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ON A VOLUME FLEXIBLE PRODUCTION POLICY IN A FAMILY PRODUCTION CONTEXT

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Abstract: A mathematical model for a volume flexible manufacturing system is developed in a family production context, assuming that there exists a dedicated production facility as well as a separate management unit for each of the items. The possibility of machine breakdowns resulting in idle times of the respective management units is taken into account. The production rates are treated as decision variables. It is also assumed that there is a limitation on the capital available for total production. An optimal production policy is derived with maximization of profit as the criterion of optimality. The results are illustrated with a numerical example. Sensitivity of the optimal solution to changes in the values of some key parameters is also studied

Keywords: Inventory, shortage, volume flexibility, family production, machine-breakdown, idle-time.

1. INTRODUCTION

In the *Classical Economic Production Lot Size* (**EPLS**) model, the amount ordered becomes available at a constant supply rate. That means, the production rate of the machine is assumed to be predetermined and inflexible [10]. A fixed rate of production is inconvenient in many respects. Firstly, a production rate much higher than the demand rate leads to rapid accumulation of inventories resulting in higher holding costs and other related problems. If the machine production rate is less than the demand rate, the management has to face stock-out situations. These inconveniences arise due to inability of the manufacturing system to adjust its production rate in keeping with the market demand variability. But the machine production rate can easily be changed [17]. The treatment of machine production rate as a decision variable is especially appropriate for automated technologies that are *volume flexible* [18].

Nowadays managers of modern manufacturing companies have mainly four systems of improving production efficiencies. These are **MRP** (*materials requirement planning*), **OPT** (*optimized production technology*), **JIT** (*just-in-time*), and **FMS** (*flexible manufacturing system*). **FMS** offers the hope of eliminating many of the weaknesses of the other approaches [2]. *Volume flexibility* (i.e., the manufacturing flexibility that is capable of adjusting the production rate with the variability in the market demand) is a major component in a **FMS**. Volume flexibility is a real necessity in many practical situations. Management may be interested in reducing machine production rates to avoid rapid accumulation of inventories. This deliberate reduction of production rates is consistent with the Just-In-Time manufacturing philosophy which has been successfully applied in many Japanese manufacturing companies. Again, reduction in the production rate may sometimes be an inevitable option for the management to cope with a declining market demand. It is, therefore, necessary that a manufacturing system should be capable of adjusting the production rates during the production runs. This requires that the production units should have automated technologies. An immediate outcome of volume flexibility is variability in unit-production-cost which varies with the production rate.

The models of Adler and Nanda [1], Sule [19],[20], Axsater and Elmaghraby [3], and Muth and Spearmann [13] were concerned with learning effects on the optimal lot size. Proteus [15], Rosenblat and Lee [16] and Cheng [4] extended the models to the imperfect production processes. Schweitzer and Seidmann [17] first enlightened the researchers about the concept of flexibility in the machine production rate and discussed optimization of processing rates for a **FMS**. Obviously, the unit production cost becomes a function of the production rate in the case of a **FMS**. Khouja and Mehrez [11] and Khouja [12] extended the **EPLS** model to an imperfect production process with a flexible production rate. Silver [21] discussed, assuming a common production cycle for all items, the effects of slowing down production in the context of a manufacturing equipment dedicated to the production of a family of items. Gallego [9] extended the model of Silver [21] by applying different production cycles for different items. Moon, Gallego and Simchi-Levi [14] discussed controllable production rates in a family production context.

In the present paper, we consider a volume flexible manufacturing system in a family production context. It is assumed that different machines $\{A_i,\ i=1,2,....n\}$ are dedicated to the production of different items i with different production rates $\{P_i,\ i=1,2,....n\}$. The management of production in machine A_i is vested with the management unit B_i,. It is assumed that a machine may become out of order during its working time. As a result, there is a mean time for every machine between its failures/breakdowns. During a breakdown of a machine, there is demand although there is no production. In such a situation, the demand is met until the inventory level falls below the quantity demanded. When inventory level becomes less than the demand, the concerned management unit B_i is rendered fully idle. This type of situation is quite likely to occur when the customer is a wholesaler having the demand of a big lot-size and the concerned management unit cannot meet this demand because the stock-size is less than the quantity demanded. We, therefore, take into account the idle time of each management unit; this idle time leads to an additional cost for the lost man-hours. It is also assumed in this model that the capital available for manufacturing the items is limited. The unit-production-cost for the machine A_i is taken to be a function of its production rate P_i and its functional form is constructed on some realistic considerations. The production rates $\{P_i, i = 1,2,....n\}$ are decision variables in the problem. We look for an optimal production policy which maximizes the total profit. Solution of the problem is illustrated with a numerical example. The algorithm for deriving the numerical solution is given in *Appendix*.

2. FUNDAMENTAL ASSUMPTIONS AND NOTATIONS

2.1. Assumptions:

1. The model is developed for multiple items.
2. Demand rate for each item is constant.
3. Production rate per unit time is considered as a decision variable.
4. Invested capital for production is limited.
5. Machine-breakdown is considered during the production period.
6. Idle time to the management unit is considered.
7. Unit production cost for the i-th item (i=1,2,...,n) is a function of the production rate.
8. Shortages are allowed during the idle-time.
9. Time horizon is infinite.

2.2 Notations:

$Q_i(t)$ - is the on-hand inventory of i-th item at time 't'.

P_i - is the production rate per unit time for the i-th item.

μ_i - is the mean time between successive breakdowns of the machines $\{A_i, i = 1,2,....n\}$.

$\psi_i(t_i)$ - is the probability density function of t_i.

m_i - is the mean time of repair of i-th machine.

τ_i - is the mean duration of a breakdown of machine $\{A_i, i = 1,2,....n\}$.

$\phi_i(\tau_i)$ - is the probability density function of τ_i.

C_h^i - is the cost of carrying one unit of i-th item in inventory per unit time.

C_s^i - is the shortage cost per unit time of i-th item.

$\eta_i(P_i)$ - is the cost for production of a unit of i-th item (i=1,2,...n).

S_p^i - is the selling price per unit of i-th item .

D_i - is the demand rate of i-th item (i=1,2,....n) per unit time.

W_i - is the cost per unit of idle time of the management unit B_i.

CAP - is the total capital available for production of all the items.

3. FORMULATION OF THE MODEL

The production cycle begins with zero stock. Production starts at time t = 0 and the stock reaches a levels $\{Q_i(t_i),\ i = 1,2,......n\}$ at times $\{t = t_i\ ,\ i = 1,2,....n\}$ after adjusting demand rates $\{D_i\ ,\ i = 1,2,....n\}$. At times $\{t = t_i\ ,\ i = 1,2,....n\}$ machines $\{A_i\ ,\ i = 1,2,....n\}$ become *out of order*. Then , repairing of machines $\{A_i,\ i = 1,2,....n\}$ starts and takes times $\{\tau_i\ ,\ i = 1,2,....n\}$ to comeback into working state . Here $\{t_i\ ,\ i = 1,2,....n\}$ and $\{\tau_i,\ i = 1,2,....n\}$ are random variables which follow probability distribution functions $\{\psi_i\ ,\ i = 1,2,....n\}$ and $\{\phi_i\ ,\ i = 1,2,....n\}$ respectively. During repairing period two cases may arise: one is *Scenario 1.a* (see Fig) which is very simple and unrealistic case, second is *Scenario 1.b* (see Fig) which is very common in the manufacturing firms or industries. Consequently, our main object is to analyze the *Scenario 1.b* (see Fig).

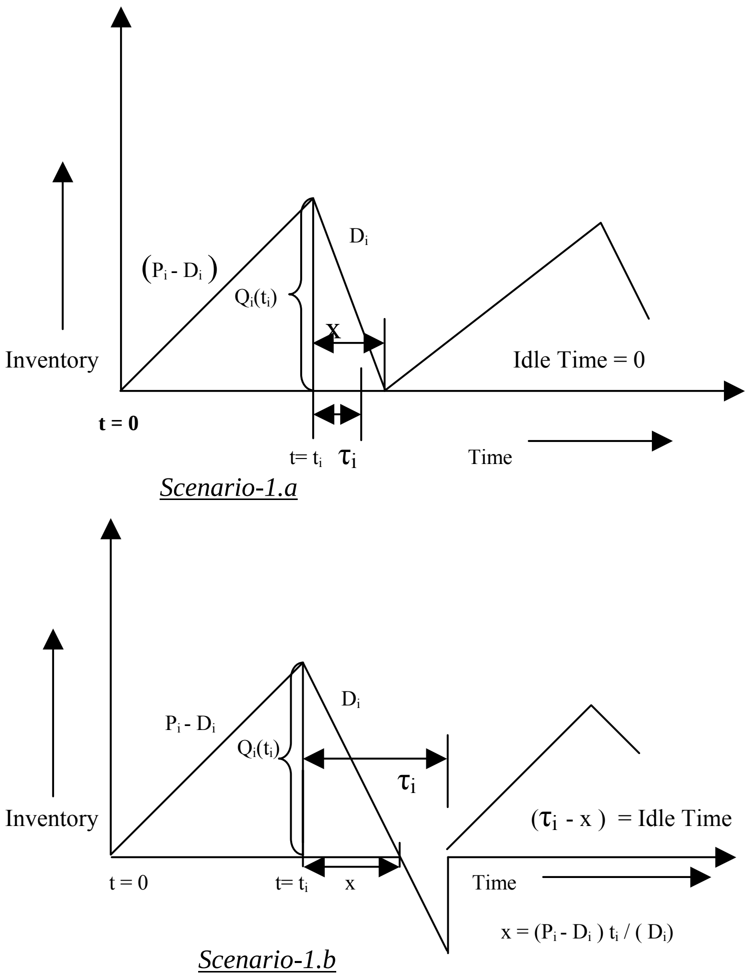

Figure. Pictorial Representation of the Model

The governing differential equations for the inventory system are:

$$\frac{dQ_i(t)}{dt} = P_i - D_i \ , \ 0 \le t \le t_i \ , with\, Q_i(0) = 0 \ ; \ for\, i = 1,2,...n. \tag{1}$$

The solution of the *Eq.*(1) is

$$Q_i(t) = (P_i - D_i)t \ \ , 0 \le t \le t_i \ \ ; for \ \ i = 1,2....n. \tag{2}$$

We can conclude that the idle times of the management units $\{B_i\, i = 1,2,...n\}$ due to a breakdown of the machines $\{A_i\,, i = 1,2,...n\}$ are (see Scenario 1.a & Scenario 1.b)

$$u_i = \begin{cases} 0 & , \ if \ \dfrac{Q_i(t_i)}{D_i} \ge \tau_i \\ \tau_i - \dfrac{Q_i(t_i)}{D_i} & , \ if \ \dfrac{Q_i(t_i)}{D_i} < \tau_i . \end{cases}$$

The expected cost per breakdown of the machine $\{A_{i,}\ i = 1,2,...n\}$, during idle time, is

$$E_{ic}^i = W_i \int_0^\infty \left\{ \int_{\frac{Q_i(t_i)}{D_i}}^{\infty} (\tau_i - \frac{Q_i(t_i)}{D_i}) \phi_i(\tau_i)\, d\tau_i \right\} \psi_i(t_i)\, dt_i \tag{3}$$

and the expected shortage cost for i – th item, during idle time, is

$$E_{SC}^i = C_S^i D_i \int_0^\infty \left\{ \int_{\frac{Q_i(t_i)}{D_i}}^{\infty} (\tau_i - \frac{Q_i(t_i)}{D_i}) \phi_i(\tau_i)\, d\tau_i \right\} \psi_i(t_i)\, dt_i . \tag{4}$$

Now, the total inventory of i-th item is

$$Inv_i(t_i) = Inventory\ during\ \ [0, t_i] + Inventiry\ during\ [0, x]$$

$$= (P_i - D_i)\int_0^{t_i} t\ dt + D_i \int_0^x t\, dt$$

$$= \frac{1}{2}(P_i - D_i)t_i^2 + \frac{1}{2}D_i x^2$$

$$= \frac{1}{2}(P_i - D_i)t_i^2 + \frac{1}{2}\frac{(P_i - D_i)^2 t_i^2}{D_i} \ , \ \therefore \ \ x = \frac{(P_i - D_i)t_i}{D_i}, see\ the\ Fig.$$

Therefore, the expected inventory cost, for i-th item , is

$$E_{inc}^{i} = \int_0^\infty Inv_i(t_i)\psi_i(t_i)\,dt_i$$

$$= \frac{1}{2}C_h^i(P_i - D_i)\int_0^\infty t_i^2\,\psi_i(t_i)\,dt_i \tag{5}$$

$$+ \frac{1}{2}\frac{C_h^i}{D_i}(P_i - D_i)^2\int_0^\infty t_i^2\psi_i(t_i)\,dt_i \ .$$

The production cost per unit of i-th item ($i=1,2,.....n$) is taken to be

$$\eta_i(P_i) = r_i + \frac{g_i}{P_i} + \alpha_i P_i \tag{6}$$

This cost is based on the following factors:

1. The material cost r_i per unit is fixed.
2. As the production rate increases, some costs like labour and energy costs are equally distributed over a large number of units. Hence the per-unit production cost (g_i /P_i) decreases as the production rate (P_i) increases.
3. The third term ($\alpha_i P_i$), associated with tool/die cost is proportional to the production rate. Empirical observations indicate [18] that the tool or die costs increase as the machine production rate is increased. In their analysis of the drilling operation, Conrad and Mc Clamrock [5] showed that "a 10% change in processing rate causes a 50% change in tool cost". Also, the probability of machine failure increases with the increase of machine production rate. Thus increased production rate accelerates the deterioration of the quality of the production process. It is, therefore, quite likely that imperfect output occurs at higher production rates. In such a situation, there are two options before the management. The imperfect items might be finished to perfect ones at additional costs or the imperfect items might be sold at a lower price causing some loss of profit. Whatever might be the situation, it is seen that tool/die costs increase at higher production rates.

Here we consider the density functions

$$\psi_i(t_i) = \frac{1}{\mu_i}e^{-t_i/\mu_i},$$

$$\phi_i(\tau_i) = \frac{1}{m_i}e^{-\tau_i/m_i}.$$

Because, reliability of spare parts of a machine follows exponential probability distribution function. Therefore the expected total profit per breakdown, including the inventory and shortage cost, is

$$ETP(P_1, P_2, ..., P_n) = Expected\ \ Revenue\ from\ selling\ items$$
$$- Expected\ \ Holding\ \ cost - Expected\ \ cost\ \ for\ idle\ time$$
$$- Expected\ \ shortage\ cost.$$

$$= \sum_{i=1}^{n} \{S_p^i - \eta_i(P_i)\} P_i \int_0^\infty t_i \psi_i(t_i)\, dt_i - \frac{1}{2} \sum_{i=1}^{n} C_h^i (P_i - D_i) \int_0^\infty t_i^2 \psi_i(t_i)\, dt_i$$

$$- \frac{1}{2} \sum_{i=1}^{n} \frac{C_h^i}{D_i} (P_i - D_i)^2 \int_0^\infty t_i^2 \psi_i(t_i)\, dt_i$$

$$- \sum_{i=1}^{n} C_s^i D_i \int_0^\infty \left\{ \int_{\frac{Q_i(t_i)}{D_i}}^{\infty} \left(\tau_i - \frac{Q_i(t_i)}{D_i}\right) \phi_i(\tau_i)\, d\tau_i \right\} \psi_i(t_i)\, dt_i$$

$$- \sum_{i=1}^{n} W_i \int_0^\infty \left\{ \int_{\frac{Q_i(t_i)}{D_i}}^{\infty} \left(\tau_i - \frac{Q_i(t_i)}{D_i}\right) \phi_i(\tau_i)\, d\tau_i \right\} \psi_i(t_i)\, dt_i \tag{7}$$

$$= \sum_{i=1}^{n} \{S_p^i - \eta_i(P_i)\} P_i \mu_i - \sum_{i=1}^{n} C_h^i (P_i - D_i) \mu_i - \sum_{i=1}^{n} C_h^i (P_i - D_i)^2 \mu_i$$

$$- \sum_{i=1}^{n} \frac{(C_s^i D_i + W_i) m_i^2 D_i}{\mu_i \{P_i + D_i (m_i / \mu_i - 1)\}}$$

Also the total expected production cost is

$$E_{prc} = \sum_{i=1}^{n} \int_0^\infty \eta_i(P_i) P_i t_i \psi_i(t_i)\, dt_i$$
$$= \sum_{i=1}^{n} \eta_i(P_i) P_i \mu_i \tag{8}$$

As the capital for manufacturing the items is limited, the constraint $\sum_{i=1}^{n} \eta_i(P_i) P_i \mu_i \le CAP$ must be satisfied.

Therefore, we have to maximize the profit function

$$ETP(P_1, P_2, ...P_n)$$
subject to the constraints:
$$\sum_{i=1}^{n} \eta_i(P_i) P_i \mu_i \le CAP, \tag{9}$$
$$P_1 \ge D_1\ ,\ P_2 \ge D_2, ...P_n \ge D_n\,.$$

The above problem can be solved by using *Interior Penalty Function* Method (see Appendix).

4. NUMERICAL EXAMPLE

Let i=1,2,3 i.e., three items, three machines and three management units are considered here. We consider the following sets of parameter values in appropriate units:

Item No.(i)	W_i	μ_i	m_i	r_i	g_i	α_i	D_i	C_h^i	C_s^i	S_p^i	CAP
1	40	8	1/2	0.8	6.25	0.01	20	0.05	2.00	1.50	
2	35	8.5	1/ 2.5	1.2	7.50	0.008	40	0.06	2.50	1.90	1500
3	30	9	1/3	1.3	8.00	0.006	35	0.03	3.00	2.10	

Solving the problem numerically with the help of computer, we find that the optimum solution is

$$P_1^* = 23.80297\ ,\ P_2^* = 42.73013\ ,\ P_3^* = 39.78868\ ,\ ETP_{\max}^* = 171.7912,$$
$$\sum E_{ic}^i = 12.7913, \sum E_{sc}^i = 28.7277\ ,\ \sum E_{inc}^i = 19.98437.$$

5. SENSITIVITY ANALYSIS

We now carry out an analysis of the sensitivity of the optimum solution to changes in the values of the parameters of the system. Changes in $P_1^*, P_2^*, P_3^*, ETP_{\max}^*$, $\sum E_{ic}^*, \sum E_{sc}^*$, and $\sum E_{inc}^*$ are shown in Table 1 for percentage changes in the values of the parameters.

From Table 1, the following points emerge:

$P_i^*\ (i = 1,2,3.)$ are more or less sensitive to changes in $W_i\ (i = 1,2,3.)$.

$P_i^*\ (i = 1,2,3.)$ are moderately sensitive to changes in $\mu_i\ (i = 1,2,3.)$.

$P_i^*\ (i = 1,2,3.)$ are fairly sensitive to changes in $m_i\ (i = 1,2,3.)$.

$\sum E_{ic}^i, \sum E_{sc}^i, \sum E_{inc}^i$ are fairly sensitive to changes in $W_i\ (i = 1,2,3.)$, $\mu_i\ (i = 1,2,3.)$, $m_i\ (i = 1,2,3.)$.

$ETP_{\max}^*$ is slightly sensitive to changes in $W_i\ (i = 1,2,3.)$, but moderately sensitive to changes in m_1, m_2 and m_3 while fairly sensitive to changes in $\mu_i\ (i = 1,2,3.)$.

Table 1: Sensitivity Analysis of the Parameters**:**

Change In %	P_1^*	P_2^*	P_3^*	ETP_{max}	$\sum E_{ic}^i$	$\sum E_{sc}^i$	E_{inc}^i
+50%	-06.71	+01.26	-01.14	-02.02	+22.52	+02.59	-08.82
+25% W_1	+00.56	-00.08	-00.12	-00.76	+08.89	+00.19	+01.11
-25%	-00.76	-00.10	+00.26	+00.75	-08.47	+00.71	-02.22
-50%	-01.34	+00.16	+00.24	+01.57	-18.94	-06.62	-02.78
+50%	-00.12	-00.26	+00.37	-01.70	+23.81	+08.93	-01.59
+25% W_2	-00.06	+00.08	-00.19	-00.87	+11.06	-05.58	-07.42
-25%	-00.42	-00.43	+00.40	+00.78	-09.42	+02.04	-02.35
-50%	-00.15	-00.50	+00.46	+01.65	-21.46	+02.14	-01.35
+50%	-00.23	-00.53	+00.96	-00.55	+09.61	+01.62	+01.07
+25% W_3	+00.01	-00.06	+00.12	-00.29	+04.26	+00.13	+02.42
-25%	+00.06	-00.62	+01.10	+00.32	-02.36	+01.70	+02.54
-50%	-00.27	-00.03	+00.50	+00.75	-07.98	-00.46	+01.60
+50%	-09.73	-04.67	-09.13	-10.91	+76.26	+89.60	-82.29
+25% μ_1	-04.59	-02.31	-04.79	-00.45	+23.22	+28.85	-47.52
-25%	+02.21	+00.65	+03.00	-04.74	-00.48	-05.35	+21.83
-50%	+06.22	+00.81	+03.49	-10.73	+05.33	-03.64	+25.13
+50%	nf	nf	nf	-------	------	---	-----
+25% μ_2	-13.27	-04.67	-11.19	-24.96	+138.91	+136.36	-93.21
-25%	+00.33	+02.20	+03.38	-09.64	-03.83	-05.24	+29.88
-50%	+00.21	+04.29	+03.41	-20.71	+01.38	+01.21	+29.18
+50%	nf	nf	nf	---	------	------	------
+25% μ_3	-13.69	-05.14	-11.19	-20.91	+153.31	+149.05	-95.47
-25%	+00.05	+00.69	+04.61	-17.76	-03.23	-04.17	+18.94
-50%	+00.35	+00.90	+07.56	-36.89	-01.22	-00.22	+17.07
+50%	+02.92	-00.86	-07.66	-05.84	+34.32	+18.62	+06.53
+25% m_1	+00.45	-00.20	-00.34	-02.88	+18.33	+09.27	-01.39
-25%	-02.80	+00.10	+01.50	+02.44	-12.53	-06.40	-07.49
-50%	-03.94	-00.20	+00.42	+04.69	-25.34	-12.26	-06.75
+50%	-00.60	+00.80	-00.48	-10.65	+35.65	+44.57	+08.06
+25% m_2	-00.30	+00.15	+00.02	-05.26	+18.40	+22.95	-08.65
-25%	+00.18	-01.34	+01.74	+04.95	-14.22	-18.30	+02.65
-50%	-00.12	-02.01	+01.97	+09.43	-28.03	-36.38	-07.27
+50%	-00.67	-01.04	+01.42	-05.59	+20.62	+29.36	-01.00
+25% m_3	-00.31	-00.82	+01.03	-02.69	+10.84	+15.20	-00.70
-25%	-00.19	-00.02	-00.14	+02.21	-01.72	-06.23	-06.23
-50%	+00.31	+00.12	+00.06	+04.08	-13.11	-19.63	-19.63

"nf" – denotes no feasible solution.

6. CONCLUSIONS

If the production rate is fixed, the following situations may arise:

1. Inventory becomes high when the production rate is high. Although the idle-time cost is low in this case, it cannot offset the inventory costs.
2. Inventory cost is low, but the idle time for the management units is high in the case of a low production rate.
3. The predetermined production rate cannot appropriately cope with the fluctuations in the market demand. In the present model, the remuneration of a management unit

depends upon its efficiency which, in turn, depends upon the kind of items it deals with. Therefore, the costs per unit of idle time are different for different management units. Hence the production rate must be adjusted so that the above costs are minimized and the profit maximized.

The following features are observed from the optimum solution in the numerical example:

1. As the mean time to repair $m_i (i = 1, 2, 3.)$ of a machine $A_i (i = 1, 2, 3.)$ decreases, the corresponding production rate $P_i (i = 1, 2, 3.)$ increases.
2. The production rate P_i of the machine A_i increases with the increase in its mean duration of a breakdown.
3. The production rate of a machine increases with the increase in the selling price of the item produced by machine.
4. The production rate of a machine increases as the idle-time cost of the concerned management unit decreases.
5. The production rate of a machine increases as the mean time between its successive breakdowns increases.

Keeping in mind the above points, this model helps owners of the family firms to produce optimal lot size which profits maximum. The ideas of the present model are of importance today as more and more volume flexible production systems are being introduced nowadays to cope with the fluctuations in the market demands arising out of globalization.

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REFERENCES

[1] Adler, G.L., and Nanda, R., "The effects of learning on optimum lot size determination - single product case", *AIIE Trans.*, 6 (1974) 14- 20.

[2] Aggarwal, S.C., "Making sense of production operations system", *Harvard Business Review,* September-October issue, (1985) 8-16.

[3] Axsater, S., and Elmaghraby, S.E., "A note on EMQ under learning and forgetting", *AIIE Trans.,* 13 (1981) 86-90.

[4] Cheng, T.C.E., "An economic order quantity model with demand dependent unit production cost and imperfect production processes", *IIE Trans.,* 23(1991) 23-28.

[5] Conrad, C.J., and McClamrock, N.H., "The drilling problem: A stochastic modelling and control example in manufacturing", *IEEE Trans. Autom.Control,* 32 (1987) 947-958.

[6] Drozda, T.J., and Wick, C. (eds.), *Tool and Manufacturing Engineers Handbook*, Society of Machanical Engineers, Dearborn, MI, 1983.

[7] Fiacco, A.V., and Mc Cormick, G.P., "Extension of SUMT for nonlinear programming: equality constraints and extrapolation", *Management Science,* 12 (1966) 816-828.

[8] Fiacco, A.V., and Mc Cormick, G.P., *Nonlinear Programming:Sequential Unconstrained Minimization Techniques*, Wiley, New York, 1968.

[9] Gallego, G., "Reduced production rates in the economic lot scheduling problem", *Int. J. Prod. Res.*, 31 (1993) 1035-1046.

[10] Hax, A.C., and Candea, D., *Production and Inventory Management*, Prentice-Hall, Englewood Cliffs, NJ, 1984.

[11] Khouja, M., and Mehrez, A., "An economic production lot size model with imperfect quality and variable production rate", *J. Opl. Res. Soc.*, 45 (1994) 1405-1417.

[12] Khouja, M., "The economic production lot size model under volume flexibility", *Computers & Ops. Res.*, 22 (1995) 515-523.

[13] Muth, E.J., and Spearmann, K., "Learning effects in economic lot size", *Mgmt Sci.*, 29 (1983) 264-269.

[14] Moon, I., Gallego, G., and Simchi-Levi, D., "Controllable production rates in a family production context", *Int.J. Prod.Res.,* 29 (1991) 2459-2470.

[15] Proteus, E.L., "Optimal lot sizing, process quality improvement, and setup cost reduction", *Ops. Res.*, 34 (1986) 137-144.

[16] Rosenblatt, M.J., and Lee, H.L., "Economic production cycles with imperfect production processes", *IIE Trans.*, 17 (1986) 48-54.

[17] Schweitzer, P.J., and Seidmann, A., "Optimizing processing rates for flexible manufacturing systems", *Mgmt. sci.*, 37 (1991) 454-466.

[18] Sethi, A.K., and Sethi, P.S., "Flexibility in manufacturing: A survey", *Int. J. Flexible Manufact. Systems*, 2 (1990) 289-328.

[19] Sule, D.R., "A note on production time variation EMQ under influence of learning and forgetting", *AIIE Trans*., 13 (1981) 91-95.

[20] Sule, D.R., "The effect of a alternate periods learning and forgetting on economic manufacturing quantity", *AIIE Trans.*, 10 (1978) 338-343.

[21] Silver, E.A., "Deliberately slowing down output in a family production context", *Int. J. Prod. Res,* 28 (1990) 17-27.

APPENDIX

The primal problem is reformulated below:
Primal Problem (General Form)

$$\min\ f(\tilde{X})$$

Such that

$$G_j(\tilde{X}) \le 0\ ,\ j = 1,2,......m.$$

where $f(\tilde{X})$, $G_j(\tilde{X})$ are continuous functions of $\tilde{X} \in R^n$.

Interior Penalty Method: (see Ref[7],[8])
This method generally deals with an unconstrained minimization problem:

$$\min\ \chi_k(\tilde{X}, r_k) = f(\tilde{X}) - r_k \sum_{j=1}^{m} \frac{1}{G_j(X)}$$

where r_k is a positive penalty parameter.

If χ_k is minimized for a decreasing sequence of values r_k, the following theorem proves that the unconstrained minima $\tilde{X}_k^*$ $(k = 1,2,......m)$ converges to the solution $\tilde{X}^*$ of the primal problem stated above.

Theorem: *If the primal problem has a solution, the unconstrained minima* $\tilde{X}_k^*$ *of* $\chi_k(\tilde{X}, r_k)$ *for a sequence of values* $r_1 > r_2 > .... > r_k$ *, converges to optimal solution of the primal problem.*

The Iterative Procedure:

Step 1. Start with an initial feasible point $\tilde{X}_1$, satisfying all the constraints with strict inequality sign,

i.e., $G_j(\tilde{X}_1) < 0$ for j= 1,2,...m. and an initial suitable value of

r_1, $r_1 = f(\tilde{X}_1) / \sum_{j=1}^{m} \frac{1}{G_j(\tilde{X}_1)}$. Set k=1.

Step 2. *Minimize* $\chi_k(\tilde{X}, r_k)$ by using any method of unconstrained minimization(we use here the *Devidon Fletcher -Powell Method*) and obtain the solution $\tilde{X}_k^*$.

Step 3. Test whether $\left| \frac{f(\tilde{X}_k^*) - f(\tilde{X}_{k+1}^*)}{f(\tilde{X}_k^*)} \right| \le \varepsilon_1 \left| \tilde{X}_k^* - \tilde{X}_{k+1}^* \right| < \varepsilon_2$ where ε_1 and ε_2 are arbitrary small positive numbers. If it is satisfied, then terminate the process; otherwise, go to the next Step.

Step 4. Find the value of next penalty parameter *r as* $r_{k+1} = Cr_k$ where 0<C<1.

Step 5. Set the new value of k=k+1, take the new starting point as $\tilde{X}_1 = \tilde{X}_k^*$ and go to *Step 2*.