

AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH EXPONENTIAL DECLINING DEMAND AND PARTIAL BACKLOGGING

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Received: March 2003 / Accepted: August 2004

Abstract: This study proposes an EOQ inventory mathematical model for deteriorating items with exponentially decreasing demand. In the model, the shortages are allowed and partially backordered. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Further, we show that the minimized objective cost function is jointly convex and derive the optimal solution. A numerical example is presented to illustrate the model and the sensitivity analysis is also studied.

Keywords: Inventory, deteriorating items, exponential declining demand, partial backlogging.

1. INTRODUCTION

In daily life, the deteriorating of goods is a common phenomenon. Pharmaceuticals, foods, vegetables and fruit are a few examples of such items. Therefore, the loss due to deterioration cannot be neglected. Deteriorating inventory models have been widely studied in recent years. Ghare and Schrader [7] were the two earliest researchers to consider continuously decaying inventory for a constant demand. Later,

Shah and Jaiswal [13] presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Aggarwal [1] developed an order-level inventory model by correcting and modifying the error in Shah and Jaiswal's analysis [13] in calculating the average inventory holding cost. Covert and Philip [5] used a variable deterioration rate of two-parameter Weibull distribution to formulate the model with assumptions of a constant demand rate and no shortages. Then, Philip [12] extended the model by considering a variable deterioration rate of three-parameter Weibull distribution. However, all the above models are limited to the constant demand.

Recently, Goyal and Giri [8] provides a detailed review of deteriorating inventory literatures. They indicated: The assumption of constant demand rate is not always applicable to many inventory items (for example, electronic goods, fashionable clothes, etc.) as they experience fluctuations in the demand rate. Many products experience a period of rising demand during the growth phase of their product life cycle. On the other hand, the demand of some products may decline due to the introduction of more attractive products influencing customers' preference. Moreover, the age of the inventory has a negative impact on demand due to loss of consumer confidence on the quality of such products and physical loss of materials. This phenomenon prompted many researchers to develop deteriorating inventory models with time varying demand pattern. In developing inventory models, two kinds of time varying demands have been considered so far: (a) continuous-time and (b) discrete-time. Most of the continuous-time inventory models have been developed considering either linearly increasing/decreasing demand or exponentially increasing/decreasing demand patterns.

Dave and Patel [6] developed an inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no-shortage. The consideration of exponentially decreasing demand for an inventory model was first proposed by Hollier and Mak [10], who obtained optimal replenishment policies under both constant and variable replenishment intervals. Hariga and Benkherouf [9] generalized Hollier and Mak's model [10] by taking into account both exponentially growing and declining markets. Wee [15, 16] developed a deterministic lot size model for deteriorating items where demand declines exponentially over a fixed time horizon. Later, Benkherouf [2] showed that the optimal procedure suggested by Wee [15] is independent of the demand rate. Chung and Tsai [4] demonstrated that the Newton's method by Wee [15] is not suitable for the first order condition of the total cost function. They decomposed it to drop the nonzero part, and then applied the Newton's method. Su et al. [14] proposed a production inventory model for deteriorating products with an exponentially declining demand over a fix time horizon.

In the mention above, most researchers assumed that shortages are completely backlogged. In practice, some customers would like to wait for backlogging during the shortage period, but the others would not. Consequently, the opportunity cost due to lost sales should be considered in the modeling. Wee [16] presented a deteriorating inventory model where demand decreases exponentially with time and cost of items. In his paper, the backlogging rate was assumed to be a fixed fraction of demand rate during the shortage period. Many researchers such as Park [11] and Hollier and Mak [10] also considered constant backlogging rates in their inventory models. In some inventory systems, however, such as fashionable commodities, the length of the waiting time for the next replenishment is the main factor in determining whether the backlogging will be accepted or not. The longer the waiting time is, the smaller the backlogging rate would be

and vice versa. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment. In a recent paper, Chang and Dye [3] investigated an EOQ model allowing for shortage. During the shortage period, the backlogging rate is variable and dependent on the length of the waiting time for the next replenishment.

In this paper, an EOQ inventory model with deteriorating items is developed, in which we assume that the demand function is exponentially decreasing and the backlogging rate is inversely proportional to the waiting time for the next replenishment. The primary problem is to minimize the total relevant cost by simultaneously optimizing the shortage point and the length of cycle. We also show that the minimized objective cost function is jointly convex and obtain the optimal solution. A numerical example is proposed to illustrate the model and the solution procedure. The sensitivity analysis of the major parameters is performed.

2. NOTATION AND ASSUMPTIONS

The mathematical model in this paper is developed on the basis of the following notation and assumptions.

Notation:

- c_1 : holding cost, \$/per unit/per unit time
- c_2 : cost of the inventory item, \$/per unit
- c_3 : ordering cost of inventory, \$/per order
- c_4 : shortage cost, \$/per unit/per unit time
- c_5 : opportunity cost due to lost sales, \$/per unit
- t_1 : time at which shortages start
- T : length of each ordering cycle
- W : the maximum inventory level for each ordering cycle
- S : the maximum amount of demand backlogged for each ordering cycle
- Q : the order quantity for each ordering cycle
- $I(t)$: the inventory level at time t

Assumptions:

1. The inventory system involves only one item and the planning horizon is infinite.
2. The replenishment occurs instantaneously at an infinite rate.
3. The deteriorating rate, θ ($0 < \theta < 1$), is constant and there is no replacement or repair of deteriorated units during the period under consideration.
4. The demand rate, $R(t)$, is known and decreases exponentially.

$$R(t) = A e^{-\lambda t}, \quad I(t) > 0, \\ = D, \quad I(t) \leq 0,$$

where $A (>0)$ is initial demand and λ ($0 < \lambda < \theta$) is a constant governing the decreasing rate of the demand.

5. During the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Hence, the proportion of customers who would like to accept backlogging at time t is decreasing with the waiting time $(T-t)$ waiting for the next replenishment. To take care of this situation we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative. The backlogging parameter δ is a positive constant, $t_1 \leq t \leq T$.

3. MODEL FORMULATION

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. The behavior of inventory system at any time is depicted in Figure 1.

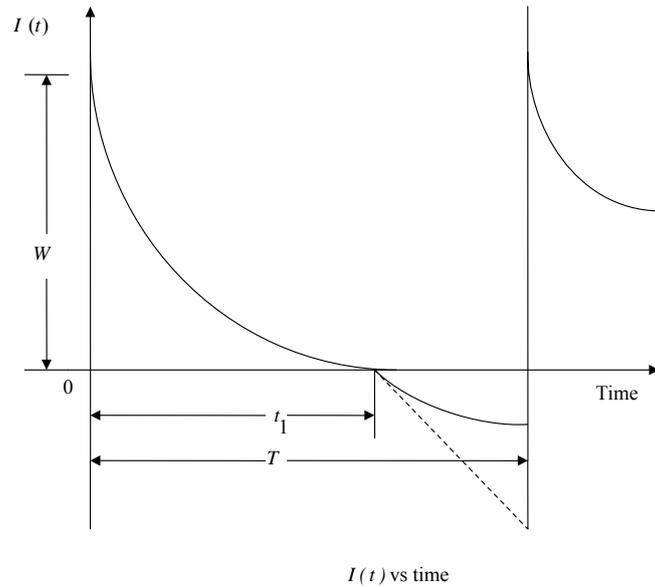


Figure 1: Inventory level $I(t)$ vs. time

Replenishment is made at time $t = 0$ and the inventory level is at its maximum, W . Due to both the market demand and deterioration of the item, the inventory level decreases during the period $[0, t_1]$, and ultimately falls to zero at $t = t_1$. Thereafter, shortages are allowed to occur during the time interval $[t_1, T]$, and all of the demand during the period $[t_1, T]$ is partially backlogged.

As described above, the inventory level decreases owing to demand rate as well as deterioration during inventory interval $[0, t_1]$. Hence, the differential equation representing the inventory status is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -A e^{-\lambda t}, \quad 0 \leq t \leq t_1, \tag{1}$$

with the boundary condition $I(0) = W$. The solution of equation (1) is

$$I(t) = \frac{A e^{-\lambda t}}{\theta - \lambda} [e^{(\theta - \lambda)(t_1 - t)} - 1], \quad 0 \leq t \leq t_1. \tag{2}$$

So the maximum inventory level for each cycle can be obtained as

$$W = I(0) = \frac{A}{\theta - \lambda} [e^{(\theta - \lambda)t_1} - 1]. \tag{3}$$

During the shortage interval $[t_1, T]$, the demand at time t is partly backlogged at the fraction $\frac{1}{1 + \delta(T - t)}$. Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dI(t)}{dt} = -\frac{D}{1 + \delta(T - t)}, \quad t_1 < t \leq T, \tag{4}$$

with the boundary condition $I(t_1) = 0$. The solution of equation (4) can be given by

$$I(t) = \frac{D}{\delta} \{ \ln [1 + \delta(T - t)] - \ln [1 + \delta(T - t_1)] \}, \quad t_1 \leq t \leq T. \tag{5}$$

Let $t = T$ in (5), we obtain the maximum amount of demand backlogged per cycle as follows:

$$S = -I(T) = \frac{D}{\delta} \ln [1 + \delta(T - t_1)]. \tag{6}$$

Hence, the order quantity per cycle is given by

$$Q = W + S = \frac{A}{\theta - \lambda} [e^{(\theta - \lambda)t_1} - 1] + \frac{D}{\delta} \ln [1 + \delta(T - t_1)]. \tag{7}$$

The inventory holding cost per cycle is

$$HC = \int_0^{t_1} c_1 I(t) dt = \frac{c_1 A}{\theta (\theta - \lambda)} e^{-\lambda t_1} [e^{\theta t_1} - 1 - \frac{\theta}{\lambda} (e^{\lambda t_1} - 1)]. \tag{8}$$

The deterioration cost per cycle is

$$\begin{aligned} DC &= c_2 [W - \int_0^{t_1} R(t) dt] \\ &= c_2 [W - \int_0^{t_1} A e^{-\lambda t} dt] \\ &= c_2 A \left\{ \frac{1}{\theta - \lambda} (e^{(\theta - \lambda)t_1} - 1) - \frac{1}{\lambda} (1 - e^{-\lambda t_1}) \right\}. \end{aligned} \tag{9}$$

The shortage cost per cycle is

$$SC = c_4 \left[-\int_{t_1}^T I(t) dt \right] = c_4 D \left\{ \frac{T-t_1}{\delta} - \frac{1}{\delta^2} \ln [1 + \delta(T-t_1)] \right\}. \tag{10}$$

The opportunity cost due to lost sales per cycle is

$$BC = c_5 \int_{t_1}^T \left[1 - \frac{1}{1 + \delta(T-t)} \right] D dt = c_5 D \left\{ (T-t_1) - \frac{1}{\delta} \ln [1 + \delta(T-t_1)] \right\}. \tag{11}$$

Therefore, the average total cost per unit time per cycle is

$$\begin{aligned} TVC &\equiv TVC(t_1, T) \\ &= (\text{holding cost} + \text{deterioration cost} + \text{ordering cost} + \text{shortage cost} + \\ &\quad \text{opportunity cost due to lost sales}) / \text{length of ordering cycle} \\ &= \frac{1}{T} \left\{ \frac{c_1 A}{\theta(\theta-\lambda)} e^{-\lambda t_1} \left[e^{\theta t_1} - 1 - \frac{\theta}{\lambda} (e^{\lambda t_1} - 1) \right] + c_2 A \left[\frac{e^{(\theta-\lambda)t_1} - 1}{\theta-\lambda} - \frac{1-e^{-\lambda t_1}}{\lambda} \right] + c_3 \right. \\ &\quad \left. + D \left(\frac{c_4}{\delta} + c_5 \right) \left[T - t_1 - \frac{\ln [1 + \delta(T-t_1)]}{\delta} \right] \right\} \\ &= \frac{1}{T} \left\{ \frac{A(c_1 + \theta c_2)}{\theta(\theta-\lambda)} \left[e^{(\theta-\lambda)t_1} - (\theta-\lambda)t_1 - 1 \right] - \frac{A(c_1 + \theta c_2)}{\theta\lambda} \left[1 - \lambda t_1 - e^{-\lambda t_1} \right] + c_3 \right. \\ &\quad \left. + \frac{D(c_4 + \delta c_5)}{\delta} \left[T - t_1 - \frac{\ln [1 + \delta(T-t_1)]}{\delta} \right] \right\}. \tag{12} \end{aligned}$$

The objective of the model is to determine the optimal values of t_1 and T in order to minimize the average total cost per unit time, TVC . The optimal solutions t_1^* and T^* need to satisfy the following equations:

$$\frac{\partial TVC}{\partial t_1} = \frac{1}{T} \left\{ \frac{A(c_1 + \theta c_2)}{\theta} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] - \frac{D(c_4 + \delta c_5)}{\delta} \left[1 - \frac{1}{1 + \delta(T-t_1)} \right] \right\} = 0, \tag{13}$$

and

$$\begin{aligned} \frac{\partial TVC}{\partial T} &= \frac{1}{T^2} \left\{ \frac{D(c_4 + \delta c_5)}{\delta} \left[\frac{(T-t_1)(\delta t_1 - 1)}{1 + \delta(T-t_1)} + \frac{1}{\delta} \ln [1 + \delta(T-t_1)] \right] \right. \\ &\quad \left. - \frac{A(c_1 + \theta c_2)}{\theta(\theta-\lambda)} \left[e^{(\theta-\lambda)t_1} - 1 \right] + \frac{A(c_1 + \theta c_2)}{\theta\lambda} \left[1 - e^{-\lambda t_1} \right] - c_3 \right\} = 0. \tag{14} \end{aligned}$$

For convenience, we let $M = \frac{A(c_1 + \theta c_2)}{\theta}$ and $N = \frac{D(c_4 + \delta c_5)}{\delta}$ and then, from (13) and (14), we get

$$T = t_1 + \left\{ \frac{1}{\delta} \frac{M}{N} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] \right\} / \left\{ 1 - \frac{M}{N} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] \right\}, \tag{15}$$

and

$$N \left\{ \frac{(T-t_1)(\delta t_1 - 1)}{1 + \delta(T-t_1)} + \frac{1}{\delta} \ln[1 + \delta(T-t_1)] \right\} - \frac{M}{\theta - \lambda} \left[e^{(\theta-\lambda)t_1} - 1 \right] + \frac{M}{\lambda} (1 - e^{-\lambda t_1}) - c_3 = 0, \text{ respectively.} \tag{16}$$

Substituting (15) into (16), we obtain

$$\frac{M}{\delta} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] (\delta t_1 - 1) - \frac{N}{\delta} \ln \left[1 - \frac{M}{N} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] \right] - \frac{M}{\theta - \lambda} \left[e^{(\theta-\lambda)t_1} - 1 \right] + \frac{M}{\lambda} (1 - e^{-\lambda t_1}) - c_3 = 0. \tag{17}$$

If we let $P = \left(1 + \frac{N}{M}\right) - \left(1 + \frac{N}{M}\right)^{-\frac{\lambda}{\theta-\lambda}}$, then we have the following results.

Theorem 1.

If $\theta > \lambda$ and $\frac{MP}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right] - \frac{N}{\delta} \ln \left[1 - \frac{M}{N} P \right] - \frac{MP(\lambda - \delta)}{\delta \lambda} - \frac{\theta}{\lambda(\theta - \lambda)} N - c_3 > 0$, then the solution to (13) and (14) not only exists but also is unique (i.e., the optimal value (t_1^*, T^*) is uniquely determined).

Proof: By assumption 5, we have $T > t_1$, and hence, from (15), we obtain

$$1 - \frac{M}{N} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] > 0, \tag{18}$$

which implies $t_1 < \hat{t}_1 \equiv \frac{1}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right]$.

Next, from (17), we let

$$F(t_1) = \frac{M}{\delta} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] (\delta t_1 - 1) - \frac{N}{\delta} \ln \left[1 - \frac{M}{N} \left[e^{(\theta-\lambda)t_1} - e^{-\lambda t_1} \right] \right] - \frac{M}{\theta - \lambda} \left[e^{(\theta-\lambda)t_1} - 1 \right] + \frac{M}{\lambda} (1 - e^{-\lambda t_1}) - c_3.$$

Taking the first derivative of $F(t_1)$ with respect to $t_1 \in (0, \hat{t}_1)$, we get

$$\frac{dF(t_1)}{dt_1} = [(\theta - \lambda)e^{(\theta - \lambda)t_1} + \lambda e^{-\lambda t_1}] \left\{ Mt_1 + \frac{M}{\delta} \frac{M}{N} [e^{(\theta - \lambda)t_1} - e^{-\lambda t_1}] \right\} / \left\{ 1 - \frac{M}{N} [e^{(\theta - \lambda)t_1} - e^{-\lambda t_1}] \right\} > 0. \tag{by equation (18)}$$

Hence, $F(t_1)$ is a strictly increasing function in $t_1 \in [0, \hat{t}_1)$.

Furthermore, we have $F(0) = -c_3 < 0$, and

$$\begin{aligned} \lim_{t_1 \rightarrow \hat{t}_1^-} F(t_1) &= \lim_{t_1 \rightarrow \hat{t}_1^-} \left\{ \frac{M}{\delta} [e^{(\theta - \lambda)t_1} - e^{-\lambda t_1}] (\delta t_1 - 1) - \frac{N}{\delta} \ln \left[1 - \frac{M}{N} [e^{(\theta - \lambda)t_1} - e^{-\lambda t_1}] \right] \right. \\ &\quad \left. - \frac{M}{\theta - \lambda} [e^{(\theta - \lambda)t_1} - 1] + \frac{M}{\lambda} (1 - e^{-\lambda t_1}) - c_3 \right\} \\ &= \frac{MP}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right] - \frac{N}{\delta} \ln \left[1 - \frac{MP}{N} \right] - \frac{MP(\lambda - \delta)}{\delta \lambda} - \frac{N\theta}{\lambda(\theta - \lambda)} - c_3, \end{aligned}$$

where $P = (1 + \frac{N}{M}) - (1 + \frac{N}{M})^{\frac{-\lambda}{\theta - \lambda}}$.

Thus, if $\theta > \lambda$ and $\frac{MP}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right] - \frac{N}{\delta} \ln \left[1 - \frac{MP}{N} P \right] - \frac{MP(\lambda - \delta)}{\delta \lambda} - \frac{\theta}{\lambda(\theta - \lambda)} N - c_3 > 0$,

we obtain $\lim_{t_1 \rightarrow \hat{t}_1^-} F(t_1) > 0$. Therefore, we can find a unique $t_1^* \in (0, \hat{t}_1)$, such that

$$F(t_1^*) = 0.$$

Once we obtain the value t_1^* , then the optimal value T^* can be uniquely determined by equation (15). This completes the proof.

Now, we can obtain the following main result.

Theorem 2.

If $\theta > \lambda$ and $\frac{MP}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right] - \frac{N}{\delta} \ln \left[1 - \frac{MP}{N} P \right] - \frac{MP(\lambda - \delta)}{\delta \lambda} - \frac{\theta}{\lambda(\theta - \lambda)} N - c_3 > 0$,

the total cost per unit time $TVC(t_1, T)$ is convex and reaches its global minimum at point (t_1^*, T^*) .

Proof: From equations (13) and (14), we have

$$\begin{aligned} \frac{\partial^2}{\partial t_1^2} TVC \Big|_{(t_1^*, T^*)} &= \frac{1}{T^*} \left\{ M [(\theta - \lambda) e^{(\theta - \lambda)t_1^*} + \lambda e^{-\lambda t_1^*}] + \frac{N\delta}{[1 + \delta(T^* - t_1^*)]^2} \right\} > 0, \\ \frac{\partial^2}{\partial t_1 \partial T} TVC \Big|_{(t_1^*, T^*)} &= \frac{-1}{T^*} \left\{ \frac{N\delta}{[1 + \delta(T^* - t_1^*)]^2} \right\}, \end{aligned}$$

and

$$\frac{\partial^2}{\partial T^2} TVC \Big|_{(t_1^*, T^*)} = \frac{1}{T^*} \frac{N \delta}{[1 + \delta(T^* - t_1^*)]^2} > 0.$$

Then,

$$\begin{aligned} & \frac{\partial^2 TVC}{\partial t_1^2} \Big|_{(t_1^*, T^*)} \times \frac{\partial^2 TVC}{\partial T^2} \Big|_{(t_1^*, T^*)} - \left[\frac{\partial^2 TVC}{\partial t_1 \partial T} \Big|_{(t_1^*, T^*)} \right]^2 \\ &= \frac{1}{T^{*2}} \left\{ M [(\theta - \lambda) e^{(\theta - \lambda) t_1^*} + \lambda e^{-\lambda t_1^*}] \frac{N \delta}{[1 + \delta(T^* - t_1^*)]^2} \right\} > 0. \end{aligned}$$

This completes the proof.

Next, by using t_1^* and T^* , we can obtain the optimal maximum inventory level and the minimum average total cost per unit time from equations (3) and (12), respectively (we denote these values by W^* and TVC^*). Furthermore, we can also obtain the optimal order quantity (we denote it by Q^*) from equation (7).

4. NUMERICAL EXAMPLE AND ITS SENSITIVITY ANALYSIS

According to the results of Section 3, we will provide an example to explain how the solution procedure works.

Suppose that there is a product with an exponentially decreasing function of demand $f(t) = Ae^{-\lambda t}$, where A and λ are arbitrary constants satisfying $A > 0$ and $\lambda > 0$. The remaining parameters of the inventory system are $A = 12$, $\theta = 0.08$, $\delta = 2$, $\lambda = 0.03$, $c_1 = 0.5$, $c_2 = 1.5$, $c_3 = 10$, $c_4 = 2.5$, $c_5 = 2$, and $D = 8$. Under the above-given parameter values, we check the condition

$$\frac{MP}{\theta - \lambda} \ln \left[1 + \frac{N}{M} \right] - \frac{N}{\delta} \ln \left[1 - \frac{M}{N} P \right] - \frac{MP(\lambda - \delta)}{\delta \lambda} - \frac{\theta}{\lambda(\theta - \lambda)} N - c_3 = 277.222 > 0,$$

and then obtain the optimal shortage point $t_1^* = 1.4775$ unit time and the optimal length of ordering cycle $T^* = 1.8536$ unit time. Thereafter, we get the optimal maximum inventory level $W^* = 18.401$ units, the optimal order quantity $Q^* = 20.1183$ units and the minimum average total cost per unit time $TVC^* = \$11.1625$.

Next, we study the effects of changes in the model parameters such as $A, \lambda, c_1, c_2, c_3, c_4, c_5, D, \theta$ and δ on the optimal shortage point, the optimal length of ordering cycle, the optimal order quantity, the optimal maximum inventory level and the minimum average total cost per unit time. The sensitivity analysis is performed by changing each of the parameters by -50 %, -25 %, +25 % and +50 % taking one parameter at a time while keeping remaining unchanged. The results are presented in Table 1.

Table 1: Sensitivity Analysis

Parameter	% change	% change in				
		t_1^*	T^*	Q^*	W^*	TVC^*
A	+50%	-22.1726%	-11.0477%	16.6083%	15.7812%	16.3351%
	+25%	-12.6640%	-6.6773%	9.0688%	8.6571%	8.9532%
	-25%	18.1536%	10.8227%	-11.1642%	-10.7793%	-11.1247%
	-50%	47.7949%	30.4855%	-25.4760%	-24.7650%	-25.5261%
λ	+50%	1.5905%	1.0612%	0.3638%	0.5032%	-0.5922%
	+25%	0.8108%	0.5449%	0.2034%	0.2750%	-0.2947%
	-25%	-0.7939%	-0.5255%	-0.1942%	-0.2674%	0.2920%
	-50%	-1.5668%	-1.0385%	-0.3851%	-0.5288%	0.5805%
c_1	+50%	-18.7783%	-9.5609%	-15.1430%	-19.3441%	13.6367%
	+25%	-10.5076%	-5.6026%	-8.5727%	-10.8570%	7.3577%
	-25%	14.0203%	8.2558%	11.7973%	14.6220%	-8.7615%
	-50%	34.2261%	21.2505%	29.4083%	35.9611%	-19.4522%
c_2	+50%	-5.3848%	-2.9418%	-4.4220%	-5.5736%	3.6838%
	+25%	-2.7810%	-1.5359%	-2.2907%	-2.8808%	1.8795%
	-25%	2.9753%	1.6929%	2.4752%	3.0917%	-1.9592%
	-50%	6.1739%	3.5423%	5.1500%	6.4214%	-4.0045%
c_3	+50%	21.0321%	25.8659%	23.0134%	21.9912%	21.4468%
	+25%	11.1364%	13.2877%	12.0705%	11.6026%	11.3371%
	-25%	-12.8575%	-14.4659%	-13.6414%	-13.2732%	-13.0330%
	-50%	-28.3648%	-30.9959%	-29.7387%	-29.1185%	-28.6834%
c_4	+50%	1.8484%	-3.1879%	-0.2998%	1.9205%	1.8804%
	+25%	1.0193%	-1.8127%	-0.1632%	1.0592%	1.0374%
	-25%	-1.2866%	2.4903%	0.2025%	-1.3325%	-1.3053%
	-50%	-2.9503%	6.0752%	0.4548%	-3.0556%	-2.9940%
c_5	+50%	2.6579%	-4.4492%	-0.4374%	2.7618%	2.7037%
	+25%	1.5357%	-2.6802%	-0.2480%	1.5961%	1.5624%
	-25%	-2.2294%	4.4697%	0.3468%	-2.3091%	-2.2620%
	-50%	-5.7171%	13.0535%	0.8627%	-5.9165%	-5.7980%
D	+50%	3.6934%	-5.9517%	2.7949%	3.8384%	3.7563%
	+25%	2.2748%	-3.8628%	1.7249%	2.3635%	2.3140%
	-25%	-4.2051%	9.0705%	-3.2067%	-4.3536%	-4.2643%
	-50%	infeasible solution				
θ	+50%	-7.3042%	-4.1875%	-3.7502%	-4.9465%	4.4345%
	+25%	-3.8037%	-2.2092%	-1.9332%	-2.5439%	2.2656%
	-25%	4.1462%	2.4811%	2.0664%	2.7009%	-2.3669%
	-50%	8.6931%	5.2703%	4.2768%	5.5752%	-4.8430%
δ	+50%	1.4917%	-1.1227%	-0.5338%	1.5499%	1.5176%
	+25%	0.8129%	-0.5859%	-0.3037%	0.8451%	0.8278%
	-25%	-1.0010%	0.6458%	0.4205%	-1.0364%	-1.0150%
	-50%	-2.2660%	1.3493%	1.0250%	-2.3472%	-2.2997%

On the basis of the results shown in Table 1, the following observations can be made.

1. t_1^* and T^* decrease while Q^* , W^* and TVC^* increase with increase in the value of the model parameter A . The obtained results show that T^* , Q^* , W^* and TVC^* are moderately sensitive to changes in the value of A . Moreover, t_1^* is highly sensitive to changes in A .
2. TVC^* decreases while t_1^* , T^* , Q^* and W^* increase with increase in the value of the model parameter λ . It is seen that t_1^* , T^* , Q^* , W^* and TVC^* are insensitive to changes in the value of the parameter λ .
3. t_1^* , T^* , Q^* and W^* decrease while TVC^* increases with increase in the value of the model parameters c_1 or c_2 . Moreover, TVC^* , t_1^* , T^* , Q^* and W^* are highly sensitive to changes in the value of the parameter c_1 and moderately sensitive to changes in the value of c_2 .
4. As the value of c_3 increases, t_1^* , T^* , Q^* , W^* and TVC^* increase. It is seen that t_1^* , T^* , Q^* , W^* and TVC^* are highly sensitive to changes in the value of c_3 .
5. T^* and Q^* decrease while t_1^* , W^* and TVC^* increase with increase in the value of the model parameters c_4 or c_5 . It is seen that t_1^* , W^* , TVC^* and T^* are lowly sensitive to changes in the values of c_4 and c_5 . However, Q^* is almost insensitive.
6. T^* decreases while t_1^* , Q^* , W^* and TVC^* increase with increase in the value of the model parameter D .
7. t_1^* , T^* , Q^* and W^* decrease while TVC^* increases with increase in the value of the model parameter θ .
8. T^* and Q^* decrease while t_1^* , W^* and TVC^* increase with increase in the value of the model parameter δ . In addition, t_1^* , W^* , TVC^* , T^* and Q^* are lowly sensitive to changes in the value of δ .

5. CONCLUSIONS

The classical economic order quantity (EOQ) model assumes a predetermined constant demand rate and no effects on shortages. In reality, however, not only demand varies with time, but also costs are affected by shortages. In the proposed model, we present an EOQ inventory model for deteriorating items with exponential declining demand and partial backlogging. The rate of deterioration is assumed to be constant and the backlogging rate is inversely proportional to the waiting time for the next replenishment. We also show that the minimized objective cost function is jointly convex and derive the optimal solution. Furthermore, a numerical example and its sensitivity analysis for parameters are provided to assess the solution procedure.

The proposed model can be extended in several ways. For instance, it could be of interest to relax the restriction of constant deterioration rate. Also, we may extend the deterministic demand function to stochastic fluctuating demand patterns. Finally, we could generalize the model to the economic production lot size model.

Acknowledgement: The authors greatly appreciate the anonymous referees for their value and helpful suggestions on an earlier version of the paper.

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