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A GROUP DECISION-MAKING AGGREGATION PROCESS

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Abstract: Within the frame of decision aid literature, decision making problems with multiple sources of information have drawn the attention of researchers from a wide spectrum of disciplines. In decision situations with multiple individuals, each one has his own knowledge of the decision problem alternatives. The use of information assessed in different domains is not a seldom situation. This non-homogeneous information can be represented by values belonging to domains with different nature as linguistic, numerical and interval valued or can be values assessed in label sets with different granularity and multigranular linguistic information.

Decision processes for solving these problems are composed by two steps: aggregation and exploitation. The main problem to deal with non-homogeneous contexts is the aggregation manner of the information assessed in these contexts. The purpose of this paper is to address this problem and establish a procedure to aggregate individual opinions into a common decision to deal with non-homogeneous contexts. This process combines at the same time numerical, interval valued and linguistic information. Since subjectivity, vagueness and imprecision enter into the assessments of experts, the 2-tuple fuzzy linguistic representation model is used to deal with the fuzziness of human judgement.

Keywords: Group decision-making, fuzzy 2-tuples, aggregation process, non-homogeneous information.

1. INTRODUCTION

The decision-making exceeds nowadays the strict framework of the traditional decision maker (DM) who isolates himself to make a decision. Several reasons can be evoked for this fact: economic and competing evolution, hierarchical structure modification, complexity control, improvement of the effectiveness, technological development, etc. Indeed, the decision-making within the organizations requires a several members synergy efforts having different interests, expertises and experiences so that each of them can use its know-how. Moreover, it is due to this synergy that the members can reach higher results than those achieved individually. For these reasons, several researches were undertaken to improve working and performance of the groups in the decision problem's resolution. These works tried first, to structure this process in different stages, usually aggregation step and exploitation step; and in addition, to elaborate a set of tools and methods allowing individual group to progress in decision problem resolution. However, most of these tools and methods do not take into account the uncertainty/inaccuracy surrounding the evaluation of the considered alternatives.

According to authors in [41], the uncertainty sources depends on: variability associated with the same evaluations, the evolution of the assumptions according to the advance of the selected decisions, the strategic choices carried out on the company and the evolution of the external context with the alternatives, etc. Most of models used for the decision-making don't take into account these uncertain measurements. However, all uncertainty types cannot be processed in the same way. The first category of uncertainty, related to the lack of information, can be solved using real values obtained in a predefined range [44]; recently other approaches based on valued intervals [29] and on the linguistic approach [11] were proposed. The major difficulty in the group decision-making problems defined in heterogeneous contexts resides mainly, in the aggregation step, i.e., how to combine these types of information. In fact, there are no standard operators making it possible to combine these different nature information types.

In this paper, to deal with the problem of group decision-making, an aggregation process based on a 2-tuple linguistic representation model is proposed. This model which allows the use of heterogeneous information in the decision problems with several DMs starts by unifying the input data in a common language, named the Basic Linguistic Term Sets (BLTS), and then by expressing the information unified by means of fuzzy sets through the BLTS. In the development of this 2-tuple model, transformation functions and operators must be defined in order to unify multigranulary information and also to transform the fuzzy sets through the linguistic BLTS into 2-tuples. Our proposal is presented in a context where information is expressed by means of numerical values, valued intervals and linguistic values belonging to different linguistic terms sets.

The proposed process in this paper for combining non-homogeneous information is based on a scheme composed by three phases: unification, aggregation and transformation into 2-tuples.

The paper is structured as follows: in the first section, the main features of the 2-tuple model are reviewed; in section 2, the importance of exploitation of the group decision making approaches is presented. In section 3, an aggregation process to combine contexts with different nature information (numerical, linguistic and of valued interval) is proposed. In section 4, a numerical problem defined in a heterogeneous context to deal with linguistic and multigranulary data is solved.

2. THE 2-TUPLE LINGUISTIC REPRESENTATION MODEL

Let $S = \{s_0, ..., s_g\}$ be a linguistic term set with granularity $g+1$. In general, the granularity of *S* should be small enough so as not to impose useless precise levels on users but big enough to allow a discrimination of the assessments in a limited number of degrees [15]. Furthermore, it is supposed that *S* satisfies the following characteristics:

(1) The set is ordered: $s_i \geq s_j$ if $i \geq j$.

(2) There is a negation: Neg $(s_i) = s_{g-i}$.

(3) There is a max operator: max $\{s_i, s_j\}$=s_i if $s_i \geq s_j$.

(4) There is a min operator: min $\{s_i, s_j\} = s_i$ if $s_i \leq s_j$.

One possibility of generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale defined a total order [38]. For example, a linguistic term sets with granularity 7 and 5, denoted as S_1 and S_2, could be given as follows:

A set of seven linguistic terms: S_1
None $(0,0,0.17)$
Very Low $(0,0.17,0.33)$
Low $(0.17,0.33,0.5)$
Medium $(0.33,0.5,0.67)$
High $(0.5,0.67,0.83)$
Very High $(0.67,0.83,1)$
Total $(0.83,1,1)$

A set of five linguistic terms: S_2
None $(0,0,0.25)$
Low $(0,0.25,0.5)$
Medium $(0.25,0.5,0.75)$
High $(0.5,0.75,1)$
Total $(0.75,1,1)$

Theoretically, the discourse universe over which the term set is defined can be random, but usually, linguistic term sets defined in the interval [0,1]. We assume that the semantics of labels are given by fuzzy numbers defined in the [0,1] interval, which are described by triangular membership functions. For example, we may assign the semantics to the set of seven and five terms, which are shown respectively in Figure 1 and 2.

We must point out that the two classical models dealing with linguistic information are

- The semantic model [6] that uses the linguistic terms just as labels for fuzzy numbers, while the computation process acts directly over those fuzzy numbers by means of the Principle of Extension; and
- The second one is the symbolic model, an ordinal scale is assumed on which linguistic assessments are to be done [8].

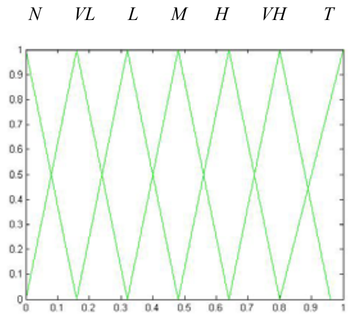

Figure 1. A set of seven linguistic terms

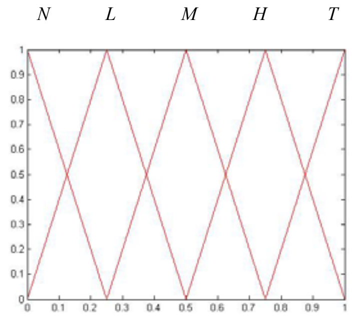

Figure 2. A set of five linguistic terms

In this paper, an extension of the second model is proposed. A new approach, called linguistic 2-tuple representation model [7, 17], where the scale is no longer purely ordinal, but still processing of linguistic information is done directly on labels, is used. It has shown itself as a good choice to manage non- homogeneous information [18, 19]. However, the method proposed does not unify the non-homogeneous information into linguistic labels directly, but into fuzzy sets over a BLTS as we have mentioned. The 2-tuple linguistic model is a kind of new information processing method. It takes 2-tuple to represent linguistic assessment information and to carry out operation.

To develop the above method, we shall define different transformation functions and operators that allow us to unify the non-homogeneous information and also to transform fuzzy sets over the BLTS into linguistic 2-tuples.

Definition 1. ([17]). *Let* $S=\{s_0,\ldots,s_g\}$ *be a linguistic term set and* β *be the result of an aggregation of the indexes of a set of labels assessed in S, i.e., the result of a symbolic aggregation operation* $\beta\in[0,g]$. *Let* $i=round(\beta)$ *and* $\alpha=\beta-i$ *be two values, such that,* $i\in[0,g]$ *and* $\alpha\in[\text{-}0.5,0.5)$ *then* α *is called a Symbolic Translation.*

From Definition 1, we can see that symbolic translation refers to a value that lies in interval [0.5,0.5). It represents the difference between β and the closest term s_i $(i=round(\beta))$ in *S*.

Definition 2. ([17]). *Let* $S=\{s_0,\ldots,s_g\}$ *be a linguistic term set and* $\beta\in[0,g]$ *a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to* β *is obtained with the following function:*

$$\Delta:[0,g]\rightarrow S\times[-0.5,0.5)$$

$$\Delta(\beta)=(s_i,\alpha),with\ \begin{cases} s_i & i=round(\beta) \\ \alpha=\beta-i & \alpha\in[-0.5,0.5) \end{cases}$$

Where, round (.) is the usual operation round, s_i *is the index of the nearest label with* β *and* α *is the value of the symbolic translation.*

Proposition 1. ([17]). *Let* $S = \{s_0, ..., s_g\}$ *be a linguistic term set and* (s_i, α) *is a 2-tuple. There is always a* Δ^{-1} *function, such that, from a 2-tuple it returns its equivalent numerical value* $\beta \in [0, g]$.

Remark. From Definition 2 and Proposition 1, it is obvious that the conversion of a linguistic term s_i into a linguistic 2-tuple by adding a value 0 as symbolic translation, results in $s_i \in S \Rightarrow (s_i, 0)$.
Based on the above definitions, we can easily give the computational models of 2-tuple. These models include comparison of 2-tuple and negation operator of 2-tuple [17].
(1) Comparison of 2-tuples: Let (s_i, α_i) and (s_j, α_j) be two 2-tuples defined in the same linguistic term set:

If $i>j$, then (s_i, α_i) is bigger than (s_j, α_j), i.e. $(s_i, \alpha_i) > (s_j, \alpha_j)$,

If $i = j$, then

- If $\alpha_i > \alpha_j$, then $(s_i, \alpha_i) > (s_j, \alpha_j)$,

- If $\alpha_i = \alpha_j$, then (s_i, α_i) and (s_j, α_j) represent the same value, i.e. $(s_i, \alpha_i) = (s_j, \alpha_j)$.

- If $\alpha_i < \alpha_j$, then (s_i, α_i) is small than (s_j, α_j), i. e. $(s_i, \alpha_i) < (s_j, \alpha_j)$.

(2) Negation operator of 2-tuples: This operator is defined as follows:

$Neg(s_i, \alpha_i) = \Delta(g - (\Delta^{-1}(s_i, \alpha_i)))$ where $s_i \in S = \{s_0, ..., s_g\}$

3. GROUP DECISION MAKING APPROACH

Most of the methodologies in decision analysis which have been discussed in the literature involve a single DM [22, 24, 37, 40]. But many complex issues inevitably concern several people, each with a different viewpoint, and thus an action that appears best from an individual perspective may not be collectively best for the organisation or group as a whole.

It has been suggested that "individual and collective decision making can be approached from the same methodological viewpoint" [47]. However, there are some deep arguments against this perception, like Arrow's Impossibility Theorem, which imply that "there is no procedure for combining individual rankings into a group ranking that does not explicitly address the question of interpersonal comparison of preferences" [24]. Accordingly, acceptable outcomes cannot be secured by mere application of a mathematical method, but through bargaining procedures aimed at a compromise solution. Therefore, systems which support group decision making must be able to handle/reflect the dynamics of a group process; in other words, the modelling framework needs to be descriptively accurate.

Each DM in a group process is characterized by his/her own personal experience, learning, situation, state of mind, and so forth, the DMs preferences may differ substantially, that is, the DMs generally use different representation formats to express their preferences for each pair of alternatives in a group decision-making problem. Herrera et al. [2, 3, 12-14, 16, 18] studied systematically the group decision making problems in which the decision information about alternatives provided by the DMs takes the form of different numerical preference structures (preference orderings [5, 36], utility functions [30, 38], fuzzy preference relations [44, 45], and multiplicative preference relations [35]). But in many real life situations, such as negotiation processes, project investment, and supply chain management, etc. [43], the decision information provided by the DMs may be presented in qualitative aspects. So, it may be difficult to qualify those using precise values. Usually, this knowledge is not precise and presents uncertainty. Early this uncertainty was expressed in the preference values by means of real values assessed in a predefined range [23, 44], soon other approaches based on interval valued [28, 29] and on the linguistic approach [11, 31] were proposed. For these reasons a decision may be made under time pressure and lack of data, the DMs have limited attention and information processing capabilities, and in group settings, all participants do not have equal expertise about problem domain [25, 26, 20, 32, 42].

This non-homogeneous information can be represented as values belonging to domains with different nature as linguistic, numerical and interval valued or can be values assessed in label sets with different granularity, multi-granular linguistic information. Therefore, the use of non-homogeneous information in decision problems with multiple experts is not an unusual situation (see [4, 9, 39] with proposals combining numerical preference representations, fuzzy preference relations, multiplicative preference relations, utility preferences, interval numerical preference, ...). However, most of the proposals for solving decision-making problems with multiple experts [5, 23] are focused on cases where all experts express their preferences by means of values from the same type, either real values, or interval values or linguistic labels in the same linguistic term set.

The process for reaching a solution of the GDM problems is composed of two steps [34]:

1. Aggregation step that combines the expert preferences into a group collective one in such a way that it summarizes or reflects the properties contained in all the individual preferences. In the literature, we can find different aggregation operators to aggregate preferences [10, 46].

2. Exploitation one that transforms the global information about the alternatives into a global ranking of them.

The main difficulty for managing GDM problems defined in non-homogeneous contexts is the aggregation step: how to aggregate this type of information?, because there are no standard operators for combining any type of non-homogeneous information.

An aggregation process for managing non-homogeneous information in GDM problems is proposed in this paper. This process unifies the input information in a unique domain. In this case, a linguistic one called basic linguistic term set (BLTS), expressing the unified information by means of fuzzy sets over the BLTS is used.

4. AGREGATION PROCESS OF HETEROGENEOUS MULTIGRANULARY INFORMATIONS

A process to carry out the aggregation step within the framework of group decision-making process is proposed by using heterogeneous information, composed of numerical values, linguistic terms and valued intervals. The proposed method consists in combining the already mentioned information from a process made up of these three stages:

- Unification of input information,
- Aggregation of individual preferences, and
- 2-tuples transformation.

In the following subsections, each stage composing this aggregation process is detailed.

4.1. Unification of input information

Heterogeneous information must be unified in a unique expression domain. To this effect, the fuzzy sets through the BLTS, noted $F(S_T)$ are proposed for the next computations.

Before transforming the input data into fuzzy sets through the BLTS, the choice of S_T is decided. It requires, then, the choice of $S_T = \{s_0, ..., s_g\}$ as BLTS (Basic Linguistic Term Set). Then, a set of terms is chosen, with a number of terms larger than the number that a person is generally able to discriminate (normally, 11 or 13). So, a BLTS with 15 terms symmetrically distributed as presented in figure 3 is chosen:

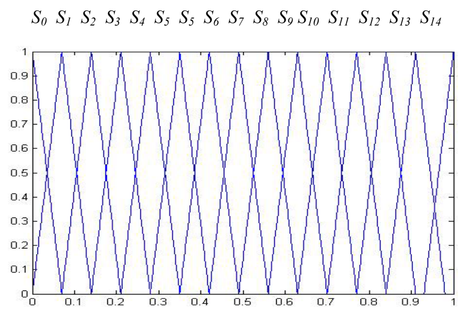

Figure: 3. A BLTS with 15 terms symmetrically distributed

Once the BLTS is selected, the transformation functions used to unify heterogeneous information are defined.

Let $F(S_T)$ be the set of the fuzzy sets in $S_T = \{s_0, ..., s_g\}$, a numerical value $\vartheta \in [0,1]$ is transformed into a fuzzy set in $F(S_T)$ by calculating the membership value of ϑ in the fuzzy number associated with the linguistic terms of S_T.

Definition 3. *[21] The function* τ_{NS_T} *transforms a numerical value into a fuzzy set in* S_T *:*

$$\tau_{NS_T} : [0,1] \rightarrow F(S_T)$$

$$\tau_{NS_T}(\vartheta) = \{(s_0, \gamma_0), ..., (s_g, \gamma_g)\}, \quad s_i \in S_T \ \ et \ \ \gamma_i \in [0,1]$$

$$\gamma_i = \mu_{s_i}(\vartheta) = \begin{cases} 0 & if \ \ \vartheta \notin \text{support } (\mu_{s_i}(x)), \\ \dfrac{\vartheta - a_i}{b_i - a_i} & if \ a_i \le \vartheta \le b_i, \\ 1 & if \ b_i \le \vartheta \le d_i, \\ \dfrac{c_i - \vartheta}{c_i - d_i} & if \ d_i \le \vartheta \le c_i. \end{cases}$$

In definition 3, the authors [21] consider membership functions $\mu_{S_i}(.)$, for the linguistic labels $s_i \in S_T$, represented by a trapezoidal parametric function (a_i, b_i, d_i, c_i). A particular case corresponds to the linguistic evaluations whose membership functions are a triangle: i.e. $b_i = d_i$.

Definition 4. *[21] Let* $S_T = \{l_0, ..., l_p\}$ *and* $S_T = \{s_0, ..., s_g\}$ *two linguistic term sets, such that,* $g \ge p$ *. Then, a linguistic transformation function,* τ_{SS_T} *, is defined as*

$$\tau_{SS_T} : [0,1] \rightarrow F(S_T),$$

$$\tau_{SS_T}(l_i) = P\{(s_k, \gamma_k^i) / k \in \{0, ..., g\}\} \ \forall l_i \in S,$$

$$\gamma_i = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\}.$$

Where, $F(S_T)$ is the set of fuzzy sets defined in S_T, $\mu_{l_i}(.)$ and $\mu_{S_k}(.)$ are related to fuzzy sets membership associated with the terms l_i and s_k respectively.

Therefore, the result of τ_{SS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

Let $I = [\underline{i}, \bar{i}]$ be a valued interval in $[0,1]$. To perform this transformation the valued interval is supposed to have a representation inspired from the fuzzy sets membership functions as follows [27]:

$$\mu_I(\vartheta) = \begin{cases} 0 & if \, \vartheta < \underline{i}, \\ 1 & if \, \underline{i} \le \vartheta \le \bar{i} \\ 0 & if \, \bar{i} < \vartheta \end{cases}$$

Where ϑ is a value in $[0,1]$.

Definition 5. [21] *Let* $S_T = \{l_0,...,l_p\}$ *a BLTS. Then, the function* τ_{IS_T} *that transforms a valued interval I in* $[0,1]$ *into fuzzy set in* S_T*, is defined as follows:*

$$\tau_{IS_T} : I \rightarrow F(S_T),$$

$$\tau_{IS_T}(I) = \{(s_k, \gamma_k^i) / k \in \{0,...,g\}\},$$

$$\gamma_i = \max_y \min\{\mu_{l_i}(y), \mu_{s_k}(y)\}$$

Where, $F(S_T)$ is the set of fuzzy sets defined in S_T, $\mu_{l_i}(.)$ and $\mu_{S_k}(.)$ are the fuzzy sets membership functions associated with the valued interval I and s_k respectively.

Therefore, the result of τ_{IS_T} for any linguistic value of S is a fuzzy set defined in the BLTS, S_T.

Once the input information was transformed into fuzzy set $F(S_T)$ in $S_T = \{s_0,...,s_g\}$, the resulting values of this stage must be aggregated, it is the goal of the next stage.

4.2. Aggregation of individual preferences

By using the already cited transformation functions, the input data are expressed by means of fuzzy sets in the BLTS, $S_T = \{s_0,...,s_{14}\}$, i.e. these data will be homogeneous (information of same nature).

Now, aggregation operators of PROMETHEE multicriteria methods [1] are used to combine the fuzzy sets in the BLTS in order to get a collective preference value for each pair of alternative which will be a fuzzy set in the BLTS.

At this step, the individual preference indexes $\tilde{\pi}^d(\tilde{a}_i, \tilde{a}_k)$ and the collective ones $\tilde{\pi}(\tilde{a}_i, \tilde{a}_k)$ are expressed by means of fuzzy sets in BLTS as follows:

$$\tilde{\pi}_{S_T}^d(a_i, a_k) = \frac{1}{\tau(W^d)} \sum_{j=1}^{n} \tau(w_j^d).\tau(P_j^d(a_i, a_k))$$

With, $d = \{\tilde{d}_1,...,\tilde{d}_D\}$ is a set of D DMs, $P_j^d(a_i, a_k) \in [0,1]$ is a preference function kept by each DM, w_j^d is the individual weight (for each DM *d*) representing the relative importance of fuzzy criterion *j* in the aggregation process $W^d = \sum_{j=1}^{n}(w_j^d)$.

Then, a collective preference index is obtained by aggregating the obtained fuzzy sets by each DM ($\pi^d(a_i, a_k)$). This collective preference value is a new fuzzy set in S_T, i.e., $\tilde{\pi}_{S_T}(a_i, a_k) = \sum_{d=1}^{D} \frac{1}{m}.(\tilde{\pi}_{S_T}^d(a_i, a_k))$, where *m* is the alternatives number.

It is clear that data were unified in fuzzy sets to be processed in the aggregation step. However, in the decision-making aid process, in particular through the exploitation step, the collective preferences will be compared to obtain the best alternative. To facilitate this comparison, these collective fuzzy sets must be transformed into linguistic 2-tuples.

4.3. Transformation of linguistic information into 2-tuples

A χ function suggested in [18] is used to transform a fuzzy set into a numerical value in the interval of granularity of S_T, $[0,\mathrm{g}]$, as follows:

$$\chi : F(S_T) \to [0,\mathrm{g}],$$

$$\chi(F(S_T)) = \chi\left(\{(s_i, \alpha_i), i = 0,...,g\}\right) = \frac{\sum_{i=0}^{g} i\alpha_i}{\sum_{i=0}^{g} \alpha_i} = \beta$$

Where, the fuzzy set $F(S_T)$, can be obtained from τ_{NS_T}, τ_{IS_T}, τ_{SS_T}.

Therefore, by applying the Δ function to β (definition 2) a collective preference relation is obtained which values are expressed by means of linguistic 2-tuples: $\Delta(\chi(\tau(\vartheta))) = \Delta(\beta) = (s_i, \alpha)$.

So, the 2-tuple collective preference index is the next: $\tilde{\pi}(\tilde{a}_i, \tilde{a}_k) =$ $= \Delta(\chi(\tilde{\pi}_{S_T}(a_i, a_k))) = \Delta(\beta_{i,k})$.

5. GROUP DECISION MAKING PROBLEM WITH HETEROGENEOUS INFORMATIONS

Let an organization that has a sum of money to invest. There are three possible options to invest this sum s_1, s_2 and s_3. In this organization, all the decisions are taken from the opinions provided by three departments DMs, R_1, R_2 and R_3: marketing department, risk analysis department and growth analysis department, on the basis of four criteria: net present value $\tilde{F}_1$, risk index $\tilde{F}_2$, environmental impact $\tilde{F}_3$ and social development $\tilde{F}_4$. Since each responsible belongs to a different knowledge field some can have more facility to express their opinions with numbers, while others prefer to express their opinions by means of linguistic expression, and even of the valued intervals.

The first responsible expresses his preferences by means of linguistic values in a linguistics terms set, *S*. The second expresses his preference relation using the numerical values in $[0,1]$. The third responsible can express his preferences in a valued interval $[0,1]$. For the fuzzy criteria $\tilde{F}_1$, $\tilde{F}_2$ and $\tilde{F}_3$ the linguistic variables given in Figure 1 are used, and for the criterion $\tilde{F}_4$ the linguistic variables of Figure 2.

The evaluations of each of the three responsible are given in the following table (Table 1):

Table1. Input data of each responsible

	R_1				R_2				R_3			
	$\tilde{F}_1$	$\tilde{F}_2$	$\tilde{F}_3$	$\tilde{F}_4$	$\tilde{F}_1$	$\tilde{F}_2$	$\tilde{F}_3$	$\tilde{F}_4$	$\tilde{F}_1$	$\tilde{F}_2$	$\tilde{F}_3$	$\tilde{F}_4$
a_1	*H*	*L*	*M*	*H*	0.51	0.12	0.2	0.6	[0.55,0.7]	[0.05,0.15]	[0.4,0.6]	[0.6,0.8]
a_2	*M*	*VL*	*M*	*L*	0.2	0.02	0.31	0.152	[0.2,0.4]	[0,0.06]	[0.4,0.6]	[0.05,0.15]
a_3	*VH*	*M*	*H*	*H*	0.76	0.31	0.68	0.77	[0.75,0.9]	[0.4,0.6]	[0.55,0.7]	[0.6,0.8]
w_j	*VH*	*H*	*VH*	*M*	0.31	0.2	0.37	0.12	[0.75,0.9]	[0.2,0.3]	[0.35,0.6]	[0.25,0.35]
Type	2	2	3	3	2	2	3	3	2	2	3	3
p_j			*L*	*VL*			0.1	0.05			[0.1,0.15]	[0.01,0.06]
q_j	*H*	*VL*			0.51	0.12			[0.55,0.7]	[0.16,0.22]		

5.1. Aggregation phase

a. Choice of the BLTS. $S_T = \{s_0, ..., s_{14}\}$ is retained.

b. Transformation of input information into $F(S_T)$. Applying the transformation functions given in definitions 3-5, the following fuzzy sets from the selected BLTS can be obtained:

Fuzzy sets of the DM R_1 :

$$\tau_{SS_T}(a_{1,1}) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, (S_7, 0.21), (S_8, 0.5), (S_9, 0.785), (S_{10}, 0.8), (S_{11}, 0.626), (S_{12}, 0.33), S_{13}, S_{14}\}$$

$$\tau_{SS_T}(a_{2,1}) = \{S_0, S_1, S_2, S_3, (S_4, 0.041), (S_5, 0.333), (S_6, 0.624), (S_7, 1), (S_8, 0.793), (S_9, 0.501), (S_{10}, 0.209), S_{11}, S_{12}, S_{13}, S_{14}\}$$

$$\tau_{SS_T}(a_{3,1}) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, (S_9, 0.082), (S_{10}, 0.374), (S_{11}, 0.665), (S_{12}, 0.826), (S_{13}, 0.751), S_{14}\}$$

$$\tau_{SS_T}(\tilde{w}_1^1) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, (S_9, 0.082), (S_{10}, 0.374), (S_{11}, 0.665), (S_{12}, 0.826), (S_{13}, 0.751), S_{14}\}$$

Fuzzy sets of the DM R_2 :

$$\tau_{NS_T}(a_{1,1}) = \tau_{NS_T}(0.51) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, (S_7, 0.875), (S_8, 0.125), S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$

$$\tau_{NS_T}(a_{2,1}) = \tau_{NS_T}(0.2) = \{S_0, S_1, (S_2, 0.143), (S_3, 0.857), S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$

$$\tau_{NS_T}(a_{3,1}) = \tau_{NS_T}(0.75) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, (S_{10}, 0.428), (S_{11}, 0.572), S_{12}, S_{13}, S_{14}\}$$

$$\tau_{NS_T}(w_1^2) = \tau_{NS_T}(0.31) = \{S_0, S_1, S_2, S_3, (S_4, 0.571), (S_5, 0.429), S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$

Fuzzy sets of the DM R_3 :

$$\tau_{IS_T}(a_{1,1}) = \tau_{IS_T}([0.55, 0.7]) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, (S_7, 0.375), (S_8, 1), (S_9, 1), (S_{10}, 0.714), S_{11}, S_{12}, S_{13}, S_{14}\}$$

$$\tau_{IS_T}(a_{2,1}) = \tau_{IS_T}([0.2, 0.4]) = \{S_0, S_1, (S_2, 0.143), (S_3, 1), (S_4, 1), (S_5, 1), (S_6, 0.714), S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$

$$\tau_{IS_T}(a_{3,1}) = \tau_{IS_T}([0.75, 0.9]) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, (S_{10}, 0.571), (S_{11}, 1), (S_{12}, 1), (S_{13}, 0.571), S_{14}\}$$

$$\tau_{IS_T}(w_1^3) = \tau_{IS_T}([0.4, 0.55]) = \{S_0, S_1, S_2, S_3, S_4, (S_5, 0.286), (S_6, 1), (S_7, 1), (S_8, 0.625), S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$

When all information are expressed by means of fuzzy sets defined in the BLTS, an aggregation operator is used to combine them. In this group decision-making problem, the individual and collective preference indexes defined in 3.2 are used.

In what follows, the presented results are limited to those obtained for the total preference index:

$$\tilde{\pi}(\tilde{a}_1,\tilde{a}_2) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, (S_7, 0.21), (S_8, 0.5), (S_9, 0.875),$$
$$(S_{10}, 0.125), S_{11}, S_{12}, S_{13}, S_{14}\}$$
$$\tilde{\pi}(\tilde{a}_1,\tilde{a}_3) = \{S_0, S_1, (S_2, 0.166), (S_3, 0.458), (S_4, 0.749), (S_5, 0.959),$$
$$(S_6, 0.667), S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$
$$\tilde{\pi}(\tilde{a}_2,\tilde{a}_1) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, (S_7, 0.187), (S_8, 0.406), (S_9, 0.625),$$
$$(S_{10}, 0.844), (S_{11}, 0.937), (S_{12}, 0.719), (S_{13}, 0.5), S_{14}\}$$
$$\tilde{\pi}(\tilde{a}_2,\tilde{a}_3) = \{S_0, S_1, S_2, S_3, (S_4, 0.041), (S_5, 0.333), (S_6, 0.624),$$
$$(S_7, 1), (S_8, 0.793), (S_9, 0.501), (S_{10}, 0.209), S_{11}, S_{12}, S_{13}, S_{14}\}$$
$$\tilde{\pi}(\tilde{a}_3,\tilde{a}_1) = \{(S_0, 0.291), (S_1, 0.583), (S_2, 0.875), (S_3, 0.834), (S_4, 0.542),$$
$$(S_5, 0.25), S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}\}$$
$$\tilde{\pi}(\tilde{a}_3,\tilde{a}_2) = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, (S_9, 0.082), (S_{10}, 0.374),$$
$$(S_{11}, 0.665), (S_{12}, 0.826), (S_{13}, 0.751), S_{14}\}$$

Now, the fuzzy sets are transformed by expressing the collective preferences into linguistic 2-tuples using the χ and Δ functions. The result of this transformation is given in Table 2.

Table 2: Collective preference indexes

Table 2: Collective preference indexes $\tilde{\pi}$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{a}_3$
$\tilde{a}_1$	-	$(S_{10}, -0.35)$	$(S_4, 0.45)$
$\tilde{a}_2$	$(S_{10}, 0.44)$	-	$(S_7, 0.29)$
$\tilde{a}_3$	$(S_2, 0.45)$	$(S_{12}, 0.03)$	-

5.2. Exploitation step

The exploitation step generates a set of alternatives (the best) for the decision problem. To this effect, this phase uses a choice function to get the alternatives set. Various choice functions were proposed in the literature of choice theory [33].

In this paper, to get the alternatives set, a function that calculates the *dominance degree* for each alternative $\tilde{a}_i$, in relation to the other alternatives is used. This function is expressed by: $\lambda(\tilde{a}_i) = \frac{1}{m-1} \sum_{i=0, i \neq j}^{n} \beta_{ik}$.

Where, *m* is the alternatives number and $\beta_{ik} = \Delta^{-1}(\tilde{\pi}(\tilde{a}_i, \tilde{a}_k))$ such as $\tilde{\pi}(\tilde{a}_i, \tilde{a}_k)$ a linguistic 2-tuple representing the collective value that expresses the preference of the alternative $\tilde{a}_i$ in relation to alternative $\tilde{a}_k$ according to DMs group. Then, a set of alternatives with highest value of dominance degree are used.

In this step, the dominance degree for these collective preference indexes is calculated (see Table 3).

Table 3: Alternatives dominance degree

a_1	a_2	a_3
$(S_7, 0.7)$	$(S_9, -0.14)$	$(S_7, 0.24)$

a_2 is the best alternative solution of the group decision making problem according to the dominance degree.

6. CONCLUSION

An aggregation process is presented to deal with heterogeneous data, in contexts composed by numerical values, valued intervals and linguistic terms, for group decision-making problems. This aggregation process is based on the information unification by means of fuzzy sets in a set of linguistic terms which were transformed into linguistic 2-tuples to facilitate the decision model exploitation step. Also, it is shown that the aggregation process can easily be extended to deal with data in a heterogeneous context in which linguistic multigranulary information appears.

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