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OPTIMAL PRICING AND LOT-SIZING DECISIONS UNDER WEIBULL DISTRIBUTION DETERIORATION AND TRADE CREDIT POLICY

S. K. MANNA

Department of Management Science, National Chiao Tung University
Hsinchu, Taiwan, R. O. C.
skmanna_5@hotmail.com

K. S. CHAUDHURI

Department of Mathematics, Jadavpur University, Kolkata 700 032, India

C. CHIANG

Department of Management Science, National Chiao Tung University
Hsinchu, Taiwan, R. O. C.

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Abstract: In this paper, we consider the problem of simultaneous determination of retail price and lot-size (RPLS) under the assumption that the supplier offers a fixed credit period to the retailer. It is assumed that the item in stock deteriorates over time at a rate that follows a two-parameter Weibull distribution and that the price-dependent demand is represented by a constant-price-elasticity function of retail price. The RPLS decision model is developed and solved analytically. Results are illustrated with the help of a base example. Computational results show that the supplier earns more profits when the credit period is greater than the replenishment cycle length. Sensitivity analysis of the solution to changes in the value of input parameters of the base example is also discussed.

Keywords: Retail price, lot-size, inventory management.

1. INTRODUCTION

Inventories of physical goods play a major role in the economy of any country because a huge amount of money is tied up in inventories. Proper management of inventories can be very worthwhile and profitable. Studies on inventory management started with the EOQ (economic order quantity) formula which is derived based on the assumption that the retailer (buyer) has to pay fully for the items as soon as he receives them from a supplier. However, as a standard practice in the real markets, a supplier usually allows a certain fixed period (known as the credit period) for settling the amount of money that the retailer owes to him for the items supplied. This TC (trade credit) policy plays an important role in the business of many products and it serves the interests of both the supplier and retailer. The supplier usually expects the profit to increase since rising sales volumes compensate the capital losses incurred during the credit period. Also, the supplier finds an efective means of price discrimination which circumvents anti-trust measures. On the other hand, the retailer earns an interest by investing the sale proceeds earned during the credit period. In addition, a TC policy develops a relationship of mutual trust between the supplier and retailer. The benefit of TC policies is more pronounced when the demand of a product depends on its retail price. Under such a marketing situation, the retailer is able to choose the selling price from a wider range of options existing in the market by utilizing the credit period ofered by the supplier. Therefore, TC policies are a common and realistic industrial policy usually followed by wholesalers. For these reasons, inventory modellers felt the need to take TC policies into consideration.

Because of the marketing significance of the TC policy, inventory modellers like Haley and Higgins (1973), Ben-Horim and Levy (1982), Kingsman (1983), Chapman *et al.* (1984), Goyal (1985), Ward and Chapman (1987), Chung (1989), Aggarwal and Jaggi (1994), etc. devoted their attention to it. One of the results of their studies is that the EOQ is independent of the length of the credit period. This result is unexpected, perhaps due to their assumption of constant demand. As for the RPLS problem, previous studies include Kunreuther and Richard (1971), Kunreuther and Schrage (1973), Ladany and Sternlieb (1974) and Shah and Jha (1991) that assume the demand to be a decreasing function of retail price. Abad (1988) and Lee (1993) studied the RPLS problem under an additional assumption that the demand is a decreasing function of price and that the supplier ofers all-unit quantity discounts. Abad (1988a) also considered the case of incremental quantity discounts. Kim *et al.* (1995) discussed the RPLS problem under the TC policy assuming the retailer's borrowing and lending rates of capital to be equal. If the buyer's borrowing and lending rate of capital are equal, then the credit period has no efect on the EOQ (Chapman *et al.* (1984), Goyal (1985), Haley and Higgins (1973)). Also, this type of assumption seems quite restrictive from the practical point of view. Recent TC related papers include Teng (2002), Huang (2003) and You (2005), etc.

All the research works mentioned above assume that inventory is depleted by consumer's demand only. This assumption is valid only for non-deteriorating items. There are numerous inventory items whose utility does not remain constant through out their life cycle. The type of products subject to on-going deterioration can be broken down into those with a maximum usable lifetime (perishable products) and those without (decaying products). Blood, medicines and certain foods are examples of perishable products with a maximum usable lifetime. Volatile liquids such as alcohol and gasoline are products which decay without a maximum lifetime. For this type of products, inventory is depleted not only by demand but also by deterioration. Ghare and Schrader (1963) derived a revised form of the economic order quantity model assuming exponential deterioration of inventory and constant demand. Cohen (1977) considered the RPLS problem for an exponentially deteriorating items. Hariga (1995) examined the inventory lot-sizing problem for decaying products. Hwang and Shinn (1997) dealt with the joint price and lot-size determination model for an exponentially deteriorating products under the condition of permissible delay in payments. Jamal *et al.* (2000) and Sarker *et al.* (2001) addressed the optimal payment time under permissible delay in payment for decaying items. Recently, Quyang *et al.* (2005) established an EOQ model for deteriorating products, in which the supplier provides not only a cash discount but also a delay in payments to customers. Barrotoni (1962) observed, while discussing the difficulties of fitting empirical data to mathematical distributions, that both leakage failure of dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distribution. Thus deterioration of physical goods modeled by a two-parameter Weibull distribution is much more realistic and generalized than the constant deterioration rate considered by the above researchers.

This paper discusses the RPLS problem for a two-parameter Weibull distribution deteriorating items as in Covert and Philip (1973). But unlike Covert and Philip's model, we assume the price-dependent demand represented by a constant price-elasticity function of retail price. In addition, this paper incorporates TC policies into our model. We relax the assumption of equal interest rates (as in Kim *et al.* (1995)) and allow the retailer's borrowing rate to be greater than or equal to his lending rate.

The rest of this paper is organized as follows. The model is first developed and solved analytically. Next, results are illustrated with the help of a base example. Sensitivity analysis of the solution to changes in the value of input parameters of the base example is also discussed.

2. ASSUMPTIONS AND NOTATIONS

We adopt the following assumptions for the model to be discussed:

- Replenishments are instantaneous with a known and constant lead time.
- The inventory system involves only one item.
- Shortages are not allowed.
- Annual demand rate is represented by a constant price elasticity function of retail price.
- The distribution of the time to deterioration follows a two parameter Weibull distribution.

Notations used in this paper are as follows:

C unit purchase cost.
S ordering cost.
H inventory carrying cost, excluding the capital opportunity cost.
R capital opportunity cost (as a percentage).
I earned interest rate (as a percentage).
t_c credit period set by the supplier.
Q lot size.
T replenishment cycle time.
P unit retail price for the product.

K scaling constant (> 0).

β_1 the constant price elasticity (> 0).

In addition, we make the following assumptions and notations:

(i) The annual demand rate $D(P) = KP^{-\beta_1}$, is a constant price elasticity function. Now, $\frac{dD(P)}{dP} = -K\beta_1 P^{-(\beta_1+1)} < 0$ implies demand is a downward sloping function of price. This is a common result in the real market.

(ii) The inventory deteriorating rate $\theta(t) = \alpha\beta t^{\beta-1}$ where $\alpha(>0)$ and $\beta(>0)$ are respectively scale and shape parameters. Deterioration rate is increasing with t for $\beta > 1$ and decreasing for $\beta < 1$. The case of constant deterioration first considered by Ghare and Schrader (1963) is also a special case when $\beta = 1$. It is clear that if the initial deterioration rate is extremely high, then the two-parameter Weibull distribution is appropriate for an item with decreasing rate of deterioration. This distribution is applicable to increasing rate of deterioration if the initial deterioration rate is approximately zero.

(iii) During the credit period proposed by supplier, sales revenue is deposited in an interest bearing account with rate I. At the end of the period, the credit is settled and the retailer starts paying the capital opportunity cost for items in stock with rate $R(R \geq I)$.

3. FORMULATION OF THE MODEL

We shall consider a continuous-review, deterministic-demand model with two-parameter Weibull distribution deterioration. Let $q(t)$ be the inventory position of the system at any time t. Demand rate $D(P)$ is assumed to be positive. Under continuous review it is logical to assume that depletion due to deterioration and depletion due to meeting demand will occur simultaneously. Accordingly, the differential equation describing the time behavior of the inventory system is

$$\frac{dq(t)}{dt} + \alpha\beta t^{\beta-1} q(t) = -D(P), \quad 0 \leq t \leq T \tag{1}$$

with the conditions $q(0) = Q, q(T) = 0$.

Solution of equation (1) is

$$q(t) = \exp(-\alpha t^{\beta})\left[q(0) - D(P)\sum_{n=0}^{\infty}\frac{\alpha^n t^{n\beta+1}}{(n\beta+1)n!}\right], \quad 0 \leq t \leq T. \tag{2}$$

Consequently, $Z(t)$, the inventory loss due to deterioration in the time interval $[0,t]$ is given by

$$Z(t) = q(0)(1-\exp(-\alpha t^{\beta})) + D(P)\exp(-\alpha t^{\beta})\sum_{n=0}^{\infty}\frac{\alpha^n t^{n\beta+1}}{(n\beta+1)n!} - D(P)t$$

Therefore, the quantity ordered per cycle is

$$Q = Z(T) + D(P)T$$
$$= q(0) - \exp(-\alpha T^{\beta})\left[q(0) - D(P)\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{(n\beta+1)n!} \right] \tag{3}$$

Using the conditions $q(0) = Q$ and $q(T) = 0$, we have

$$Q = D(P)\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{(n\beta+1)n!} \tag{4}$$

Using (4), we have from (2)

$$q(t) = D(P)\exp(-\alpha t^{\beta})\left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{(n\beta+1)n!} - \sum_{n=0}^{\infty} \frac{\alpha^n t^{n\beta+1}}{(n\beta+1)n!} \right],\ 0 \le t \le T. \tag{5}$$

The retailer's objective is to maximize the annual net profit $\Pi(P,T)$ from the sales of the products. The inventory cost $C^T(T,P)$ in the first cycle consisting of four components can be expressed as

$C^T(T,P)$ = Ordering cost + Purchasing cost + Inventory carrying cost
+ Capital opportunity cost.

We have the annual ordering cost $= \dfrac{S}{T}$.

The annual purchasing cost

$$= \frac{CQ}{T}$$
$$= CD(P)\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta}}{(n\beta+1)n!}.$$

The annual inventory carrying cost

$$= \frac{H}{T}\int_0^T q(t)$$
$$= \frac{HD(P)}{T}\int_0^T \left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{(n\beta+1)n!} - \sum_{n=0}^{\infty} \frac{\alpha^n t^{n\beta+1}}{(n\beta+1)n!} \right] \exp(-\alpha t^{\beta})dt$$
$$= \frac{HD(P)}{T}\int_0^T \left[\sum_{n=0}^{\infty} \frac{\alpha^n T^{n\beta+1}}{(n\beta+1)n!} - \sum_{n=0}^{\infty} \frac{\alpha^n t^{n\beta+1}}{(n\beta+1)n!} \right] \sum_{m=0}^{\infty} \frac{(-1)^m \alpha^m t^{m\beta}}{m!} dt$$
$$= \frac{HD(P)}{T}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^m \alpha^{n+m} T^{(n+m)\beta+2}}{(n\beta+1)(m\beta+1)n!m!} - \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^m \alpha^{n+m} T^{(n+m)\beta+2}}{(n\beta+1)((n+m)\beta+2)n!m!} \right]$$
$$= \frac{HD(P)}{T}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^m \alpha^{n+m} T^{(n+m)\beta+2}}{(m\beta+1)((n+m)\beta+2)n!m!} \right].$$

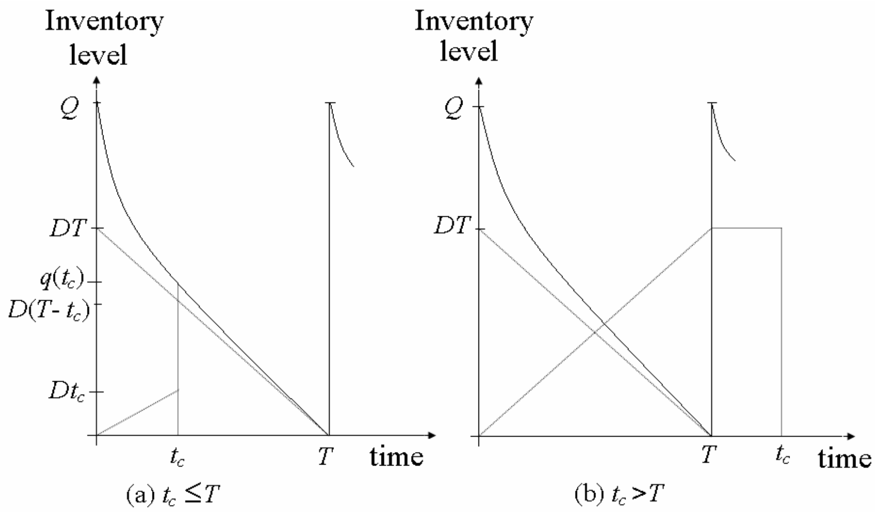

Figure 1: Credit period vs. replenishment cycle time

Annual capital opportunity cost:

(i) Case 1 $(t_c \le T)$:(see Fig. 1(a)) During the credit period t_c, the sales revenue is used to earn interest with annual rate I. Average number of products in stock earning interest during time $(0, t_c)$ is $\frac{D(P)t_c}{2}$ and the interest earned per order becomes $\frac{D(P)t_c^2 CI}{2}$. After settling the credit, the product still in stock is assumed to be financed with annual rate R. Interest payable per order can be expressed as $CR\int_{t_c}^{T} q(t)dt.$ Therefore, the annual capital opportunity cost

$$
=\frac{1}{T}\left[CR\int_{t_c}^{T} q(t)dt-\frac{1}{2}CID(P)t_c^2\right]
$$

$$
=\frac{D(P)}{T}\left[CR\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{n\beta+1}(T^{m\beta+1}-t_c^{m\beta+1})}{(n\beta+1)(m\beta+1)n!m!}\right.
$$

$$
\left.-\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}(T^{(n+m)\beta+2}-t_c^{(n+m)\beta+2})}{(n\beta+1)((n+m)\beta+2)n!m!}-\frac{1}{2}CIt_c^2\ \right]
$$

(ii) Case 2 $(t_c > T):(see\,Fig.\ \ 1(b))$ In the case $t_c > T$, all the sales revenue is used to earn interest with annual rate I during the credit period t_c. The average number of products in stock earning interest during time $(0,T)$ and (T,t_c) become $\frac{D(P)T}{2}$ and $D(T)T$ respectively. Therefore, the annual capital opportunity cost

$$=-\frac{1}{T}(\frac{1}{2}D(P)TTCI+D(P)T(t_c-T)CI)$$
$$=\frac{1}{2}CID(P)T-CID(P)t_c$$

The annual inventory cost $C(T,P)=\frac{C^T(T,P)}{T}$

The annual net profit

$$\Pi(P,T)=\text{Annual sales revenue - Annual inventory cost}$$
$$=PD(P)-C(T,P)\,. \tag{6}$$

Let $\Pi_1(P,T)$ and $\Pi_2(P,T)$ be the annual net profits when $t_c \le T$ and $t_c > T$ respectively.

Using (6), we have

$$\Pi_1(P,T)=KP^{-\beta_1}\left[P-C\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{(n\beta+1)n!}\right]$$

$$-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{(n+m)\beta+1}}{(m\beta+1)((n+m)\beta+2)n!m!}\right]$$

$$-CR\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{n\beta}(T^{m\beta+1}-t_c^{m\beta+1})}{(n\beta+1)(m\beta+1)n!m!}$$

$$+\frac{1}{T}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}\left[T^{(n+m)\beta+2}-t_c^{(n+m)\beta+2}\right]}{(n\beta+1)((n+m)\beta+2)n!m!}+\frac{1}{2}(CIt_c^2)\,]\,\right]-\frac{S}{T}, \tag{7}$$

$$\Pi_2(P,T)=KP^{-\beta_1}\left[P-C\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{(n\beta+1)n!}\right.$$

$$\left.-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{(n+m)\beta+1}}{(m\beta+1)((n+m)\beta+2)n!m!}\right]-\frac{1}{2}CIT+CIt_c\,\right]-\frac{S}{T}. \tag{8}$$

The necessary conditions to maximize $\Pi_1(P,T)$ and $\Pi_2(P,T)$ are respectively given by the equations

$$\frac{\partial \Pi_1(P,T)}{\partial P} = 0 \text{ and } \frac{\partial \Pi_1(P,T)}{\partial T} = 0\,, \tag{9}$$

$$\frac{\partial \Pi_2(P,T)}{\partial P} = 0 \text{ and } \frac{\partial \Pi_2(P,T)}{\partial T} = 0. \tag{10}$$

Equation (9) yields the results

$$\begin{aligned}
&1-\frac{\beta_1}{P}\Big[\, P - C\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{(n\beta+1)n!} \\
&-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{(n+m)\beta+1}}{(m\beta+1)((n+m)\beta+2)n!m!}\right] \\
&-CR\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{n\beta}(T^{m\beta+1}-t_c^{m\beta+1})}{(n\beta+1)(m\beta+1)n!m!} \\
&+\frac{1}{T}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}(T^{(n+m)\beta+2}-t_c^{(n+m)\beta+2})}{(n\beta+1)((n+m)\beta+2)n!m!}+\frac{1}{2}CIt_c^2\right]\Big] \\
&=0\,.
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
&KP^{-\beta_1}\Bigg[-C\sum_{n=0}^{\infty}\frac{n\beta\alpha^n T^{n\beta-1}}{(n\beta+1)n!} \\
&-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}((n+m)\beta+1)T^{(n+m)\beta}}{(m\beta+1)((n+m)\beta+2)n!m!}\right] \\
&-CR\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}\left[((n+m)\beta+1)T^{(n+m)\beta}-(n\beta)T^{n\beta-1}t_c^{m\beta+1}\right]}{(n\beta+1)(m\beta+1)n!m!} \\
&+\frac{1}{T}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{(n+m)\beta+1}}{(n\beta+1)n!m!}\right] \\
&-\frac{1}{T^2}\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}(T^{(n+m)\beta+2}-t_c^{(n+m)\beta+2})}{(n\beta+1)((n+m)\beta+2)n!m!}\right. \\
&\left.+\frac{1}{2}CIt_c^2\,\right]\Bigg]+\frac{1}{T^2}S=0.
\end{aligned} \tag{12}$$

Similarly equation (10) gives

$$1-\frac{\beta_1}{P}\left[P-C\sum_{n=0}^{\infty}\frac{\alpha^n T^{n\beta}}{(n\beta+1)n!}\right.$$
$$\left.-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}T^{(n+m)\beta+1}}{(m\beta+1)((n+m)\beta+2)n!m!}\right]-\frac{1}{2}CIT+CIt_c\right]=0 \tag{13}$$

And

$$KP^{-\beta_1}\left[-C\sum_{n=0}^{\infty}\frac{n\beta\alpha^n T^{n\beta-1}}{(n\beta+1)n!}\right.$$
$$\left.-H\left[\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^m\alpha^{n+m}((n+m)\beta+1)T^{(n+m)\beta}}{(m\beta+1)((n+m)\beta+2)n!m!}\right]-\frac{1}{2}CI\right]+\frac{S}{T^2}=0. \tag{14}$$

The solutions of the equations (11) and (12) give the optimal values $P=P^*$ and $T=T^*$ which maximize $\Pi_1(P,T)$, provided they satisfy the sufficient conditions

$$\frac{\partial^2\Pi_1}{\partial P^2}\frac{\partial^2\Pi_1}{\partial T^2}-\left(\frac{\partial^2\Pi_1}{\partial P\partial T}\right)^2>0,\ \frac{\partial^2\Pi_1}{\partial P^2}<0 \text{ and } \frac{\partial^2\Pi_1}{\partial T^2}<0. \tag{15}$$

Also for $\Pi_2(P,T)$, optimal solution P^*, T^* of the equations (13) and (14) must satisfy the sufficient conditions

$$\frac{\partial^2\Pi_2}{\partial P^2}\frac{\partial^2\Pi_2}{\partial T^2}-\left(\frac{\partial^2\Pi_2}{\partial P\partial T}\right)^2>0,\ \frac{\partial^2\Pi_2}{\partial P^2}<0 \text{ and } \frac{\partial^2\Pi_2}{\partial T^2}<0. \tag{16}$$

4. COMPUTATIONAL RESULTS

Profit functions for the two cases are the functions of P and T. Our objective is to determine P and T which maximize the profit functions. Using Second-Order Newton-Raphson method in programming language Fortran-77, we solve the highly non-linear equations (11) and (12) to obtain P and T for given input parameters. Optimal profit and order size for case 1 are calculated from expressions (7) and (4) respectively. Using the same programming language, we solve equations (13) and (14) for P and T. Then optimal profit and order size for case 2 are determined from expressions (8) and (4) respectively.

To illustrate, consider the base example: $S=90, K=60000, C=5, H=0.5, R=0.14,$ $I=0.1, \beta=1.5, \beta_1=2, t_c=0.4$ and $\alpha=0.2$ in appropriate units. The optimal solution for case 1 is P^* = 10.34, X^* = 0.47, and corresponding optimal profit $\Pi_1^*(P^*,T^*)=$ 2709.54 and optimal order size $Q^*=271.99$ For $t_c=0.8$ in base example, the optimal solution for case 2 is $P^*=9.92, T^*=0.46, \Pi_2^*(P^*,T^*)=2828.24$ and $Q^*=289.56$.

Comparing the results of the two numerical examples, we find that for a cent percent increase in t_c, the profit increases by 4.38%, order quantity increases by 6.45%, price decreases by 4.04% and cycle time decreases by 1.69%.

The sensitivity of each of the decision variables P, T, Π_1, Π_2 and Q to changes in each of the parameters $S, K, C, H, R, I, \beta_1, t_c$ and α are examined in Tables 1-2. The sensitivity analysis is performed by changing each of the parameters by -25%, -10%, 10% and 25%, taking one parameter at a time and keeping the remaining parameters unchanged. It is difficult to exhibit the sensitivity results analytically in respect of even one parameter because of the complicated nature of the profit functions Π_1 and Π_2. From Tables 1-2, we see that the optimal profit is insensitive to changes in R and β_1. The optimal profit is slightly sensitive to changes in the values of the parameters S, H, I, β, t_c and α in both the cases $t_c \le T$ and $t_c > T$. It is quite sensitive to changes in the parameter K. It is also sensitive to larger changes in C. The optimal order quantity is almost insensitive to changes in the parameters H, I, R, β_1 and slightly sensitive to changes in β, t_c and α. It has low sensitivity to S and moderate sensitivity to K. It is quite sensitive to larger changes in C.

Table 1: Sensitivity analysis for case $1(t_c \le T)$

changing parameters	(%) change	P^*	T^*	Π_1^*	Q^*	(%) change in Π_1^*	(%) change in Q_1^*
S	+25	10.43	0.51	2658.70	294.35	-1.87	+8.22
	+10	10.38	0.49	2688.15	281.51	-0.79	+3.49
	-10	10.30	0.45	2731.89	261.88	+0.82	-3.71
	-25	10.24	0.42	2766.15	246.01	+2.08	-9.55
K	+25	10.26	0.43	3444.16	313.84	+27.11	+15.38
	+10	10.30	0.45	3002.80	289.11	+10.82	+6.29
	-10	10.38	0.49	2417.24	254.28	-10.78	-6.51
	-25	10.46	0.53	1980.99	226.34	-26.88	-16.78
C	+25	12.44	0.50	2233.15	202.88	-17.58	-25.41
	+10	10.34	0.47	2709.54	271.99	0.00	0.00
	-10	10.34	0.47	2709.54	271.99	0.00	0.00
	-25	8.25	0.43	3424.50	388.65	+26.38	+42.88
H	+25	10.40	0.47	2692.52	269.61	-0.62	-0.87
	+10	10.36	0.47	2702.72	271.04	-0.25	-0.35
	-10	10.31	0.47	2716.38	272.96	+0.25	+0.35
	-25	10.28	0.47	2726.68	274.42	+0.63	+0.89
R	+25	10.33	0.46	2709.91	268.86	+0.01	-1.15
	+10	10.33	0.46	2709.70	270.68	0.00	-0.48
	-10	10.34	0.47	2709.38	273.41	0.00	+0.52
	-25	10.35	0.47	2709.11	275.75	-0.01	+1.38
I	+25	10.27	0.45	2723.71	267.12	+0.52	-1.79
	+10	10.31	0.46	2715.13	270.09	+0.20	-0.70
	-10	10.37	0.47	2704.06	273.85	-0.20	+0.68
	-25	10.41	0.48	2696.01	276.53	-0.50	+1.66

Table 1 (continued): Sensitivity analysis for case $1(t_c \le T)$

changing parameters	(%) change	P^*	T^*	Π_1^*	Q^*	(%) change in Π_1^*	(%) change in Q_1^*
β	+25	10.28	0.49	2733.59	284.04	+0.88	+4.42
	+10	10.31	0.47	2720.55	276.89	+0.40	+1.79
	-10	10.38	0.46	2696.11	267.10	-0.49	-1.79
	-25	10.46	0.45	2669.87	260.20	-1.46	-4.33
β_1	+25	10.34	0.47	2709.54	271.99	0.00	0.00
	+10	10.34	0.47	2709.54	271.99	0.00	0.00
	-10	10.34	0.47	2709.54	271.99	0.00	0.00
	-25	10.34	0.47	2709.54	271.99	0.00	0.00
t_c	+25	10.25	0.47	2737.40	281.05	+1.02	+3.32
	+10	10.30	0.47	2720.93	275.51	+0.42	+1.29
	-10	10.38	0.46	2697.81	268.63	-0.43	-1.23
	-25	10.44	0.46	2679.57	263.86	-1.10	-2.98
α	+25	10.35	0.44	2694.67	257.49	-0.54	-5.33
	+10	10.34	0.46	2703.50	265.83	-0.22	-2.26
	-10	10.33	0.48	2715.71	278.72	+0.22	+2.47
	-25	10.33	0.50	2725.21	290.07	+0.57	+6.64

Table 2: Sensitivity analysis for case $2(t_c > T)$

changing parameters	(%) change	P^*	T^*	Π_2^*	Q^*	(%) change in Π_2^*	(%) change in Q^*
S	+25	10.01	0.512	2775.81	315.31	-1.85	+8.89
	+10	9.96	0.48	2806.12	300.56	-0.78	+3.79
	-10	9.88	0.44	2851.44	277.79	+0.82	-4.06
	-25	9.82	0.40	2887.25	259.14	+2.08	-10.50
K	+25	9.84	0.41	3594.88	331.39	+27.10	+14.44
	+10	9.88	0.44	3134.21	306.78	+10.81	+5.94
	-10	9.96	0.48	2523.35	271.56	-10.78	-6.21
	-25	10.04	0.52	2068.49	242.85	-26.86	-16.12
C	+25	11.94	0.50	2331.68	217.29	-17.55	-24.95
	+10	9.92	0.46	2828.24	289.56	0.00	0.00
	-10	9.92	0.46	2828.24	289.56	0.00	0.00
	-25	7.91	0.42	3574.06	410.66	+26.37	+41.82
H	+25	9.98	0.46	2810.08	286.94	-0.64	-0.90
	+10	9.95	0.46	2820.96	288.51	-0.25	-0.36
	-10	9.90	0.46	2835.54	290.61	+0.25	+0.36
	-25	9.86	0.46	2846.52	292.20	+0.64	+0.91
R	+25	9.92	0.46	2828.24	289.56	0.00	0.00
	+10	9.92	0.46	2828.24	289.56	0.00	0.00
	-10	9.92	0.46	2828.24	289.56	0.00	0.00
	-25	9.92	0.46	2828.24	289.56	0.00	0.00
I	+25	9.73	0.43	2876.58	284.65	+1.70	-1.69
	+10	9.85	0.45	2847.27	287.49	+0.67	-0.71
	-10	10.00	0.47	2809.60	291.76	-0.65	+0.76
	-25	10.11	0.48	2782.36	295.33	-1.62	+1.99

Table 2 (continued): Sensitivity analysis for case $2(t_c > T)$

changing parameters	(%) change	P^*	T^*	Π_2^*	Q^*	(%) change in Π_2^*	(%) change in Q^*
β	+25	9.87	0.48	2853.53	304.49	+0.89	+5.15
	+10	9.89	0.47	2839.84	295.64	+0.41	+2.10
	-10	9.96	0.45	2814.04	283.42	-0.50	-2.11
	-25	10.03	0.44	2786.18	274.45	-1.48	-5.21
β_1	+25	9.92	0.00	2828.00	289.56	0.00	0.00
	+10	9.92	0.46	2828.24	289.56	0.00	0.00
	-10	9.92	0.46	2828.24	289.56	0.00	0.00
	-25	9.92	0.46	2828.24	289.56	0.00	0.00
t_c	+25	9.71	0.45	2891.81	296.75	+2.24	+2.48
	+10	9.83	0.46	2853.33	292.39	+0.88	+0.97
	-10	10.01	0.46	2803.58	286.78	-0.87	-0.95
	-25	10.14	0.47	2767.39	282.72	-2.15	-2.36
α	+25	9.93	0.43	2813.14	272.30	-0.53	-5.96
	+10	9.92	0.45	2822.10	282.19	-0.21	-2.54
	-10	9.92	0.47	2834.50	297.67	+0.22	+2.80
	-25	9.92	0.50	2844.14	311.58	+0.56	+7.60

5. CONCLUSIONS

This paper deals with the RPLS problem under a TC policy when the item in stock deteriorates over time. The deterioration rate is time-dependent and follows a two-parameter Weibull distribution. This type of deterioration, first considered by Covert and Philip (1973), is more realistic than the exponential decay in the RPLS models of Cohen (1977). In the present paper, introduction of the two-parameter Weibull distribution deterioration creates much mathematical and computational difficulties. The expression of the profit function involves double-summation infinite series whose convergence is critically dependent on the choice of the values of the parameters α and β, and $0 < \alpha < 1, \beta > 1$ are appropriate choices. $\beta > 1$ represents an increasing rate of deterioration when the initial rate is approximately zero. For $0 < \alpha < 1, \beta > 1,$ the contribution of the infinite series becomes negligible after a few terms. Computational results show that supplier earns more profits when the credit period is greater than the replenishment cycle length.

The proposed model can be extended in several ways. For instance, a three-parameter (instead of two-parameter) Weibull distribution can be assumed. Also, demand can be a function of both selling price and time. Finally, quantity discounts and inflation may be incorporated into the model.

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