Yugoslav Journal of Operations Research
15 (2005), Number 2, 243-258

FUZZY MULTI-CRITERIA APPROACH TO ORDERING POLICY RANKING IN A SUPPLY CHAIN

Danijela TADIĆ
Faculty of Mechanical Engineering, University of Kragujevac
Kragujevac, Serbia & Montenegro

Received: October 2003 / Accepted: January 2005

Abstract: In this paper, a new fuzzy multi-criteria mathematical model for the selection of the best among a finite number of ordering policy of raw material in a supply chain is developed. The problem treated is a part of the purchasing plan of a company in an uncertain environment and it is very common in business practice. Optimization criteria selected describe the performance measures of ordering policies and generally their relative importance is different. It is assumed that the values of the optimization criteria are vague and imprecise. They are described by discrete fuzzy numbers and by linguistic expressions. The linguistic expressions are modelled by discrete fuzzy sets. The measures of belief that one ordering policy is better than another are defined by comparing fuzzy numbers. An illustrative example is given.

Keywords: Supply chain, ordering policy, fuzzy data, fuzzy rank, multi-criteria optimization.

1. INTRODUCTION

The design and management of supply chains (SC) are nowadays one of the most active research topics of management decision making in SC. In the literature, there is no quantitative and precise definition of SC. Usually it is defined as an integrated process where several business entities, such as suppliers, production plants, warehouses and customers work together to plan, coordinate and control raw materials, in-process products, and end products from suppliers to customers [17].

Social, economic, technological and other environmental changes in the business world request changes in the operation of SC. The complexity of decision-making in coordinated management is permanently increasing. By applying coordinated management and control the total costs in SC can be significantly reduced [4].

A lot of different management control problems exist in the SC. It is difficult to make a unique classification of the management and control problems because most researches address a single business entity of the SC ([14], [1], [5]). According to classifications [15], [1] and [5], treated problem belongs to supplier-production system, tactical level and Buyer-Seller relations, respectively.

In the literature, the term "supply management" has not yet been unanimously defined. Purchasing function has recently emerged as an important area of management and decision-making. There is no doubt that the most important activity of purchasing function is the choice of the ordering policy ([1], [3], [4]). Nowadays, the SC draws attention to quality, cost reduction, customer satisfaction, and partnerships. Strategic sourcing, supplier partnership, risk analysis in such partnership are the main topics in field of buyer-seller relation.

In manufacturing theory and purchasing practice of SC, single and multiple sources are used regardless of the question whether the purchase refers to one or more kinds of raw material. A single source implies supply of raw material from one reliable supplier. In this case, the highest level of supplier partnering can be achieved. It is denoted as "network supplier", which means that SC and supplier have identical aims and resources [10]. In purchasing environments where no single reliable supplier is available to deliver in small lot sizes on time and in the quantity required, multiple source offers an attractive alternative. In the literature one can find the papers which treat supply issues with two or more suppliers of raw material.

Kelle and Miller [8] considered the following problem: under which circumstances is single or dual sourcing preferable if the objective of the decision is to minimize the stockout risk. They assume that: (1) demand for the item is known, (2) both suppliers have random lead times with different characteristics. The optimal split rate of dual sourcing is provided in the form of exact formulae and simple approximations. The research presented is considered to be a step towards better risk evaluation and decision support in ordering strategically important items.

In the literature, there are a lot of papers where purchasing problems are treated as single criterion optimization tasks. Many researchers considered lead time as optimization criterion for determining the best ordering policy. Lau and Lau [9] developed a mathematical model to determine the optimal ordering policy when two suppliers have to be engaged. From the three possible strategies: "using supplier-1 only", "using supplier-2, only" and "using both suppliers", the lowest cost policy is identified. They assume that one supplier offers a lower unit price but has a poorer lead time than the other one. Unit prices, offered by both suppliers, are deterministic. The lead time of each supplier is assumed to be a stochastic variable. The optimal order-split rate depends on the particular combination of the cost and demand parameters (e.g., shortage cost per unit, holding cost per unit per year, standard deviation of lead time, etc.). By applying the proposed procedure the optimal ordering policy for any given combination of inventory parameters can be determined. In [7] a fuzzy mathematical model for determining the best supply strategy is developed. Each supply strategy is described by a number of attributes, which characterize the performances of suppliers. The attributes are either cardinal or linguistic expressions. Uncertainties, if any, are modeled by fuzzy numbers. The algorithm for choosing the best supply strategy is based on the fuzzification of the analytic hierarchy process [6].

This paper looks at the problem of choosing the best ordering policy when it comes to one particular type of raw material which is of crucial importance in the SC. In practice, the consumption volume of considered kind of raw material is determined in operational production plan. This quantity is input data for purchasing function. The ordering quantity, q, is described as deterministic variable or imprecise variable. We assume that ova ordering quantity can be ordered by S competing suppliers offer different unit price, lead-time performances and methods of payment. Purchasing managers define the participation of each considered supplier in purchasing of considered item. Determining of split rate is based on: (1) evidence and (2) subjective judgments of experts. The percentage split rate of supply among all suppliers belongs to an integer interval [0,100]. This statement of problem corresponds to realistic situations in SC. In other words, purchasing managers define different ordering alternatives of considered kind of raw material, which is called ordering policies. In terms of importance, this is a decision of high priority because some SCs spend a very large part of their total costs on raw materials and service sourced outside the SC.

In this paper we suppose the following:

1. We considered solely the ordering policies relevant for one item
2. The number of ordering policies defined by purchasing managers is finite
3. The determining of ordering policy is a multi-criteria optimization task. Optimization criteria have different relative importance.
4. The optimization criteria have imprecise values for each ordering policy. This assertion is based on the fact that relations between elements of SC critically depend on human activities. This fact is one of the main reasons why emergent SC systems require fuzzy system modeling [13]. The values of uncertain optimization criteria can be described by discrete fuzzy numbers. The fuzzy approach to treating uncertainties has some advantages over the stochastic approach:
   - Calculating of probability distributions for each stochastic variable requests a lot of evidence,
   - Combining of different uncertainties leads to a complex probability distribution, this results in very complex mathematical expressions.

In real problems like the one we have been considering, there are a lot of imprecise data. Turk and Fazel Zarandi [15] have summarized advantages of the phases in modeling uncertain values:

- Fuzzy system models are conceptually easy to understand
- Fuzzy system models are flexible, and with any given system, it is easy to manage it with system models or layer more functionality on top of it without starting again from scratch
- Fuzzy system models can capture most nonlinear functions of arbitrary complexity
- Fuzzy system models are tolerant of imprecise data
- Fuzzy system models can be built on top of the experience of experts
- Fuzzy system models can be blended with conventional control techniques
- Fuzzy system models are based on natural languages
- Fuzzy system models provide better communication between experts and managers

According to the same authors [15], SC as a complex system with imprecise parameters and conditions can be analyzed and modelled with the application of fuzzy set theory more appropriately.

5. The solution of the problem treated can be found by using the measures of belief that one ordering policy is better than the others [16], which is calculated by comparing discrete fuzzy numbers.

This paper is organized in the following way: in Section 2, the problem statement of ranking ordering policies is presented. In Section 3, the optimization criteria are defined and they are described by discrete fuzzy numbers. In Section 4, a new procedure for determination of the best ordering policy is presented. The proposed procedure is illustrated by an example given in Section 5.

2. PROBLEM STATEMENT

The mathematical model for choosing the best ordering policy in SC is developed under the following assumptions:

1. Only one item of raw material is considered.
2. In general, purchasing of main item is performed by S suppliers, so that $S = \{1,...,s,...,S\}$, which operate in an uncertain environment. Following the literature, we shall assume $S < 5$ [12]. Generally, we consider I different ordering policies: $I = \{1,...,i,...,I\}$. If we assume that the considered kind of raw material purchase comes from S suppliers, then a ordering policy $i \in I$ is defined as: $\sum_{s=1}^{S} a_s^i \cdot q$

where:

q is the total quantity of orders

a_s^i is the percentage illustrating the part an s-supplier plays in the total quantity of orders for i-ordering policy

It should be mentioned that number of suppliers - S and numbers of ordering policies - I are defined by supply managers. Most frequently S does not equal I.

3. Each supplier is described by attributes such as: unit price of raw material, quality of raw material, lead time, method of payment, etc. Supply managers define all the attributes which describe each supplier. Their values can be either deterministic or imprecise. The optimization criteria in choosing ordering policy are calculated from supplier attributes. In general, we consider K optimization criteria, i.e. $K = \{1,...k,...K\}$.

In this paper, three optimization criteria are associated: unit price of raw material, lead time and method of payment. The procedure for optimization criteria calculation is presented in Section 3.

4. As it is known, the optimization criteria can be either of benefit or cost type. Yoon and Hwang [18] define two criteria types:
(a) Benefit optimization criteria are positively correlated with utility or the preferences of decision maker, which means: if the criteria values increase, so does the utility of decision maker,

(b) Cost optimization criteria are negatively correlated with the utility of decision maker, which means: if the optimization criteria values increase, the utility of the decision maker decreases.

According to classification which is given in [18], unit price of raw material and lead time are cost optimization criteria. Method of payment is benefit optimization criterion.

5. In general, the relative importance of each optimization criterion $k \in K$, w_k $(k = \overline{1, K})$ is different. Determination of criteria weight is a difficult task which presents a problem to itself. There are a number of techniques to assess the weights of optimization criteria [18]. They are normalized, non-normalized or linguistic expressions.

In this paper, the comparison pair matrix of relative criteria importance $W = [w_k / w_{k'}]_{KxK}$ is subjectively constructed. Elements of this matrix, $w_k / w_{k'}$ (k = 1,…,K) and $(k' = 1,...,K)$, are importance of optimization criterion k ($k \in K$) with respect to optimization criterion k' ($k' \in K$). The values of this matrix are positive and are within the interval [1,9]. The value 1 marks that the optimization criteria k ($k \in K$) and k' ($k' \in K$) are equally important. Value 9 shows that the optimization criterion k ($k \in K$) is extremely more important than optimization criterion k' ($k' \in K$). Elements of this matrix have the following properties:

- Elements of the main diagonal are not defined
- Values of off-diagonal terms are reciprocal to each other
- Consistency index provides a way of measuring how many errors were made when making judgments

Here, the optimization criteria weighted vector is calculated by applying eigenvector method [12]. Optimization criteria weights are ordinal numbers.

3. MODELLING OF OPTIMIZATION CRITERIA VALUES

The following optimization criteria are assumed: unit price of raw material, lead time and method of payment. It seems that these optimization criteria have the greatest importance in purchasing problem of SC [7]. In this Section, modelling of optimization criteria is based on discrete fuzzy numbers. Why we opted for discrete fuzzy numbers? We used discrete membership function in order to avoid analytic considerations in order to apply "digital way of thinking". Also, uncertainties which appear in organizational problems in SC can be naturally described by discrete fuzzy numbers.

The value of criterion k ($k \in K$) (k=1,..,K) for ordering policy i ($i \in I$) (I = 1,…,I) is described by discrete fuzzy number $\overline{f}_{ik}$ [19]. The method of calculation of these fuzzy numbers is shown further on.

3.1. Unit price of raw material

Unit price of raw material depends on: consumption of raw material in a given time period and agreement between production system and supplier. It should be noticed that this parameter can be different for each supplier.

The consumption volume of considered kind of raw material can be described by fuzzy number $\tilde{q}$:

$$\tilde{q} = \{q_g, \mu_{\tilde{q}}(q_g)\} \tag{3.1}$$

where:

The volume of consumption is denoted by q_g .It is a discrete value in the domain of fuzzy number $\tilde{q}$, $q_g \in D_1$, $D_1 = \{q_1,...,q_g,...,q_G\}$. The number of discrete values in D_1 depends on the discretization step. Upper and bottom values in domain D_1 of discrete fuzzy number $\tilde{q}$ can be determined by expert assessment.

$\mu_{\tilde{q}}(q_g)$ is a membership function of fuzzy number $\tilde{q}$. Each value of the domain is paired with exact probability value, calculated from registered data.

The unit price of raw material offered by a supplier s $(s = \overline{1,S})$ can be described by an empirical expression which shows the dependence of unit price and ordering quantity. This dependence can be described by decreasing function:

$$c_s = f_s(d_g) \tag{3.2}$$

Expressions (3.1) and (3.2) enable determination of the fuzzy value of unit price of raw material for each supplier. It is defined by fuzzy number $\overline{c}_s$ $(s = \overline{1,S})$:

$$\tilde{c}_s = \{c_y, \mu_{\tilde{c}_s}(c_y)\} \tag{3.3}$$

where:
The value of unit price of raw material is denoted by c_y. It is a discrete value of domain of fuzzy number $\tilde{c}_s$ $(s = \overline{1,S})$. This value is $c_y = f_s(d_g)$

$\mu_{\tilde{c}_s}(c_y)$ is the value of membership function of fuzzy number $\tilde{c}_s$ $(s = \overline{1,S})$ which is obtained by expression $\mu_{\tilde{c}_s}(c_y) = \mu_{\tilde{q}}(q_g)$.

The value of unit price of raw material for each ordering policy-i $(i \in I)$ is uncertain variable, described by discrete fuzzy number $\tilde{f}_{i1}$, so that:

$$\tilde{f}_{i1} = \sum_{s=1}^{S} a_s^i \cdot \tilde{c}_s \tag{3.4}$$

3.2. Lead time

The lead time of a supplier is the time from the moment when the supplier receives an order to the moment when the supplier is ready to make the delivery. According to the definition of lead time, transportation time is not included in lead time. The value of lead time can be either deterministic or uncertain, while in a special case lead time is equal to zero. For purchasing problem of SC, which operates in real environment, it is supposed that this variable is uncertain. In some papers, the lead time is supposed to be a stochastic variable with either normal, or Gamma or Poisson distribution [9].

It is assumed that lead time is given by linguistic expressions: "small", “medium”, "large", etc. The values of these descriptors are determined by fuzzy numbers:

$$\tilde{L}_m = \{l_n^m, \mu_{\tilde{L}_m}(l_n^m)\} \tag{3.5}$$

where:

l_n^m is value in domain of discrete fuzzy number $\tilde{L}_m$ ($n = 1,\dots,N$) and ($m = 1,\dots,M$). Total number of discrete value in domain of discrete fuzzy number, $\tilde{L}_m$ depends on the discretization step. It is reasonable to assume that the discretization step one day. Upper and bottom values are determined on basis of evidence data.

$\mu_{\tilde{L}_m}(l_n^m)$ is membership function of discrete fuzzy number $\tilde{L}_m$. Its value is determined by subjective judgments of purchasing managers.

Total number of linguistic descriptors for describing the lead times is M. Generally, this number M differs from the total number of suppliers S. Lead time of each supplier is described by discrete fuzzy number $\tilde{L}_s$, so that $\tilde{L}_s = \tilde{L}_m$. The total number of fuzzy numbers $\tilde{L}_s$ is S. Generally, discrete fuzzy numbers $\tilde{L}_s$ differ from each other, but sometimes all of them or some of them are identical.

In this paper, it is assumed that value of lead time for ordering policy-i ($i \in I$) equals the longest delivery time of the supplier of the raw material in i-ordering policy. The value of lead time for each ordering policy-i ($i \in I$) is described by discrete fuzzy number $\tilde{f}_{i2}$, so that:

$$\tilde{f}_{i2} = \max_{i,s}(\tilde{L}_s) \tag{3.6}$$

The procedure for calculating of the discrete fuzzy number $\tilde{f}_{i2}$ (3.6) is based on measures of belief that lead time of one supplier is longer than lead times of other suppliers for ordering policy-i ($i \in I$). These measures of belief are calculated by comparing discrete fuzzy numbers.

3.3. Method of payment

The method of payment has a very high priority when it comes to ranking and choosing the ordering policy, especially when SC is operating in a business environment with a lot of uncertainty.

The method of payment is a financial criterion. There are a lot of methods of payment, which are used, in economic theory and practice. They are the results of an agreement between the production system and supplier. There is no unique classification of this criterion.

In this paper, three methods of payment are considered: (1) partial advanced payment and the rest in cash, (2) partial advanced and rest on credit and (3) on credit. They can be described by three preferential linguistic expressions: "unfavorable", "medium favorable" and "favorable" which are described by fuzzy numbers, $\tilde{P}_1, \tilde{P}_2, \tilde{P}_3$, respectively.

For example the linguistic expression "medium favorable" is modelled by fuzzy number $\tilde{P}_2$:

$$\tilde{P}_2 = \{p_j, \mu_{\tilde{P}_3}(p_j)\} \tag{3.7}$$

where:

p_j is a discrete value in the domain of fuzzy number $\tilde{P}_2$. The values of domain are determined by scale of measures, for example “school’s” scale of measures. These values are integer.

$\mu_{\tilde{P}_2}(p_j)$ is a membership function of fuzzy number $\tilde{P}_2$.

In this paper, the discrete fuzzy number $\tilde{P}_2$ can be defined:

$$\tilde{P}_2 = \{(1,0.25),(2,0.5),(3,1),(4,0.5),(5,0.25)\} \tag{3.8}$$

Linguistic expressions "favorable" and "unfavorable" are modelled by fuzzy numbers $\tilde{P}_1$ and $\tilde{P}_3$, respectively. Let us define these fuzzy numbers:

$$\tilde{P}_1 = \{p_j, \mu_{\tilde{P}_1}(p_j)\} \tag{3.9}$$

$$\tilde{P}_3 = \{p_j, \mu_{\tilde{P}_3}(p_j)\} \tag{3.10}$$

They are obtained by applying simple operations, concentration (Con) and dilation (Dil), respectively, used to modify membership function.

Here, we suppose that fuzzy number $\tilde{P}_1$ is concentrated, its membership functions become more concentrated around points with higher membership grades as, in this case:

$$\tilde{P}_1 = \text{Con } \tilde{P}_2 \tag{3.11}$$

and $\mu_{\tilde{P}_1}(p_i) = \mu^2_{\tilde{P}_2}(p_i)$

Dilation has the opposite effect from concentration and is produced by modifying the membership function through the transformation:

$$\tilde{P}_3 = \text{Dil } \tilde{P}_2 \tag{3.12}$$

and $\mu_{\tilde{P}_3}(p_i) = \mu_{\tilde{P}_2}^{1/2}(p_i)$

From the points discussed above it transpires that the method of payment for supplier s (s = 1,…,S) is described by discrete fuzzy number $\tilde{P}_s = \cup(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3)$.

The value of method of payment for each ordering policy-i ($i \in I$) is described by discrete fuzzy number $\tilde{f}_{i3}$, so that:

$$\tilde{f}_{i3} = \sum_{s=1}^{S} a_s^i \cdot \tilde{P} \tag{3.13}$$

4. RANKING OF ORDERING POLICIES

The problem treated is stated as a multi-criteria optimization task. It is formally presented by $I \times K$ matrix, $F = \left[\tilde{f}_{ik}\right]_{I \times K}$. The matrix element $\tilde{f}_{ik}$ $(i = \overline{1,I}, k = \overline{1,K})$ is the value of optimization criterion $k \in K$ for the ordering policy (alternative) $i \in I$. If the criteria values for some $k \in K$ are imprecise and uncertain, then all elements of k-th column are fuzzy numbers. If the criteria values for some $k \in K$ are certain, then all elements of the k-th column are determinate i.e. certain numbers.

Under assumptions given in Section 2, the matrix element $\tilde{f}_{ik}$ $(i = \overline{1,I}, k = \overline{1,K})$ is a discrete fuzzy number. Procedure of obtaining the discrete fuzzy numbers $\tilde{f}_{ik}$ $(i = \overline{1,I}, k = \overline{1,K})$ is presented in Section 3.

In this Section, we consider the following sub problems:

1. The problem of normalization of discrete fuzzy numbers through which the various criterion dimensions are transformed into non-dimensional criteria. The normalization is necessary so that the different values of the criteria can become comparable.
2. The problem of determining the influence the weight of optimization criteria has on the choice if the best ordering policy.
3. The essence of the method developed is the determination of the discrete fuzzy number $\tilde{A}_i$ $(i = \overline{1,I})$ which is allocated to each ordering policy discussed here i ($i \in I$). The best ordering policy with respect to all optimization criteria, taking into consideration their different weights is the policy most likely to have the allocated discrete fuzzy number $\tilde{A}_i$ $(i = \overline{1,I})$ which is smaller than all the other discrete fuzzy numbers $\tilde{A}_i$ $(i = \overline{1,I})$. Probability measure is determined by comparison of the discrete fuzzy numbers.

The algorithm for determining the best ordering policy is as follows.

Step 1. Construct the normalized matrix $F_n = \left[\left(\tilde{f}_{ik}\right)_n\right]_{I\times K}$. Transform all the cardinal criteria values f_{ik} into $(f_{ik})_n$ defined on a common scale [0,1] by applying linear transformation [11]:
(a) for a benefit type criterion k ($k \in K$):

$$\left(f_{ik}\right)_n = \frac{f_{ik}}{f_k^{\max}} \tag{4.1}$$

where:
$f_k^{\max}$ is the max value of support of fuzzy numbers $\tilde{f}_{ik}, k = \overline{1,K}$ for $\mu(f_{ik}) \neq 0$.
(b) for a cost type criterion k (k$\in K$):

$$\left(f_{ik}\right)_n = 1 - \frac{f_{ik} - f_k^{\min}}{f_k^{\max}} \tag{4.2}$$

where:
$f_k^{\min}$ is the min value of support of fuzzy numbers $\tilde{f}_{ik}, k = \overline{1,K}$ for $\mu(f_{ik}) \neq 0$.
The values of membership function of each discrete fuzzy numbers $(\tilde{f}_{ik})_n$ can be obtained according to expression $\mu_{\tilde{f}_{ik}}(f_{ik}) = \mu_{(\tilde{f}_{ik})_n}((f_{ik})_n)$.

Step 2. Construct the weighted matrix. The elements are calculated by multiplying each column of the matrix $F_n = \left[\left(\tilde{f}_{ik}\right)_n\right]_{I\times K}$ with its associated weights:

$$F_1 = \left[w_k \cdot (\tilde{f}_{ik})_n\right]_{I\times K} \tag{4.3}$$

Step 3. Map each row of weighted matrix into a fuzzy number $\tilde{A}_i, i = \overline{1,I}$ [2]:

$$\tilde{A}_i = \sum_{k=1}^{K} w_k \cdot (\tilde{f}_{ik})_n \tag{4.4}$$

Step 4. Determine the measure of belief that the fuzzy number $\tilde{A}_i$ is better than all other fuzzy numbers. It is computed by expression

$$Poss\ (\tilde{A}_i \geq \tilde{A}_{i'}) \quad i = \overline{1,I}, i' = \overline{1,I}, i \neq i' \tag{4.5}$$

In this paper, calculating a measure belief $Poss\ (\tilde{A}_i \geq \tilde{A}_{i'})\ i = \overline{1,I}, i' = \overline{1,I}, i \neq i'$ is based on the procedure in [16]. In this way, we determine preference relation between ordering policies $(i,i') \in I$. By using expression (4.5) the graph of preference relation among ordering policies is determined.

5. AN ILLUSTRATIVE EXAMPLE

Assume the following:

- The main kind of raw material is considered.
- There are two suppliers in SC, S=2. The three different ordering policies are defined by purchasing managers. These are denoted by a set of indices $I = \{1,2,3\}$, so that: 1-means only the first supplier is chosen, 2-means only the second supplier is chosen, 3-combined supply 50% from the first supplier and 50% from the second supplier are purchased.
- The optimization criteria are: unit price, lead time and method of payment The relative importance of two criteria are judged and the judgments are presented by matrix: $\begin{bmatrix} - & 2 & 5 \\ & - & 3 \\ & & - \end{bmatrix}$
- The volume of consumption of raw material are described by discrete fuzzy number $\tilde{q} = \{(40,0.2),(65,0.6),(90,1),(115,0.6),(140,0.2)\}$
- The dependence of unit price for order quantity is shown in Fig. 5.1.

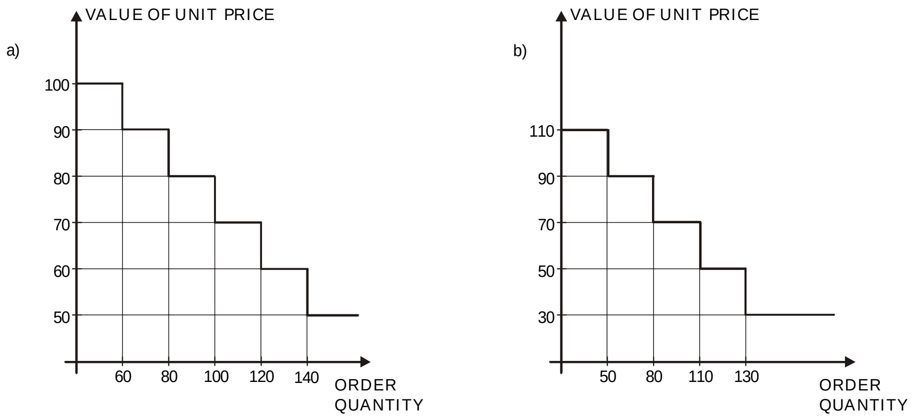

Figure 5.1: Value of unit price which offer:
(a) the first supplier and (b) the second supplier

- Here, let us suppose that lead time of first supplier is not higher than five days. The lead time of second supplier can be described by linguistic expression "about ten days". These linguistic descriptors are modeled by discrete fuzzy numbers:

"not higher five days" $= \tilde{L}_1 = \{(1,0.2),(2,0.4),(3,0.6),(4,0.8),(5,1)\}$

"about ten days"$= \tilde{L}_2 = \{(8,0.3),(9,0.7),(10,1),(11,0.7),(12,0.3)\}$

- The first supplier is asking partial advanced payment and the rest in cash. This is an unfavorable way of payment, which is modeled by fuzzy number $\tilde{P}_1$. The second supplier enables sale on credit. It is described as a favorable way of payment, which is modeled by fuzzy number $\tilde{P}_3$.

5.1. Determining values of data entry

From pair wise comparison of relative importance of criteria arranged in square matrix the weights of criteria are calculated: $w_1 = 0.582, w_2 = 0.309$ and $w_3 = 0.109$.

Further on the calculation of all the criteria values is given:

1. Value of unit price- $\tilde{f}_1 \left(i = \overline{1,3}\right)$

The values of unit prices of raw material offered by the first and the second supplier is presented by fuzzy numbers $\bar{c}_1, \bar{c}_2$, respectively:

$$\tilde{c}_1 = \{(100, 0.2), (90, 0.6), (80, 1), (70, 0.6), (50, 0.2)\}$$

$$\tilde{c}_1 = \{(110, 0.2), (90, 0.6), (70, 1), (50, 0.6), (30, 0.2)\}$$

The values of unit prices of raw material for each ordering policy is:

$$\tilde{f}_{11} = \bar{c}_1, \ \tilde{f}_{11} = \bar{c}_1$$

$$\tilde{f}_{31} = 0.5 \cdot \tilde{c}_1 + 0.5 \cdot \tilde{c}_2 = \\ = \{(105, 0.2), ..., (90, 0.6), ..., (75, 1), (70, 0.6), ..., (60, 0.2), ..., (40, 0.2)\}$$

2. Value of lead time - $\tilde{f}_{i2} \ (i = \overline{1,3})$ is given as

$$\tilde{f}_{12} = \tilde{L}_1, \ \tilde{f}_{22} = \tilde{L}_2$$

$$\tilde{f}_{32} = \max_s(\tilde{L}_1, \tilde{L}_2) = \tilde{L}_2$$

3. Value of method of payment - $\tilde{f}_{i3} \ (i = \overline{1,3})$

$$\tilde{f}_{13} = \tilde{P}_1 = \{(1, 0.0625), (2, 0.25), (3, 1), (4, 0.25), (5, 0.0625)\}$$

$$\tilde{f}_{23} = \tilde{P}_3 = \{(1, 0.5), (2, 0.71), (3, 1), (4, 0.71), (5, 0.5)\}$$

$$\tilde{f}_{33} = 0.5 \cdot \tilde{P}_1 + 0.5 \cdot \tilde{P}_3 = \begin{Bmatrix} (1, 0.0625), (1.5, 0.25), (2, 0.5), (2.5, 0.71), (3, 1), (3.5, 0.71), \\ (4, 0.5), (4.5, 0.25), (5, 0.0625) \end{Bmatrix}$$

5.2. Ranking of ordering policies

Ranking of ordering policies is realized by algorithm, which is presented in Section 4.

Step 1. The normalized values $\left(\tilde{f}_{ik}\right)_n, i=1,2,3$ and $k=1,2,3$ are calculated:

$$\left(\tilde{f}_{11}\right)_n = \{(0.5,0.2),(0.6,0.6),(0.7,1),(0.8,0.6),(1,0.2)\}$$

$$\left(\tilde{f}_{21}\right)_n = \{(0.27,0.2),(0.45,0.6),(0.64,1),(0.82,0.6),(1,0.2)\}$$

$$\left(\tilde{f}_{31}\right)_n = \begin{Bmatrix}(0.38,0.2),(0.43,0.2),(0.48,0.2),(0.57,0.6),(0.62,0.6),(0.67,1),\\(0.71,0.6),(0.77,0.6),(0.81,0.6),(0.86,0.2),(0.9,0.2),(1,0.2)\end{Bmatrix}$$

$$\left(\tilde{f}_{12}\right)_n = \{(1,0.2),(0.8,0.4),(0.6,0.6),(0.4,0.8),(0.2,1)\}$$

$$\left(\tilde{f}_{22}\right)_n = \left(\tilde{f}_{32}\right) = \{(1,0.3),(0.92,0.6),(0.83,1),(0.75,0.6),(0.67,0.3)\}$$

$$\left(\tilde{f}_{13}\right)_n = \{(0.2,0.0625),(0.4,0.25),(0.6,1),(0.8,0.25),(1,0.0625)\}$$

$$\left(\tilde{f}_{23}\right)_n = \{(0.2,0.5),(0.4,0.71),(0.6,1),(0.8,0.71),(1,0.5)\}$$

$$\left(\tilde{f}_{33}\right)_n = \begin{Bmatrix}(0.2,0.0625),(0.3,0.25),(0.4,0.5),(0.5,0.71),(0.6,1),(0.7,0.71),\\(0.8,0.5),(0.9,0.25),(1,0.0625)\end{Bmatrix}$$

Step 2. The values of elements of matrix F_1 are:

$$F_1 = \left[0.582\cdot\left(\tilde{f}_{i1}\right)_n \; 0.309\cdot\left(\tilde{f}_{i2}\right)_n \; 0.109\cdot\left(\tilde{f}_{i3}\right)_n\right]_{3x3},\; i=\overline{1,3}\,.$$

Step 3. The values of fuzzy numbers $\left(\tilde{A}_i\right), i=\overline{1,3}$ are presented in following figures:

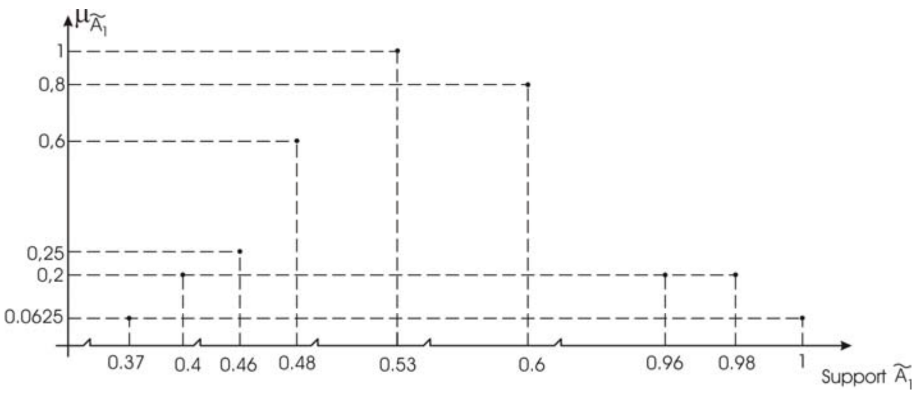

Figure 5.2: Discrete fuzzy number $\tilde{A}_1$

Support $\tilde{A}_1 = \{0.4, 0.42, 0.44, ..., 0.52, 0.53, 0.54, ..., 0.96, 0.98, 1\}$ with 47 elements

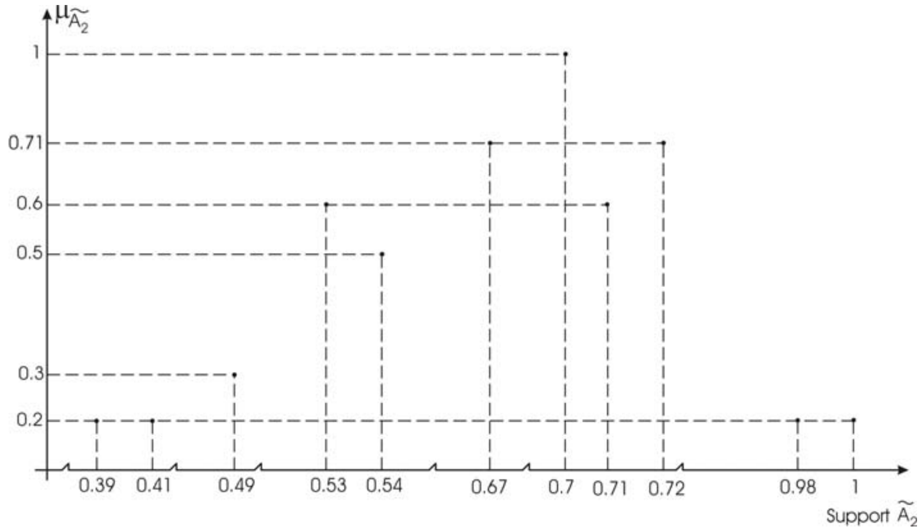

Figure 5.3: Discrete fuzzy number $\tilde{A}_2$

Support $\tilde{A}_2 = \{0.39, 0.41, ..., 0.69, 0.7, 0.71, 0.72, ..., 0.98, 1\}$ with 53 elements

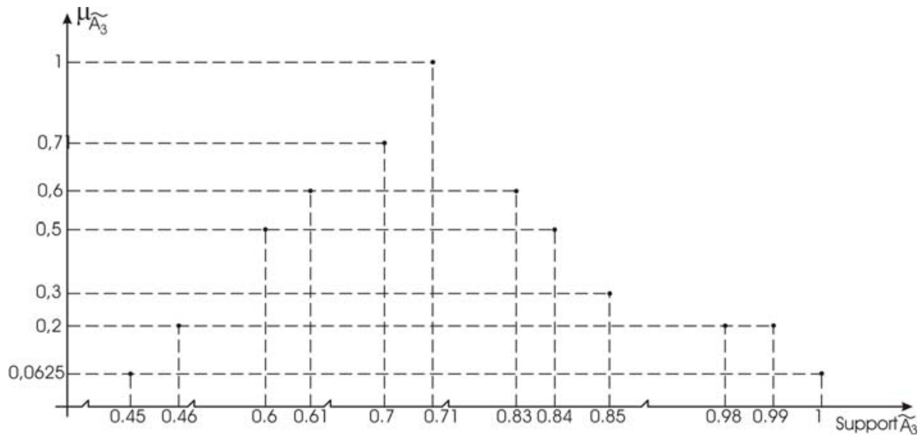

Figure 5.4: Discrete fuzzy number $\overline{A}_3$

Support $\overline{A}_3 = \{0.39, 0.41, 0.43..., 0.69, 0.7, 0.71..., 0.97, 0.98, 1\}$ with 53 elements

Step 4. The measure of belief that the fuzzy number $\tilde{A}_i$, $i = \overline{1,3}$ is better than all others fuzzy numbers:

$$\text{Poss}\left(\tilde{A}_1 \geq \tilde{A}_2\right) = 0.44, \ \text{Poss}\left(\tilde{A}_1 \geq \tilde{A}_3\right) = 0.38 \ \text{and} \ \text{Poss}\left(\tilde{A}_2 \geq \tilde{A}_3\right) = 0.42$$

The preference relation among the ordering policies can be presented in Fig. 5.5

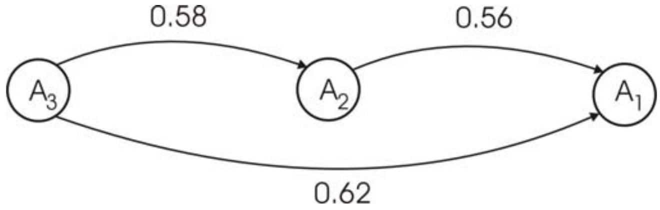

Figure 5.5: Graph of preference

The best ordering policy is ordering policy with number 3.

6. CONCLUSION

In this paper, a new fuzzy model for ranking the ordering policies of raw material in SC is presented. The problem of choosing the best ordering policy is the part of the management material flow problem in SC. The advantages of developed model according to literal sources are shown, primary, in the more realistic statement of the problem. Supply managers define different supply alternatives needed quantity of considered kind of raw material. By developing model, the best alternative with respect of multi-criteria is found. Also, the developed model is flexible according to the possibility of number change, kind of optimization criteria change and also importance of optimization criteria change. Considering available literature, methodology defined and developed this way has numerous advantages and lot less limits.

The following conclusion is made:

(i) It is possible to describe the problem of solving the best ordering policy as multi-criteria optimization task by formal language that enables to look for the solution by exact method.
(ii) The uncertainties which exist in the model can be described by discrete fuzzy numbers.
(iii) The chosen ordering policy is extending through whole SC, so it is clear to see importance of this decision. Changing values of optimization criteria, also as changing of their importance are created as a consequence of environmental changes. All the changes can be easily incorporated into the model, so in this way, exactness of input data increases as well as the correctness of the decision.
(iv) Ranking of finite number of supply strategies, with respect to many optimization criteria, simultaneously, is obtained by comparing fuzzy numbers.
(v) The rank of supply strategies is determined by using the measures of belief that one strategy is better than all others.
(vi) The developed methodology gives the possibilities through simulation to get the answer if there would be the result change if the input data change.
(vii) The developed methodology of choice of the best ordering policy is illustrated by numerical example. The values of input data and possible alternatives (ordering policies) are arbitrary defined.

REFERENCES

[1] Ballou, H.R., *Basic Business Logistics*, Prentice-Hall, Inc.Englewood Cliffs, USA, 1992.

[2] Bass, M.S., and Kwakernaak, H., "Rating and ranking of multiple aspect alternatives using fuzzy sets", *Automatic*, 13 (1997) 47-58.

[3] Dobler, D.W., *Purchasing and Material Management: Text and Cases*, McGraw-Hill, USA, 1990.

[4] Dooley, K., "Purchasing and supply", *Operational Research*, 8 (1995) 21-25.

[5] Fazel Zarandi, H.M., et al, "Supply chain: crips and fuzzy aspects", *Int. J. Appl. Math. Comput. Sci,*, 12 (3) (2002) 423-435.

[6] Galović, D., "Multicriteria optimization in choosing the supply strategy of the complex production distribution system", Ph.D Dissertation, University of Belgrade, 1999.

[7] Galović, D., and Zrnić, Đ., "Multicriteria approach to the problem of the integration of suppliers into a production-distribution system", YUJOR, 9 (2) (1999) 207-222.

[8] Kelle, P., and Miller, P.A., "Stockout risk and order splitting", *Proc. 10th ISIR, Budapest*, 1998.

[9] Lau, H.S., and Lau, A.H., "Coordinating two suppliers with offsetting lead time and price performance", *Journal of Operations Management*, 11 (1994) 327-337.

[10] Mackey, M., "Achieving inventory reduction trough the use of partner-shipping", *Ind. Eng.*, 24 (1992) 36-38.

[11] Petrović, R., and Petrović, D., "Multicriteria ranking of inventory replenishment policies in the presence of uncertainty in customer demand", *International Journal of Production Economics*, 71 (2001) 439-446.

[12] Saaty, T.L., "How to make a decision: The analytic hierarchy process", *EJOR*, 48 (1990) 9-26.

[13] Sugeno, M., and Yasukawa, T. "A fuzzy-logic-based approach to qualitative modeling", *IEEE Trans. Fuzzy Syst*., 1 (1) (1993) 7-31.

[14] Thomas, J., and Griffin, M.P., "Coordinated supply chain management", *EJOR*, 94 (1996) 1-15.

[15] Türk sen, I.B., and Fazel Zarandi, M.H., "Production planning and scheduling: Fuzzy and crip approaches", in: *Practical Applications of Fuzzy Technologies,* H.J. Zimmerman (ed.), Kluwer Academic Publisher, Boston, 1999, 479-529.

[16] Vincke, P., *Multicriteria Decision-Aid*, John Wiley& Sons, 1992.

[17] Werner, D., and Albers, S., "Supply chain management in the global context", Working Paper No.102, University of Koeln, 2002.

[18] Yoon, K.P., and Hwang, C.L., "Multiple attribute decision making an introduction", *Series: Quantitative Applications in the Social Sciences 104, Sage University Paper*, Thousand Oaks, California, 1995.

[19] Zimmermann, H.J., *Fuzzy Set Theory and its Applications*, Boston, Kluwer Academic Publishing, 1996.