

POSSIBLE CLASSIFICATION OF THE METHODS OF OPERATIONAL RESEARCH APPLICABLE IN THE FIELD OF DEFENSE

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Abstract: The overall dynamic development of operational research in various fields of human activities urges the need for a clearer and mathematically more explicit classification of its methods. This need is also very urgent in the field of defense, particularly because of the complications of modern conflicts, as well as of new security requirements. One of the possible classifications of methods based on the theory of games as a mathematical model for solving conflict situations is presented in this paper. The connections between methods and their mathematical description are underlined.

Keywords: Classification of methods, conflict, games, military applications.

1. INTRODUCTION

Operational research as a set of scientific, mathematical, and statistical methods for solving problems in various areas of human activities has been developed intensely within the last fifty years, [1], [5], [7], [14], [14]. Defense is regarded as the cradle of operational research, and has found its own place in almost all fields of human activities. The relevancy of the theory of games was also approved by the fact that the Nobel Prize for economics in 1994 and again in 2005 was awarded to the scientist who has mostly worked in this field [1].

The complex environment of various systems of defense, especially under war conditions, emphasizes the requirement for a special methodology to be used for solving acute problems [13]. Such methodology has to rely on the basis of mathematics, statistics, and heuristics. This paper presents an attempt to classify operational research methods used for solving problems in defense, with respect to the mentioned scientific

basis. As a starting point for this classification, we consider the phenomenon of conflict (conflict situation) as a general problem that can be modeled and solved as a game.

The problem of classification has already been discussed [15], but only one criterion has been applied. The contribution of this paper is classification of multicriteria problems, rendering their mathematical description, and giving an example of application.

A conflict situation is defined in section 2, used as a basis for the operational research methods classification. In addition, special attention has been paid to the description of multicriteria games, giving for the first time the procedure for their solution as seen in the example in section 4.

2. DESCRIPTION OF A CONFLICT SITUATION AND A GAME

The dominant problem in defense particularly in war conditions is connected with a conflict situation. In general, a conflict situation can be described through four characteristics, as follows:

1. Two sides (coalitions) at least are required for a conflict. Each side has its own interests, marked as K_i .
2. Opposite sides involved in a conflict have different (usually opposite) interests, and they are called the participants – the players. A participant can be a single player or a group of players' coalition.
3. Participants of a conflict situation tend to realize their goals by using some of their strategies. Strategy assumes certain rules of action that lead to the realization of a particular result (phase) through solving a conflict situation. The group of all theoretically possible strategies for all coalitions is marked as K_{ol} , while those which are practically possible are marked as K , thus $K \in K_d$. The group of all strategies in a considered conflict situation we mark as S_k . A playing coalition choose one strategy, then makes calculation considering limitations and conditions, and, in doing so, defines a possible situation S , where S is a subset of the Cartesian product S_k ;
4. A conflict participant takes the advantage over its opposite player (v_k), for the determined coalition interests ($K \in K_i$), if a strategy S' satisfies the higher coalition interests than a strategy S'' , marked as $S' >_k S''$. This relationship between advantages under particular conditions is called the dominance relationship for coalition K .

A strategy can be defined as a group of all alternatives available to a player when he makes decisions. Many problems in conflict situations can be solved by using the theory of games when strategies are determined. When the opposite players are replaced by neutral sides which strictly follow the determined strategies, their neutral natures could be simulated by a computer. In that case a game is realized at the desk table instead on the battlefield, or at the sport field, at the market, etc.

A real conflict situation in which decisions are made, is usually so complicated as to be completely expressed by mathematics, thus instead of a real situation we study model, neglecting its secondary characteristics.

A game is a mathematical model of a real conflict situation and is performed according the rules that define a behavior of each player. The following definition [15] gives a formal description of a game:

A game is a system:

$$G = \left\langle K_d, \{S_k\}_{K \in K_d}, S, K_i, \left\{ \begin{matrix} \times \\ K \end{matrix} \right\}_{K \in K_i} \right\rangle \quad (1)$$

where: $K_d, K_i, S_k (K \in K_d)$ – any groups such as $S \subseteq \prod S_k$, a $\times (K \in K_i)$ any binary relation on S .

3. CLASSIFICATION OF THE METHODS

When the group of coalition interests is empty, $K_i = 0$, then this is a case of pure descriptive tasks, which have no practical importance in the defense branch. If $K_i \neq 0$, then we have optimization tasks, but for $|K_i| = 1$ it is an optimization task outside the conflict, and finally a task with the conflict when $K_i \geq 2$.

Optimization tasks outside the conflict can be arranged according to the solving method, to: linear, nonlinear, dynamic and discrete programming; and by a number of criteria to: single criterion and multi criteria programming.

Optimization tasks in a conflict include non strategic games, $|K_d| = 1$, and strategic games, $|K_d| \geq 2$, as Figure 1 shows (see details [11], [15].)

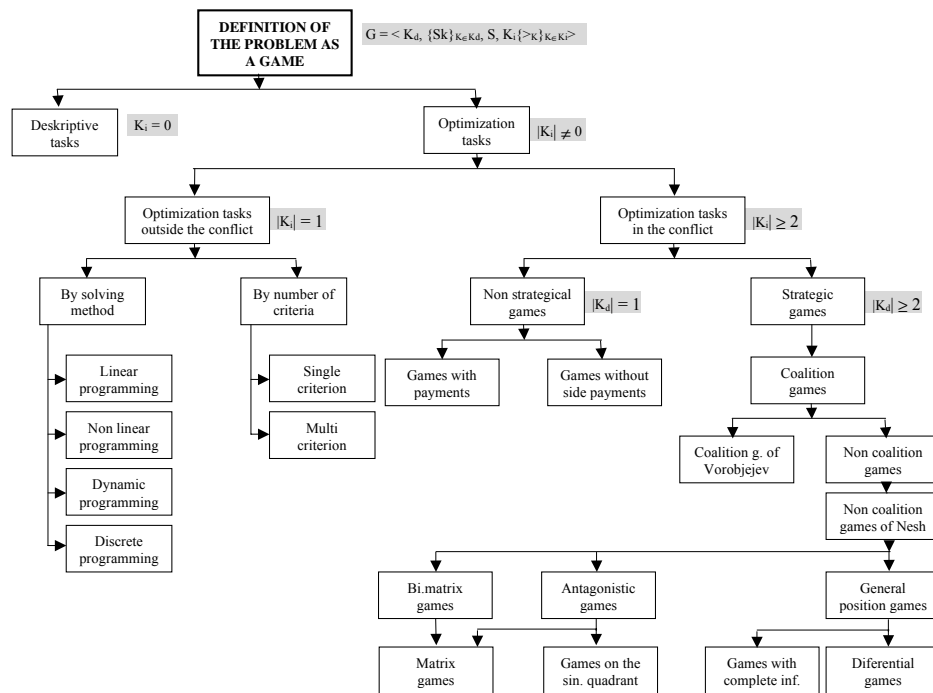


Figure 1: Classification of the optimization tasks

If the game in a form (1) has only one strategy corresponding to a group of situations S , every coalition of interests $K \in K_i'$ corresponds the group $W(K) \subset S$. Let for every $\alpha \in W(K)$ a group $K(\alpha) \subset S$ contains all valid situations β , i.e., $\alpha \underset{K}{\propto} \beta$.

This set is characterized by the fact that strategic decision for the set does not play any role.

Let us suppose that the set K_i' is a subset of the player's set I , then for every player $i \in I$ corresponds to Euclid space E^i and is marked for every $K \in K_d$ multiplication $\prod_{i \in k} E^i / E^k$, and multiplication $\prod_{i \in k} E^i / E^i$. Let $S \subset E^i$. Thus, in the game situations are presented as real vectors whose components correspond to the players. Those vectors are called "payments", and the games of this type are called the games with the payments. The games without side payments can be obtained if in the game with payments, for every $K \in K_i$ replace:

$$V(K) \subset E^K, \quad (2)$$

$$W(K) = (V(K) \times E^{I/K}) \cap S. \quad (3)$$

Let a K_d and K_i substrate of set I , elements of which we called players. Assume that for every player $i \in I$ there is corresponding set S_i (set of individual strategies of each player i). Let for every $K \in K_d$:

$$S_k = \prod_{i \in k} S_i, \quad (4)$$

$$S_k = \prod_{i \in K} S_i \quad (5)$$

For every $k \in K_i$ on the group of all situation s correspond the pay off function N_k . It is accepted if $S' \underset{K}{\propto} S''$, for $k \in K$, then $N_k(s') > N_k(s'')$. Games with above characteristics are called coalitions games. If the formula (3) is replaced with equality $S = \prod_{i \in I} S_i$, then it is the coalition games of Vorobjev. If in coalition game with forbidden situations holds $K_d = K_i = I$, then the game G gets the form of non coalition games. If in the non coalition game holds that $S_k = \prod_{i \in K} S_i$, then the game gets the form of non coalition game of Nash. If in the non coalition game of Nash, there are two players I i II and for every situation holds $s \in S_I \times S_{II}$, than follows:

$$N_I(s) = -N_{II}(S), \quad (6)$$

and then, this is the antagonistic game.

By receding from equation (4) in the case of antagonistic game, and if we accept S_I and S_{II} as finite, and then this is bi-matrix game. If in the case of antagonistic game and S_I i S_{II} considered as finite (or if in the case of bi-matrix game include equation (4)), then we get the matrix game. In practice, this is the most often case for description of conflict situation.

The base for solving previous problems is "geometry" of one theorem by M. Taskovic, [10], in the form:

$$\max_{x \in X, y \in Y} \min \{x, y, g(x, y)\} = \min_{y \in Y, x \in X} \max \{x, y, g(x, y)\} \quad (7)$$

where $g: P^2 \rightarrow P$ comes as declining function, and X, Y are subgroups of group P .

If in the case of antagonistic game holds: $S_I = S_{II} = [0,1]$, then it is the case of a single quadratic game. Sub-kinds of a single quadratic game are numerous.

For the problem involving two players with many criteria, however, the following stands true:

$$\begin{aligned} &\mu_{R_1 \circ R_2 \circ \dots \circ R_n}(x, y; z_1, z_2, \dots, z_n) = \tag{8} \\ &= \min_{x, y, z_1, \dots, z_n \in [0,1]} \max [\mu_{R_1}(x, y; z_1), \dots, \mu_{R_n}(x, y; z_n), \dots, g(\mu_{R_1}(x, y; z_1), \dots, \mu_{R_n}(x, y; z_n))] = \\ &= \max_{x, y, z_1, \dots, z_n \in [0,1]} \min [\mu_{R_1}(x, y; z_1), \dots, \mu_{R_n}(x, y; z_n), \dots, g(\mu_{R_1}(x, y; z_1), \dots, \mu_{R_n}(x, y; z_n))] = \end{aligned}$$

where $\mu_{R_i}(x, y; z_i)$ function which to show value matrix effective, a $g(\mu_{R_i}(x, y; z_i))$ function which to show value matrix effective and criterion.

With multi criteria conflict situations, the function $g(X)$ is defined through priority order vector I and priority value V :

$$g(X) = g(x_1, x_2, \dots, x_k), \quad g(x_1, x_2, \dots, x_k) = \Lambda, \tag{9}$$

where Λ is a vector of weight coefficient.

This vector is calculated on the basis of priority vector: $V(v_1, v_2, \dots, v_n)$, for $v_i \in [0,1]$, form the area R^n and determined in accordance with priority order vector: $I(i_1, i_2, \dots, i_n)$.

There e_q, e_{q+1} denotes the importance of criteria. Some equations stand true as follows:

$$V_q = \frac{e_q}{e_{q+1}}, \quad 0 \leq \lambda_q < 1, \quad q \in [1, 2, \dots, k], \tag{10}$$

$$\sum_{q=1}^k \lambda_q = \lambda_1 + \lambda_2 + \dots + \lambda_k = 1, \quad \text{for } \lambda_q \geq \lambda_{q+1}. \tag{11}$$

The vectors Λ and V are related as follows:

$$v_q = \left(\frac{\lambda_q}{\lambda_{q+1}} \right) \tag{12}$$

This relationship is the basis for determining of the function $g(X)$ and the vector Λ . It is shown by the following expression:

$$\lambda_q = \frac{\prod_{i=q}^k v_i}{\sum_{q=1}^k \prod_{j=q}^k v_i} \tag{13}$$

The given expression can be derived as follows:

$$v_1 = \lambda_1 / \lambda_2, \quad v_2 = \lambda_2 / \lambda_3, \quad \dots, v_{k-1} = \lambda_{k-1} / \lambda_k, \quad v_k = 1 \tag{14}$$

resulting in:

$$\prod_{i=1}^{q-1} v_i = v_1 \cdot v_2 \cdot \dots \cdot v_{q-1} = \frac{\lambda_1 \cdot \lambda_2 \cdot \dots \cdot \lambda_{q-1}}{\lambda_2 \cdot \dots \cdot \lambda_{q-1} \cdot \lambda_q} = \frac{\lambda_1}{\lambda_q} \tag{15}$$

$$1 + \sum_{i=q}^k \frac{1}{\prod_{i=1}^{q-1} v_i} = \frac{\sum_{q=1}^k \prod_{i=q}^k v_i}{\prod_{i=q}^k v_i} \tag{16}$$

resulting in:

$$\lambda_1 = \frac{\prod_{i=q}^k v_i}{\sum_{q=1}^k \prod_{j=q}^k v_i} \tag{17}$$

the relationship between Λ and V .

4. EXAMPLE

The previous statements are illustrated here for the case of conflict situation, (see details in [7], [8], [9], [10], [12]), involving two players (X, Y) and five criteria (z_1, z_2, z_3, z_4, z_5). The Y unit has to destroy a facility having four possibilities of attacking it (y_1, y_2, y_3, y_4). The X unit can defend the said facility in two ways (x_1, x_2). The values of the game are given according to five criteria (z_1, z_2, z_3, z_4, z_5). They are the suffering of goods kept in the said facility.

Solution: Let us impose priority order $I = (i_1, i_2, i_3, i_4, i_5)$ and payment matrix for each criteria (z_1, z_2, z_3, z_4, z_5) in accordance with the possibilities of players X and Y and priority order:

For z_1, z_2 , and z_3 criteria, payment matrix is in Table 1.

Table 1: Payment matrix

For z_1 criteria:					For z_2 criteria:					For z_3 criteria:				
A/B	y_1	y_2	y_3	y_4	A/B	y_1	y_2	y_3	y_4	A/B	y_1	y_2	y_3	y_4
x_1	3	1	4	5	x_1	4	0	6	1	x_1	3	5	2	4
x_2	1	2	0	1	x_2	3	1	1	2	x_2	2	1	5	3

Step 1: The solution of a game per each criteria:

Results:

$$\begin{bmatrix} 3 & 1 & 4 & 5 \\ 1 & 2 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 4 & 0 & 6 & 1 \\ 3 & 1 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 3 & 5 & 2 & 4 \\ 2 & 1 & 5 & 3 \end{bmatrix}$$

$$X_{I1/2} = (0,40; 0,60) \quad X_{III1/2} = (0,00; 1,00) \quad X_{III1/2} = (0,75; 0,25)$$

$$Y_{I1/2} = (0,80; 0,20) \quad Y_{III1/2} = (0,83; 0,17) \quad Y_{III1/2} = (0,75; 0,25)$$

$$V_{I1/2} = \mu_R(x, y, z_1) = 1,60000 \quad V_{III1/2} = \mu_R(x, y, z_2) = 1,0000 \quad V_{III1/2} = \mu_R(x, y, z_3) = 1,2500$$

The game has been used for solving problems of decision making in defense by iterative method, for which the software tool has been developed in the programming language Clipper 87 and Turbo Pascal (see details in [8]).

Step 2: Linking of criteria – determination of $g(X)$ function.

The $g(X)$ function is determined as described above: the priority vector $V(2,4,1)$.

$$v_i = \frac{e_{q1}}{e_q}; \quad \lambda_q = \frac{\prod_{i=q}^k v_i}{\sum_{q=1}^k \prod_{j=q}^k v_i} \quad (18)$$

$$\lambda_1 = 8/13 = 0,6154, \quad \lambda_2 = 4/13 = 0,3077, \quad \lambda_3 = 1/13 = 0,0777. \quad (19)$$

Step 3: Reducing the problems to one matrix:

$$\begin{bmatrix} 3 & 1 & 4 & 5 \\ 1 & 2 & 0 & 1 \end{bmatrix} \cdot \lambda_1 = \begin{bmatrix} 1,85 & 0,62 & 2,46 & 3,08 \\ 0,62 & 1,24 & 0,00 & 0,62 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 6 & 1 \\ 3 & 1 & 1 & 2 \end{bmatrix} \cdot \lambda_2 = \begin{bmatrix} 1,23 & 0,00 & 1,85 & 0,31 \\ 0,92 & 0,31 & 0,31 & 0,62 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 5 & 2 & 4 \\ 2 & 1 & 5 & 3 \end{bmatrix} \cdot \lambda_3 = \begin{bmatrix} 0,23 & 0,38 & 0,15 & 0,31 \\ 0,15 & 0,08 & 0,38 & 0,23 \end{bmatrix}$$

Adding up matrixes using uniform criteria (common matrix):

$$\begin{bmatrix} 2,3131 & 0,9980 & 4,4619 & 3,7169 \\ 1,6928 & 1,6370 & 0,6913 & 1,4723 \end{bmatrix}$$

$$X_{z1/2} = (0,20; 0,80) \quad Y_{z1/2} = (0,86; 0,14) \quad (20)$$

$$V_{z1/2} = 1,4739 = \mu_{R1.R2.R3}(x, y; z_1, z_2, z_3).$$

The shown methodology could be used for solving problems in economy [3], [4], defense [5], as well as in politics and other fields. Special attention should be paid to acquiring information [2] since it is in the direct connection with the quality of any decision making.

5. CONCLUSIONS

As a conclusion this paper shows the results as follows:

- The performed classification of operational research methods. This classification uses the theory of game as a base for mathematical modeling and solving of conflict situations, with a focal point on multi-criteria decision making.
- The connection between methods and their mathematical description is established.
- The min-max theorem of professor Taskovic was applied for the first time in solving of multi-criteria conflict situation, as a very effective one for defining, analyzing and solving problems in the process of decision making.
- The shown classification can be used in practice for fast comparing of the results of various operational research methods, and for choosing the optimal one.

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