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MULTI-ITEM FUZZY INVENTORY MODEL FOR DETERIORATING ITEMS WITH FINITE TIME-HORIZON AND TIME-DEPENDENT DEMAND

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Abstract: This paper develops a finite time-horizon fuzzy multi-deteriorating inventory model with/without shortage, where the demand increases linearly with time. Here, the total profit is to be maximized under the limitation on investment. In this problem, total profit, total investment cost and the time-horizon are fuzzy in nature. The impreciseness in the above objective and constraint goals have been expressed by fuzzy linear/non-linear membership functions and vagueness in time-horizon by different types of fuzzy numbers. Results are illustrated with numerical examples.

Keywords: Fuzzy inventory, deteriorating items, backlogged shortages, dynamic demand, finite time-horizon.

1. INTRODUCTION

Most of the classical deterministic inventory models consider the demand rate to be constant, independent of time '*t*'. However, for certain types of inventory, particularly consumer goods (viz. food grains, oilseeds, etc.), the demand rate increases with time. In real life, the harvest of food grains is periodical. A large number of landless people of developing countries have a constant demand of food grains throughout the year. Marginal farmers or landless labourers produce food-grain in their own piece of land or in the land of their land-lords by share cropping. Due to various reasons, many of them are bound to sale a part of their food grains immediately after production. This section of people cannot produce enough food grains to meet their need for the entire production cycle after the partial sale. As a result, the demand of food grains remains partly constant and partly increases with time for a fixed time-horizon. Stanfel and Sivazlian (1975) discussed a finite time-horizon inventory problem for time dependent demands. Silver and Meal (1973) developed an approximate solution technique of a deterministic inventory model with time varying demand. Donaldson (1977) first developed an exact solution procedure for items with a linearly increasing demand rate over a finite planning horizon. However, his solution procedure was computationally complicated. Removing the complexity, many other researchers have proposed various other techniques to solve the same inventory problem. In recent years, Dave (1989), Goyal et. al. (1992) and Datta and Pal (1992) developed models with shortages assuming demand to be time proportional. All these models are based on the assumption that there is no deterioration effect on inventory.

The most important assumption in the exiting literature is that life time of an item is infinite while it is in storage. But in reality, many physical goods deteriorate due to dryness, spoilage, vaporisation etc. and are damaged due to staying longer than their normal storage period. The deterioration also depends on preserving facilities and environmental conditions of warehouse/storage. So, due to deterioration effect, a certain fraction of the items are either damaged or decayed and are not in a perfect condition to satisfy the future demand of customers for good items. Deterioration for such items is continuous and constant or time-dependent and/or dependent on the on-hand inventory. A number of research papers have already been published on above type of items by Dave and Patel (1981), Sachan (1989), Goswami and Chowdhury (1991), Kang and Kim(1983).

It has been recognised that one’s ability to make precise and significant statement concerning an inventory model diminishes with increasing complexities of the marketing situation. Generally, in inventory systems only linguistic (vague) statements are used to describe the model and it may not be possible to state the objective function and the constraints in precise mathematical form. It may not also be possible to express the objective in certain terms because the objective goal is not definable precisely. Similarly, length of time-horizon and average storage cost may be imprecise in nature. Here, the phenomena of such model may be described in a fuzzy way.

The theory of fuzzy sets was developed for a domain in which description of activities and observations are fuzzy in the sense that there are no well defined boundaries of the set of activities or observations to which the description is applied. The theory was initiated by Zadeh (1965) and later applied to different practical systems by several researchers. Zadeh showed the intention to accommodate uncertainty in the non-stochastic sense rather than the presence of random variables. Bellman and Zadeh (1970) first applied fuzzy set theory in decision-making processes. Zimmerman (1976) used the concept of fuzzy set in decision-making processes by considering the objective and constraints as fuzzy goals. He first applied fuzzy set theory with suitable choice of membership functions and derived a fuzzy linear programming problem. Currently, the fuzzy programming techniques are applied to solve linear as well as non-linear programming problems (Trappgy, et-al (1988), Carlsson and Korhonen (1986), etc.). Lai and Hwang (1992, 1994) described the application of fuzzy sets to several operation research problems in two well-known books.

However, as far as we know, fuzzy set theory has been used in few inventory models. Sommer (1981) applied fuzzy dynamic programming to an inventory and production-scheduling problem. Kacprzyk and Staniewski (1982) considered a fuzzy inventory problem in which, instead of minimizing the total average cost, they reduced it to a multi-stage fuzzy-decision-making problem and solved by a branch and bound algorithm. Park (1987) examined the EOQ formula with fuzzy inventory costs represented by Trapezoidal fuzzy number (TrFN). Recently, Lam and Wong (1996) solved the fuzzy model of joint economic lot size problem with multiple price breaks. Roy and Maiti (1995) solved the classical EOQ model in a fuzzy environment with fuzzy goal, fuzzy inventory costs and fuzzy storage area by FNLP method using different types of membership functions for inventory parameters. They (1997) examined the fuzzy EOQ model with demand dependent unit price and imprecise storage area by both fuzzy geometric and non-linear programming methods. They (1997, 1998) also discussed single and multi-period fuzzy inventory models using fuzzy numbers.

It may be noted that none has considered the time-horizon as fuzzy number and attacked the fuzzy optimization problem directly using fuzzy non-linear programming techniques. Till now, no literature is available for the multi-item inventory models for finite time-horizon in fuzzy environment with or without shortages.

In this paper, we have developed a multi-item inventory model incorporating the constant rate of deterioration effect assuming the demand to be a linearly increasing function with time and shortages to be allowed for the prescribed finite time-horizon. In the exiting literature of inventory models, the time-horizon is assumed to be fixed. But in reality, time-horizon is normally limited but imprecise, uncertain and flexible. This may be better represented by some fuzzy numbers. The problem is reduced to a fuzzy optimization problem associating fuzziness with the time-horizon, objective and constraint goal. The fuzzy multi-item inventory problem is solved for different fuzzy numbers and fuzzy membership functions. The model is illustrated with a numerical example and the results for the fuzzy and crisp model are compared.

2. MODEL AND ASSUMPTIONS

We use the following notations in proposed model:

n = numbers of items,

B = total investment for replenishment.

For i-th. item (i = 1, 2, 3,…..,n)

$D_i(t)$ = demand rate (function of time),

C_{1i} = inventory holding cost per unit item per unit time,

p_i = cost price per unit item,

s_i = selling price per unit item,

C_{2i} = shortage cost per unit item per unit time,

θ_i = constant rate of deterioration,

H_i = prescribed time-horizon,

m_i = total number of replenishment to be made during the prescribed time-horizon H_i, i.e. m_i must be a positive integer (decision variable),

T_i = length of each cycle i.e., $T_i = H_i/m_i$

q_{ij} = inventory level at time t for j-th cycle, (j = 1, 2, …, m_i)

Q_{ij} = lot size for j-th cycle, (j = 1, 2, …, m_i)
R_{ij} = backlogged quantity for j-th cycle, (j = 1, 2, …, m_i)
F_{ij} = replenishment cost for j-th cycle, (j = 1, 2, …, m_i)
k_i = fraction of scheduled period T_i of which no shortage occur, i.e. k_i be a real number in [0, 1] (decision variable),
$PF(m, k)$ = Total profit of the system.
(Where m and k are the vectors of n decision variables m_i (i = 1, 2, …,n) and k_i (i = 1, 2, …,n) respectively.)

The basic assumptions about the model are:

(i) Replenishment rate is instantaneous,
(ii) Shortages are allowed and fully backlogged,
(iii) Lead time is zero,
(iv) The demand rate $D_i(t)$ at any instant 't' is a linear function of t such that

$$D_i(t) = a_i + b_i t, \quad a_i, b_i \geq 0, \qquad 0 \leq t \leq H_i,$$

(v) The replenishment cost F_{ij} for j-th cycle (j = 1, 2, …., m_i) is linearly dependent on time and is of the following form

$$F_{ij} = A_i + r_i(j-1)T_i, \quad j = 1, 2, \ldots, m_i$$

where $A_i > 0$ and r_i is the additional replenishment cost per unit of items,
(vi) We assume that the period for which there is no shortage in each interval $[(j-1)T_i, jT_i]$ is a fraction of the scheduling period T_i and is equal to k_iT_i ($0 < k_i < 1$). Shortages occur at times $(k_i+j-1)T_i$, (j = 1, 2, …, m_i-1) where $(j-1)T_i < (k_i + j-1)T_i < jT_i$, ($j$ = 1, 2, ….m_i-1). Last replenishment occurs at time $(m_i-1)T_i$ and shortages are not allowed in the last period $[(m_i-1)T_i, H_i]$.

Our problem is to derive the optimal reorder and shortage points and hence to determine the optimal values of m_i and k_i (i = 1, 2,……., n) which maximize the total profit over the time-horizon [0, H_i].

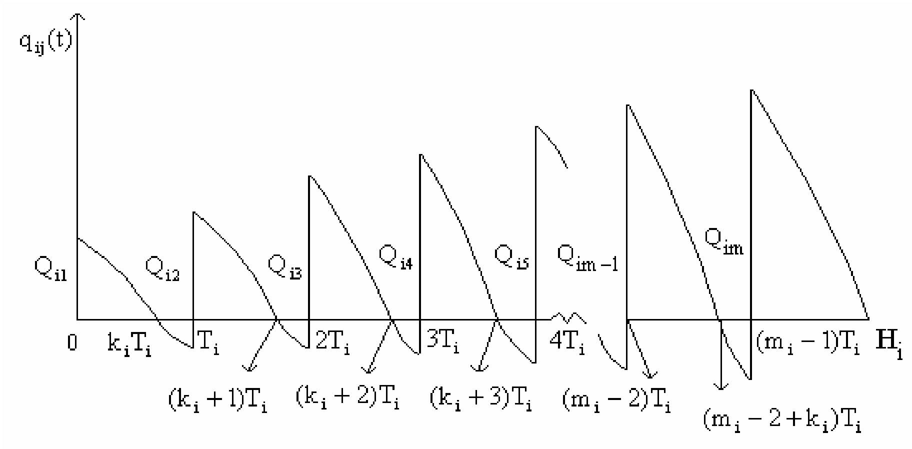

Fig-1. Pictorial representation of the inventory system

3. MATHEMATICAL FORMULATION OF THE PROBLEM

Let $q_{ij}(t)$ be the amount in inventory at time t during the jth cycle $[(j-1)T_i \le t \le jT_i$, $j = 1, 2, 3, \ldots, m_i]$. Then the differential equations describing the system during the jth cycle are

$$\frac{dq_{ij}(t)}{dt} + \theta_i q_{ij}(t) + D_i(t) = 0\ , \ (j-1)T_i \le t \le (k_i+j-1)T_i, \tag{1}$$

$$\frac{dq_{ij}(t)}{dt} + D_i(t) = 0\ , \quad (k_i+j-1)T_i \le t \le jT_i,\ \ j = 1, 2, \ldots, (m_i-1), \tag{2}$$

and the differential equation governing the stock status for the last replenishment cycle $[(m_i-1)T_i \le t \le H_i]$ is

$$\frac{dq_{im_i}(t)}{dt} + \theta_i q_{im_i}(t) + D_i(t) = 0\ , \quad (m_i-1)T_i \le t \le H_i, \tag{3}$$

with the boundary conditions $q_{ij}(t) = 0$ at $t = (k_i+j-1)T_i$, $j = 1, 2, \ldots, (m_i-1)$ and $q_{im_i}(t) = 0$ at $t = H_i$.

The solutions of (1), (2), and (3) are

$$q_{ij}(t) = \left(a_i - \frac{b_i}{\theta_i}\right)\frac{e^{\theta_i\{(k_i+j-1)T_i - t\}} - 1}{\theta_i} + \frac{b_i}{\theta_i}\left[(k_i+j-1)T_i e^{\theta_i\{(k_i+j-1)T_i - t\}} - t\right] \tag{4}$$
$$(j-1)T_i \le t \le (k_i+j-1)T_i,$$

$$q_{ij}(t) = a_i\left\{(k_i+j-1)T_i - t\right\} + \frac{b_i}{2}\left\{(k_i+j-1)^2 T_i^2 - t^2\right\},\quad (k_i+j-1)T_i \le t \le jT_i, \tag{5}$$
$$j = 1, 2, \ldots, (m_i-1),$$

$$q_{im_i}(t) = \left(a_i - \frac{b_i}{\theta_i}\right)\frac{e^{\theta_i\{H_i - t\}} - 1}{\theta_i} + \frac{b_i}{\theta_i}\left[H_i e^{\theta_i\{H_i - t\}} - t\right] \quad (m_i-1)T_i \le t \le H_i. \tag{6}$$

If Q_{ij} be the inventory level in time $(j-1)T_i$, then

$$Q_{ij} = \left(a_i - \frac{b_i}{\theta_i}\right)\frac{e^{\theta_i k_i T_i} - 1}{\theta_i} + \frac{b_i}{\theta_i}\left[(k_i+j-1)T_i e^{\theta_i k_i T_i} - (j-1)T_i\right] \quad j = 1, 2, \ldots, (m_i-1), \tag{7}$$

and

$$Q_{im_i} = \left(a_i - \frac{b_i}{\theta_i}\right)\frac{e^{\theta_i T_i} - 1}{\theta_i} + \frac{b_i}{\theta_i}\left[H_i e^{\theta_i T_i} - (m_i - 1)T_i\right]. \tag{8}$$

Let R_{ij} be the total amount of backlogged quantities over the period $[(k_i+j-1)T_i, jT_i]$, $j = 1, 2, \ldots, (m_i-1)$. Then

$$R_{ij} = -\int_{(k_i+j-1)T_i}^{jT_i} q_{ij}(t)dt = \frac{1}{2}(K_i - 1)^2 T_i^2 \left[a_i + b_i T_i \left\{j + \frac{2}{3}(K_i - 1)\right\}\right], j = 1,2,\ldots,(m_i-1). \tag{9}$$

Holding cost in j-th $[j = 1, 2, \ldots, (m_i-1)]$ cycle is $V_{ij} = C_{1i}G_{ij}$

$$G_{ij} = \int_{(j-1)T_i}^{(k_i+j-1)T_i} q_{ij}(t)dt \tag{10}$$

$$= \left[\frac{a_i - \frac{b_i}{\theta_i}}{\theta_i}\left\{\frac{e^{\theta_i k_i T_i} - 1}{\theta_i} - k_i T_i\right\} + \frac{b_i}{\theta_i}\left\{(k_i + j - 1)T_i \frac{e^{\theta_i k_i T_i} - 1}{\theta_i} - \frac{k_i^2 + 2(j-1)k_i}{2} T_i^2\right\}\right]$$

$$j = 1, 2, \ldots, (m_i-1). \tag{11}$$

Holding cost in the last cycle is

$$V_{im_i} = C_{1i}\int_{(m_i-1)T_i}^{H_i} q_{im_i}(t)dt = C_{1i}G_{im_i}, \tag{12}$$

where

$$G_{im_i} = \int_{(m_i-1)T_i}^{H_i} q_{im_i}(t)dt$$

$$= \left[\frac{a_i - \frac{b_i}{\theta_i}}{\theta_i}\left\{\frac{e^{\theta_i T_i} - 1}{\theta_i} - T_i\right\} - \frac{b_i}{\theta_i}\left\{H_i \frac{1 - e^{\theta_i T_i}}{\theta_i} + \frac{(2m_i - 1)}{2} T_i^2\right\}\right]. \tag{13}$$

Total number of deteriorating items over the period $[(j-1)T_i, jT_i]$, $j = 1, 2, \ldots, (m_i-1)$, is

$$S_{D_{ij}} = \theta_i G_{ij}, \quad j = 1, 2, \ldots, (m_i-1), \tag{14}$$

and total number of deteriorating items in the last cycle is

$$S_{D_{im_i}} = \theta_i G_{im_i} . \tag{15}$$

Therefore, the total profit of the system for the entire time horizon H_i is

$$PF_i(m_i, k_i) = \sum_{j=1}^{m_i-1} \left\{ (s_i - p_i)(Q_{ij} + R_{ij}) - V_{ij} - F_{ij} - s_i S_{D_{ij}} - C_{2i} R_{ij} \right\}$$

$$+ \left\{ (s_i - p_i) Q_{im_i} - V_{im_i} - F_{im_i} - s_i S_{D_{im_i}} \right\} .$$

Crisp model:

Hence, our object is to maximize the total profit subject to the limitation on investment cost, i.e.

$$\text{Max PF(m, k)} = \sum_{i=1}^{n} PF_i(m_i, k_i) \tag{16}$$

$$= \sum_{i=1}^{n} \left\{ (s_i - p_i) Q_i - F_i - (C_{1i} + \theta_i s_i) G_i + (s_i - p_i - C_{2i}) R_i \right\}$$

subject to

$$\sum_{i=1}^{n} p_i(Q_i) \le B$$

$Q_i > 0$, $i = 1, 2, \ldots\ldots, n$,

where,

$$Q_i = \sum_{j=1}^{m_i} Q_{ij} =$$

$$(m_i - 1)\left[\frac{e^{\frac{\theta_i k_i H_i}{m_i}} - 1}{\theta_i} \left(a_i - \frac{b_i}{\theta_i} \right) + \frac{b_i}{2\theta_i} \left\{ e^{\frac{\theta_i k_i H_i}{m_i}} \left(2k_i + m_i - 2 \right) - \left(m_i - 2 \right) \frac{H_i}{m_i} \right\} \right]$$

$$+ \frac{e^{\theta_i \frac{H_i}{m_i}} - 1}{\theta_i} \left(a_i - \frac{b_i}{\theta_i} \right) + \frac{b_i}{\theta_i} \left[e^{\theta_i \frac{H_i}{m_i}} H_i - (m_i - 1) \frac{H_i}{m_i} \right] \quad i = 1, 2, \ldots\ldots, n.$$

$$F_i = \sum_{j=1}^{m_i} F_{ij} = m_i\left[A_i + \frac{r_i(m_i-1)H_i}{2m_i}\right] \qquad i = 1, 2, \ldots\ldots, n.$$

$$G_i = \sum_{j=1}^{m_i} G_{ij}$$

$$= (m_i-1)\left[\left(a_i - \frac{b_i}{\theta_i}\right)\left\{\frac{e^{\frac{\theta_i k_i H_i}{m_i}}-1}{\theta_i} - \frac{k_i H_i}{m_i}\right\} + \frac{H_i b_i}{2m_i\theta_i}\left\{\frac{e^{\frac{\theta_i k_i H_i}{m_i}}-1}{\theta_i}\left(2k_i + m_i - 2\right) - k_i\left(k_i + m_i - 2\right)\frac{H_i}{m_i}\right\}\right] +$$

$$\left[\frac{a_i - \frac{b_i}{\theta_i}}{\theta_i}\left\{\frac{e^{\theta_i\frac{H_i}{m_i}}-1}{\theta_i} - T_i\right\} - \frac{b_i}{\theta_i}\left\{H_i\,\frac{1-e^{\theta_i\frac{H_i}{m_i}}}{\theta_i} + \frac{(2m_i-1)}{2}\left(\frac{H_i}{m_i}\right)^2\right\}\right]\right]$$

$$i = 1, 2, \ldots, n.$$

$$R_i = \sum_{j=1}^{m_i-1} R_{ij} = \frac{1}{2}(K_i - 1)^2(m_i - 1)T_i^2\left[a_i + b_i T_i\left\{\frac{m_i}{2} + \frac{2}{3}(K_i - 1)\right\}\right] \qquad i = 1, 2, \ldots, n.$$

Fuzzy Objective and Constraint goal:

In most of the programming model, the decision maker is not able to articulate a precise aspiration level to an objective or constraint. However, it is possible for him to state the desirability of achieving an aspiration level in an imprecise interval around it. An objective with inexact target value (aspiration level) is termed as a fuzzy goal. Similarly, a constraint with imprecise aspiration level is also treated as a fuzzy goal.

Fuzzy Decision:

A fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the objective goal and the constraints. More formally, given a fuzzy goal $\tilde{G}$ and a fuzzy constraint $\tilde{C}$ in the space alternatives *X*, a decision $\tilde{D}$ is defined as the fuzzy set, $\tilde{G} \cap \tilde{C}$. The membership function of the fuzzy decision $\mu_{\tilde{D}}$ is given by min($\mu_{\tilde{G}}, \mu_{\tilde{C}}$).

Mathematical Formulation of the fuzzy model:

When the above profit goal, average storage cost and total time horizon becomes fuzzy then the said crisp model (16) is transformed to

$$M\tilde{a}x\ \ PF(m,k)=\sum_{i=1}^{n}P\tilde{F}_i(m_i,k_i)$$

$$=\sum_{i=1}^{n}\left\{(s_i-p_i)\tilde{Q}_i-\tilde{F}_i-(C_{1i}+\theta_i s_i)\tilde{G}_i+(s_i-p_i-C_{2i})\tilde{R}_i\right\} \tag{17}$$

subject to

$$\sum_{i=1}^{n}p_i\tilde{Q}_i\ \le\ \tilde{B}$$

where,

$$\tilde{Q}_i=(m_i-1)\left[\frac{e^{\frac{\theta_i k_i\tilde{H}_i}{m_i}}-1}{\theta_i}\left(a_i-\frac{b_i}{\theta_i}\right)+\frac{b_i}{2\theta_i}\left\{e^{\frac{\theta_i k_i\tilde{H}_i}{m_i}}\left(2k_i+m_i-2\right)-\left(m_i-2\right)\frac{\tilde{H}_i}{m_i}\right\}\right]$$

$$+\frac{e^{\theta_i\frac{\tilde{H}_i}{m_i}}-1}{\theta_i}\left(a_i-\frac{b_i}{\theta_i}\right)+\frac{b_i}{\theta_i}\left[e^{\theta_i\frac{\tilde{H}_i}{m_i}}H_i-(m_i-1)\frac{\tilde{H}_i}{m_i}\right]$$

$$\tilde{G}_i=(m_i-1)\left[\left(a_i-\frac{b_i}{\theta_i}\right)\left\{\frac{e^{\frac{\theta_i k_i\tilde{H}_i}{m_i}}-1}{\theta_i}-\frac{k_i\tilde{H}_i}{m_i}\right\}+\frac{\tilde{H}_i b_i}{2m_i\theta_i}\left\{\frac{e^{\frac{\theta_i k_i\tilde{H}_i}{m_i}}-1}{\theta_i}\left(2k_i+m_i-2\right)\right.\right.$$

$$\left.\left.-k_i\left(k_i+m_i-2\right)\frac{\tilde{H}_i}{m_i}\right\}\right]$$

$$+\left[\frac{a_i-\frac{b_i}{\theta_i}}{\theta_i}\left\{\frac{e^{\theta_i\frac{\tilde{H}_i}{m_i}}-1}{\theta_i}-\frac{\tilde{H}_i}{m_i}\right\}-\frac{b_i}{\theta_i}\left\{\tilde{H}_i\frac{1-e^{\theta_i\frac{\tilde{H}_i}{m_i}}}{\theta_i}+\frac{(2m_i-1)}{2}\left(\frac{\tilde{H}_i}{m_i}\right)^2\right\}\right]$$

$$\tilde{F}_i=\sum_{j=1}^{m_i}F_{ij}=m_i\left[A_i+\frac{r_i(m_i-1)\tilde{H}_i}{2m_i}\right]$$

$$\tilde{R}_i = \sum_{j=1}^{m_i-1} R_{ij} = \frac{1}{2}(K_i - 1)^2 (m_i - 1)\left(\frac{\tilde{H}_i}{m_i}\right)^2 \left[a_i + b_i \frac{\tilde{H}_i}{m_i}\left\{\frac{m_i}{2} + \frac{2}{3}(K_i - 1)\right\}\right].$$

(where wavy bar (~) represents the fuzzy characterisation).

In this fuzzy model, the fuzzy objective goal and fuzzy investment cost constraint are represented by their membership functions, which may be linear or non-linear. Here, μ_{PF} and μ_B are assumed to be non-decreasing and non-increasing continuous linear/non-linear membership function for objective profit goal and storage cost constraint as follows:

$$\mu_{PF}(x) = \begin{cases} 1 & \text{for} \quad x > PF \\ 1 - \dfrac{PF - x}{P_{\text{PF}}} & \text{for} \quad PF - P_{PF} < x < PF \\ 0 & \text{for} \quad x < PF - P_{PF} \end{cases}$$

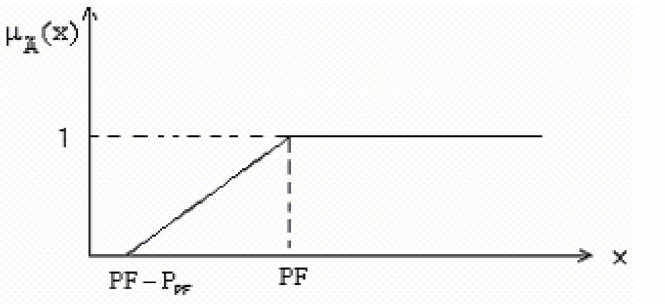

Fig.-2 : Fuzzy linear profit function

$$\mu_{PF}(x) = \begin{cases} 1 & \text{for} \quad x > PF \\ 1 - \left(\dfrac{PF - x}{P_{\text{PF}}}\right)^2 & \text{for} \quad PF - P_{PF} < x < PF \\ 0 & \text{for} \quad x < PF - P_{PF} \end{cases}$$

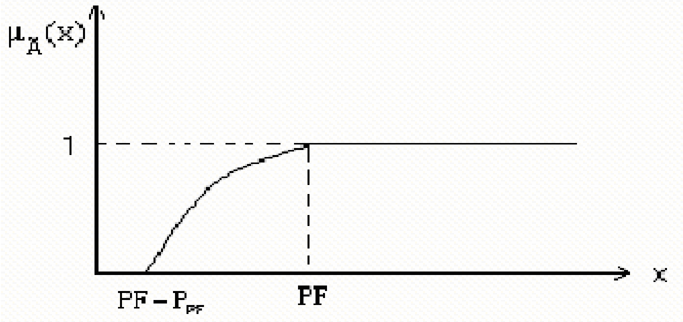

Fig.-3 : Fuzzy Parabolic profit function

$$\mu_B(x) = \begin{cases} 1 & \text{for} \quad x < B \\ 1 - \dfrac{x - B}{P_B} & \text{for} \quad B < x < B + P_B \\ 0 & \text{for} \quad x > B + P_B \end{cases}$$

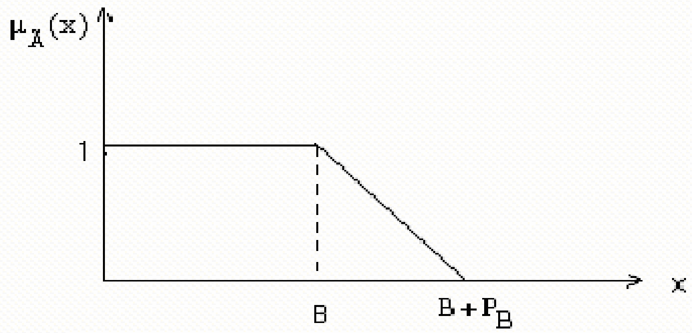

Fig.-4 : Fuzzy linear investment cost function

$$\mu_B(x) = \begin{cases} 1 & \text{for } x < B \\ 1 - \left(\dfrac{x-B}{P_B}\right)^2 & \text{for } B < x < B + P_B \\ 0 & \text{for } x > B + P_B \end{cases}$$

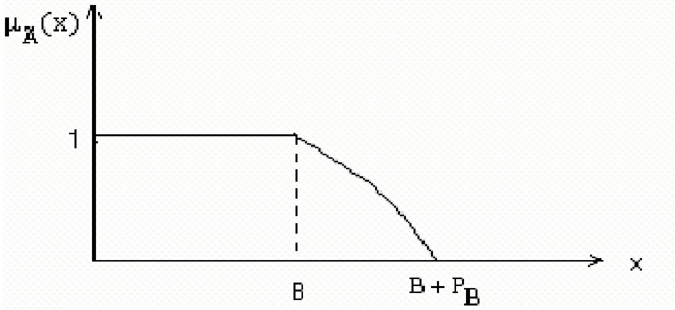

Fig.-5 : Fuzzy parabolic investment cost function

Again, as the time-horizon be near about H_i (i = 1, 2, …., n), these fuzzy coefficients may be represented by different type of fuzzy numbers (e.g. TFN, TrFN, PFN and PrFN) and μ_{H_i} (i = 1, 2, …., n) are represented the membership functions of these coefficients.

Let μ_D be the membership function of the fuzzy set 'decision' of the model. Since, μ_{H_i} (i = 1, 2,…., n), μ_B are the membership functions of fuzzy coefficients, constraint goal and μ_{PF} is the membership function of fuzzy objective goal. The decision space in fuzzy environment is the intersection of fuzzy sets corresponding to the fuzzy profit goal and fuzzy constraint goals.

Hence our problem is

$$\text{Max } \mu_D(x) \tag{18}$$

subject to

$$\mu_{PF} \geq \mu_D(x),$$

$$\mu_B \geq \mu_D(x),$$

where m, k are the decision vectors as in (18),

$$\mu_D(x) = \text{Min}\,[\mu_{PF}, \mu_B, \mu_{H_1}, \mu_{H_2}, ..., \mu_{H_n}], \quad i = 1, 2, …., n.$$

Defining α for μ_D(x) and using the expressions for the membership functions μ_{PF}, μ_B, μ_{H_i} (Considering $\tilde{H}_i$ as TFN), the following equivalent crisp problem can be defined as a mixed integer non-linear programming problem:

$$\text{Max } \alpha \tag{19}$$

subject to

$PF(m, k) > \mu_{PF}^{-1}(\alpha)$

$B(m, k) > \mu_B^{-1}(\alpha)$

$\mu_{L_i}^{-1}(\alpha) \le \mu_{H_i} \le \mu_{U_i}^{-1}(\alpha) \quad i = 1, 2, \ldots, n.$

where *m*, *k* are the decision vectors as in (18),

$\mu_{PF}^{-1}(\alpha) = PF - (1-\alpha)^{1/n_0} P_{PF}$

$\mu_B^{-1}(\alpha) = B - (1-\alpha)^{1/n_0} P_B$

$\mu_{L_i}^{-1}(\alpha) = H_{1i} + \alpha(H_{2i} - H_{1i})$

$\mu_{U_i}^{-1}(\alpha) = H_{3i} - \alpha(H_{2i} - H_{1i})$

$n_0 - 1$ or 2, $\alpha \in [0, 1]$, for TFN $\tilde{H}_i$ (= $[H_{1i}, H_{2i}, H_{3i}]$).

Similarly, for TrFN $\tilde{H}_i$ we can express the equivalent crisp problem.

The problems in (16) and (19) are solved using a mixed integer-programming algorithm in FORTRAN-77.

4. NUMERICAL EXAMPLE

To illustrate the above crisp model (16) and the corresponding fuzzy model (17) we assume the following input data shown in table-1 and present the results for crisp and fuzzy models in table 2, and table 3, 4, 5, 6 respectively. Different fuzzy models are due to different fuzzy membership functions and fuzzy numbers for total profit, total investment cost and time horizon respectively.

Table 1: Input data for crisp and fuzzy numbers

Items	S_I($)	p_i($)	a_i	θ_i	b_i	A_i($)	C_{1i}($)	C_{2i}($)	H_i	r_i	w_i
1	10	7.5	20	0.005	2	100	0.5	1.5	12	0.1	0.8
2	8	5	18	0.008	2.5	120	0.6	2	14	0.1	0.6
3	6	4	18	0.006	2	100	0.5	1.5	11	0.1	0.8

B = $ 6720

Table 2: Results for crisp model

Cases	PF($)	m_1	m_2	m_3	k_1	k_2	k_3	B($)
With Shortages	693.97	4	4	3	0.9297	0.8819	0.7740	6135.70
Without Shortages	677.17	3	3	2	1	1	1	6707.09

Table 3: Optimal result for fuzzy model-1

When the membership function of total profit *PF* and total investment cost are linear and time-horizon $H_i (i = 1, 2, 3.)$ are triangular fuzzy number, i.e., $\tilde{PF}$ = (\$650, \$800), $\tilde{B}$ = (\$6000, \$7500), $\tilde{H}_1$ = (10, 12, 15), $\tilde{H}_2$ = (12, 14, 16), $\tilde{H}_3$ = (8, 11, 14).

Cases	α	PF($)	m_1	m_2	m_3	k_1	k_2	k_3	H_1	H_2	H_3	B($)
With Shortages	0.777	766.59	4	5	3	0.875	0.980	0.730	12.67	14.45	11.66	6629.69
Without Shortages	0.705	741.09	4	5	3	1	1	1	12.88	14.59	9.55	6941.84

Table 4: Optimal result for fuzzy model-2

When the membership function of total profit *PF*, total investment cost are linear and time-horizon $H_i (i = 1, 2, 3.)$ are trapezoidal fuzzy number, i.e, where $\tilde{PF}$ = (\$650, \$800), $\tilde{B}$ = (\$6000, \$7500), $\tilde{H}_1$ = (8, 11, 13, 15), $\tilde{H}_2$ = (12, 14, 15, 17), $\tilde{H}_3$ = (8, 11, 12, 14).

Cases	α	PF($)	m_1	m_2	m_3	k_1	k_2	k_3	H_1	H_2	H_3	B($)
With Shortages	0.814	772.31	3	5	3	0.864	0.991	0.752	12.61	14.29	11.26	6432.31
Without Shortages	0.732	750.34	3	5	3	1	1	1	12.34	13.77	10.63	6874.12

Table 5: Optimal result for fuzzy model-3

When the membership function of total profit *PF*, total investment cost are parabolic and time-horizon $H_i (i = 1, 2, 3.)$ are triangular fuzzy number, i.e, where $\tilde{PF}$ = (\$650, \$800), $\tilde{B}$ = (\$6000, \$7500), $\tilde{H}_1$ = (8, 11, 13, 15), $\tilde{H}_2$ = (12, 14, 15, 17), $\tilde{H}_3$ = (8, 11, 12, 14).

Cases	α	PF($)	m_1	m_2	m_3	k_1	k_2	k_3	H_1	H_2	H_3	B($)
With Shortages	0.850	741.97	4	4	3	0.887	0.863	0.744	12.49	14.29	11.50	6272.71
Without Shortages	0.673	714.23	4	4	3	1	1	1	10.81	15.13	10.02	6857.73

Table 6: Optimal result for fuzzy model-4

When the membership function of total profit *PF* is parabolic, investment cost is linear and total time-horizon H_1, H_2 are triangular fuzzy number but H_3 is trapezoidal fuzzy number, i.e, where $\tilde{PF}$ = (\$650, \$800), $\tilde{B}$ = (\$6000, \$7500), $\tilde{H}_1$ = (10, 12, 15), $\tilde{H}_2$ = (12, 14, 16) and $\tilde{H}_3$ = (8, 11, 12, 14).

Cases	α	PF($)	m_1	m_2	m_3	k_1	k_2	k_3	H_1	H_2	H_3	B($)
With Shortages	0.658	754.64	4	5	3	0.821	0.978	0.786	12.32	15.02	10.54	6643.42
Without Shortages	0.594	704.37	4	5	3	1	1	1	11.61	14.81	9.78	6609.70

5. CONCLUSION

In this paper, we have solved a time-horizon inventory problem for deteriorating items having a linear time-dependent demand under storage cost constraint in fuzzy environment. The model permits inventory shortage in each cycle (except the last cycle), which is completely backlogged within the cycle itself. Optimal results of both the crisp and fuzzy models for two cases (with shortages and without shortages) for different fuzzy numbers are presented in tables 2-6. Here it is observed that the results of the fuzzy model are better than the respective crisp ones.

We also mentioned herewith that in most of the fixed time-horizon problems, optimum number of replenishment is evaluated by trial and error method, i.e., substituting m = 1, 2, 3, ….. and then choosing that value of m for which cost is minimum or profit is maximum. In this paper, number of replenishment has been taken as a decision variable (integer) and the optimum value has been evaluated using the mixed integer-programming algorithm.

Still, there is a lot of scope to make the inventory problems much more realistic by considering some parameters of the objective/constraints are probabilistic, and other are imprecise. These fuzzy stochastic problems are to convert into an equivalent deterministic/crisp problem using probability distribution and fuzzy membership functions and then solved by different programming methods.

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