

A COMPARATIVE ANALYSIS OF THE DEA-CCR MODEL AND THE VIKOR METHOD

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Abstract: Data Envelopment Analysis (DEA) introduces a model for weights determination maximizing efficiency of the decision-making units. The primary focus of the DEA model is to compare decision-making units (alternatives) in terms of their efficiency in converting inputs into outputs. The multicriteria decision making (MCDM) method VIKOR uses a common set of weights expressing a decision maker's preferences. In contrast, the CCR model of DEA does not provide a common set of weights that could express the preferences of a decision maker. The weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the real aspects of decision making, such as the preference structure. A comparison of DEA and MCDM shows that DEA resembles MCDM, but the results differ. In spite of these differences, DEA could be used as a supplement for screening alternatives within MCDM. An application of DEA and MCDM is illustrated by an example of hydropower system planning.

Keywords: Multi criteria, decision making, data envelopment analysis, compromise.

1. INTRODUCTION

Among the numerous approaches available for conflict management, one of the most prominent is multicriteria decision making (MCDM). MCDM is a complex and

dynamic process including one managerial level and one engineering level [9]. The managerial level defines the goals, and chooses the final “optimal” alternative. Thus multicriteria nature of decisions is emphasized at this level, in which public officials called “decision makers” have the power to accept or reject solutions proposed by the engineering level. The decision maker (DM) who provides the preference structure, is “off line” from the optimization procedure done at the engineering level. Very often, the preference structure is based on political rather than solely on technical criteria. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of no inferior and/or compromise solutions. The compromise ranking method (called VIKOR) has been introduced as one applicable technique to implement within MCDM [19].

For engineering, the main effort is in generating and evaluating the alternatives. These efforts are different for different projects, since projects vary in the types of needs they meet or the related problems they solve [28]. The physical, environmental, and social setting in which planning takes place also differs from one location to another. Therefore, a solution can be developed for a particular project in a particular way by applying MCDM. The CCR model of Data Envelopment Analysis could be applied to screen alternatives in a multiple objective sense without DM’s preference among criteria [23]. Factor Analysis (performing a principal component solution and orthogonal rotation of a factor matrix) could be used to study relationship among dependent criterion functions in order to discover something about the nature of the independent variables that affect them, even though those independent variables, called factors, are not within the set of criteria.

The CCR model of Data Envelopment Analysis (DEA), developed by Charnes et al. [6], is a linear programming technique used to estimate the relative efficiency of decision-making units (DMUs), considering the multiple inputs that they consume, and multiple outputs that they produce. A standard formulation of DEA creates a separate linear programming model for each DMU, in which the unknown variables are the weights associated with inputs and outputs. The basic result of DEA is an envelopment surface (efficient frontier) consisting of the “best practice” decision-making units, as well as an efficient measure that reflects the distance from each DMU to the frontier [10], [25]. Tone introduced a slacks-based measure of efficiency in DEA, including input excesses and output shortfalls of the DMU [27].

A relationship between DEA and multicriteria decision making was considered by Stewart [24] who concluded that the fields of MCDM and DEA have developed, to a large extent, independently of each other. The criteria in MCDM can be divided into costs (inputs) and benefits (outputs) which gives the methodological connection between DEA and MCDM [8],[26]. A DMU within DEA is usually called an alternative within MCDM. Bouyssou [5] considers proposals to use DEA as a tool for MCDM, concluding with some remarks on the possible areas of interaction between DEA and MCDM. Adler and Golany [1] consider the use of principal components to reduce the course of dimensionality that occurs in DEA when there is an excessive number of inputs and outputs in relation to the number of decision-making units, and that can improve the discrimination power of DE

An intention of this paper is to compare the DEA method (CCR model) and a MCDM method, VIKOR, as well as to apply DEA as a supplement within multicriteria decision making. DEA provides an efficiency measure that does not rely on the

application of a common weighting of the inputs and outputs. The DEA-CCR model could be applied to screen alternatives in a multiple objective sense without DM's preference among criteria. In contrast, a multicriteria decision making approach is based on the assumption that a common set of weights must be applied across all units (alternatives). An example of hydropower system planning illustrates possible application of DEA within MCDM. Finally, it is concluded that DEA does not provide a common set of weights that could express the preferences of a decision maker. DEA is an approach different from the MCDM method, VIKOR, and the DEA results were not useful within MCDM.

2. DATA ENVELOPMENT ANALYSIS – DEA

Data Envelopment Analysis (DEA) is an operations research technique for measuring the performance of decision-making units (DMUs), with multiple non commensurable inputs and outputs. The basic result of DEA is an envelopment surface (efficient frontier) consisting of the “best practice” DMUs. DEA introduces a model for relative efficiency measures incorporating multiple inputs and outputs. The following formula for efficiency (Eff) is used:

$$Eff_j = \sum_k u_k y_{kj} / \sum_i v_i x_{ij}, \quad \forall j \quad (1)$$

where y_{kj} is the amount of output k from j -th DMU, u_k is the weight associated with output k , x_{ij} is the amount of input i into j -th DMU, v_i is the weight associated with input i , and Eff_j is the efficiency of j -th DMU. The DEA-CCR model is formulated as the following problem:

$$M1 : \max_{(u,v)} (Eff_m = \sum_k u_k y_{km} / \sum_i v_i x_{im})$$

subject to :

$$\sum_k u_k y_{kj} / \sum_i v_i x_{ij} \leq 1, \quad \forall j$$

$$u_k, v_i \geq 0, \quad \forall k, \forall i$$

which maximizes the efficiency of m -th DMU, subject to the efficiency of all DMUs being ≤ 1 .

The linear version of the model M1 is the following linear programming (DEA-LP) model:

$$M2: \max_{(u,v)} \sum_k u_k y_{km}$$

subject to:

$$\sum_i v_i x_{im} = b$$

$$\sum_k u_k y_{kj} - \sum_i v_i x_{ij} \leq 0, \quad \forall j$$

$$u_k, v_i \geq \varepsilon \text{ or } \varepsilon), \quad \forall k, \quad \forall i$$

where b is a constant, here $b=100$, and the small positive quantity ε could be introduced in order to avoid any input or output being totally ignored in determining the efficiency.

The efficiency of the target DMU (m -th) can be obtained by solving model $M2$ (DEA-LP model). The obtained weights (u, v) are the most favourable ones from the point of view of the m -th DMU. Because the objective function is varying for different DMUs, the weights obtained for each target DMU clearly may be different.

Within DEA analysis, a DMU can appear efficient simply because of its pattern of inputs and outputs. A minimum limit to weight for any input and output would ensure that each of them played some part in the determination of the efficiency. Also, a maximum limit could avoid any input and output being over-represented. This analysis leads to a compromise between weights flexibility and a common set [10]. Charnes et al. [6] recognized the difficulty in seeking a common set of weights to determine relative efficiency, and the difficulty in bounding the weights was considered by Stewart [24]. Chiang and Tzeng [7] applied a multiple objective programming method in determining a common set of weights. They considered every DMU's efficiency as one objective function to be maximized, and the solution is determined by maximizing minimum DMUs' efficiency. Joro et al. [16] presented a structural comparison between classical DEA models and the reference point approach in multiple objective linear programming.

A review of methods for increasing discrimination between efficient DMUs in DEA is presented in [4], classifying the methodologies into two groups: the first group comprises those methods that incorporate a priori information provided by a decision maker into the model, while the second group of methods does not require such a priori information. Within the first group three streams are considered: weight restrictions [14],[21], preference structure [29] and Value Efficiency Analysis [15],[17]. Within the second group three methodologies are presented: super efficiency [3], cross-evaluation [13], and a multiple objective linear programming approach [18]. Besides discrimination, these approaches resolve unfitness of the weighting scheme, which frequently can be unreal by the CCR model, giving a big weight to variables with less importance or giving a small (or zero) weight to important variables. The relative importance of criteria (inputs and outputs) is associated with decision maker's preference.

3. THE VIKOR METHOD

The VIKOR method has been developed to solve the following problem

$$mco\{(f_{ij}(A_j), j = 1, \dots, J), i = 1, \dots, n\}$$

where: J is the number of feasible alternatives; $A_j = \{x_1, x_2, \dots\}$ is the j -th alternative obtained (generated) with certain values of system variables x ; f_{ij} is the value of the i -th criterion function for the alternative A_j ; n is the number of criteria; mco denotes the operator of a multi criteria decision making procedure for selecting the best (compromise) alternative in multi criteria sense. The extended VIKOR method in

comparison with three multicriteria decision making methods TOPSIS, PROMETHEE, and ELECTRE is presented in the work of Opricovic and Tzeng [19].

The algorithm VIKOR has the following steps:

(i) Determine the best f_i^* and the worst f_i^- values of all criterion functions, $i = 1, 2, \dots, n$;

$$f_i^* = \max_j f_{ij}, \quad f_i^- = \min_j f_{ij}, \quad \text{if the } i\text{-th function represents a benefit};$$

$$f_i^* = \min_j f_{ij}, \quad f_i^- = \max_j f_{ij}, \quad \text{if the } i\text{-th function represents a cost}.$$

(ii) Compute the values S_j and R_j , $j=1, 2, \dots, J$, by the relations

$$S_j = \sum_{i=1}^n w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-) \quad (2)$$

$$R_j = \max_i [w_i (f_i^* - f_{ij}) / (f_i^* - f_i^-)] \quad (3)$$

where w_i are the weights of criteria, expressing the DM's preference as the relative importance of the criteria.

(iii) Compute the values Q_j , $j = 1, 2, \dots, J$, by the relation

$$Q_j = v(S_j - S^*) / (S^- - S^*) + (1-v)(R_j - R^*) / (R^- - R^*) \quad (4)$$

where $S^* = \min_j S_j$, $S^- = \max_j S_j$, $R^* = \min_j R_j$, $R^- = \max_j R_j$; and v is introduced as a weight for the strategy of "the majority of criteria" (or "the maximum group utility"), whereas $1-v$ is the weight of the individual regret. These strategies could be compromised by $v = 0.5$, and here v is modified as $v = (n+1) / 2n$ (from $v + 0.5(n-1)/n = 1$) since the criterion (1 of n) related to R is included in S , too.

(iv) Rank the alternatives, sorting by the values S , R and Q in decreasing order. The results are three ranking lists.

(v) Propose as a compromise solution the alternative ($A^{(1)}$) which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

C1. "Acceptable Advantage":

$$Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$$

where: $A^{(2)}$ is the alternative with second position in the ranking list by Q ;

$$DQ = 1/(J-1).$$

C2. "Acceptable Stability in decision making":

The alternative $A^{(1)}$ must also be the best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or "by consensus" $v \approx 0.5$, or "with veto" ($v < 0.5$). Here, v is the weight of decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

-Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
 -Alternatives $A^{(1)}, A^{(2)}, \dots, A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the positions of these alternatives are “in closeness”).

(vi) Determine the weight stability interval $[w_i^L, w_i^U]$ for each (i -th) criterion, separately, with the initial (given) values of weights. The compromise solution obtained with initial weights $(w_i, i=1, \dots, n)$, will be replaced at the highest ranked position if the value of a weight is out of the stability interval. The stability interval is only relevant concerning one-dimensional weighting variations.

(vii) Determine the trade-offs, $tr_{ik} = |(D_i w_k) / (D_k w_i)|, k \neq i, k = 1, \dots, n$, where tr_{ik} is the number of units of the i -th criterion evaluated the same as one unit of the k -th criterion, and $D_i = f_i^* - f_i^-, \forall i$. The index i is given by the VIKOR user.

(viii) The decision maker may give a new value of $tr_{ik}, k \neq i, k = 1, \dots, n$ if he or she does not agree with computed values. Then, VIKOR performs a new ranking with new values of weights $w_k = |(D_k w_i tr_{ik}) / D_i|, k \neq i, k = 1, \dots, n; w_i = 1$ (or previous value). VIKOR normalizes weights, with the sum equal to 1. The trade-offs determined in step (vii) could help the decision maker to assess new values, although that task is very difficult.

(ix) The VIKOR algorithm ends if the new values are not given in step (viii).

The results by the VIKOR method are rankings by S , R , and Q , proposed compromise solution (one or a set), weight stability intervals for a single criterion, and the trade-offs introduced by VIKOR.

The VIKOR method focuses on ranking and selecting from a set of alternatives, and determines compromise solutions for a problem with conflicting criteria, which can help the decision makers to reach a final decision. The obtained compromise solution could be accepted by the decision makers because it provides a maximum group utility of the “majority” (represented by $\min S$, Equation (2)), and a minimum individual regret of the “opponent” (represented by $\min R$). The compromise solutions could be the base for negotiation, involving the decision makers’ preference by criteria weights. The trade-offs determined in step (vii) could help the decision maker to assess new values, although that task is very difficult. Trade-off assessment is the most difficult issue in MCDM, and many methods have been developed to alleviate this problem.

4. A COMPARISON OF DEA AND VIKOR

The focus of the VIKOR method is in selecting the best (compromise) alternative from a set of alternatives in the presence of conflicting criteria, by analyzing the criteria space. Assuming that each alternative is evaluated according to each criterion function, the VIKOR method ranks alternatives by comparing the measure of closeness to an ideal alternative.

DEA introduces a linear programming model for weights determination, individually maximizing the efficiency of the decision-making units (DMUs). Therefore

the DMUs cannot be ranked with these weights, which vary from unit to unit. Charnes et al. recognized the difficulty in seeking a common set of weights to determine relative efficiency [6]. An approach to determine a common set of weights to be used within a MCDM method is presented in this paper (Eqs (5)). However, such common set of weights has no relation with the preference of decision maker who is competent to assess the relative importance of the criteria within MCDM. We conclude that DEA does not provide weights that can express the preference of a decision maker. This is because DEA provides an efficiency measure that does not rely on the application of a common weighting of the inputs and outputs. On the contrary, a multicriteria decision making approach is based on the assumption that a common set of weights must be applied across all alternatives (decision-making units).

DEA correlates with some MCDM methods in a decision environment where discrete alternatives with multiple dimensions exist [2], but there is no generally accepted approach for making a comparison of DEA and MCDM methods [20],[22]. We have analyzed similarity of DEA and the VIKOR method of MCDM in order to find a way of using DEA results for multicriteria decision making, particularly in assessing criteria weights. However, we found that the DEA approach was different from a MCDM method and the DEA results were not as useful within MCDM as expected.

The fields of MCDM and DEA have developed, to a large extent, independently of each other [8], [24]. However, decision-making units, inputs and outputs in DEA can be considered as alternatives, costs and benefits in MCDM, respectively. This relationship provides a basis for a comparison of DEA and the VIKOR method of MCDM. From this standpoint a similarity is evident, although there are essential differences within.

We did several numerical experiments (such as the application in Section 5) that compare DEA and VIKOR methods, and the findings are summarized below.

- *Efficiency and Pareto optimality*: The concept of Pareto optimality is the core of DEA and MCDM. However, the frontier by DEA is in the space of output/input ratios, and Pareto optimality in MCDM is considered in criteria space. This is why the positions of DMUs (alternatives) are different in these spaces.

- *Decision criterion*: In DEA this is the ratio of multiple outputs and multiple inputs, while in VIKOR it is the aggregated function (distance function) of all criteria. Any DMU, that performs the best on one particular ratio of an output to an input, is found to be efficient by DEA; while a noninferior solution in MCDM is any DMU with at least one input or one output as the best [12]. By definition the alternative A_j is a noninferior solution if and only if there is no A_k for which $f_{ik} \succcurlyeq f_{ij}$, $i \in n$, and $f_{ik} > f_{ij}$

for at least one i ; where \succcurlyeq and $>$ mean \geq and $>$, respectively, for benefit criteria (outputs in DEA); or \leq and $<$ for cost criteria (inputs in DEA). There is no solution better than a noninferior solution according to all criteria. Each noninferior solution could be the best solution in MCDM with certain weights.

- *Solution*: The set of efficient units determined by DEA has no relationship with noninferior solutions within MCDM, whereas the compromise solution by VIKOR is a noninferior solution (Pareto-optimal). An efficient unit is a noninferior solution in the space of output/input ratios considered by DEA. A noninferior solution within MCDM could be inefficient unit by DEA. An efficient unit determined by DEA could be the best compromise solution by VIKOR, although an inefficient unit by DEA also could be the

best compromise solution by VIKOR. In many cases, efficient units by DEA are highly ranked by VIKOR, and very inefficient units by DEA are given low rankings by VIKOR, although the exception could be the alternative with the extreme value of certain criterion.

- *Weights*: The values of weights (u, v) determined by DEA are not related to the decision makers' preference; whereas in MCDM the criteria weights are assessed or given by decision makers.

- *Usefulness*: DEA determines the efficient DMUs and generates potential improvements for inefficient DMUs. In contrast, the VIKOR method ranks alternatives by comparing the measure of closeness to the ideal alternative, and then selects the best (compromise) alternative from a set of alternatives in the presence of conflicting criteria.

The potential improvements for inefficient units by DEA (obtainable by *Frontier Analyst* software [11]) show how a DMU needs to decrease its inputs or increase its outputs in order to become efficient. This is very useful result within DEA application, but it is of less interest within MCDM.

In spite of these differences, DEA could be considered as a pre process in MCDM, providing a substantial screening of alternatives for MCDM. Because DEA determines the efficient DMUs without any information of the relative importance of inputs and outputs, it could be a useful tool in MCDM, particularly when a decision maker is not able to express a preference at the beginning of system design or planning. However, DEA can not replace MCDM in selecting the best (compromise) solution for the MCDM problem.

5. AN APPLICATION: HYDROPOWER SYSTEM PLANNING

5.1. Hydropower System on Drina River

Previous studies of hydropower potential for the Drina River, in the former Yugoslavia, have selected potential dam sites for reservoirs to provide hydropower. In addition, comprehensive analysis was required to resolve conflicting technical, social and environmental features. Even if the topographic surveys confirm that the required reservoir capacity is available, a hydrological solution may conflict with environmental, social, and cultural features.

The VIKOR method was applied to evaluate alternative hydropower systems on the Drina River. The alternatives were generated by varying two system parameters, dam site and dam height. The following six alternatives were selected for multicriteria optimization.

A_1 – Hydropower system (HPS) Gorazde, one reservoir, normal level at 375 m.a.s.l

A_2 - HPS Gorazde 383

A_3 - Cascade HPS: Gorazde 352, Sadba 362, Ustikolina 373, Paunci 384

A_4 - Cascade HPS: Gorazde 375, Paunci 384

A_5 - Cascade HPS: Gorazde 362, Ustikolina 373, Paunci 384

A_6 - Cascade HPS: Sadba 362, Ustikolina 373, Paunci 384

The systems consist of from one (A_1 and A_2) to four reservoirs (A_3). The dam site Gorazde is at river km 298, Sadba at km 301 (upstream), Ustikolina at km 307, and

Paunci at km 315. The dams within a system with more than one reservoir form a cascade. The designed reservoir systems are evaluated according to the following criteria:

- f_1 - Profit (10^6 Dinar, Yugoslav currency)
- f_2 - Costs (10^6 Dinar)
- f_3 - Total energy produced (GWh/year)
- f_4 - Peak energy produced (GWh/year)
- f_5 - Number of homes to be relocated
- f_6 - Area flooded by reservoirs (ha)
- f_7 - Number of villages to displace (even partially)
- f_8 - Environmental protection (grades 1 to 5).

Table 1: Performance matrix

Criteria	Alternatives								
	Name	Unit	Extrem	A_1	A_2	A_3	A_4	A_5	A_6
f_1	Profit	10^6 Din	Max	4184.3	5211.9	5021.3	5566.1	5060.5	4317.9
f_2	Costs	10^6 Din	Min	2914.0	3630.0	3920.5	3957.9	3293.5	2925.9
f_3	Total energy produced	GWh	Max	407.2	501.7	504.0	559.5	514.1	432.8
f_4	Peak energy produced	GWh	Max	251.0	308.3	278.6	335.3	284.2	239.3
f_5	Homes to be relocated	Num.	Min	195	282	12	167	69	12
f_6	Reservoirs area	Ha	Min	244	346	56	268	90	55
f_7	Villages to displace	Num.	Min	15	21	3	16	7	3
f_8	Environmental protect.	Grade	Max	2.41	1.41	4.42	3.36	4.04	4.36

The values of criterion functions are obtained by a comprehensive study of this reservoir system on the Drina River, and the results are presented in Table 1. The multicriteria optimization task is to maximize the criterion functions f_1 , f_3 , f_4 , and f_8 , and to minimize functions f_2 , f_5 , f_6 , and f_7 .

5.2. DEA application

The decision-making units (DMUs) are : $A_1, A_2, A_3, A_4, A_5, A_6$;

The inputs are : f_2 (costs), f_5 , f_6 , and f_7 (social impacts, land resources);

The outputs are : f_1 (profit), f_3 , and f_4 (energy produced), and f_8 (lower environmental impact evaluated higher). The criterion functions to be maximized in MCDM are considered as outputs in DEA.

Linear programming problem $M2$ could be solved by a linear programming program package. The obtained results, with input data from Table 1, are presented in Table 2.

Table 2 : The results using DEA

DMU	Eff.	$u_1(f_1)$	$v_1(f_2)$	$u_2(f_3)$	$u_3(f_4)$	$v_2(f_5)$	$v_3(f_6)$	$v_4(f_7)$	$u_4(f_8)$
A_1	0.998	0.0	0.0343	0.0	0.3977	0.0	0.0	0.0	0.0
A_2	0.984	0.0	0.0275	0.0	0.3192	0.0	0.0	0.0	0.0
A_3	1.0	0.0	0.0133	0.1984	0.0	0.0	0.8512	0.0	0.0
A_4	0.982	0.0	0.0253	0.0	0.2928	0.0	0.0	0.0	0.0
A_5	1.0	0.0	0.0304	0.1564	0.0	0.0	0.0	0.0	4.8521
A_6	1.0	0.0	0.0342	0.0	0.0	0.0	0.0	0.0	22.936

Efficient DMUs (alternatives) are: A_3, A_5, A_6 ($Eff = 1$), and inefficient DMUs are: A_1, A_2, A_4 ($Eff < 1$). Ranking based on efficiency (Table 2) is as follows: $A_3 \approx A_5 \approx A_6, A_1, A_2, A_4$.

The cross-efficiency, presented in Table 3, is determined with the weights from Table 2. For example, column " A_3 " shows the efficiency of A_3 determined with the weights obtained for the DMU in the first column. Ranking based on cross-efficiency average values (Table 3) is as follows: $A_6, A_5, A_3, A_4, A_1, A_2$.

Table 3: Cross-efficiency

$(u,v) \setminus Eff.$	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0.99820	0.98424	0.82352	0.98175	1.0	0.94780
A_2	0.99821	0.98425	0.82352	0.98176	1.0	0.94781
A_3	0.32763	0.29023	1.0	0.39511	0.84599	1.0
A_4	0.99820	0.98423	0.82351	0.98175	1.0	0.94780
A_5	0.85189	0.77392	0.84229	0.86375	1.0	1.0
A_6	0.55500	0.26066	0.75657	0.56969	0.82317	1.0
Mean	0.78819	0.71292	0.84490	0.79564	0.94486	0.97390

Potential improvements (target and $\Delta f = target - actual$) for inefficient DMUs are determined by *Frontier Analyst* (software) [11], as presented in Table 4. An inefficient DMU could be efficient if the actual values are improved to the target values, by increasing the outputs and decreasing the inputs.

Table 4: Potential improvements

	A_1			A_2			A_4			Total
	Actual	Target	Δf	Actual	Target	Δf	Actual	target	Δf	%
f_1	4184.3	4469.3	+285.0	5211.9	5489.6	+277.7	5566.1	5970.4	+404.3	2.1
f_2	2914.0	2908.8	-5.2	3630.0	3572.8	-57.2	3957.9	3885.7	-72.2	-0.4
f_3	407.2	454.0	+46.8	501.7	557.7	+56	559.5	606.5	+47.0	3.4
f_4	251.0	251.0	0.0	308.3	308.3	0.0	335.3	335.3	0.0	0.0
f_5	195	61	-134	282	75	-207	167	81	-86	-21.0
f_6	244	80	-164	346	98	-248	268	106	-162	-21.7
f_7	15	6	-9	21	8	-13	16	8	-8	-18.6
f_8	2.41	3.57	+1.16	1.41	4.38	+2.97	3.36	4.77	+1.41	32.7

The set of alternatives are extended to $\{A_1, A_2, A_3, A_4, A_5, A_6, A^*, A^-\}$, including the ideal A^* (F^* in VIKOR) and negative-ideal A^- . In this case, the efficient units are A^* and A_3 . Ranking based on efficiency is as follows: $A^*(1.0)$, $A_3(1.0)$, $A_6(0.986)$, $A_5(0.813)$, $A_1(0.752)$, $A_2(0.752)$, $A_4(0.736)$, $A^-(0.554)$. Potential improvements (target and $\Delta f = \text{target} - \text{actual}$) are determined using *Frontier Analyst* and presented in Table 5. The alternative A_3 is efficient, but with reference to the ideal F^* there are potential improvements.

The results presented in Tables 4 and 5 show that greater improvements are needed for inputs: f_2 - costs, f_5 - number of homes to be relocated, f_6 - area flooded by the reservoirs, f_7 - number of villages to displace, and for the output: f_8 - environmental protection. An interesting observation is that greater total potential improvements (Table 4) are for the criteria f_5 , f_6 , f_7 and f_8 representing social interests. In many cases local residents oppose hydropower system from the point of view of these social interests.

Table 5: Potential improvements (reference F^*)

	A_1		A_2		A_3		A_4		A_5		A_6	
	Target	Δf	target	Δf	target	Δf	Target	Δf	target	Δf	target	Δf
f_1	4184.3	0.0	5211.9	0.0	5566.1	544.8	5566.1	0.0	5114.4	54.0	5490.5	1173.
f_2	2190.6	-723.4	2728.6	-901.4	2914.0	-1006.	2914.0	-1044.	2677.5	-616.0	2874.4	-51.5
f_3	420.6	13.4	523.9	22.2	559.5	55.5	559.5	0.0	514.1	0.0	551.9	119.1
f_4	252.1	1.1	314.0	5.7	335.3	56.7	335.3	0.0	308.1	23.9	330.7	91.5
f_5	9.0	-186	11.2	-271	12.0	0	12.0	-155	11.0	-58	11.8	-1
f_6	41.3	-203	51.5	-295	55.0	-1	55.0	-213	50.5	-40	54.2	-1
f_7	2.26	-13	2.81	-18	3.0	0	3.0	-13	2.76	-4	2.96	-1
f_8	3.32	0.91	4.14	2.73	4.42	0.0	4.42	1.06	4.06	0.02	4.36	0.0

This application of DEA indicates the set $\{A_3, A_6, A_5\}$ as good alternatives, selecting them as candidates for the best solution within MCDM. This is DEA's main usefulness for multicriteria decision making.

5.3. Common set of weights

The obtained weights (u, v) by DEA are the most favourable ones from the point of view of each DMU separately, and usually they are different (see Table 2). The weights w for the multicriteria ranking by VIKOR are obtained using the relations introduced in this paper, as follows.

$$w_{1,j} = u_{1,j}(f_1^* - f_1^-)$$

$$w_{2,j} = v_{1,j}(f_2^- - f_2^*)$$

$$w_{i,j} = u_{i-1,j} (f_i^* - f_i^-) , \text{ for } i=3,4 \quad (5)$$

$$w_{i,j} = v_{i-3,j} (f_i^- - f_i^*) , \text{ for } i=5,6,7$$

$$w_{8,j} = u_{4,j} (f_8^* - f_8^-)$$

The obtained values are presented in Table 6, and the mean values can represent a common set of weights. The “norm” values are normalized weights with $\sum w_i = 1$. This procedure above of transforming DEA weights is introduced here to obtain a common set of weights, which are used for multicriteria ranking by VIKOR. The weight values “nonnorm” in Table 6 are determined as follows: $w_3 = 1$ (for minimum u or v different than zero), $w_{i,nonnorm} = [w_i/w_3]_{norm}$, for $i=2,4,6,8$.

Table 6: Weights by DEA for MCDM

	A_1	A_2	A_3	A_4	A_5	A_6	mean	norm.	nonnorm
$w_1(u_1)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$w_2(v_1)$	35.82	28.76	13.93	26.38	31.70	35.68	28.71	0.2632	3.19
$w_3(u_2)$	0.0	0.0	30.22	0.0	23.82	0.0	9.01	0.0826	1.
$w_4(u_3)$	38.18	30.65	0.0	28.11	0.0	0.0	16.16	0.1481	1.79
$w_5(v_2)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$w_6(v_3)$	0.0	0.0	247.7	0.0	0.0	0.0	41.29	0.3784	4.58
$w_7(v_4)$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$w_8(u_4)$	0.0	0.0	0.0	0.0	14.60	69.04	13.94	0.1278	1.56

5.4. VIKOR application

Alternatives $A_1, A_2, A_3, A_4, A_5, A_6$ are ranked using the VIKOR method with the data from Table 1 and six sets of weights values. The obtained results are presented in Table 7. The even criteria weights, unorganized values $W1 = \{w_i = 1, \forall i\}$, represent indifference of the decision maker. The criteria weights $W2 = \{w_i = 2, i = 1, 2, 3, 4 \text{ and } w_i = 1, i = 5, 6, 7, 8\}$ express an economic preference. The weights $W3 = \{w_i = 1, i = 1, 2, 3, 4 \text{ and } w_i = 2, i = 5, 6, 7, 8\}$ express preference for social attributes and environment, and $W4 = \{w_i = 1, i = 1, 2, 3, 4 \text{ and } w_i = 3.2, i = 5, 6, 7, 8\}$ emphasizes more social criteria. The weights obtained from the results by DEA (Table 6) $W5 = \{0.0, 3.19, 1.0, 1.79, 0.0, 4.58, 0.0, 1.56\}$ express the primary preference for a minimum area flooded by reservoirs and minimum costs; and these values are used as a common set to rank alternatives by VIKOR. The weights $W1 - W4$ were proposed in order to analyze the preference stability of the compromise solution. The weights $W6$ are obtained from given new trade-offs in Table 10.

The ranking results in Table 7 indicate alternative A_5 as the best ranked. It has a good advantage for the weight sets $W1, W2$, and $W6$. With the weights $W3, W4$ and $W5$ the compromise sets are obtained $\{A_5, A_3, A_6\}, \{A_3, A_5, A_6\}, \{A_5, A_6\}$, respectively. In

these cases the first ranked alternative has no advantage to be a single solution (see step (v)). If the weights of social criteria are increased, such as W4, the alternative A_3 moves to the first place.

Table 7: Ranking by VIKOR

Weights			A_1	A_2	A_3	A_4	A_5	A_6
W1	Equal $w_i = 1, \forall i$	Q_j	0.989	1.0	0.417	0.628	0.0	0.526
		S_j	0.692	0.7	0.29	0.423	0.28	0.346
		R_j	0.125	0.125	0.121	0.125	0.067	0.125
W2	Economics $w_i = 2, i \leq 4$	Q_j	1.0	0.559	0.505	0.508	0.0	0.646
		S_j	0.701	0.6	0.386	0.365	0.317	0.459
		R_j	0.167	0.114	0.161	0.167	0.089	0.167
W3	Social $w_i = 2, i \geq 5$	Q_j	0.700	1.0	0.129	0.543	0.046	0.175
		S_j	0.683	0.8	0.193	0.48	0.243	0.232
		R_j	0.113	0.167	0.08	0.122	0.044	0.083
W4	"More social" $w_i = 3.2, i \geq 5$	Q_j	0.679	1.0	0.044	0.580	0.066	0.073
		S_j	0.678	0.857	0.138	0.513	0.222	0.167
		R_j	0.129	0.190	0.057	0.139	0.042	0.060
W5	DEA "Common set"	Q_j	0.568	1.0	0.402	0.660	0.040	0.081
		S_j	0.544	0.760	0.372	0.585	0.261	0.222
		R_j	0.245	0.378	0.254	0.277	0.096	0.148
W6	From Table 10	Q_j	0.989	0.962	0.423	0.586	0.0	0.463
		S_j	0.691	0.664	0.331	0.424	0.301	0.383
		R_j	0.152	0.154	0.143	0.149	0.073	0.137

The weight stability intervals in Table 8 (for W1) show the stability of alternative A_5 as the highest ranked for small weight values, although it will loose the first place if some of the criteria is relatively highly preferred. The alternative A_5 is a real compromise. The first position of alternative A_3 is stable with higher values of weights for criteria, f_5, f_6, f_7 , and f_8 ("social" criteria), but only for a small value of w_2 for cost (see results for W4 in Table 8).

Table 8: Weight Stability Intervals [w^L, w^U]

	Weights W1			Weights W4		
	Initial	w^L	w^U	Initial	w^L	w^U
w_1	0.125	0.0	0.177	0.06	0.0	0.399
w_2	0.125	0.1	0.194	0.06	0.0	0.066
w_3	0.125	0.0	0.182	0.06	0.0	0.324
w_4	0.125	0.0	0.159	0.06	0.024	0.293
w_5	0.125	0.0	0.172	0.19	0.0	0.857
w_6	0.125	0.0	0.175	0.19	0.0	0.918
w_7	0.125	0.0	0.172	0.19	0.047	0.752
w_8	0.125	0.0	0.175	0.19	0.0	1.0

The trade-offs values determined by VIKOR are presented in Table 9, showing how many 10^6 Din are evaluated as one unit of k -th criterion, for example, the tr_{25} (for W1) shows that one home (average) is $3.87 \cdot 10^6$ Din, whereas for W4 it is $12.37 \cdot 10^6$ Din.

Table 9: Trade-offs by VIKOR

Weights	$tr_{2k}, k = 1, \dots, n$ (10^6 Din/ \vec{k})							
	$\vec{1}$	$\vec{2}$	$\vec{3}$	$\vec{4}$	$\vec{5}$	$\vec{6}$	$\vec{7}$	$\vec{8}$
W1 $w_i = 1, \forall i$	0.76	1	6.85	10.87	3.87	3.59	57.99	346.8
W4 $w_i = 3.2, i \geq 5$	0.76	1	6.85	10.87	12.37	11.48	185.6	1109.8

The trade-offs values obtained by VIKOR match most economic trade-offs that existed in the region, and only tr_{28} seems too high.

Table 10: New Trade-offs and new weights

	$\vec{1}$	$\vec{2}$	$\vec{3}$	$\vec{4}$	$\vec{5}$	$\vec{6}$	$\vec{7}$	$\vec{8}$
$tr_{2k}, k = 1, \dots, n$	0.66	1	7	10	4	2	60	100
New weights	0.130	0.149	0.152	0.137	0.154	0.083	0.154	0.043
New weights ($w_2 = 1$)	0.87	1	1.02	0.92	1.03	0.56	1.03	0.29

The new trade-offs values were given by the decision maker, as presented in Table 10, and VIKOR determined the new weights. The ranking list by VIKOR is $A_5, A_3, A_6, A_4, A_2, A_1$ and the compromise solution with these new weights is alternative A_5 .

Factor analysis indicates two factors (performing a principal component solution and orthogonal rotation of a factor matrix). Each factor underlies four criteria, the first one for f_5, f_6, f_7 , and f_8 , and the second one for f_1, f_2, f_3 , and f_4 . These two factors could be called the social factor and the economic factor, respectively. Local residents in many cases oppose hydropower systems due to the social factor.

5.5. A comparison of DEA and VIKOR results

The weight sets W1, W2, W3, and W4 (Table 7) were proposed by the decision maker for the Drina project in order to analyze the preference stability of the compromise solution. The weights W5 obtained from DEA results are very different from weights W1, W2, W3, or W4. It was pointed out that the values of weights (u, v) determined by DEA are not related to the decision makers' preference.

The preference of a decision maker (DM) regarding the relative importance of criteria is not included within a DEA application. This preference could strongly affect the selection of an alternative as a final (preferred) solution. Inclusion of DM preference is one of the main differences between DEA and VIKOR.

All six alternatives A_1, \dots, A_6 are noninferior solutions within MCDM. Three of these alternatives, $\{A_3, A_5, A_6\}$, are efficient by DEA, and three are inefficient DMUs, $\{A_1, A_2, A_4\}$. Efficient DMUs $\{A_3, A_5, A_6\}$ are highly ranked by VIKOR, and inefficient DMUs $\{A_1, A_2, A_4\}$ are mainly low ranked. Alternative A_5 (or A_3) is the best ranked by VIKOR (see Table 7).

Alternative A_3 has the best ratios of f_8/f_5 and f_8/f_7 , it is an efficient DMU, and it is the best compromise solution by VIKOR for certain weights. Also, it is efficient in the extended set $\{A_1, A_2, A_3, A_4, A_5, A_6, A^*, A^-\}$, although it is inferior to the ideal alternative A^* .

Introducing ideal A^* ($F^* = (f_1^*, \dots, f_n^*)$ in VIKOR) provides better screening of alternatives by DEA, and differentiates the efficient DMUs $\{A_3, A_5, A_6\}$. Although this result may not be considered as ranking in the MCDM sense, it could be useful information for MCDM.

Alternative A_4 is inefficient by DEA since it has no single best ratio output/input. However, it is the best according to f_1 , f_3 , and f_4 (all outputs), and it is the best as ranked by VIKOR with the weights $W7 = \{3, 1, 3, 3, 1, 1, 1, 2\}$.

5.6. Discussion and Proposed Solution

The results by both methods, DEA and VIKOR, indicate the set $\{A_3, A_5, A_6\}$ as good alternatives. As an alternative for a final solution, alternative A_5 could be considered the best compromise. A comparison of alternatives A_5 and A_3 is presented in Table 11, where d_{ij} denotes a normalized distance of j -th alternative to the ideal F^* according to i -th criterion, $d_{ij} = (f_i^* - f_{ij}) / (f_i^* - f_i^-)$.

Alternative A_5 is closer to the ideal according to the “economic” criteria f_1, f_2, f_3, f_4 . The economic factor underlies these criteria. The alternative A_3 has an additional “defect” in that it is more expensive, although it would be preferred from the social point of view.

It may be concluded that three alternatives $\{A_3, A_5, A_6\}$ indicated as good solutions. The alternatives A_5 and A_6 are similar three-reservoir systems, where two reservoirs are the same. The alternative A_3 is a system of four small reservoirs. The decision makers for the Drina project prefer alternative A_5 , which could be developed in two phases. The first phase develops the system of two reservoirs, and the second phase adds the third reservoir, with a different dam site that could be analyzed later (alternatives A_5 and A_6).

Table 11: Comparison of alternatives A_5 and A_3

Name	Criteria			Comparison				
	Unit	Extreme	A_5	A_3	d_{i5}	d_{i3}	$A_5 \succ A_3 ?$	
f_1 Profit	10 ⁶ Din	Max	5060.5	5021.3	0.366	0.394	$\succ \approx$	
f_2 Costs	10 ⁶ Din	Min	3293.5	3920.5	0.364	0.964	$\succ \succ$	
f_3 Total energy produced	GWh	Max	514.1	504.0	0.298	0.364	\succ	
f_4 Peak energy produced	GWh	Max	284.2	278.6	0.532	0.591	\succ	
f_5 Homes to relocate	num.	Min	69	12	0.211	0.0	\succ	
f_6 Reservoirs area	Ha	Min	90	56	0.120	0.003	\succ	
f_7 Villages to displace	num.	Min	7	3	0.222	0.0	\succ	
f_8 Environmental protect.	Grade	Max	4.04	4.42	0.126	0.0	\succ	

6. CONCLUSIONS

Data Envelopment Analysis (DEA) is an approach to relative efficiency measurement where there are multiple noncommensurable inputs and outputs. DEA provides an efficiency measure that does not rely on the application of common weighting of the inputs and outputs. In contrast, a multicriteria decision making approach is based on the assumption that a common set of weights has to be applied across all alternatives.

By the compromise ranking method, VIKOR, a compromise solution is determined which could be accepted by the decision makers because it provides a maximum “group utility” for the “majority”, and a minimum of individual regret for the “opponent”. The focus of the VIKOR method is that of selecting from a set of alternatives in the presence of conflicting criteria by analyzing the criteria space. The weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model actual aspects of decision making.

The primary focus of DEA model is that of comparing decision-making units (alternatives) from the point of view of their efficiency in converting inputs into outputs. DEA introduces a model for weights determination individually maximizing efficiency of the decision making units. Therefore the DMUs cannot be ranked with these weights that vary from unit to unit. An approach to determine a common set of weights to be used within the VIKOR method is presented in this paper. However, such common set of weights has no relation with the preference of decision maker who is competent to assess the relative importance of the criteria. Thus, DEA could be a preprocess in MCDM, providing screening of alternatives, particularly when the decision maker is not able to express preferences at the beginning of system design or planning.

Finally, DEA resembles MCDM, but it provides different results. In spite of these differences, DEA could be useful in screening alternatives, and identifying efficient units as candidates for the best solution within MCDM. Further research on DEA modifications could bring DEA closer to MCDM.

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