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MULTI-ITEM FUZZY INVENTORY PROBLEM WITH SPACE CONSTRAINT VIA GEOMETRIC PROGRAMMING METHOD

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Abstract: In this paper, a multi-item inventory model with space constraint is developed in both crisp and fuzzy environment. A profit maximization inventory model is proposed here to determine the optimal values of demands and order levels of a product. Selling price and unit price are assumed to be demand-dependent and holding and set-up costs sock dependent. Total profit and warehouse space are considered to be vague and imprecise. The impreciseness in the above objective and constraint goals has been expressed by fuzzy linear membership functions. The problem is then solved using modified geometric programming method. Sensitivity analysis is also presented here.

Keywords: Inventory, fuzzy programming, and modified geometric programming.

1. INTRODUCTION

In many inventory problems, the unit price and selling price of a product are considered as independent in nature. But when the demand of a product is very high, then to meet the demand, the production is increased. Therefore the total cost of manufacturing is then spread over a large number of items and this will result lower average unit production cost as well as lower selling price for an item. Hence the unit production cost and selling price are assumed inversely related to the demand of the item. Cheng [3, 4] developed some inventory models considering the demand - dependent unit cost and solved by geometric programming (GP) method. Jung and Klein [7] extended the EOQ model with the same assumptions and solved by GP method for a single item.

In classical inventory models, inventory costs, unit holding cost and set - up cost are supposed to be constant. But in reality, these inventory costs are dependent on the amount produced / purchased. Hariri and Abou-el-ata [6], Abou-el-ata and Kotb [1] developed some inventory models with variable inventory costs and solved by GP method.

Geometric programming method is a relatively new technique to solve a non-linear programming problem. Duffin, Peterson and Zener [5] first developed the idea on GP method. Kotchenberger [8] was the first to use this method on inventory problems. Later on, Worrall and Hall [15] analyzed a multi-item inventory model with several constraints using polynomial GP method. Recently, Shawky and Abou-el-ata [12] formed a constrained production lot-size model with trade policy and solved by GP method, comparing with Lagrange non-linear method.

In many inventory models the objective goals and constraint goals are assumed to be deterministic. But in real life situations, the above said goals might not be exactly known are somewhat imprecise in nature. In this situation, the inventory problems along with the constraints may be realistically represented formulating the model in fuzzy environment, which is solved by different fuzzy programming methods. L. A. Zadeh [16] first introduced the fuzzy set theory. Bellman and Zadeh [2], Zimmerman [17] solved fuzzy decision-making problems using fuzzy programming techniques. Very few inventory problems was developed and solved using fuzzy set theory. Sommer [13] used fuzzy dynamic programming to an inventory problem. Park [9] applied fuzzy set theoretic approach on the EOQ model. Roy and Maiti [10] solved the classical EOQ model for a single item in fuzzy environment. They [11] also examined the fuzzy EOQ model with demand - dependent unit price and imprecise storage area by both fuzzy geometric and non - linear programming methods.

In this paper, we have formulated a multi - item profit maximization inventory model under limited storage area with demand - dependent unit price and selling price and stock - dependent holding and set- up costs of the item. Here objective and constraint goals are expressed in fuzzy environment. After fuzzification, the problem is solved by fuzzy additive geometric programming (FAGP) method. The crisp model is also solved by Geometric Programming method. The model is illustrated numerically and the results obtained from both the environments (crisp and fuzzy) are compared. Sensitivity analysis has also been discussed here.

2. MODEL FORMULATION

A multi-item inventory model with infinite rate of replenishment, no shortages and limited storage space is developed under the following assumptions.

n = number of items,

W = total available storage area,

p_W = tolerance limit of W,

c_{PF} = profit goal,
p_{PF} = tolerance limit of c_{PF},

Parameters for $i\ (=1,2,...,n)$ -th items are

D_i = demand of each product (decision variable),
Q_i = order level (decision variable),
s_i = selling price of each product
= $s_{0i}D_i^{-\alpha_i}$ where $s_{0i} > 0$ and $0 < \alpha_i < 1$,
p_i = unit price of each product
= $p_{0i}D_i^{-\beta_i}$ where $p_{0i} > 0$ and $0 < \beta_i < 1$,
c_{1i} = holding cost of each product
= $c_{01i}Q_i^{-\gamma_i}$ where $c_{01i} > 0$ and $0 < \gamma_i < 1$,
c_{3i} = set up cost of each product
= $c_{03i}Q_i^{-\delta_i}$ where $c_{03i} > 0$ and $0 < \delta_i < 1$.

3. MATHEMATICAL FORMULATION

3.1. Crisp model

Total profit = total revenue - production cost - holding cost - set up cost.

$$\begin{aligned}\max PF(D_i, Q_i) &= \sum_{i=1}^{n}[s_iD_i - p_iD_i - 0.5c_{1i}Q_i - c_{3i}D_iQ_i^{-1}] \\ &= \sum_{i=1}^{n}[s_{0i}D_i^{(1-\alpha_i)} - p_{0i}D_i^{(1-\beta_i)} - 0.5c_{01i}Q_i^{\gamma_i+1} - c_{03i}D_iQ_i^{\delta_i-1}]\end{aligned} \quad (1)$$

subject to $$\sum_{i=1}^{n} w_iQ_i \le W$$

$$D_i > 0,\ Q_i > 0,\ (i = 1,2,...,n).$$

3.2. Fuzzy model

When the profit goal and storage area are flexible i.e. fuzzy in nature, the said crisp model (1) is transformed to

$$\widetilde{\max} PF(D_i, Q_i) = \sum_{i=1}^{n}[s_{0i}D_i^{1-\alpha_i} - p_{0i}D_i^{1-\beta_i} - 0.5c_{01i}Q_i^{\gamma_i+1} - c_{03i}D_iQ_i^{\delta_i-1}] \quad (2)$$

subject to $$\sum_{i=1}^{n} w_iQ_i \le \tilde{W}$$

$$D_i > 0,\ Q_i > 0,\ (i = 1,2,...,n).$$

4. MATHEMATICAL ANALYSIS

The multi item inventory models (1) and (2) are solved by crisp and fuzzy environment respectively. In fuzzy set theory, the imprecise objectives and constraints are defined by their membership functions, which may be linear and / or non-linear. For simplicity, we assume here $\mu_{PF}(D_i, Q_i)$ and $\mu_w(Q_i)$ to be linear membership functions for the objective and constraint. They are

$$\mu_{PF}(D_i, Q_i) = \begin{cases} 0 & \text{for } PF(D_i, Q_i) < c_{PF} - p_{PF} \\ 1 + \dfrac{PF(D_i, Q_i) - c_{PF}}{p_{PF}} & \text{for } c_{PF} - p_{PF} \le PF(D_i, Q_i) < c_{PF} \\ 1 & \text{for } PF(D_i, Q_i) > c_{PF} \end{cases}$$

Pictorial representation of $\mu_W(Q_i)$ is

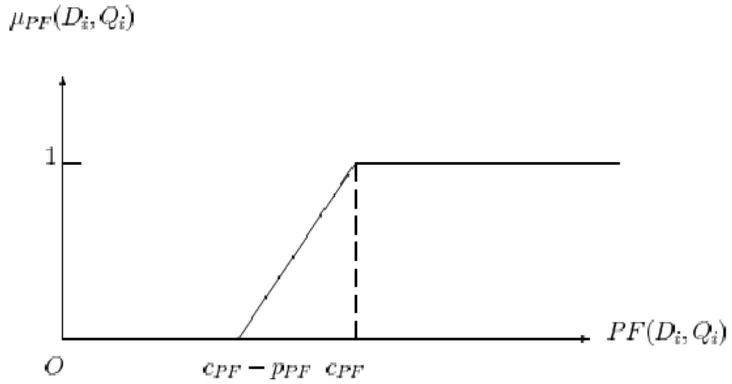

Figure 1: Membership function of $PF(D_i, Q_i)$

and

$$\mu_W(Q_i) = \begin{cases} 1 & \text{for } \sum_{i=1}^{n} w_i Q_i < W \\ 1 - \dfrac{\sum_{i=1}^{n} w_i Q_i - W}{p_W} & \text{for W} \le \sum_{i=1}^{n} w_i Q_i < W + p_W \\ 0 & \text{for } \sum_{i=1}^{n} w_i Q_i > W + p_W \end{cases}$$

Pictorial representation of $\mu_W(Q_i)$ is

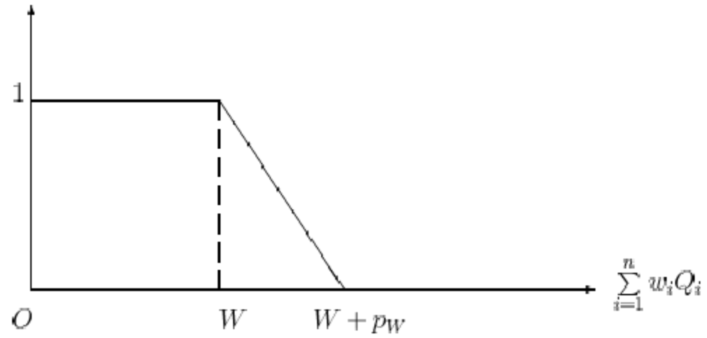

Figure 2: Membership function of $\sum_{i=1}^{n} w_i Q_i \le W$

The problem (2) can be formulated with the fuzzy additive goal programming (Tiwari, Dharmar and Rao (1987)) problem as

$$\max\ (\mu_{PF}(D_i, Q_i) + \mu_W(Q_i)) \tag{3}$$

subject to

$$\mu_{PF}(D_i, Q_i) = 1 + \frac{PF(D_i, Q_i) - c_{PF}}{p_{PF}}, \text{ and}$$

$$\mu_W(Q_i) = 1 - \frac{\sum_{i=1}^{n} w_i Q_i - W}{p_W},$$

$$D_i > 0,\ Q_i > 0,\ 0 \le \mu_{PF}(D_i, Q_i),\ \mu_W(Q_i) \le 1, \quad (i = 1, 2, ..., n).$$

Which is equivalent to

$$\max\ (\mu_{PF}(D_i, Q_i) + \mu_W(Q_i)) = 2 - \frac{c_{PF}}{p_{PF}} + \frac{W}{p_W} + \min U(D_i, Q_i)$$

subject to

$$D_i > 0,\ Q_i > 0, \quad (i = 1, 2, ..., n).$$

Here $U(D_i, Q_i) = \left(-\frac{PF(D_i, Q_i)}{p_{PF}} + \frac{\sum_{i=1}^{n} w_i Q_i}{p_W} \right).$

The first three terms are constants as they are independent of decision variables hence at present they can be omitted from the analysis. So (3) reduces to

$$\min U(D_i, Q_i) = \sum \left[-\frac{p_{0i} D_i^{1-\beta_i}}{p_{PF}} + \frac{c_{01i} Q_i^{\gamma_i + 1}}{2 p_{PF}} + \frac{c_{03i} D_i Q_i^{\delta_i - 1}}{p_{PF}} + \frac{w_i Q_i}{p_W} - \frac{s_{0i} D_i^{1-\alpha_i}}{p_{PF}} \right] \tag{4}$$

subject to

$D_i > 0,\ Q_i > 0, \quad (i = 1, 2, ..., n)$.

5. SOLUTION OF THE PROPOSED INVENTORY MODELS

5.1. Crisp model

Multiplying both sides of (1) by (-1) we get the standard signomial GP form as follows

$$\min PF'(D_i, Q_i) = \sum_{i=1}^{n} [\, p_{0i} D_i^{1-\beta_i} + 0.5 c_{01i} Q_i^{\gamma_i + 1} + c_{03i} D_i Q_i^{\delta_i - 1} - s_{0i} D_i^{1-\alpha_i} \,] \tag{5}$$

subject to $\sum_{i=1}^{n} w_i Q_i \le W$,

$D_i > 0,\ Q_i > 0,\ (i = 1, 2, ..., n)$.

Here $PF'(D_i, Q_i) = -PF(D_i, Q_i)$.

Here the primal function is a constrained signomial function with (3n–1) degree of difficulty. It is difficult to solve by formulating its dual problem for more items. Therefore, we use the Modified Geometric Programming (MGP) method outlined by Hariri and Abou-el-Ata [6] for the solution. Under this formulation, we have the corresponding dual function is

$$\max d(w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i})$$
$$= -\left[\prod_{i=1}^{n} \left\{ \left(\frac{p_{0i}}{w_{1i}} \right)^{w_{1i}} \left(\frac{c_{01i}}{2 w_{2i}} \right)^{w_{2i}} \left(\frac{c_{03i}}{w_{3i}} \right)^{w_{3i}} \left(\frac{s_{0i}}{w_{4i}} \right)^{-w_{4i}} \left(\frac{\lambda w_i}{W w_{5i}} \right)^{w_{5i}} \right\} \right]^{-1} \tag{6}$$

subject to the normality and orthogonality conditions

$$-w_{1i} - w_{2i} - w_{3i} + w_{4i} = 1,$$
$$(1 - \beta_i) w_{1i} + w_{3i} - (1 - \alpha_i) w_{4i} = 0, \quad (i = 1, 2, ..., n)$$
$$(\gamma_i + 1) w_{2i} + (\delta_i - 1) w_{3i} + w_{5i} = 0.$$

where $\lambda = \sum_{i=1}^{n} w_{5i}$ and the dual variables $w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}$ are all non-negative for $i = 1, 2, ..., n$. Expressing the dual variables w_{2i}, w_{3i}, w_{5i} in terms of w_{1i}, w_{4i} and then the dual function (6) becomes

$$\max d(w_{1i}, w_{4i})$$
$$= -\left[\prod_{i=1}^{n}\left\{\left(\frac{p_{0i}}{w_{1i}}\right)^{w_{1i}}\left(\frac{c_{01i}}{2w_{2i}}\right)^{w_{2i}}\left(\frac{c_{03i}}{w_{3i}}\right)^{w_{3i}}\left(\frac{s_{0i}}{w_{4i}}\right)^{-w_{4i}}\left(\frac{\lambda w_i}{Ww_{5i}}\right)^{w_{5i}}\right\}\right]^{-1} \quad (7)$$

where

$$w_{2i} = \alpha_i w_{4i} - \beta_i w_{1i} - 1,$$
$$w_{3i} = (1-\alpha_i)w_{4i} - (1-\beta_i)w_{1i},$$
$$w_{5i} = \{(1-\delta_i)(1-\alpha_i) - \alpha_i(\gamma_i+1)\}w_{4i} + \{\beta_i(\gamma_i+1) - (1-\delta_i)(1-\beta_i)\}w_{1i} + (\gamma_i+1).$$

Now optimizing the objective function (7) we get the optimal values of dual variables w_{1i}^*, w_{4i}^*. The optimal values of decision variables are obtained using the theorem of geometric programming as follows:

$$\frac{p_{0i}D_i^{*(1-\beta_i)}}{w_{1i}^*} = \frac{s_{0i}D_i^{*(1-\alpha_i)}}{w_{4i}^*},$$

$$\frac{w_iQ_i^*}{W} = \frac{\{(1-\delta_i)(1-\alpha_i) - \alpha_i(\gamma_i+1)\}w_{4i}^* + \{\beta_i(\gamma_i+1) - (1-\delta_i)(1-\beta_i)\}w_{1i}^* + (\gamma_i+1)}{\sum_{i=1}^{n}[\{(1-\delta_i)(1-\alpha_i) - \alpha_i(\gamma_i+1)\}w_{4i}^* + \{\beta_i(\gamma_i+1) - (1-\delta_i)(1-\beta_i)\}w_{1i}^* + (\gamma_i+1)]}$$

Solving the above relations, we get,

$$D_i^* = \left(\frac{s_{0i}w_{1i}^*}{p_{0i}w_{4i}^*}\right)^{\frac{1}{(\alpha_i-\beta_i)}},$$

$$Q_i^* = \frac{W[\{(1-\delta_i)(1-\alpha_i) - \alpha_i(\gamma_i+1)\}w_{4i}^* + \{\beta_i(\gamma_i+1) - (1-\delta_i)(1-\beta_i)\}w_{1i}^* + (\gamma_i+1)]}{w_i\sum_{i=1}^{n}[\{(1-\delta_i)(1-\alpha_i) - \alpha_i(\gamma_i+1)\}w_{4i}^* + \{\beta_i(\gamma_i+1) - (1-\delta_i)(1-\beta_i)\}w_{1i}^* + (\gamma_i+1)]}$$

Using the optimal values of decision variables say, D_i^* and Q_i^* we get the optimal value of the objective function $PF^*(D_i^*, Q_i^*)$ from (1).

5.2. Fuzzy model

The primal function (4) is an unconstrained signomial GP problem with (3*n*–1) degree of difficulty. The corresponding dual function according to Hariri and Abou-el-Ata [6] is as follows

$$\max dw(w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i})$$
$$= -\left[\prod_{i=1}^{n}\left\{\left(\frac{p_{0i}}{p_{PF}w_{1i}}\right)^{w_{1i}}\left(\frac{c_{01i}}{2p_{PF}w_{2i}}\right)^{w_{2i}}\left(\frac{c_{03i}}{p_{PF}w_{3i}}\right)^{w_{3i}}\left(\frac{s_{0i}}{p_w w_{4i}}\right)^{-w_{4i}}\left(\frac{\lambda w_i}{p_{PF}Ww_{5i}}\right)^{w_{5i}}\right\}\right]^{-1} \quad (8)$$

subject to the normality and orthogonality conditions

$$-w_{1i} - w_{2i} - w_{3i} - w_{4i} + w_{5i} = 1,$$
$$(1-\beta_i)w_{1i} + w_{3i} - (1-\alpha_i)w_{5i} = 0, \quad (i = 1, 2, ..., n)$$
$$(\gamma_i + 1)w_{2i} + (\delta_i - 1)w_{3i} + w_{4i} = 0.$$
$$w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i} > 0.$$

Expressing the dual variables w_{2i}, w_{3i}, w_{4i} in terms of w_{1i}, w_{5i} the dual function (8) becomes,

$$\max d(w_{1i}, w_{5i})$$
$$= -\left[\prod_{i=1}^{n}\left\{\left(\frac{p_{0i}}{p_{PF}w_{1i}}\right)^{w_{1i}}\left(\frac{c_{01i}}{2p_{PF}w_{2i}}\right)^{w_{2i}}\left(\frac{c_{03i}}{p_{PF}w_{3i}}\right)^{w_{3i}}\left(\frac{s_{0i}}{p_w w_{4i}}\right)^{-w_{4i}}\left(\frac{\lambda w_i}{p_{PF}Ww_{5i}}\right)^{w_{5i}}\right\}\right]^{-1} \tag{9}$$

where

$$w_{2i} = \frac{1}{\gamma_i} + \frac{(1-\alpha_i)(1-\beta_i)-\alpha_i}{\gamma_i}w_{5i} - \frac{(1-\beta_i)(1-\delta_i)-\beta_i}{\gamma_i}w_{1i},$$
$$w_{3i} = (1-\alpha_i)w_{5i} - (1-\beta_i)w_{1i},$$
$$w_{4i} = -\frac{\gamma_i+1}{\gamma_i} + \left[(1-\alpha_i)(1-\beta_i) - \frac{\gamma_i+1}{\gamma_i}\{(1-\delta_i)(1-\alpha_i)-\alpha_i\}\right]w_{5i}$$
$$+\left[\frac{\gamma_i+1}{\gamma_i}\{(1-\beta_i)(1-\delta_i)-\beta_i\} - (1-\beta_i)(1-\delta_i)\right]w_{1i}.$$

Now optimizing the dual function (9) we get the optimal values of dual variables w_{1i}^*, w_{5i}^* and hence $w_{2i}^*, w_{3i}^*, w_{4i}^*$. The optimal values of decision variables are obtained using the theorem of geometric programming as follows:

$$\frac{p_{0i}D_i^{*(1-\beta_i)}}{p_{PF}w_{1i}^*} = \frac{s_{0i}D_i^{*(1-\alpha_i)}}{p_{PF}w_{5i}^*}, \text{ and}$$
$$\frac{c_{01i}Q_i^{*(\gamma_i+1)}}{2p_{PF}w_{2i}^*} = \frac{w_i Q_i^*}{p_w w_{4i}^*}. \tag{10}$$

From the relations (10), we get,

$$D_i^* = \left(\frac{s_{0i}w_{1i}^*}{p_{0i}w_{5i}^*}\right)^{\frac{1}{(\alpha_i-\beta_i)}}, \text{ and}$$
$$Q_i^* = \frac{2p_{PF}w_i w_{2i}^*}{c_{01i}p_w w_{4i}^*}. \tag{11}$$

Using the optimal values of decision variables say, D_i^* and Q_i^* we get the optimal value of the objective function $PF^*(D_i^*, Q_i^*)$ from (2).

6. NUMERICAL EXAMPLES

For numerical illustration, we consider an inventory model for two items with the following data.

Table 1: Input data for Crisp and Fuzzy model ((1)&(2))

i	s_{0i} (\$)	p_{0i} (\$)	c_{01i} (\$)	c_{03i} (\$)	α_i	β_i	γ_i	δ_i	w_i (m^2)	W (m^2)	p_W	c_{PF}	p_{PF}
1	100	10	0.5	50	0.4	0.2	0.6	0.5	4	195	10	545	10
2	120	12	0.4	60	0.5	0.6	0.4	0.55	2				

Following the analysis outlined in section 5, the optimum values of demand, order quantities and maximum profit under crisp and fuzzy environment are evaluated and presented in table 2.

Table 2: Optimal results

Model	i	D_i^*	Q_i^*	PF^* (\$)
Crisp model	1 2	47.25568 23.17970	29.96363 37.57371	534.51036
Fuzzy model	1 2	48.47515 23.78689	30.70790 38.65906	539.7391

7. SENSITIVITY ANALYSIS

A sensitivity analysis of fuzzy model has been performed with respect to the percentage change of index parameters $\alpha_i, \beta_i, \gamma_i$ and δ_i which are presented in tables - 3, 4, 5, 6. In the formulation, unit cost and selling price have been expressed as some quantities power to the demand. Similarly unit holding cost and set-up cost are some quantities power to the order level. Hence, the said indexes are very effective in determining optimal demand, order level and maximum profit. Thus the sensitivity analysis results are in aggrement as it was expected. If the selling price, holding cost and set-up cost increase (decrease) obviously the profit will decrease (increase). But, it will have the reverse effect in the case of unit cost.

This phenomenon is depicted by the tables from 3, 5, 6. It is observed that all the decision variables and the objective value diminished when the index parameters α_i, γ_i and δ_i increase. But in table 4 decision variables and the objective value increase when the index parameter β_i increases.

(A) Percentage change of α_i :

Table 3: Effect on decision variables & objective function

% change of α_i	i	D_i^*	Q_i^*	PF^* (\$)
–6	1	145.4996	56.73451	947.8058
	2	57.34578	66.74807	
–4	1	97.20933	45.39107	768.3402
	2	41.68793	54.81035	
–2	1	67.49814	37.01799	637.4998
	2	31.12685	45.71429	
2	1	35.85952	25.86223	465.1176
	2	18.56387	33.09916	
4	1	27.22847	22.07883	407.0774
	2	14.74891	28.64743	
6	1	21.15610	19.08003	361.2004
	2	11.90737	25.03984	

(B) Percentage change of β_i :

Table 4: Effect on decision variables & objective function

% change of α_i	i	D_i^*	Q_i^*	PF^* (\$)
–6	1	42.76117	28.58911	513.3044
	2	22.19483	37.16807	
–4	1	44.56648	29.27656	521.9720
	2	22.81204	37.66050	
–2	1	46.47365	29.98136	530.8015
	2	23.30938	38.16986	
2	1	50.57522	31.45484	548.7758
	2	24.25183	39.12581	
4	1	52.77913	32.22367	557.9232
	2	24.69537	39.57229	
6	1	55.09257	33.01354	567.1821
	2	25.11518	39.99992	

(C) Percentage change of γ_i :

Table 5: Effect on decision variables & objective function

% change of α_i	i	D_i^*	Q_i^*	PF^* ($)
–6	1	53.12392	33.54557	566.4128
	2	25.11205	41.02047	
–4	1	51.52657	32.57038	557.1338
	2	24.62820	40.15675	
–2	1	49.97781	31.62521	548.3538
	2	24.21111	39.40363	
2	1	47.01990	29.81972	531.2987
	2	23.37274	37.92421	
4	1	45.60960	28.95965	523.0424
	2	22.97314	37.20531	
6	1	44.24531	28.12783	514.9608
	2	22.57366	36.49675	

(D) Percentage change of δ_i :

Table 6: Effect on decision variables & objective function

% change of α_i	i	D_i^*	Q_i^*	PF^* ($)
–6	1	77.07675	38.74571	690.4541
	2	40.47468	51.86214	
–4	1	65.96059	35.88196	637.1922
	2	34.25389	47.35938	
–2	1	56.50761	33.20457	585.0164
	2	28.22829	42.55833	
2	1	41.65210	28.38400	498.7511
	2	20.16589	35.17747	
4	1	35.85666	26.22586	461.6203
	2	17.18349	32.06103	
6	1	30.93107	24.22548	427.9535
	2	14.72634	29.26195	

8. CONCLUSION

Here we have formulated a multi-item profit maximization model with limited storage area in crisp and fuzzy environment. The problem is then solved by modified geometric programming method. It is observed that in fuzzy environment the model gives the better optimum result than the crisp environment. The beauty of this approach is that a crisp / fuzzy inventory problem with large degree of difficulty can be easily solved by the method presented here and decision variables can be determined. This method can be applied in other typical decision-making problems in the areas of structural analysis, environment etc. where once they are formulated as geometric programming problem with very large degree of difficulty.

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