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OPTIMAL BATCH PRODUCTION STRATEGIES UNDER CONTINUOUS PRICE DECREASE AND TIME DISCOUNTING

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Abstract: Single price discount in unit cost for bulk purchasing is quite common in reality as well as in inventory literature. However, in today's high-tech industries such as personal computers and mobile industries, continuous decrease in unit cost is a regular phenomenon. In the present paper, an attempt has been made to investigate the effects of continuous price decrease and time-value of money on optimal decisions for inventoried goods having time-dependent demand and production rates. The proposed models are developed over a finite time horizon considering both shortages and without shortages in inventory. Numerical examples are taken to illustrate the developed models and to examine the sensitivity of model parameters.

Keywords: Inventory, continuous price change, time value of money, shortage.

1. INTRODUCTION

Most of the traditional inventory models in inventory management literature do not take into account the factors like time-value of money, continuous price decrease etc. In reality, these factors do have significant effects on the EOQ (economic order quantity) of any inventory system. Since the resource of an industry is highly correlated to the return of investment and it depends very much on the time of use, therefore, taking account of the time value of money is very critical in managerial decisions. Buzacott [2] was the first who introduced the concept of inflation in inventory modelling. Misra [10] considered internal as well as external inflation rates in his model and analyzed the influence of interest on replenishment strategies. Chandra and Bahner [3], and Sarker and Pan [13] developed infinite/finite replenishment models with shortages, considering inflation and time value of money. Following the approach of Misra [9, 10], Dutta and Pal [4] investigated a finite time horizon inventory model with time-dependent demand rate, shortages, inflation and time value of money. Bose et al. [1] developed an economic order quantity model for deteriorating items with linear time dependent demand and shortages, incorporating the effects of inflation and time value of money.

A great deal of researches have so far been undertaken to relax some of the original assumptions used in the EOQ model of Harris [6]. Among them, one of the key assumptions is that all costs in the model do not change during the foreseeable horizon. But in the cases where the inflation rate is high or the price increase or decrease is highly expected, this assumption does not hold good. For this reason, several extensions have already been made in the literature where ordering cost, unit purchase cost, holding cost etc. are assumed to be not fixed. In today's high-tech industries, in particular, personel computers and mobile industries, we observe that the component cost is decreasing at a sustained and significant rate.

Existing EOQ models that allow price change can be summarized based on two criteria: (i) finite horizon vs. infinite horizon and (ii) continuous price change vs. single announced price change. Naddor [11] was the first who developed an EOQ model with a single price change over an infinite planning horizon. Later, Peterson and Silver [12] studied this model by assuming that the price change occurs at the end of an EOQ cycle. Later, Taylor and Bradly [14] revised Peterson and Silver's [12] model by considering a situation where the price increase does not coincide with the end of EOQ cycle. Although many EOQ extensions have been made in the literature by considering price change, only a few of them considered the possibility of price decrease and most of them, namely Goyal et al. [5], Lev and Weiss [8], were limited to the single price change case. Recently, Khouja and Park [7] have made an attempt to study the effect of continuous price decrease in the optimal replenishment policy.

The purpose of this paper is to develop EOQ models for products whose price decreases continuously with time, taking into account the time value of money. We focus our study on a production inventory system prescribed over a finite planning horizon. The paper is organized as follows: The assumptions and notations used throughout this paper are given in the next section. The model without shortage is developed in Section 3. Section 4 treats the proposed model with shortages. In Section 5, the models are illustrated with numerical examples. Finally, the paper is concluded in Section 6.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

166

The following basic assumptions are made for the proposed models:

- 1. The production-inventory system consists of only one product and it operates for a prescribed planning horizon.
- 2. The demand and production rates are deterministic and are known functions of time.
- 3. The unit price of the product decreases continuously with time.
- 4. The discount rate regarding the time value of money is known.

2.2. Notations

The following notations are used throughout the paper:

$$D(t) = \text{demand rate} = A_1 e^{b_1 t}, A_1 > 0, b_1 \ge 0$$

$$P(t) = \text{production rate} = A_2 e^{b_2 t}, A_2 > A_1, b_2 \ge b_1$$

$$I_j(t) = \text{inventory level at any time } t, j = 1, 2$$

- H = length of a finite time horizon
- r_1 = inventory holding cost per unit per unit time
- $C_s = \text{set up cost per set up}$
- C_0 = price per unit of the product at time t = 0
- u = percent decrease in unit price per unit time
- C(t) = price per unit of the product at time t
- δ = discount rate
- n = number of production-inventory cycles completed during the planning horizon H

3. THE MODEL WITHOUT SHORTAGE: MODEL 1

We suppose that *n* production-inventory cycles are completed during the planning horizon *H*. For modeling simplicity, we divide the planning horizon *H* into *n* equal parts so that the length of each cycle is H/n. Consider the *i*-th cycle $[T_{i-1}, T_i]$, i = 1,2,...,n, where $T_i = iH/n$. The production starts at time T_{i-1} and stops at time t_{i1} ($T_{i-1} < t_{i1} < T_i$). As time passes, the inventory level increases with a rate P(t)-D(t) in the interval $[T_{i-1}, t_{i1}]$ and attains the maximum level at time t_{i1} . During the time period $[t_{i1}, T_i]$, the inventory level decreases in order to meet up demand and ultimately reaches to zero level at time T_i , i = 1,2,...,n. A schematic diagram of the production-inventory process is shown in Figure 1.



Figure 1: Schemetic diagram of the model without shortage

The instantaneous states of the inventory level at any time t ($T_{i-1} \le t \le T_i$), i = 1,2, ..., n can be represented by the following differential equations:

$$\frac{dI_{1}(t)}{dt} = P(t) - D(t), \quad T_{i-1} \le t \le t_{i1}$$
with $I_{1}(T_{i-1}) = 0;$
(1)

(2)

and

$$\frac{dI_2(t)}{dt} = -D(t), \quad t_{i1} \le t \le T_i$$

with $I_2(T_i) = 0.$

The solutions of the differential equations (1) and (2) are given by

$$I_{1}(t) = \frac{A_{2}}{b_{2}} \left(e^{b_{2}t} - e^{b_{2}T_{i-1}} \right) - \frac{A_{1}}{b_{1}} \left(e^{b_{1}t} - e^{b_{1}T_{i-1}} \right), \ T_{i-1} \le t \le t_{i1}$$
(3)

and

$$I_{2}(t) = \frac{A_{1}}{b_{1}} \left(e^{b_{1}T_{i}} - e^{b_{1}t} \right), \quad t_{i1} \le t \le T_{i,}$$

$$\tag{4}$$

respectively. Since the total quantity produced during the time period $[T_{i-1}, t_{i1}]$ satisfies the total demand during the period $[T_{i-1}, T_i]$, therefore, we have

$$\int_{T_{i-1}}^{t_{i1}} P(t) dt = \int_{T_{i-1}}^{T_i} D(t) dt.$$

After integrating and simplifying, we get

$$t_{i1} = \frac{1}{b_2} \log \left[\frac{A_1 b_2}{A_2 b_1} \left\{ e^{i b_1 H / n} - e^{(i-1)b_1 H / n} \right\} + e^{(i-1)b_2 H / n} \right]$$
(5)

The total cost in this model includes the set up cost, inventory holding cost and production cost. Since the set up cost is needed at the beginning of each cycle, therefore, the present value of the set up costs for n setups during the planning horizon H is given by

$$(OC)_{PV} = \sum_{i=0}^{n-1} c_s e^{-\delta T_i} = c_s \left[\frac{1 - e^{-\delta H}}{1 - e^{-\delta H/n}} \right].$$
 (6)

The present value of the holding costs for *n* cycles is

$$(HC)_{\rm PV} = \sum_{i=1}^{n} r_{\rm I} \left[\int_{T_{i-1}}^{t_{i1}} I_{1}(t) e^{-\hat{\sigma}t} dt + \int_{t_{i1}}^{T_{i}} I_{2}(t) e^{-\hat{\sigma}t} dt \right]$$

$$= \sum_{i=1}^{n} r_{\rm I} \left[\frac{A_{2}}{b_{2}m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} + \frac{A_{2}e^{(i-1)b_{2}H/n}}{b_{2}\delta} \left\{ e^{\hat{\sigma}_{i1}} - e^{-(i-1)\hat{\sigma}H/n} \right\} - \frac{A_{1}}{m_{1}\delta} \left\{ e^{im_{1}H/n} - e^{(i-1)m_{1}H/n} \right\} + \frac{A_{1}e^{-\hat{\sigma}_{i1}}}{b_{1}\delta} \left\{ e^{ib_{1}H/n} - e^{(i-1)b_{1}H/n} \right\} \right],$$
(7)

168

where $m_j = b_j - \delta$ for j = 1, 2.

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Let us suppose that $c(t) = c_0 e^{-ct}$, where c = -log(1-u/100). In the *i*-th cycle $[T_{i-1}, T_i]$, i = 1, 2, ..., n, the production starts at time $t = T_{i-1}$ and stops at $t = t_{i1}$, therefore, the present value of the production costs for *n* cycles is given by

$$(PC)_{PV} = \sum_{i=1}^{n} \int_{T_{i-1}}^{t_{i1}} c(t)P(t)e^{-\delta t} dt = \sum_{i=1}^{n} \frac{c_0 A_2}{m_2} \left\{ e^{m_2 t_{i1}} - e^{(i-1)m_2 H/n} \right\}$$
(8)

Hence from equations (6), (7) and (8) the present worth of the total cost over the finite time horizon H is given by

$$TC_{1}(n) = c_{s} \left[\frac{1 - e^{-\partial H}}{1 - e^{-\partial H/n}} \right] + \sum_{i=1}^{n} \frac{c_{0}A_{2}}{m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} \\ + \sum_{i=1}^{n} r_{1} \left[\frac{A_{2}}{b_{2}m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} + \frac{A_{2}e^{(i-1)b_{2}H/n}}{b_{2}\delta} \left\{ e^{\delta_{i1}} - e^{-(i-1)\delta H/n} \right\} \\ - \frac{A_{1}}{m_{1}\delta} \left\{ e^{im_{1}H/n} - e^{(i-1)m_{1}H/n} \right\} + \frac{A_{1}e^{-\delta_{i1}}}{b_{1}\delta} \left\{ e^{ib_{1}H/n} - e^{(i-1)b_{1}H/n} \right\} \right].$$
(9)

Our objective is to find the optimal value of *n* which minimizes $TC_1(n)$. Since *n* is a discrete variable, the minimum value of $TC_1(n)$ can be obtained by satisfying the condition $\Delta TC_1(n-1) < 0 < \Delta TC_1(n)$ where $\Delta TC_1(n) = TC_1(n+1) - TC_1(n)$.

4. THE MODEL WITH SHORTAGE: MODEL 2

We now extend the model developed in the previous section by allowing shortages to occur in inventory. The following additional assumption and notation are adopted to develop the model:

- (i) Unsatisfied demands are not backordered and are assumed to be lost completely.
- (ii) r_2 is the cost of running out one unit of the product for a unit time
- (iii) $I_3(t)$ is the inventory level at any time t.

Let us consider the *i*-th production-inventory cycle $[T_{i-1}, T_i]$, i = 1, 2, ..., n, where the production starts at time T_{i-1} stops at time t_{i1} ($T_{i-1} < t_{i1} < T_i$). The inventory level decreases from the maximum level at time t_{i1} to the zero level at time t_{i2} ($T_{i-1} < t_{i1} < t_{i2} < T_i$). The inventory remains in the negative level during the time period $[t_{i2}, T_i]$, i = 1, 2, ..., n because of continuous demand for the product. The maximum shortage level occurs at time T_i , where the production again starts for the next cycle. The graphical representation of the inventory system is shown in Figure 2.

Let k (0 < k <1) represent the fraction of each length during which inventory is carried for each cycle. Then we have,

$$t_{i1} = (k + i - 1)H/n, \quad i = 1, 2, ..., n.$$



Figure 2: Schematic of diagram of the model with shortage

The inventory level at any time t ($T_{i-1} \le t \le T_i$) can be described by the following differential equations:

$$\frac{dI_{1}(t)}{dt} = P(t) - D(t), \quad T_{i-1} \le t \le t_{i1}$$
(10)

with
$$I_1(T_{i-1}) = 0;$$

 $\frac{dI_2(t)}{dt} = -D(t), \quad t_{i1} \le t \le t_{i2}$
(11)

with
$$I_2(t_{i2}) = 0$$
,
 $\frac{dI_3(t)}{dt} = -D(t), \quad t_{i2} \le t \le T_i$
(12)

with
$$I_3(t_{i2}) = 0$$
.

The solutions of equations (10), (11) and (12) are respectively

$$I_{1}(t) = \frac{A_{2}}{b_{2}} \left(e^{b_{2}t} - e^{b_{2}T_{i-1}} \right) - \frac{A_{1}}{b_{1}} \left(e^{b_{1}t} - e^{b_{1}T_{i-1}} \right), \ T_{i-1} \le t \le t_{i1}$$
(13)

$$I_2(t) = \frac{A_1}{b_1} \left(e^{b_1 t_{i2}} - e^{b_1 t} \right), \quad t_{i1} \le t \le t_{i2}, \tag{14}$$

$$I_{3}(t) = \frac{A_{1}}{b_{1}} \left(e^{b_{1}t_{i2}} - e^{b_{1}t} \right), \quad t_{i2} \le t \le T_{i.}$$
(15)

Since $I_1(t_{i1}) = I_2(t_{i1})$, therefore, we have from equations (13) and (14)

$$t_{i2} = \frac{1}{b_1} \log \left[\frac{A_2 b_1}{A_1 b_2} \left\{ e^{b_2 m_3 H / n} - e^{(i-1)b_2 H / n} \right\} + e^{(i-1)b_1 H / n} \right]$$
(16)

170

where $m_3 = k + i - 1$, i = 1, 2, ..., n. The present value of the holding costs for *n* cycles is given by

$$(HC)_{pv} = \sum_{i=1}^{n} r_{1} \left[\int_{T_{i-1}}^{t_{i1}} I_{1}(t) e^{-\delta t} dt + \int_{t_{i1}}^{T_{i}} I_{2}(t) e^{-\delta t} dt \right]$$

$$= \sum_{i=1}^{n} r_{1} \left[\frac{A_{2}}{b_{2}m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} + \frac{A_{2}e^{(i-1)b_{2}H/n}}{b_{2}\delta} \left\{ e^{\delta t_{i1}} - e^{-(i-1)\delta H/n} \right\} - \frac{A_{1}}{m_{1}\delta} \left\{ e^{m_{1}t_{i2}} - e^{(i-1)m_{1}H/n} \right\} + \frac{A_{1}e^{-m_{3}\delta H/n}}{b_{1}\delta} \left\{ e^{b_{1}t_{i2}} - e^{(i-1)b_{1}H/n} \right\} \right],$$
(17)

where $m_j = b_j - \delta$ for j = 1, 2.

Similarly, the present value of the shortage costs for n cycles is

$$(SC)_{\rm PV} = \sum_{i=1}^{n} \int_{t_{i2}}^{T_i} -r_3 I_3(t) e^{-\delta t} dt$$

= $\sum_{i=1}^{n} \frac{r_1 A_2}{b_1} \Big[\frac{1}{\delta} \Big\{ e^{b_1 t_{i2} - i \delta H / n} - e^{m_1 t_{i2}} \Big\} + \frac{1}{m_1} \Big\{ e^{i m_1 H / n} - e^{m_1 t_{i2}} \Big\} \Big].$ (18)

Hence, the present value of the total cost over the entire time horizon H is

$$TC_{2}(n) = c_{s} \left[\frac{1 - e^{-\partial H}}{1 - e^{-\partial H/n}} \right] + \sum_{i=1}^{n} \frac{c_{0}A_{2}}{m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} \\ + \sum_{i=1}^{n} r_{1} \left[\frac{A_{2}}{b_{2}m_{2}} \left\{ e^{m_{2}t_{i1}} - e^{(i-1)m_{2}H/n} \right\} + \frac{A_{2}e^{(i-1)b_{2}H/n}}{b_{2}\delta} \left\{ e^{\partial_{i1}} - e^{-(i-1)\partial H/n} \right\} \\ - \frac{A_{1}}{m_{1}\delta} \left\{ e^{m_{1}t_{i2}} - e^{(i-1)m_{1}H/n} \right\} + \frac{A_{1}e^{-m_{3}\partial H/n}}{b_{1}\delta} \left\{ e^{b_{1}t_{i2}} - e^{(i-1)b_{1}H/n} \right\} \right] \\ + \sum_{i=1}^{n} \frac{r_{1}A_{2}}{b_{1}} \left[\frac{1}{\delta} \left\{ e^{b_{1}t_{i2}-i\partial H/n} - e^{m_{1}t_{i2}} \right\} + \frac{1}{m_{1}} \left\{ e^{im_{1}H/n} - e^{m_{1}t_{i2}} \right\} \right].$$
(19)

Since the above cost function is a function of a continuous variable k and a discrete variable n, it is very difficult to find the optimal values of k and n simultaneously. So, for any given n, necessary condition for optimal $TC_2(n, k)$ is

$$\frac{dTC_2(n,k)}{dk} = 0. \tag{20}$$

A solution k^* of equation (20) would be a minimizer of $TC_2(n, k)$ provided that

$$\left\lfloor \frac{d^2 T C_2(n,k)}{dk^2} \right\rfloor_{k=k^*} > 0.$$

Applying the line search technique on *n* one can find n^* , the optimal value of *n* and the corresponding k^* which jointly determine the minimum total discounted cost $TC_2(n^*, k^*)$.

171

5. NUMERICAL EXAMPLES

5.1. Example 1

Let us consider the parameter values of Model 1 as given in the following: $A_1 = 20$, $A_2 = 30$, $b_1 = 0.01$, $b_2 = 0.02$, $r_1 = 20$, $c_s = 400$, u = 40, $c_0 = 200$, $\delta = 0.03$, H = 14 in appropriate units.

We apply the line search technique on *n*. Table 1 shows that $TC_1(n)$ is convex in *n* and the minimum value 12944.6 is obtained for n = 8. As *n* increases, the present value of set up cost increases whereas the present values of holding cost and production cost decrease. Table 2 shows the optimal results of Model 1 for different lengths of the time horizon *H*. The number of production-inventory cycles as well as the total discounted cost increase with *H*, as expected.

п	$(OC)_{\rm PV}$	$(HC)_{\rm PV}$	$(PC)_{PV}$	$TC_1(n)$
1	400.000	11725.00	11421.13	23546.0
2	724.234	6138.62	10747.40	17610.2
3	1050.060	4155.46	10077.30	15282.8
4	1376.280	3140.44	9596.89	14113.6
5	1702.660	2523.85	9256.89	13483.4
6	2029.120	2109.62	9008.51	13147.2
7	2355.630	1812.17	8820.67	12988.5
8^*	2682.170	1588.23	8674.24	12944.6*
9	3008.720	1413.55	8557.16	12979.4
10	3335.290	1273.48	8461.54	13070.3

Table 1: Optimal results in Model 1

 Table 2: Optimal results in Model 1 for different values of H

Tuble 2. Optimiliar results in whoden i for different values of <i>H</i>					
Н	n^*	$TC_1(n^*)$			
10	6	11800.4			
12	7	12394.4			
14	8	12944.6			
16	9	13465.9			
18	10	13964.1			
20	11	1442.2			

5.2. Example 2

We consider the following data for Model 2: $A_1 = 20$, $A_2 = 30$, $b_1 = 0.01$, $b_2 = 0.02$, $r_1 = 20$, $r_2 = 80$, $c_s = 400$, u = 40, $c_0 = 100$, $\delta = 0.03$, H = 14 in appropriate units.

Table 3 shows that the minimum value of $TC_2(n,k)$ is 11120.1 and it is obtained for n = 10 and k = 0.412773. Comparing the results in Table 1 and 3 we find that Model 2 provides lower cost than Model 1. Like Model 1, both the number of productioninventory cycles and the total discounted cost increase as the length of the time horizon increases, see Table 4.

n	k	$TC_2(n,k)$
1	0.595319	22400.1
2	0.567733	16671.1
3	0.544458	14190.8
4	0.523368	12867.1
5	0.503609	12099.5
6	0.484671	11638.5
7	0.466261	11363.0
8	0.448205	11207.7
9	0.430399	11134.6
10^{*}	0.412773^{*}	11120.1^{*}
11	0.395281	11148.7
12	0.377893	11210.0

Table 3: Optimal results in Model 2

Table 4 Optimal results in Model 2 for different values of H

Н	n^*	k^{*}	$TC_{2}(n^{*},k^{*})$
10	10	0.250377	9611.42
12	10	0.354908	10474.10
14	10	0.412773	11120.10
16	11	0.433619	11681.40
18	11	0.457352	12179.70
20	12	0.463805	12640.80

Table 5 Influence of δ on the total discounted cost in Model 2

δ	n^*	k^{*}	$TC_2(n^*,k^*)$	% change in $TC_2(n^*,k^*)$
0.00	9	0.452558	12643.70	0.00000
0.01	9	0.445580	12102.70	-4.27881
0.02	9	0.438198	11601.00	-8.24679
0.03	10	0.412773	11120.10	-12.05030
0.04	10	0.403499	10670.40	-15.60700
0.05	11	0.374118	10246.10	-18.96280
0.06	11	0.362713	9837.64	-22.19330
0.07	12	0.328932	9448.79	-25.26880
0.08	12	0.315146	9073.31	-28.23850
0.09	13	0.276512	8705.95	-31.14400
0.10	14	0.234664	8346.13	-33.98980

и	n^*	k^{*}	$TC_2(n^*,k^*)$
30	13	0.267852	12781.80
32	12	0.312935	12444.70
34	11	0.352775	12099.50
36	11	0.368346	11763.40
38	10	0.401049	11432.30
40	10	0.412773	11120.10
42	10	0.423487	10827.10
44	9	0.449015	10543.00
46	9	0.457227	10274.40
48	9	0.464824	10021.80
50	9	0.471879	9783.66

Table 6 Sensitivity analysis with respect to *u* in Model 2

Table 5 shows the effects of time value of money on the total cost. When δ varies from 1% to 10%, the total cost varies from 4% to 34%. Table 6 shows the sensitivity analysis with respect to the model parameter *u* in Model 2. The total discounted cost decreases significantly with the increase in percentage increase in unit price. For a 25% decrease in the value of *u* (*u* = 40 to *u* = 30), $TC_2(n^*,k^*)$ increases 15% whereas for 25% increase in the value of *u* (*u* = 40 to *u* = 50), $TC_2(n^*,k^*)$ decreases 12%.

6. CONCLUSIONS

In this paper, we have developed inventory models for items whose demand and production rates are dependent on time and unit price decreases continuously with time. We have obtained optimal decisions by taking into account the time value of money over a finite planning horizon. Some realistic features that are highlighted in the developed models are likely to be associated with an inventory of electronic goods for which the assumption of continuous price decrease is quite appropriate. It is a well known fact that the prices of electronic goods are becoming increasingly lower each year. So the idea of continuous price decrease for these products can not be ignored. Moreover, the assumption of time dependent production and demand rates is also realistic. In any industry, the production rate and demand rate can not remain constant for long time; it might vary with time. In that sense, the idea of time dependent production and demand rate is more appropriate than the idea of constant production and demand rates. Furthermore, the occurrence of shortages in inventory is a natural phenomenon in real situations. Finally, the effect of time value of money is taken into account as it may be observed that today's economy of many countries is in the grip of a large scale inflation and a consequent sharp decline in the purchasing power of money.

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