_t^*-\frac{\psi_t^* \omega_t^{*\alpha^*} \xi_t^{*1-\alpha^*}}{\left(r-\mu\right) A_t^*}\times\left[\left(\frac{\alpha^*}{1-\alpha^*}\right)^{-\alpha^*}+\left(\frac{\alpha^*}{1-\alpha^*}\right)^{1-\alpha^*}\right]\right]=0\ ,\end{aligned} \quad (14)$$

where **the value of export-oriented manufacturer's foreign production is** $V^f\left(R_t\right)$

$$V^f\left(R_t\right)=R_t\left\{\frac{1}{\left(r-\mu\right)}\left[\psi_t^* P_t^*-\frac{\psi_t^* \omega_t^{*\alpha^*} \xi_t^{*1-\alpha^*}}{A_t^*}\left[\left(\frac{\alpha^*}{1-\alpha^*}\right)^{-\alpha^*}+\left(\frac{\alpha^*}{1-\alpha^*}\right)^{1-\alpha^*}\right]\right]\right\}. \quad (15)$$

This result is similar to that for the static economy in Definition 1. Similarly, the value of the option to transfer the location of production is, $F^f(R_t)$

$$F^f(R_t)=A_1 R_t^{\beta_1}+A_2 R_t^{\beta_2}\ , \quad (16)$$

where the two roots are defined as

$$\beta_1 = \frac{1}{\sigma^2}\left\{-(r-\frac{1}{2}\sigma^2)+\sqrt{(r-\frac{1}{2}\sigma^2)^2+2r\sigma^2}\right\} > 1,$$

$$\beta_2 = \frac{1}{\sigma^2}\left\{-(r-\frac{1}{2}\sigma^2)+\sqrt{(r-\frac{1}{2}\sigma^2)^2+2r\sigma^2}\right\} < 0\,.$$

$A_2 R_t^{\beta_2}$ diverges if the real exchange rate, $R_t$, approaches zero. An export-oriented manufacturer will relocate his domestic production immediately and the waiting-time value approaches zero. Consequently, $A_2 = 0$ must be set and Eq. (16) must be corrected to $F^d\left(R_t\right) = A_1 R_t^{\beta_1}$. Consequently the **market value of the export-oriented manufacturer's foreign production is then defined as,**

$$V^F\left(R_t\right) = V^f\left(R_t\right) + A_1 R_t^{\beta_1}$$

$$= R_t\left\{\frac{1}{\left(r-\mu\right)}\left[\psi_t^* P_t^* - \frac{\psi_t^* \omega_t^{*\alpha^*} \xi_t^{*1-\alpha^*}}{A_t^*}\left[\left(\frac{\alpha^*}{1-\alpha^*}\right)^{-\alpha^*} + \left(\frac{\alpha^*}{1-\alpha^*}\right)^{1-\alpha^*}\right]\right]\right\} + A_1 R_t^{\beta_1}\,. \quad (17)$$

Notably, the market value of an export-oriented manufacturer's foreign production is a nonlinear function of the real exchange rate. If a local export-oriented manufacturer chooses to switch production from domestic (foreign) to foreign (domestic), then the cost of transferring production location $E^*$ ($l^*$) should be paid in foreign currency. The market value related with the export-oriented manufacturer of domestic and foreign production is defined as,

$$V^D(R_t^L) = V^F(R_t^L) - E^* R_t^L\,, \quad (18)$$

$$V^F(R_t^H) = V^D(R_t^H) - l^* R_t^H\,. \quad (19)$$

The method by which a production location is decided is now considered. Dixit and Pindyck (1994) outlined the conditions as follows.

$$V^D(R_t^L) = V^F(R_t^L) - E^* R_t^L\,, \quad (20)$$

$$V_R^D(R_t^L) = V_R^F(R_t^L) - E^*\,, \quad (21)$$

$$V^F(R_t^H) - l^* - l^* R_t^H\,, \quad (22)$$

$$V_R^F(R_t^H) = V_R^D(R_t^H) - l^*\,. \quad (23)$$

Eq. (20)-(23), are nonlinear equations to which a closed-form solution can not be determined.

## 3. COMPARING THE DECISION METHOD

**Theorem 1.** *The optimal entry (exit) threshold value of the export-oriented manufacturer deciding to transfer their location using the NPV method,* $R_{NPV}^{L}$ *(* $R_{NPV}^{H}$ *).*

$$R_{NPV}^{L} = \frac{H}{\left[G+H^{*}+E^{*}\right]},\ R_{NPV}^{H} = \frac{H}{\left[G+H^{*}-l^{*}\right]}.$$

**Proof:** From traditional NPV method, Eq. (20)-(23) should correct to Eq. (24)-(27)

$$V^{d}(R_{NPV}^{L}) = V^{f}(R_{NPV}^{L}) - E^{*}R_{NPV}^{L}, \tag{24}$$

$$V_{R}^{d}(R_{NPV}^{L}) = V_{R}^{f}(R_{NPV}^{L}) - E^{*}, \tag{25}$$

$$V^{f}(R_{NPV}^{H}) = V^{D}(R_{NPV}^{H}) - l^{*}R_{NPV}^{H}, \tag{26}$$

$$V_{R}^{f}(R_{NPV}^{H}) = V_{R}^{d}(R_{NPV}^{H}) - l^{*}. \tag{27}$$

From Eq. (24)-(27), we can determine optimal entry (exit) threshold value of the export-oriented manufacturer deciding to transfer their location using the NPV method, $R_{NPV}^{L}$ ( $R_{NPV}^{H}$ ).

$$R_{NPV}^{L} = \frac{H}{\left[G+H^{*}+E^{*}\right]},\ R_{NPV}^{H} = \frac{H}{\left[G+H^{*}-l^{*}\right]},$$

where

$$H \equiv \frac{\psi_t \omega_t^{\alpha} \xi_t^{1-\alpha}}{rA_t}\left[\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right],$$

$$H^{*} \equiv \frac{\psi_t^{*} \omega_t^{*\alpha^{*}} \xi_t^{*1-\alpha^{*}}}{(r-\mu)A_t^{*}}\left[\left(\frac{\alpha^{*}}{1-\alpha^{*}}\right)^{-\alpha^{*}} + \left(\frac{\alpha^{*}}{1-\alpha^{*}}\right)^{1-\alpha^{*}}\right],$$

$$G \equiv \frac{1}{(r-\mu)} P_t^{*}[(1-\tau)\psi_t - \psi_t^{*}].$$

Q.E.D.

Sensitive analysis of other parameters is follow below

| | $\omega$ | $\omega^{*}$ | $\xi$ | $\xi^{*}$ | $A$ | $A^{*}$ | $\tau$ | $E^{*}$ | $l^{*}$ | $\mu$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_{NPV}^{L}$ | + | - | + | - | - | + | - | - | No | + | No |
| $R_{NPV}^{H}$ | + | - | + | - | - | + | - | No | + | + | No |

**Theorem 2.** *If the entry and exit costs of the export-oriented manufacturer deciding to transfer their location, which are both to zero, then optimal entry and exit threshold value of the export-oriented manufacturer deciding to transfer their location using the NPV method, which will converge to* $R^*_{NPV} = \dfrac{H}{\left[G+H^*\right]}$.

**Proof:** Because $R^H_{NPV} > 0$, thus $G + H^* > l^* > 0$.

$$If \;\; E^* = l^* = 0 \;,\; then \; R^*_{NPV} = R^H_{NPV} = R^L_{NPV} = \frac{H}{\left[G+H^*\right]} .$$

Q.E.D.

**Theorem 3.** *If the entry and exit costs of the export-oriented manufacturer deciding to transfer their location, which are both to zero, then optimal entry and exit threshold value of the export-oriented manufacturer deciding to transfer their location using the real options analysis, which will converge to* $R^*_{ROA} = \left(\dfrac{r}{r-\mu}\right) \times \left(\dfrac{H}{G+H^*}\right)$.

**Proof:** If $E^* = l^* = 0$, then Eq. (20)-(23) should correct to Eq. (28)-(31)

$$V^D(R^L_{ROA}) = V^F(R^L_{ROA}) , \tag{28}$$

$$V^D_R(R^L_{ROA}) = V^F_R(R^L_{ROA}) , \tag{29}$$

$$V^F(R^H_{ROA}) = V^D(R^H_{ROA}) , \tag{30}$$

$$V^F_R(R^H_{ROA}) = V^D_R(R^H_{ROA}) . \tag{31}$$

From Eq. (28)-(31), we know $R^H_{ROA} = R^L_{ROA} = R^*_{ROA}$. The value-matching and smooth-pasting conditions at $R^*_{ROA}$ are given respectively by

$$A_1\left(R^*_{ROA}\right)^{\beta_1} = A_2\left(R^*_{ROA}\right)^{\beta_2} + R^*_{ROA}\left[G+H^*\right] - H , \tag{32}$$

$$\beta_1 A_1\left(R^*_{ROA}\right)^{\beta_1 - 1} = \beta_2 A_2\left(R^*_{ROA}\right)^{\beta_2 - 1} + \left[G+H^*\right]. \tag{33}$$

Solving for the unknown constants $A_2$ and $A_1$, the following expression are derived

$$A_2 = -\frac{\left(R^*_{ROA}\right)^{-\beta_2}}{\left(\beta_1 - \beta_2\right)}\left\{\left(\beta_1 - 1\right)\left[G+H^*\right]R^*_{ROA} - \beta_1 H\right\} ,$$

$$A_1 = -\frac{\left(R^*_{ROA}\right)^{-\beta_1}}{\left(\beta_1 - \beta_2\right)}\left\{\left(\beta_2 - 1\right)\left[G+H^*\right]R^*_{ROA} - \beta_2 H\right\} .$$

The value function $V^F\left(R_t\right)$ can then be written as

$$V^F\left(R_t\right)=V^f\left(R_t\right)-R_tE^*+A_1\left(R^*_{ROA}\right)R_t^{\beta_1}.$$

The first order condition with respect to $R^*_{ROA}$, ensuring optimality, is given by $V^F_{R^*_{ROA}}(R_t)=0$, for any arbitrary non-zero value of $R_t$, requires $\dfrac{\partial A_1\left(R^*_{ROA}\right)}{\partial R^*_{ROA}}=0$. Thus,

$$\frac{\partial B_1\left(R^*_{ROA}\right)}{\partial R^*_{ROA}}=-\frac{\left(R^*_{ROA}\right)^{\beta_1-1}}{\left(\beta_1-\beta_2\right)}\left\{R^*_{ROA}\left(\beta_1-1\right)\left(\beta_2-1\right)\left[G+H^*\right]-\beta_1\beta_2H\right\}=0.$$

Solving the first-order condition, the following expression for the optimal switching point $R^*_{ROA}$ is obtained. The second-order condition is negative, ensuring that the point is a maximum.

Q.E.D.

**Remarks:** Several results follow immediately from Theorem 1, 2, 3. (a) If the drift of real exchange rate goes to zero, then $R^*_{ROA}$ and $R^*_{NPV}$ are equal. (b) If the drift of real exchange rate rises, then $R^*_{ROA}$ and $R^*_{NPV}$ will increase. Consequently, the real exchange rate volatility won't affect $R^*_{ROA}$ and $R^*_{NPV}$.

Sensitive analysis of other parameters is follow below

| | $\omega$ | $\omega^*$ | $\xi$ | $\xi^*$ | $A$ | $A^*$ | $\tau$ |
|---|---|---|---|---|---|---|---|
| $R^*_{ROA}$ | + | - | + | - | - | + | - |
| $R^*_{NPV}$ | + | - | + | - | - | + | - |

## 4. CONCLUSIONS

The model is established using a single export-oriented manufacturer who produces domestically, and for one that produces at the overseas point of sale (foreign). Either the domestic or the foreign production location can be selected, and an export-oriented manufacturer can then freely transfer local production overseas and obtain an optimal location at which production can be achieved for less than the market value of the existing production location. This investigation yields the optimal threshold value for the export-oriented manufacturer's decisions to switch the location of production. A sensitivity analysis of the relevant internal and external factors is also presented. The above-mentioned result can provide an export-oriented manufacturer with a reference for decision-making. No matter whether we use real options or traditional NPV method, as a local export oriented manufacturer decides to transfer their production from domestic (foreign) to foreign (domestic) and the cost of transferring production location $E^*$ ($l^*$) and the drift of real exchange rate are both equal to zero, then optimal entry and exit threshold value will converge to the same value.

The authors hope that this model represents a useful beginning of the important examination of the effects of exchange rate fluctuations on industry in which sunk costs are important. At various points, possible extensions of the model in future research are indicated. Building a Syscom On-Line EERP (Extended Enterprise Resource Planning) system can help in choosing the optimal production locations (more than one) anywhere in the world. This basic build to order production model can help domestic industry become international industry.

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## APPENDIX A

If $\frac{K_t}{L_t} = \frac{(1-\alpha)\omega_t}{\alpha\xi_t}$, then Eq. (7) is hold.

**Proof:**

Step 1: (first order condition)

Object function: $\underset{\{L_t, K_t\}}{Max}\left[(1-\tau)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t\right]$

By Lagrange Multipliers, we let $f(L_t, K_t) = A_t L_t^{\alpha} K_t^{1-\alpha}$ and we can get

$$\begin{aligned} \amalg(L_t, K_t, \lambda) &= [(1-\tau)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t] + \lambda_t\left[A_t L_t^{\alpha} K_t^{1-\alpha} - \psi_t\right] \\ &= [(1-\tau)R_t\psi_t P_t^* - \omega_t L_t - \xi_t K_t] + \lambda_t\left[f - \psi_t\right], \end{aligned} \tag{A.1}$$

$$\frac{\partial \amalg(L_t, K_t, \lambda)}{\partial L_t} = -\omega_t + \lambda_t\left[\alpha A_t L_t^{\alpha-1} K_t^{1-\alpha}\right] = 0, \tag{A.2}$$

$$\frac{\partial \amalg(L_t, K_t, \lambda)}{\partial K_t} = -\xi_t + \lambda_t\left[(1-\alpha)A_t L_t^{\alpha} K_t^{-\alpha}\right] = 0, \tag{A.3}$$

$$\frac{\partial \amalg(L_t, K_t, \lambda)}{\partial \lambda_t} = A_t L_t^{\alpha} K_t^{1-\alpha} - \psi_t = 0. \tag{A.4}$$

From Eqs. (A.2)-(A.3), we can get Eq. (A.5)

$$\frac{\omega_t}{\xi_t} = \frac{\lambda_t\left[\alpha A_t L_t^{\alpha-1} K_t^{1-\alpha}\right]}{\lambda_t\left[(1-\alpha)A_t L_t^{\alpha} K_t^{-\alpha}\right]} = \frac{\alpha K_t}{(1-\alpha)L_t}. \tag{A.5}$$

By Eqs. (A.2)-(A.5), we can get the optimal solution ( $\tilde{L}_t, \tilde{K}_t, \tilde{\lambda}$ )

$$\tilde{L}_t = \frac{\psi_t}{A_t}\left(\frac{\alpha\xi_t}{(1-\alpha)\omega_t}\right)^{(1-\alpha)}, \tag{A.6}$$

$$\tilde{K}_t = \frac{\psi_t}{A_t}\left(\frac{\alpha\xi_t}{(1-\alpha)\omega_t}\right)^{-\alpha}, \tag{A.7}$$

$$\tilde{\lambda}_t = \frac{\omega_t}{\left[\alpha A_t L_t^{\alpha-1} K_t^{1-\alpha}\right]} = \frac{\xi_t}{\left[(1-\alpha)A_t L_t^{\alpha} K_t^{-\alpha}\right]} > 0. \tag{A.8}$$

Step 2: (second order condition)

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial L_t \partial L_t} = \lambda_t f_{LL}(L_t,K_t) = \lambda_t\left[\alpha(\alpha-1)A_t L_t^{\alpha-2}K_t^{1-\alpha}\right], \tag{A.9}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial L_t \partial K_t} = \lambda_t f_{LK}(L_t,K_t) = \lambda_t\left[\alpha(1-\alpha)A_t L_t^{\alpha-1}K_t^{-\alpha}\right], \tag{A.10}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial L_t \partial \lambda_t} = f_L(L_t,K_t) = \alpha A_t L_t^{\alpha-1}K_t^{1-\alpha}, \tag{A.11}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial K_t \partial K_t} = \lambda f_{KK}(L_t,K_t) = \lambda_t\left[(1-\alpha)(-\alpha)A_t L_t^{\alpha}K_t^{-\alpha-1}\right], \tag{A.12}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial K_t \partial L_t} = \lambda f_{KL}(L_t,K_t) = \lambda_t\left[\alpha(1-\alpha)A_t L_t^{\alpha-1}K_t^{-\alpha}\right], \tag{A.13}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial K_t \partial \lambda} = f_K(L_t,K_t) = (1-\alpha)A_t L_t^{\alpha}K_t^{-\alpha}, \tag{A.14}$$

$$\frac{\partial^2 \coprod(L_t,K_t,\lambda)}{\partial \lambda \partial \lambda} = 0. \tag{A.15}$$

Substituting Eqs. (A.6)-(A.14) into determine $\Delta$:

$$\Delta = \begin{vmatrix} \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial L_t \partial L_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial L_t \partial K_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial L_t \partial \lambda_t} \\ \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial K_t \partial L_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial K_t \partial K_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial K_t \partial \lambda_t} \\ \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial \lambda_t \partial L_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial \lambda_t \partial K_t} & \frac{\partial^2 \coprod(\tilde{L}_t,\tilde{K}_t,\tilde{\lambda}_t)}{\partial \lambda_t \partial \lambda_t} \end{vmatrix}$$

$$= \begin{vmatrix} \tilde{\lambda} f_{LL}(\tilde{L}_t,\tilde{K}_t) & \tilde{\lambda} f_{LK}(\tilde{L}_t,\tilde{K}_t) & f_L(\tilde{L}_t,\tilde{K}_t) \\ \tilde{\lambda} f_{KL}(\tilde{L}_t,\tilde{K}_t) & \tilde{\lambda} f_{KK}(\tilde{L}_t,\tilde{K}_t) & f_K(\tilde{L}_t,\tilde{K}_t) \\ f_L(\tilde{L}_t,\tilde{K}_t) & f_K(\tilde{L}_t,\tilde{K}_t) & 0 \end{vmatrix}$$

$$= \tilde{\lambda}\left[2 f_L f_K f_{LK} - (f_L)^2 f_{KK} - (f_K)^2 f_{LL}\right] = \tilde{\lambda}\left[-\alpha(1-\alpha)A_t^3 L_t^{3\alpha-2}K_t^{1-3\alpha}\right] > 0.$$

Step 3: by step 1 and 2. Substituting Eq.(A.5) into object function, then the proof is finished.

Q.E.D.

## APPENDIX B

When we calculate the value of the firm's option to transfer the production from domestic (foreign) to foreign (domestic), we have consider the hedge of the real exchange by foreign currencies and this calculating process is as follows: (see Dixit and Pindyck, 1994)

Let $F = F^{d(f)}\left(R_t\right)$ be the value of the firm's option to transfer the production from domestic (foreign) to foreign (domestic). Consider the following portfolio: Hold the option to transfer the product location, which is worth $F$, and go $a$ units short in the $R$. The value of this portfolio is $\Phi = F - aR$. So we have $\Phi = F - aR_{10}$ immediately and $F_R = a$. Note that this portfolio is dynamic; as $R$ changes, $F_R$ may change from one short interval of time to the next, so that the composition of the portfolio will be change. However, over each short interval of length $dt$, we hold $a$ fixed. The short position in this portfolio will require a payment of $\delta RF_R$ dollars per time period, where $\delta$ is the difference between $r$ and $\mu$, that is, $\delta = r - \mu$; otherwise no rational firm will entry into the long side of the transaction. A firm holding a long position in the project will demand the risk-adjusted return $rR$, which equals the capital gain $\mu R$ plus the dividend stream $\delta R$. Since the short position includes $F_R$ unit of the project, it will require paying out $\delta RF_R$. Taking this payment into account, the total return from holding the portfolio over a short time interval $dt$ is

$$d\Phi = d\left(F - aR\right) = d\left(F - aR\right) = dF - F_R dR - \delta RF_R dt .$$

To obtain an expression for $dF$, use Ito's Lemma:

$$dF = F_R dR + \frac{1}{2} F_{RR}\left(dR\right)^2 .$$

Hence the total return on the portfolio is

$$d\Phi = \frac{1}{2} F_{RR}\left(dR\right)^2 - \delta RF_R dt .$$

From equation (1) for $dF$, we know that $\left(dR\right)^2 = \sigma^2\left(R\right)^2 dt$ so the return on the portfolio becomes

$$\frac{1}{2}\sigma^2 R^2 F_{RR} dt - \delta RF_R dt .$$

Note that this return is risk-free. Hence to avoid arbitrage possibilities, it must equal $r\Phi dt = r\left[F - F_R R\right]dt$:

$$\frac{1}{2}\sigma^2 R^2 F_{RR} dt - \delta RF_R dt = r\left[F - F_R R\right]dt .$$

Dividing through by $dt$ and rearranging gives the following differential equation that $F$ must satisfy:

$$\frac{1}{2}\sigma^2 R^2 F_{RR} + (r-\delta) R F_R - rF = 0 .$$

We know $\delta = r - \mu$, so we have

$$\frac{1}{2}\sigma^2 R^2 F_{RR} + \mu R F_R - rF = 0 .$$

The solution is given by,

$$F^{d(f)}(R_t) = A_1 (R_t)^{\beta_1} + A_2 (R_t)^{\beta_2} , \tag{B.1}$$

where the characteristic equation is $Q \equiv 0.5 \times \sigma^2 \beta(\beta-1) + \mu\beta - r = 0$ and the two roots are defined as

$$\beta_1 = \frac{1}{\sigma^2}\left\{ -(\mu - \frac{1}{2}\sigma^2) + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2} \right\} > 1 ,$$

$$\beta_2 = \frac{1}{\sigma^2}\left\{ -(\mu - \frac{1}{2}\sigma^2) - \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2} \right\} < 0 .$$

$A_1 (R_t)^{\beta_1}$ will diverge if the real exchange rate, $R_t$, approaches infinity. The exporter won't transfer production location from locally to the other country perpetually and the waiting–time's value approaches to zero. Therefore, $A_1 = 0$ must be set and Eq. (A.1) must be corrected to $F^d (R_t) = A_2 (R_t)^{\beta_2}$. Otherwise, $A_2 (R_t)^{\beta_2}$ will diverge if the real exchange rate, $R_t$, approaches to zero. The exporter won't transfer production location from the other country to locally perpetually and the waiting–time value approaches zero. Therefore, $A_2 = 0$ must be set and Eq. (B.1) must be corrected to $F^f (R_t) = A_1 (R_t)^{\beta_1}$.