RETAILER'S REPLENISHMENT POLICIES UNDER CONDITIONS OF PERMISSIBLE DELAY IN PAYMENTS

Yung-Fu HUANG

Department of Business Administration, Chaoyang University of Technology, Taichung, Taiwan, R.O.C.

Abstract: Goyal (1985) is frequently cited when the inventory systems under conditions of permissible delay in payments are discussed. Goyal implicitly assumed that: 1. The unit selling price and the unit purchasing price are equal; 2. At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold.

But these assumptions are debatable in real-life situations. The main purpose of this paper is to modify Goyal's model to allow the unit selling price and the unit purchasing price not necessarily be equal to reflect the real-life situations. Furthermore, this paper will adopt different payment rule. We assume that the retailer uses sales revenue during the permissible credit period to make payment to the supplier at the end of the credit period. If it is not enough to pay off the purchasing cost of all items, the retailer will pay off the remaining balance by taking loan from the bank. So, the retailer starts paying for the interest charges on the amount of loan from the bank after the account is settled. Then the retailer will return money to the bank at the end of the inventory cycle. Under these conditions, we model the retailer's inventory system as a cost minimization problem to determine the retailer's optimal cycle time and optimal order quantity. Four cases are developed to efficiently determine the optimal cycle time and the optimal order quantity. Numerical examples are given to illustrate these cases. Comparing with Goyal's model, we also find that the optimal cycle times in this paper are not longer than those of Goyal's model.

Keywords: EOQ, permissible delay in payments, trade credit.

1. INTRODUCTION

The classical economic order quantity (EOQ) model assumes that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items are received. However, in practice the supplier will offer the retailer a fixed delay period, which is the trade credit period in settling the accounts. Before the end of trade credit period, the retailer can sell the goods, accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of trade credit period. In the real world, the supplier often makes use of this policy to promote his commodities. Several published papers have appeared in the literature which treat inventory problems under varying conditions. Some of these papers are discussed below.

Goyal [13] established a single-item inventory model under permissible delay in payments. Chand and Ward [3] analyzed Goyal's problem [13] under assumptions of the classical economic order quantity model, obtaining different results. Chung [9, 10] developed an alternative approach to determine the economic order quantity under condition of permissible delay in payments. Shah [22], Aggarwal and Jaggi [1] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Chang et al. [5] extended this issue to the varying rate of deterioration. Chu et al. [8] and Chung et al. [12] also investigated the deteriorating items under this condition and developed efficient approach to determine the optimal cycle time. Liao et al. [19] and Sarker et al. [20] investigated this topic with inflation. Jamal et al. [15] and Chang and Dye [4] extended this issue with allowable shortage. Chung [11] developed an alternative approach to modify Shah's [22] solution. Chang et al. [6] extended this issue with linear trend demand. Chen and Chuang [7] investigated light buyer's inventory policy under trade credit by the concept of discounted cash flow. Kim et al. [18] developed an optimal credit policy to increase wholesaler's profits with price-dependent demand functions. Hwang and Shinn [14] modeled an inventory system for retailer's pricing and lot sizing policy for exponential deteriorating products under the condition of permissible delay in payment. Jamal et al. [16] and Sarker et al. [21] addressed the optimal payment time under permissible delay in payment with deterioration. Khouja and Mehrez [17] investigated the effect of four different supplier credit policies on the optimal order quantity within the EOQ framework. Shawky and Abou-El-Ata [23] investigated the production lot-size model with both restrictions on the average inventory level and trade-credit policy using geometric programming and Lagrange approaches. Teng [25] assumed that the selling price was not equal to the purchasing price to modify the inventory model under permissible delay in payments. Shinn and Hwang [24] determined the retailer's optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period was a function of the retailer's order size, and also the demand rate was a function of the selling price. Arcelus et al. [2] modeled the retailer's profit-maximizing retail promotion strategy, when confronted with a vendor's trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise.

From the above literature review, we understand that Goyal [13] is well known in the study of the inventory systems under conditions of permissible delay in payments. Goyal [13] implicitly makes the following assumptions:

1. The unit selling price and the unit purchasing price are assumed to be equal. However, in practice, the unit selling price is not lower than the unit purchasing price in general. Consequently, the viewpoint of Goyal [13] is debatable sometimes.
2. At the end of the credit period, the account is settled. The retailer starts paying for higher interest charges on the items in stock and returns money of the remaining balance immediately when the items are sold. What the above statement describes is just one of the ways how the capital of enterprises is arranged. Based on considerations of profits, costs and developments of enterprises, enterprises may invest their capitals to the best advantage. Hence, the arrangement of capital of an enterprise is an important issue to the enterprise itself.

This paper tries to consider some alternatives to move capital to match the policy of enterprise. According to the given arguments, this paper will make the following assumptions to modify Goyal's model.

a) The unit selling price and the unit purchasing price are not necessarily equal to match the practical situations.
b) The retailer uses sales revenue during the permissible credit period to make payment to the supplier at the end of the credit period. If it is not enough to pay off the purchasing cost of all items, the retailer will pay off the remaining balance by taking loan from the bank. So, the retailer starts paying for the interest charges on amount of loan from the bank after the account is settled. In addition, this paper also assumes that the retailer does not return money to the bank until the end of the inventory cycle.

Incorporating the above assumptions (a) and (b), we tried to develop model of the inventory systems under conditions of permissible delay in payments.

2. MODEL FORMULATION AND CONVEXITY

The following notation and assumptions will be used throughout:

Notation:

D = annual demand
A = cost of placing one order
c = unit purchasing price per item
s = unit selling price per item
h = unit stock holding cost per item per year excluding interest charges
I_e = interest which can be earned per \$ in a year
I_p = interest charges per \$ investment in inventory per year
M = the trade credit period in years
T = the cycle time in years
$TVC(T)$ = the total variable cost per unit time when $T > 0$
T^* = the optimal cycle time of $TVC(T)$
Q^* = the optimal order quantity = DT^*.

Assumptions:

1) Demand rate is known and constant.
2) Shortages are not allowed.
3) Time period is infinite.
4) The lead time is zero.
5) $s \geq c$ and $I_p \geq I_e$.
6) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. If the credit period is less than the cycle length, the retailer continues to accumulate revenue and earn interest for the rest of the inventory cycle. At the end of credit period, the retailer pays off the remaining balance by taking loan from the bank if necessary. The retailer does not return money to the bank until the end of inventory cycle.

The total annual variable cost consists of the following elements.

(1) Annual ordering cost $= \dfrac{A}{T}$.

(2) Annual stock holding cost (excluding interest charges) $= \dfrac{DTh}{2}$.

(3) Based on the above assumptions (5) and (6), the sales revenue is DsM at the end of the credit period and the remaining balance will be DcT–DsM if the sales revenue is less than the total purchasing cost. Therefore, the remaining balance will be paid off by taking loan from the bank and must be financed at higher interest rate I_p until the end of inventory cycle. But the interest payable will be zero if DsM–$DcT \geq 0$. That is, the total sales revenue during the credit period exceeds the total purchasing cost. Hence the interest payable for two cases is obtained as follows:

(i): $T \geq \dfrac{sM}{c}$

Annual interest payable $= I_p(DcT$–$DsM)(T$–$M) / T$.   (1)

(ii): $T \leq \dfrac{sM}{c}$

In this case, no interest charges are paid for the items.

(4) According to assumption (6), the interest earned during the credit period and beyond the credit period until the cycle time T. Hence the interest earned for three cases are obtained as follows:

(i): $T \geq \dfrac{sM}{c}$, shown in Figure 1.

In this case, the interest earned must consider two periods. One is the trade credit offered by the supplier within M. At M, the retailer pays the sales revenue DsM to the supplier. Beyond M, the retailer still accumulates the interest earned until the end of inventory cycle.

Annual interest earned $= \left[\dfrac{DsM^2}{2} + \dfrac{Ds(T-M)^2}{2} \right] I_e \ / \ T$.   (2)

(ii): $M \leq T \leq \dfrac{sM}{c}$, shown in Figure 2.

In this case, the interest earned is similar to the above case (i). But the total sales revenue during the credit period exceeds the total purchasing cost. Hence the interest earned from the profit $DsM - DcT$ can be accumulated on [M, T].

Annual interest earned

$$= \left[\frac{DsM^2}{2} + (DsM - DcT)(T - M) + \frac{Ds(T - M)^2}{2} \right] I_e \; / \; \mathrm{T} \,. \tag{3}$$
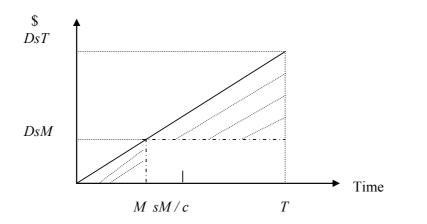

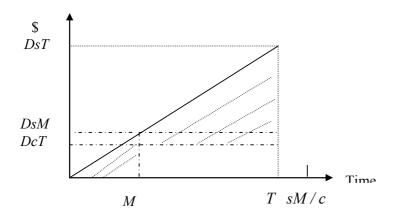**Figure 1:** The total accumulation of interest earned when $sM / c \leq T$

Figure 2: The total accumulation of interest earned when $M \leq T \leq sM / c$

(iii): $T \leq M$, shown in Figure 3.

In this case, the retailer can sell the items and earn interest until the end of the credit period.

$$\text{Annual interest earned} = \left[\frac{DsT^2}{2} + DsT(M-T)\right]I_e \ / \ \text{T} . \tag{4}$$
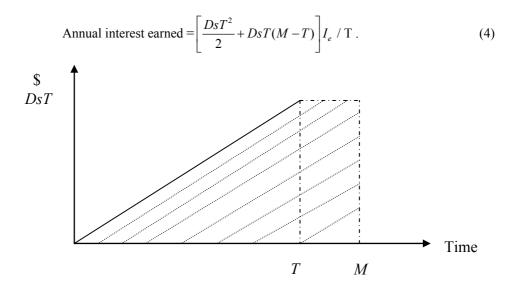

Figure 3: The total accumulation of interest earned when $T \le M$

From the above arguments, the annual total variable cost for the retailer can be expressed as

$TVC(T)$ = ordering cost + stock-holding cost + interest payable − interest earned

We show that the annual total variable cost, $TVC(T)$, is given by

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if} & T \ge \dfrac{sM}{c} & (5a) \\[2mm] TVC_2(T) & \text{if} & M \le T \le \dfrac{sM}{c} & (5b) \\[2mm] TVC_3(T) & \text{if} & T \le M & (5c) \end{cases}$$

where

$$TVC_1(T) = \frac{A}{T} + \frac{DTh}{2} + [I_p(DcT - DsM)(T-M)]/T$$
$$- \left[\frac{DsM^2}{2} + \frac{Ds(T-M)^2}{2}\right]I_e/T, \tag{6}$$

$$TVC_2(T) = \frac{A}{T} + \frac{DTh}{2} - \left[\frac{DsM^2}{2} + (DsM - DcT)(T-M) + \frac{Ds(T-M)^2}{2}\right]I_e/T \tag{7}$$

and

$$TVC_3(T) = \frac{A}{T} + \frac{DTh}{2} - \left[\frac{DsT^2}{2} + DsT(M-T)\right]I_e/T .$$ (8)

At $T = \frac{sM}{c}$, we find $TVC_1\left(\frac{sM}{c}\right) = TVC_2\left(\frac{sM}{c}\right)$. Similarly, $TVC_2(M) = TVC_3(M)$.

Hence $TVC(T)$ is continuous and well-defined. All $TVC_1(T), TVC_2(T), TVC_3(T)$ and $TVC(T)$ are defined on $T > 0$. Equations (6), (7) and (8) yield

$$TVC_1'(T) = -\left[\frac{A + DsM^2(I_p - I_e)}{T^2}\right] + D(\frac{h + 2cI_p - sI_e}{2}) ,$$ (9)

$$TVC_1''(T) = \frac{2[A + DsM^2(I_p - I_e)]}{T^3} > 0 ,$$ (10)

$$TVC_2'(T) = \frac{-A}{T^2} + D(\frac{h + 2cI_e - sI_e}{2}) ,$$ (11)

$$TVC_2''(T) = \frac{2A}{T^3} > 0 ,$$ (12)

$$TVC_3'(T) = \frac{-A}{T^2} + D(\frac{h + sI_e}{2})$$ (13)

and

$$TVC_3''(T) = \frac{2A}{T^3} > 0 .$$ (14)

Equations (10), (12) and (14) imply that all $TVC_1(T), TVC_2(T)$ and $TVC_3(T)$ are convex on $T > 0$. Then, there are two situations to occur:

(1) If $s = c$, then $TVC_1'(M) = TVC_2'(M) = TVC_3'(M)$ and $TVC(T)$ is convex on $T > 0$.

(2) If $s > c$, then $TVC_1'\left(\frac{sM}{c}\right) \neq TVC_2'\left(\frac{sM}{c}\right)$ and $TVC_2'(M) \neq TVC_3'(M)$ in general.

Consequently, $TVC(T)$ is piecewise convex but not convex.

3. THE DETERMINATION OF THE OPTIMAL CYCLE TIME T^*

Let $TVC_i'(T_i^*) = 0$ for all $i = 1, 2, 3$. We can obtain

$$T_1^* = \sqrt{\frac{2[A + DsM^2(I_p - I_e)]}{D(h + 2cI_p - sI_e)}} \quad \text{if} \quad h + 2cI_p - sI_e > 0 ,$$ (15)

$$T_2* = \sqrt{\frac{2A}{D[h + I_e(2c - s)]}} \quad \text{if} \quad h + I_e(2c - s) > 0 \tag{16}$$

and

$$T_3* = \sqrt{\frac{2A}{D(h + sI_e)}} \,. \tag{17}$$

Then we have the following results.

Case 1:

a) Suppose that $h + 2cI_p < sI_e$. Then $T* = \infty$. (When $T* = \infty$, it means that the retailer prefers to keep money of the remaining balance and does not return money to the bank.)

b) Suppose that $h + 2cI_p = sI_e$. Then

      (i) If $T_3* \geq M$, then $T* = \infty$.

      (ii) If $T_3* < M$ and $TVC(T_3*) \leq -DM[cI_p + s(I_p - I_e)]$, then $T* = T_3*$.

      (iii) If $T_3* < M$ and $TVC(T_3*) > -DM[cI_p + s(I_p - I_e)]$, then $T* = \infty$.

Case 2: Suppose that $h + 2cI_p > sI_e$ and $h + 2cI_e \leq sI_e$. Then

a) If $T_3* \leq M$ and $T_1* \geq \dfrac{sM}{c}$, then $TVC(T*)=\min\{TVC(T_1*), TVC(T_3*)\}$. Hence $T*$ is T_1* or T_3* associated with the least cost.

b) If $T_3* \leq M$ and $T_1* < \dfrac{sM}{c}$, then $TVC(T*)=\min\{TVC(T_3*), TVC(\dfrac{sM}{c})\}$. Hence $T*$ is T_3* or $\dfrac{sM}{c}$ associated with the least cost.

c) If $T_3* > M$ and $T_1* \geq \dfrac{sM}{c}$, then $T* = T_1*$.

d) If $T_3* > M$ and $T_1* < \dfrac{sM}{c}$, then $T* = \dfrac{sM}{c}$.

Based on Case 1 and 2, from now on, we assume $h + 2cI_e > sI_e$. Hence $h + 2cI_p > sI_e$. Consequently, both T_1* and T_2* are well-defined. By the convexity of $TVC_i(T)$ ($i = 1, 2, 3$), we see

$$TVC_1'(T) \begin{cases} < 0 & \text{if} \quad T < T_1* & \text{(18a)} \\ = 0 & \text{if} \quad T = T_1* & \text{(18b)} \\ > 0 & \text{if} \quad T > T_1* & \text{(18c)} \end{cases}$$

$$TVC_2'(T) \begin{cases} < 0 & \text{if } T < T_2 * & \text{(19a)} \\ = 0 & \text{if } T = T_2 * & \text{(19b)} \\ > 0 & \text{if } T > T_2 * & \text{(19c)} \end{cases}$$

and

$$TVC_3'(T) \begin{cases} < 0 & \text{if } T < T_3 * & \text{(20a)} \\ = 0 & \text{if } T = T_3 * & \text{(20b)} \\ > 0 & \text{if } T > T_3 *. & \text{(20c)} \end{cases}$$

Equations 18 (a, b, c), 19 (a, b, c) and 20 (a, b, c) imply that $TVC_i(T)$ is decreasing on $(0, T_i^*]$ and increasing on $[T_i^*, \infty)$ for all $i = 1, 2, 3$. Equations (9), (11) and (13) yield that

$$TVC_1'\left(\frac{sM}{c}\right) = \frac{-2A + D\left(\frac{sM}{c}\right)^2 \left\{h + 2cI_p\left(1 - \frac{c}{s}\right) + sI_e\left[2\left(\frac{c}{s}\right)^2 - 1\right]\right\}}{2\left(\frac{sM}{c}\right)^2}, \qquad (21)$$

$$TVC_2'\left(\frac{sM}{c}\right) = \frac{-2A + D\left(\frac{sM}{c}\right)^2 [h + I_e(2c - s)]}{2\left(\frac{sM}{c}\right)^2}, \qquad (22)$$

$$TVC_2'(M) = \frac{-2A + DM^2[h + I_e(2c - s)]}{2M^2} \qquad (23)$$

and

$$TVC_3'(M) = \frac{-2A + DM^2(h + sI_e)}{2M^2}. \qquad (24)$$

Furthermore, we let

$$\Delta_1 = -2A + D\left(\frac{sM}{c}\right)^2 \left\{h + 2cI_p\left(1 - \frac{c}{s}\right) + sI_e\left[2\left(\frac{c}{s}\right)^2 - 1\right]\right\}, \qquad (25)$$

$$\Delta_2 = -2A + D\left(\frac{sM}{c}\right)^2 [h + I_e(2c - s)], \qquad (26)$$

$$\Delta_3 = -2A + DM^2[h + I_e(2c - s)] \qquad (27)$$

and

$$\Delta_4 = -2A + DM^2(h + sI_e). \qquad (28)$$

Then, there are two situations to occur:

(I) If $s > c$, equations (25), (26), (27) and (28) yield that $\Delta_1 \geq \Delta_2 > \Delta_3$ and $\Delta_4 > \Delta_3$.

(II) If $s = c$, equations (25), (26), (27) and (28) yield that $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 =$

$$= -2A + DM^2(h + cI_e).$$

The determination of T^* of situation (I) is discussed in this section. However, the determination of T^* of situation (II) will be discussed in the next section.

Case 3: Suppose that $s > c$. Then

(1) If $\Delta_3 \geq 0$, $\Delta_4 > 0$, $\Delta_2 > 0$ and $\Delta_1 > 0$, then $TVC(T^*) = TVC(T_3^*)$ and $T^* = T_3^*$.

(2) If $\Delta_3 < 0$, $\Delta_4 \geq 0$, $\Delta_2 \geq 0$ and $\Delta_1 \geq 0$, then $TVC(T^*) = \min\{TVC(T_2^*), TVC(T_3^*)\}$. Hence T^* is T_2^* or T_3^* associated with the least cost.

(3) If $\Delta_3 < 0$, $\Delta_4 \geq 0$, $\Delta_2 < 0$ and $\Delta_1 \geq 0$, then $TVC(T^*) = \min\{TVC(T_3^*), TVC(\frac{sM}{c})\}$.

Hence T^* is T_3^* or $\frac{sM}{c}$ associated with the least cost.

(4) If $\Delta_3 < 0$, $\Delta_4 \geq 0$, $\Delta_2 < 0$ and $\Delta_1 < 0$, then $TVC(T^*) = \min\{TVC(T_3^*), TVC(T_1^*)\}$. Hence T^* is T_3^* or T_1^* associated with the least cost.

(5) If $\Delta_3 < 0$, $\Delta_4 < 0$, $\Delta_2 \geq 0$ and $\Delta_1 \geq 0$, then $TVC(T^*) = TVC(T_2^*)$ and $T^* = T_2^*$.

(6) If $\Delta_3 < 0$, $\Delta_4 < 0$, $\Delta_2 < 0$ and $\Delta_1 \geq 0$, then $TVC(T^*) = TVC(\frac{sM}{c})$ and $T^* = \frac{sM}{c}$.

(7) If $\Delta_3 < 0$, $\Delta_4 < 0$, $\Delta_2 < 0$ and $\Delta_1 < 0$, then $TVC(T^*) = TVC(T_1^*)$ and $T^* = T_1^*$.

4. COMPARISON WITH GOYAL'S MODEL

In this section, we assume that $s = c$. Then $h + 2cI_e > cI_e$ and $h + 2cI_p > cI_e$. Hence, equations (15), (16) and (17) can be rewritten as

$$T_1^* = \sqrt{\frac{2[A + DcM^2(I_p - I_e)]}{D(h + 2cI_p - cI_e)}} \tag{29}$$

and

$$T_2^* = T_3^* = \sqrt{\frac{2A}{D(h + cI_e)}}. \tag{30}$$

Furthermore, equations 5 (a, b, c), (9), (11) and (13) can be reduced to

$$TVC(T) = \begin{cases} TVC_1(T) & \text{if} & T \geq M & \text{(31a)} \\ TVC_3(T) & \text{if} & T \leq M & \text{(31b)} \end{cases}$$

and

$$TVC_1'(M) = TVC_2'(M) = TVC_3'(M) = \frac{-2A + DM^2(h + cI_e)}{2M^2}, \tag{32}$$

respectively. Recall $TVC(T)$ to be convex if $s = c$. Then, we have the following result.

Case 4: Suppose that $s = c$ and $\Delta = -2A + DM^2(h + cI_e)$. Then

(1) If $\Delta > 0$, then $T^* = T_3^*$.
(2) If $\Delta < 0$, then $T^* = T_1^*$.
(3) If $\Delta = 0$, then $T^* = T_1^* = T_3^* = M$.

Cases 3 and 4 immediately determine the optimal cycle time T^* after computing the numbers Δ_1, Δ_2, Δ_3, Δ_4 and Δ. Then, we can calculate optimal order quantity by DT^*. Cases 3 and 4 are very efficient solution procedures.

$$\text{Let } \overline{T_1}^* = \sqrt{\frac{2A + DcM^2(I_p - I_e)}{D(h + cI_p)}} \quad \text{and} \quad \overline{T_3}^* = \sqrt{\frac{2A}{D(h + cI_e)}} \ .$$

Moreover, we let $\overline{T}^*$ denote the optimal cycle time of Goyal's model.

Theorem 1 in Chung [9] determines the optimal cycle time of Goyal's model can be described as follows:

Case 5:

(1) If $\Delta > 0$, then $\overline{T}^* = \overline{T_3}^*$.
(2) If $\Delta < 0$, then $\overline{T}^* = \overline{T_1}^*$.
(3) If $\Delta = 0$, then $\overline{T}^* = \overline{T_1}^* = \overline{T_3}^* = M$.

Then we have the following result.

Case 6: $T^* \leq \overline{T}^*$. In fact, we have
(1) If $\Delta > 0$, then $T^* = \overline{T}^* = T_3^* = \overline{T_3}^*$.
(2) If $\Delta = 0$, then $T^* = \overline{T}^* = M$.
(3) If $\Delta < 0$, then $T^* \leq \overline{T}^*$.

Case 6 explains that the optimal cycle times in this paper are not longer than those of Goyal's model.

5. NUMERICAL EXAMPLES

To illustrate all results, let us apply the proposed method to efficiently solve the following numerical examples. For convenience, the numbers of the parameters are selected randomly. The optimal cycle time and optimal order quantity are summarized in Table 1, Table 2, Table 3, and Table 4, respectively.

Table 1: The optimal cycle time and optimal order quantity using Case 1

A	D	s	c	I_p	I_e	h	M	$h+2cI_p-sI_e$	Other judgment	Case	Optimal cycle time, T^*	Optimal order quantity, Q^*
100	3500	180	50	0.15	0.13	1	0.1	<0		1-(a)	∞	
200	1000	200	63.33	0.15	0.12	5	0.1	=0	$T_3^*=0.1174>M$	1-[b(i)]	∞	
10	1000	85	40	0.1	0.1	0.5	0.25	=0	$T_3^*=0.0471<M$, $TVC(T_3^*)<-DM[cI_p+s(I_p-I_e)]$	1-[b(ii)]	$T_3^*=0.0471$	47
200	2000	200	63.33	0.15	0.12	5	0.1	=0	$T_3^*=0.083<M$, $TVC(T_3^*)>-DM[cI_p+s(I_p-I_e)]$	1-[b(iii)]	∞	

Table 2: The optimal cycle time and optimal order quantity using Case 2

A	D	s	c	I_p	I_e	h	M	$h+2cI_p-sI_e$	$h+I_e(2c-s)$	Other judgment	Case	Optimal cycle time, T^*	Optimal order quantity, Q^*
300	4500	160	70	0.15	0.1	1	0.09	>0	<0	$T_3^*=0.0886<M, T_1^*=0.2093>sM/c=0.2057, TVC(T_1^*)<TVC(T_3^*)$	2-(a)	$T_1^*=0.2093$	942
200	3000	160	70	0.16	0.1	1	0.09	>0	<0	$T_3^*=0.0886<M, T_1^*=0.1976<sM/c=0.2057, TVC(sM/c)<TVC(T_3^*)$	2-(b)	$sM/c=0.2057$	617
300	3500	160	70	0.15	0.1	1	0.1	>0	<0	$T_3^*=0.1004>M, T_1^*=0.235>sM/c=0.2268$	2-(c)	$T_1^*=0.235$	823
200	3500	160	70	0.16	0.1	1	0.08	>0	<0	$T_3^*=0.082>M, T_1^*=0.179>sM/c=0.1829$	2-(d)	$sM/c=0.1829$	640

Table 3: The optimal cycle time and optimal order quantity using Case 3

A	D	s	c	I_p	I_e	h	M	$h+I_e(2c-s)$	Δ_1	Δ_2	Δ_3	Δ_4	Other judgment	Case	Optimal cycle time, T^*	Optimal order quantity, Q^*
170	5000	150	75	0.15	0.07	7	0.1	>0	>0	>0	>0	>0		3-(1)	$T_3^*=0.0623$	312
200	3000	140	120	0.15	0.1	1	0.1	>0	>0	>0	>0	>0	$TVC(T_2^*)<TVC(T_3^*)$	3-(2)	$T_2^*=0.1101$	330
250	3000	160	125	0.15	0.1	1	0.1	>0	>0	<0	<0	>0	$TVC(sM/c)<TVC(T_3^*)$	3-(3)	$sM/c=0.128$	384
100	1000	150	100	0.15	0.13	1	0.1	>0	<0	<0	<0	>0	$TVC(T_1^*)<TVC(T_3^*)$	3-(4)	$T_1^*=0.1504$	150
300	3500	140	130	0.15	0.11	1.7	0.1	>0	>0	>0	<0	<0		3-(5)	$T_2^*=0.1073$	376
180	2000	140	100	0.15	0.12	1	0.1	>0	>0	<0	<0	<0		3-(6)	$sM/c=0.14$	280
200	1000	140	120	0.15	0.08	5	0.1	>0	<0	<0	<0	<0		3-(7)	$T_1^*=0.1414$	141

Table 4: The optimal cycle time and optimal order quantity using Case 4

A	D	s=c	I_p	I_e	h	M	Δ	Case	Optimal cycle time, T^*	Optimal order quantity, Q^*
200	3000	100	0.15	0.13	3	0.1	>0	4-(1)	$T_3^*=0.0913$	274
200	1800	100	0.15	0.12	5	0.1	<0	4-(2)	$T_1^*=0.1108$	199
212.5	2500	100	0.15	0.12	5	0.1	=0	4-(3)	$M=0.1$	250

From the Table 1, Table2, Table 3 and Table 4, we know that we can easily and quickly find the optimal cycle time and optimal order quantity for the retailer using the cases developed in this paper depending on Δ_1, Δ_2, Δ_3, Δ_4 and Δ. These efficient procedures of the determination of the optimal cycle time are the major contributions in this paper. In the real-life business situations, the decision-makers can easily make the right replenishment policies using these efficient procedures.

6. SUMMARY

This paper is to modify Goyal's model to allow the unit selling price and the unit purchasing price not necessarily be equal and adopts different payment rule to develop the retailer's inventory model within the EOQ framework to reflect the realistic business situations. We develop four cases to help the retailer in accurately and quickly determining the optimal replenishment decisions under minimizing the annual total variable cost. Case 1 gives the decision rule of the optimal cycle time when $h + 2cI_p < sI_e$, then $T^* = \infty$. This result implies that the retailer ought to lengthen the loan period to the bank as possible. Thus, the retailer can get most benefit from keeping money. When $h + 2cI_p = sI_e$, then T^* is ∞ or T_3^* associated with the least cost. Case 2 does the decision rule of the optimal cycle time when $h + 2cI_p > sI_e$ and $h + 2cI_e \leq sI_e$. Furthermore, Case 3 gives the decision rule of the optimal cycle time when $h + 2cI_e > sI_e$ and $s > c$, the determination of T^* depends on Δ_1, Δ_2, Δ_3, and Δ_4. Moreover, Case 4 does the decision rule of the optimal cycle time when $s = c$, the determination of T^* depends on Δ. These efficient procedures of the determination of the optimal cycle time are the major contributions in this paper. Finally, comparing with Goyal's model, we can obtain that the optimal cycle times in this paper are not longer than those of Goyal's model. Numerical examples are given to illustrate all cases developed in this paper.

A future study will further incorporate the proposed model into more realistic assumptions, such as probabilistic demand, deteriorating items, allowable shortages and a finite rate of replenishment.

REFERENCES

[1] Aggarwal, S.P., and Jaggi, C.K., "Ordering policies of deteriorating items under permissible delay in payments", *Journal of the Operational Research Society*, 46 (1995) 658-662.

[2] Arcelus, F.J., Shah, N.H., and Srinivasan, G., "Retailer's pricing, credit and inventory policies for deteriorating items in response to temporary price/credit incentives", *International Journal of Production Economics*, 81-82 (2003) 153-162.

[3] Chand, S., and Ward, J., "A note on economic order quantity under conditions of permissible delay in payments", *Journal of Operational Research Society*, 38 (1987) 83-84.

[4] Chang, H.J., and Dye, C.Y., "An inventory model for deteriorating items with partial backlogging and permissible delay in payments", *International Journal of Systems Science*, 32 (2001) 345-352.

[5] Chang, H.J., Dye, C.Y., and Chuang, B.R., "An inventory model for deteriorating items under the condition of permissible delay in payments", *Yugoslav Journal of Operations Research*, 12 (2002) 73-84.

[6] Chang, H.J., Hung, C.H., and Dye, C.Y., "An inventory model for deteriorating items with linear trend demand under the condition of permissible delay in payments", *Production Planning and Control*, 12 (2001) 274-282.

[7] Chen, M.S., and Chuang, C.C., "An analysis of light buyer's economic order model under trade credit", *Asia-Pacific Journal of Operational Research*, 16 (1999) 23-34.

[8] Chu, P., Chung, K.J., and Lan, S.P., "Economic order quantity of deteriorating items under permissible delay in payments", *Computers and Operations Research*, 25 (1998) 817-824.

[9] Chung, K.J., "A theorem on the determination of economic order quantity under conditions of permissible delay in payments", *Computers and Operations Research*, 25 (1998) 49-52.

[10] Chung, K.J., "Economic order quantity model when delay in payments is permissible", *Journal of Information & Optimization Sciences*, 19 (1998) 411-416.

[11] Chung, K.J., "The inventory replenishment policy for deteriorating items under permissible delay in payments", *Opsearch*, 37 (2000) 267-281.

[12] Chung, K.J., Chang, S.L., and Yang, W.D., "The optimal cycle time for exponentially deteriorating products under trade credit financing", *The Engineering Economist*, 46 (2001) 232-242.

[13] Goyal, S.K., "Economic order quantity under conditions of permissible delay in payments", *Journal of the Operational Research Society*, 36 (1985) 35-38.

[14] Hwang, H., and Shinn, S.W., "Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments", *Computers and Operations Research*, 24 (1997) 539-547.

[15] Jamal, A.M.M., Sarker, B.R., and Wang, S., "An ordering policy for deteriorating items with allowable shortages and permissible delay in payment", *Journal of the Operational Research Society*, 48 (1997) 826-833.

[16] Jamal, A.M.M., Sarker, B.R., and Wang, S., "Optimal payment time for a retailer under permitted delay of payment by the wholesaler", *International Journal of Production Economics*, 66 (2000) 59-66.

[17] Khouja, M., and Mehrez, A., "Optimal inventory policy under different supplier credit policies", *Journal of Manufacturing Systems*, 15 (1996) 334-339.

[18] Kim, J.S., Hwang, H., and Shinn, S.W., "An optimal credit policy to increase wholesaler's profits with price dependent demand functions", *Production Planning and Control*, 6 (1995) 45-50.

[19] Liao, H.C., Tsai, C.H., and Su, C.T., "An inventory model with deteriorating items under inflation when a delay in payment is permissible", *International Journal of Production Economics*, 63 (2000) 207-214.

[20] Sarker, B.R., Jamal, A.M.M., and Wang, S., "Supply chain model for perishable products under inflation and permissible delay in payment", *Computers and Operations Research*, 27 (2000) 59-75.

[21] Sarker, B.R., Jamal, A.M.M., and Wang, S., "Optimal payment time under permissible delay in payment for products with deterioration", *Production Planning and Control*, 11 (2000) 380-390.

[22] Shah, N.H., "A lot-size model for exponentially decaying inventory when delay in payments is permissible", *Cahiers du CERO*, 35 (1993) 115-123.

[23] Shawky, A.I., and Abou-El-Ata, M.O., "Constrained production lot-size model with trade-credit policy: 'a comparison geometric programming approach via Lagrange' ", *Production Planning and Control*, 12 (2001) 654-659.

[24] Shinn, S.W., and Hwang, H., "Optimal pricing and ordering policies for retailers under order-size-dependent delay in payments", *Computers and Operations Research*, 30 (2003) 35-50.

[25] Teng, J.T., "On the economic order quantity under conditions of permissible delay in payments", *Journal of the Operational Research Society*, 53(2002) 915-918.