A PRODUCTION INVENTORY MODEL WITH DETERIORATING ITEMS AND SHORTAGES

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Abstract: A continuous production control inventory model for deteriorating items with shortages is developed. A number of structural properties of the inventory system are studied analytically. The formulae for the optimal average system cost, stock level, backlog level and production cycle time are derived when the deterioration rate is very small. Numerical examples are taken to illustrate the procedure of finding the optimal total inventory cost, stock level, backlog level and production cycle time. Sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system.

Keywords: Deteriorating item, shortage, economic order quantity model.

1. INTRODUCTION

In recent years, the control and maintenance of production inventories of deteriorating items with shortages have attracted much attention in inventory analysis because most physical goods deteriorate over time. The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item can not be used for its original purpose. Food items, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Research in this direction began with the work of Whitin [16] who considered fashion goods deteriorating at the end of a prescribed storage period. Ghare and Schrader [7] developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal [15]. Aggarwal [1] reconsidered this model by rectifying the error in the work of Shah and Jaiswal [15] in calculating the average inventory holding cost. In all these models, the demand rate and the deterioration

rate were constants, the replenishment rate was infinite and no shortage in inventory was allowed.

Researchers started to develop inventory systems allowing time variability in one or more than one parameters. Dave and Patel [5] discussed an inventory model for replenishment. This was followed by another model by Dave [4] with variable instantaneous demand, discrete opportunities for replenishment and shortages. Bahari-Kashani [2] discussed a heuristic model with time-proportional demand. An Economic Order Quantity (EOQ) model for deteriorating items with shortages and linear tend in demand was studied by Goswami and Chaudhuri [8]. On all these inventory systems, the deterioration rate is a constant.

Another class of inventory models has been developed with time-dependent deterioration rate. Covert and Philip [3] used a two-parameter Weibull distribution to represent the distribution of the time to deterioration. This model was further developed by Philip [13] by taking a three-parameter Weibull distribution for the time to deterioration. Mishra [11] analyzed an inventory model with a variable rate of deterioration, finite rate of replenishment and no shortage, but only a special case of the model was solved under very restrictive assumptions. Deb and Chaudhuri [6] studied a model with a finite rate of production and a time-proportional deterioration rate, allowing backlogging. Goswami and Chaudhuri [9] assumed that the demand rate, production rate and deterioration rate were all time dependent. Detailed information regarding inventory modelling for deteriorating items was given in the review articles of Nahmias [12] and Rafaat [14]. An order-level inventory model for deteriorating items without shortages has been developed by Jalan and Chaudhuri [10].

In the present paper we have developed a continuous production control inventory model for deteriorating items with shortages. It is assumed that the demand rate and production rate are constants and the distribution of the time to deterioration of an item follows the exponential distribution. The main focus is on the structural behaviour of the system. The convexity of the cost function is established to ensure the existence of a unique optimal solution. The optimum inventory level is proved to be a decreasing function of the deterioration rate where the deterioration rate is taken as very small and the cycle time is taken as constant. The formulae for the optimal average system cost, stock level, backlog level and production cycle time are derived when the deterioration rate is very small. Numerical examples are taken and the sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system.

2. NOTATIONS AND MODELLING ASSUMPTIONS

The following notations and assumptions are used for developing the model.

(i) a is the constant demand rate.
(ii) $p\ (> a)$ is the constant production rate.
(iii) C_1 is the holding cost per unit per unit time.
(iv) C_2 is the shortage cost per unit per unit time.
(v) C_3 is the cost of a deteriorated unit.
       (C_1, C_2 and C_3 are known constants)
(vi) C is the total inventory cost or the average system cost.

(vii) $Q(t)$ is the inventory level at time t (≥ 0).
(viii)   Replenishment is instantaneous and lead time is zero.
(ix) T is the fixed duration of a production cycle.
(x)   Shortages are allowed and backlogged.
(xi)   The distribution of the time to deterioration of an item follows the exponential distribution $g(t)$ where

$$g(t) = \begin{cases} \theta e^{-\theta t}, & \text{for } t > 0, \\ 0, & \text{otherwise.} \end{cases}$$

θ is called the deterioration rate; a constant fraction θ ($0 < \theta << 1$) of the on-hand inventory deteriorates per unit time. It is assumed that no repair or replacement of the deteriorated items takes place during a given cycle.

Here we assume that the production starts at time $t = 0$ and stops at time $t = t_1$. During $[0, t_1]$, the production rate is p and the demand rate is a ($< p$). The stock attains a level Q_1 at time $t = t_1$. During $[t_1, t_2]$, the inventory level gradually decreases mainly to meet demands and partly for deterioration. The stock falls to the zero level at time $t = t_2$. Now shortages occur and accumulate to the level Q at time $t = t_3$. The production starts again at a rate p at $t = t_3$ and the backlog is cleared at time $t = T$ when the stock is again zero. The cycle then repeats itself after time T.

This model is represented by the following diagram:
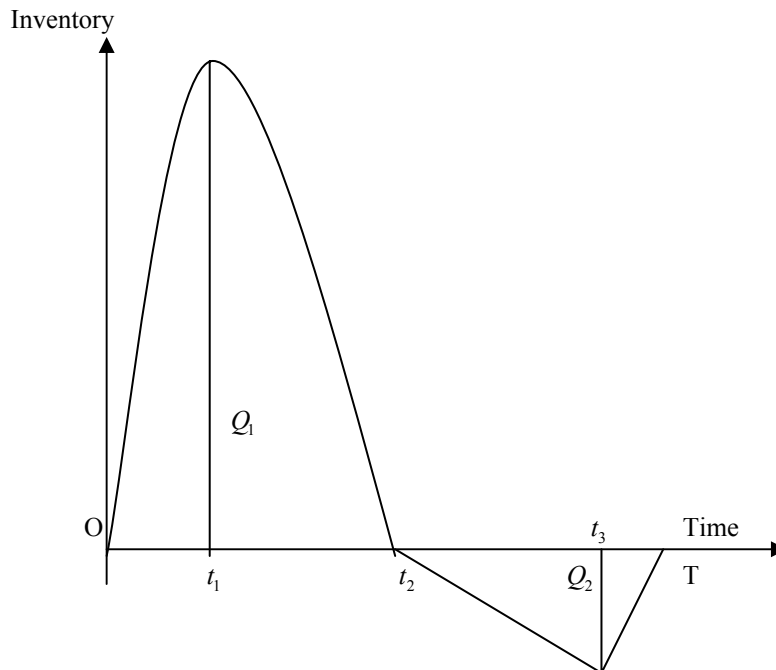
3. THE MATHEMATICAL MODEL AND ITS ANALYSIS

Let $Q(t)$ be the on-hand inventory at time t ($0 \leq t \leq T$). Then the differential equations governing the instantaneous state of $Q(t)$ at any time t are given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = p - a, \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dQ(t)}{dt} + \theta Q(t) = -a, \quad t_1 \leq t \leq t_2 \tag{2}$$

$$\frac{dQ(t)}{dt} = -a, \quad t_2 \leq t \leq t_3 \tag{3}$$

$$\frac{dQ(t)}{dt} = p - a, \quad t_3 \leq t \leq T \tag{4}$$

The boundary conditions are

$$Q(0) = 0, \; Q(t_1) = Q_1, \; Q(t_2) = 0, \; Q(t_3) = -Q_2, \; Q(T) = 0 \tag{5}$$

The solutions of equations (1) – (4) are given by

$$Q(t) = \frac{1}{\theta}(p - a)(1 - e^{-\theta t}), \quad 0 \leq t \leq t_1 \tag{6}$$

$$= -\frac{a}{\theta} + (Q_1 + \frac{a}{\theta})e^{\theta(t_1 - t)}, \quad t_1 \leq t \leq t_2 \tag{7}$$

$$= a(t_2 - t), \quad t_2 \leq t \leq t_3 \tag{8}$$

$$= (p - a)(t - t_3) - Q_2, \quad t_3 \leq t \leq T \tag{9}$$

From (5) and (6), we have

$$Q_1 = Q(t_1) = \frac{1}{\theta}(p - a)(1 - e^{-\theta t_1})$$

$$\Rightarrow e^{\theta t_1} = [1 - \frac{\theta Q_1}{(p - a)}]^{-1}$$

$$\Rightarrow t_1 = \frac{1}{\theta}\log[1 + \{\frac{\theta Q_1}{(p - a)} + \frac{\theta^2 Q_1^2}{(p - a)^2}\}] \tag{10a}$$

$$= \frac{Q_1}{p - a} + \frac{\theta Q_1^2}{2(p - a)^2} \tag{10b}$$

(neglecting higher powers of θ, $0 < \theta \ll 1$) .

Again from (5) and (7), we have

$$0 = Q(t_2) = -\frac{a}{\theta} + (Q_1 + \frac{a}{\theta})e^{\theta(t_1 - t_2)}$$

$$\Rightarrow t_2 - t_1 = \frac{1}{\theta}\log(1 + \frac{\theta Q_1}{a}) \tag{11}$$

$$\Rightarrow t_2 = \frac{1}{\theta}\log[(1 + \frac{\theta Q_1}{a})\{1 + \frac{\theta Q_1}{(p-a)} + \frac{\theta^2 Q_1^2}{(p-a)^2}\}] \quad \text{(using (10))} \tag{12}$$

Using the condition $Q(t_3) = -Q_2$, we have from (8)

$$a(t_2 - t_3) = -Q_2$$

$$\Rightarrow t_3 = \frac{Q_2}{a} + t_2 \tag{13}$$

$$\Rightarrow t_3 = \frac{Q_2}{a} + \frac{1}{\theta}\log[(1 + \frac{\theta Q_1}{a})\{1 + \frac{\theta Q_1}{(p-a)} + \frac{\theta^2 Q_1^2}{(p-a)^2}\}] \tag{14}$$

From (9) and $Q(T) = 0$, we have

$$(p-a)(T - t_3) = Q_2 \tag{15}$$

Therefore, total deterioration in $[0, T]$

$$= \{(p-a)t_1 - Q_1\} + \{Q_1 - a(t_2 - t_1)\}$$

$$= [\frac{(p-a)}{\theta}\log\{1 + \frac{\theta Q_1}{p-a} + \frac{\theta^2 Q_1^2}{(p-a)^2}\} - Q_1] + [Q_1 - \frac{a}{\theta}\log(1 + \frac{\theta Q_1}{a})]$$

$$= \frac{p}{\theta}\log\{1 + \frac{\theta Q_1}{p-a} + \frac{\theta^2 Q_1^2}{(p-a)^2}\} - \frac{a}{\theta}\log[\{1 + \frac{\theta Q_1}{p-a} + \frac{\theta^2 Q_1^2}{(p-a)^2}\}(1 + \frac{\theta Q_1}{a})]$$

$$= \frac{p}{\theta}\{\frac{\theta Q_1}{p-a} + \frac{\theta^2 Q_1^2}{(p-a)^2} - \frac{\theta^2 Q_1^2}{2(p-a)^2}\}$$

$$- \frac{a}{\theta}\{\frac{\theta Q_1}{p-a} + \frac{\theta^2 Q_1^2}{(p-a)^2} + \frac{\theta Q_1}{a} + \frac{\theta^2 Q_1^2}{a(p-a)} - \frac{\theta^2 Q_1^2 p^2}{2a^2(p-a)^2}\}$$

(Neglecting higher powers of θ)

$$= \frac{\theta Q_1^2 p}{2a(p-a)} \tag{16}$$

The deterioration cost over the period $[0, T]$

$$= \frac{C_3 \theta p Q_1^2}{2a(p-a)} \tag{17}$$

The shortage cost over the period $[0, T]$

$$= C_2 \int_{t_2}^{T} \{-Q(t)\} \, dt$$

$$= -C_2 [\int_{t_2}^{t_3} a(t_2 - t) dt + \int_{t_3}^{T} \{(p - a)(t - t_3) - Q_2\} \, dt] \quad \text{(by (8) and (9))}$$

$$= \frac{C_2 Q_2^2 p}{2a(p - a)} \quad \text{(by using (13) and (15))} \tag{18}$$

The inventory carrying cost over the cycle $[0, T]$

$$= C_1 \int_{0}^{t_2} Q(t) \, dt$$

$$= C_1 [\int_{0}^{t_1} \frac{(p - a)}{\theta} (1 - e^{-\theta t}) dt + \int_{t_1}^{t_2} \{-\frac{a}{\theta} + (Q_1 + \frac{a}{\theta}) e^{\theta(t_1 - t)}\} \, dt] \tag{19}$$

Now, $\quad \int_{0}^{t_1} \frac{(p - a)}{\theta} (1 - e^{-\theta t}) \, dt$

$$= \frac{(p - a)}{2} t_1^2 (1 - \frac{\theta t_1}{3}) \quad \text{(neglecting higher powers of } \theta \text{)}$$

$$= \frac{Q_1^2}{2(p - a)} + \frac{\theta Q_1^3}{3(p - a)^2} \quad \text{(using (10) and neglecting higher powers of } \theta \text{)} \tag{20}$$

$$\int_{t_1}^{t_2} \{-\frac{a}{\theta} + (Q_1 + \frac{a}{\theta}) e^{\theta(t_1 - t)}\} \, dt = \frac{a}{\theta}(t_1 - t_2) + (Q_1 + \frac{a}{\theta})\frac{1}{\theta}\{1 - e^{-\theta(t_2 - t_1)}\}$$

$$= \frac{a}{\theta} \cdot \frac{1}{\theta} \log(1 + \frac{\theta Q_1}{a})^{-1} + (Q_1 + \frac{a}{\theta})\frac{1}{\theta}\{1 - (1 + \frac{\theta Q_1}{a})^{-1}\} \quad \text{(by 11))}$$

$$= \frac{a}{\theta^2} \log\{1 - (\frac{\theta Q_1}{a} - \frac{\theta^2 Q_1^2}{a^2} + \frac{\theta^3 Q_1^3}{a^3})\} + (Q_1 + \frac{a}{\theta})\frac{1}{\theta}\{1 - (1 - \frac{\theta Q_1}{a} + \frac{\theta^2 Q_1^2}{a^2} - \frac{\theta^3 Q_1^3}{a^3})\}$$

$$= \frac{a}{\theta^2}\{-\frac{\theta Q_1}{a} + \frac{\theta^2 Q_1^2}{a^2} - \frac{\theta^3 Q_1^3}{a^3} - \frac{1}{2}(\frac{\theta^2 Q_1^2}{a^2} - \frac{2\theta^3 Q_1^3}{a^3})\} + (Q_1 + \frac{a}{\theta})(\frac{Q_1}{a} - \frac{\theta Q_1^2}{a^2} + \frac{\theta^2 Q_1^3}{a^3})$$

$$= \frac{Q_1^2}{2a} \quad \text{(neglecting higher powers of } \theta \text{)} \tag{21}$$

Therefore, the inventory carrying cost over the cycle $[0, T]$

$$= C_1\{\frac{Q_1^2}{2(p - a)} + \frac{\theta Q_1^3}{3(p - a)^2} + \frac{Q_1^2}{2a}\} = C_1\{\frac{Q_1^2 p}{2a(p - a)} + \frac{\theta Q_1^3}{3(p - a)^2}\} \tag{22}$$

Hence the total inventory cost of the system (using (17), (18) & (22))

$$= C\,(Q_1,\,Q_2)$$

$$= \frac{C_1}{T}\{\frac{Q_1^2\,p}{2a(p-a)} + \frac{\theta Q_1^3}{3(p-a)^2}\} + \frac{C_2\,p\,Q_2^2}{2aT(p-a)} + \frac{C_3\,\theta\,p\,Q_1^2}{2aT(p-a)} \tag{23}$$

From (14) and (15), we have

$$Q_2 = \frac{aT(p-a)}{p} - Q_1 - \frac{\theta}{2}Q_1^2\,\frac{(2a-p)}{a(p-a)} \tag{24}$$

Therefore, using (23) and (24), the total inventory cost of the system

$$= C(Q_1) = \frac{C_1}{T}\{\frac{Q_1^2\,p}{2a(p-a)} + \frac{\theta Q_1^3}{3(p-a)^2}\} + \frac{C_2\,p}{2aT(p-a)}\{\frac{aT(p-a)}{p} - Q_1$$

$$- \frac{\theta}{2}Q_1^2\,\frac{(2a-p)}{a(p-a)}\}^2 + \frac{C_3\,\theta\,p\,Q_1^2}{2aT(p-a)} \tag{25}$$

Theorem 1: *The average system cost function $C(Q_1)$ is strictly convex when $0<\theta<<1$.*
Proof: Using (25), we have

$$\frac{dC(Q_1)}{dQ_1} = \frac{C_1}{T}\{\frac{Q_1\,p}{a(p-a)} + \frac{\theta Q_1^2}{(p-a)^2}\}$$

$$- \frac{C_2\,p\,Q_2}{aT(p-a)}\{1 + \frac{\theta Q_1(2a-p)}{a(p-a)}\} + \frac{C_3\,\theta\,p\,Q_1}{aT(p-a)} \tag{26}$$

$$\frac{d^2C(Q_1)}{dQ_1^2} = \frac{C_1}{T}\{\frac{p}{a(p-a)} + \frac{2\theta Q_1}{(p-a)^2}\} + \frac{C_2\,p}{aT(p-a)}[\{1 + \frac{\theta Q_1(2a-p)}{a(p-a)}\}^2$$

$$- \frac{\theta Q_2(2a-p)}{a(p-a)}] + \frac{C_3\,\theta\,p}{aT(p-a)} > 0$$

$$\text{(as } 0<\theta<<1 \text{ and } p>a) \tag{27}$$

Therefore $C(Q_1)$ is strictly convex when $0<\theta<<1$.

As $C(Q_1)$ is strictly convex in Q_1, there exists an unique optimal stock level Q_1^*

that minimizes $C(Q_1)$. This optimal Q_1^* is the solution of the equation $\dfrac{dC}{dQ_1} = 0$.

We, therefore, find from (26) that Q_1^* is the unique root of the following equation in Q_1:

$$\frac{C_1}{T}\{\frac{Q_1\,p}{a(p-a)} + \frac{\theta Q_1^2}{(p-a)^2}\} - \frac{C_2\,p\,Q_2}{aT(p-a)}\{1 + \frac{\theta Q_1(2a-p)}{a(p-a)}\} + \frac{C_3\,\theta\,p\,Q_1}{aT(p-a)} = 0 \tag{28}$$

where Q_2 is given by (24).

After some calculations, neglecting higher powers of θ, we have

$$Q_1^* = \frac{a(p-a)C_2T}{p(C_1+C_2)}[1 - \frac{\{C_1C_2T(p-a)^2 + C_3p^2(C_1+C_2)\}}{p^2(C_1+C_2)^2}\theta] \tag{29}$$

which is a decreasing function of θ , where $0 < \theta << 1$. From (24), the optimal backlog level Q_2^* is given by (for fixed T) :

$$Q_2^* = \frac{aT(p-a)C_1}{p(C_1+C_2)} + \frac{a(p-a)C_2T}{p(C_1+C_2)}[\frac{\{C_1C_2T(p-a)^2 + C_3p^2(C_1+C_2)\}}{p^2(C_1+C_2)^2}$$
$$+ \frac{(p-2a)C_2T}{2p(C_1+C_2)}]\theta \tag{30}$$

Therefore Q_2^* is an increasing or decreasing function of θ if

$$\frac{C_1C_2T(p-a)^2 + C_3p^2(C_1+C_2)}{p^2(C_1+C_2)^2} + \frac{(p-2a)C_2T}{2p(C_1+C_2)} > \quad \text{or} \quad < 0 \text{ respectively.}$$

If Q_1 is fixed and T varies, then Q_2 also vary and is given by (24). In this case the average system cost is a function of T alone and given by

$$C(T) = \frac{C_1}{T}\{\frac{Q_1^2 p}{2a(p-a)} + \frac{\theta Q_1^3}{3(p-a)^2}\} + \frac{C_2 p}{2aT(p-a)}\{\frac{aT(p-a)}{p}$$
$$-Q_1 - \frac{\theta Q_1^2}{2}\frac{(2a-p)}{a(p-a)}\}^2 + \frac{C_3\theta pQ_1^2}{2aT(p-a)} \tag{31}$$

Theorem 2: *The average system cost function $C(T)$, given by* (31)*, is strictly convex when* $0 < \theta << 1$.
Proof: Here

$$\frac{dC(T)}{dT} = -\frac{C_1}{T^2}\{\frac{Q_1^2 p}{2a(p-a)} + \frac{\theta Q_1^3}{3(p-a)^2}\}$$
$$- \frac{C_2 p}{2aT^2(p-a)}\{\frac{a(p-a)T}{p} - Q_1 + \frac{\theta(p-2a)}{2a(p-a)}Q_1^2\}^2 \tag{32}$$
$$+ \frac{C_2}{T}\{\frac{a(p-a)T}{p} - Q_1 + \frac{\theta(p-2a)}{2a(p-a)}Q_1^2\} - \frac{C_3\theta pQ_1^2}{2aT^2(p-a)}$$

and

$$\frac{d^2C(T)}{dT^2} = \frac{2C_1}{T^3}\{\frac{Q_1^2 p}{2a(p-a)} + \frac{\theta Q_1^3}{3(p-a)^2}\}$$
$$+ \frac{C_2 pQ_1^2}{aT^3(p-a)}\{1 - \frac{\theta Q_1(p-2a)}{a(p-a)}\}$$
$$+ \frac{C_3 pQ_1^2}{aT^3(p-a)}\theta > 0$$
$$(\text{as } 0 < \theta << 1 \text{ and } p > a) \tag{33}$$

Hence $C(T)$ is strictly convex when $0<\theta<<1$.

Since $C(T)$ is strictly convex in T, there exists an unique optimal cycle time T^* that minimizes $C(T)$. This optimal cycle time T^* is the solution of the equation $\dfrac{dC}{dT}=0$.

Therefore, the optimal cycle time T^* is the unique root of the following equation in T (using (32)) :

$$-\frac{C_1}{T^2}\{\frac{Q_1^2 p}{2a(p-a)}+\frac{\theta Q_1^3}{3(p-a)^2}\}-\frac{C_2 p}{2aT^2(p-a)}\{\frac{a(p-a)T}{p}-Q_1$$

$$+\frac{\theta(p-2a)}{2a(p-a)}Q_1^2\}^2+\frac{C_2}{T}\{\frac{a(p-a)T}{p}-Q_1+\frac{\theta(p-2a)}{2a(p-a)}Q_1^2\}-\frac{C_3\theta p Q_1^2}{2aT^2(p-a)}=0 \tag{34}$$

After some calculations, neglecting higher powers of θ , we have

$$T^*=\frac{pQ_1}{a(p-a)\sqrt{C_2}}[(C_1+C_2)+\frac{\theta}{3}\{\frac{2}{p}C_1Q_1a^2+3C_2Q_1(2a-p)+3C_3a(p-a)\}]^{1/2} \tag{35}$$

Therefore, we conclude that T^* is an increasing or decreasing function of θ if

$$2C_1Q_1a^2+3C_2Q_1p(2a-p)+3C_3ap(p-a)> \text{ or } <0 \text{ respectively.}$$

4. NUMERICAL EXAMPLES

Here we have calculated optimal stock level Q_1^* , optimal backlog level Q_2^* ,and the minimum average system cost C^* for given values of production cycle length T and other parameters and T^* , Q_2^* and C^* for given values of Q_1 and other parameters by considering two examples.

Example 1: Let $\theta=0.0004$, $C_1=4$, $C_2=20$, $C_3=40$, $p=20$, $a=8$, and $T=80$ in appropriate units .Based on these input data , the computer outputs are as follows :

$Q_1^*=319.2747$, $Q_2^*=65.5748$ and $C^*=646.5293$

Example 2: Here we have taken $\theta=0.0004$, $C_1=4$, $C_2=20$, $C_3=40$, $p=20$, $a=8$ and $Q_1=60$ in appropriate units . The computer outputs are as follows :

$Q_2^*=5.5109$, $T^*=13.6418$ and $C^*=115.0922$

5. SENSITIVITY ANALYSIS

I. Here we have studied the effects of changes in the values of the parameters θ , C_1, C_2, C_3, p, a and T on the optimal total inventory cost, stock level and backlog level derived by the proposed method .The sensitivity analysis is performed by changing the value of each of the parameters by –50%, –25%, 25%, and 50%, taking one parameter at a time and keeping the remaining six parameters unchanged. Example 1 is used. On the basis of the results shown in table 1, the following observations can be made.

Table 1: Sensitivity analysis

Parameter	% change	% change in Q_1^*	% change in Q_2^*	% change in C^*
θ	-50	0.113	-1.199	-0.506
	-25	0.057	-0.599	-0.253
	+25	-0.057	0.599	0.254
	+50	-0.114	1.196	0.508
C_1	-50	9.155	- 44.327	- 45.167
	-25	4.375	- 21.187	- 21.589
	+25	- 4.021	19.475	19.843
	+50	- 7.728	37.438	38.144
C_2	-50	-14.391	69.722	-14.483
	- 25	- 5.309	25.718	- 5.347
	+25	3.484	-16.871	3.511
	+50	5.946	-28.792	5.993
C_3	-50	0.033	-0.162	-0.165
	-25	0.017	-0.081	-0.082
	+25	-0.017	0.081	0.082
	+50	-0.033	0.162	0.165
p	-50	-66.619	-68.628	-67.013
	-25	-22.172	-23.805	-22.542
	+25	13.282	14.757	13.649
	+50	22.125	24.798	22.806
a	-50	-33.416	-31.207	-32.838
	-25	-12.550	-11.122	-12.192
	+25	4.217	2.568	3.838
	+50	0.089	-3.026	-0.593
T	-50	-49.959	-50.518	-50.170
	-25	-24.969	-25.388	-25.128
	+25	24.949	25.647	25.214
	+50	49.879	51.552	50.516

It is seen from table 1 that the solution is insensitive to changes in the parameters θ and C_3, while it is considerably sensitive to changes in the parameters C_1, C_2, p, a and T.

II.   We now study the effects of changes in the values of the parameters θ, C_1, C_2, C_3, p, a, and Q_1 on the optimal total inventory cost , cycle time and backlog level by using example 2 .

Table 2: Sensitivity analysis

Parameter	% change	%change in T^*	% change in Q_2^*	% change in C^*
θ	-50	0.187	1.960	0.244
	-25	0.094	0.981	- 0.122
	+25	-0.094	-0.981	0.123
	+50	-0.188	-1.974	0.248
C_1	-50	-4.515	-53.646	-48.639
	-25	-2.231	-26.513	-42.056
	+25	2.183	25.934	23.561
	+50	4.319	51.326	46.657
C_2	-50	11.400	135.458	-3.823
	-25	3.939	46.805	-1.400
	+25	-2.439	-28.989	0.973
	+50	-4.100	-48.725	1.731
C_3	-50	-1.625	-19.311	0.028
	-25	-0.809	- 9.616	-0.024
	+25	0.803	9.538	0.097
	+50	1.599	19.002	0.264
p	-50	222.603	84.082	2.465
	-25	33.414	43.977	0.600
	+25	-15.165	-45.479	1.040
	+50	-24.595	-92.649	4.122
a	-25	8.876	-55.540	1.555
	+25	0.141	50.706	0.800
	+50	8.256	97.018	3.158
	+75	27.850	139.245	6.520
Q_1	-50	-49.107	-39.536	-49.973
	-25	-24.328	-17.116	-25.012
	+25	23.869	11.741	25.135
	+50	47.275	18.030	50.469

It is observed from table 2 that the solution is insensitive to changes in the parameter θ, slightly sensitive to changes in the parameter C_3 while it is considerably sensitive to changes in the parameters C_1, C_2, p, a and Q_1.

Therefore the above sensitivity analysis indicates that sufficient care should be taken to estimate the parameters C_1, C_2, p, a and T(or Q_1) in market studies.

6. CONCLUDING REMARKS

In the present paper, we have dealt with a continuous production control inventory model for deteriorating items with shortages. It is assumed that the demand and production rates are constant and the distribution of the time to deterioration of an item follows the exponential distribution. This model is applicable for food items, drugs, pharmaceuticals etc. Here we have studied the structural properties of this inventory system. The sensitivity analysis shows that sufficient care should be taken to estimate the parameters C_1, C_2, p, a and T (or Q_1) in market studies.

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