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THE DISTRIBUTION OF TIME FOR CLARK'S FLOW AND RISK ASSESSMENT FOR THE ACTIVITIES OF PERT NETWORK STRUCTURE

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Abstract: This paper presents the ways of quantification of flow time qualifications that can be used for planning or other stochastic processes by employing Clark's methods, central limit theorem and Monte Carlo simulation. The results of theoretical researches on superponed flow time quantification for complex activities and events flow in PERT network for project management are also presented. By extending Clark's research we have made a generalization of flow models for parallel and ordinal activities and events and specifically for their critical and subcritical paths. This can prevent planning errors and decrease the project realization risk. The software solution is based on Clark's equations and Monte Carlo simulation. The numerical experiment is conducted using Mathcad Professional.

Keywords: Simulation, mathematical model, risk assessment.

1. INTRODUCTION

Figure 1 represents the basic submodel of network diagram for activities and events for which Clark's equation of resulting activity is defined. It consists of an oriented graph with two parallel activities with a common start (r) and a terminal event

(*k*). Complex network with parallel-ordinal activities like this one, for the sake of further analysis must be divided into sub networks with only ordinal or parallel activities.

Parallel activities may be independent (locally autonomous) until their realization (e.g. in *k* moment). However, there can exist dependencies between them, in case of which we have Clark's equations [1]. In this paper we have compared the results of Clark's equations with Monte Carlo numerical simulation. The basic model of Figure 1 has a key role in network planning, and not only that. Both methods, the analytical and the numerical one, have got enough power to study and solve different processes based on graph models, no matter if they are activity flows or the flows of resources, energy, mass queueing, fluids, the reliability of technical systems etc. According to [1] those problems are stochastic and using analytical methods for their solving without some level of approximation is often impossible. Inspired by researches [1], [2] and [7] we defined and solved one PERT (Program Evaluation and Review Technique) network model as an addition to the algorithm development for solving the general model of critical time flows based on the ordinal-parallel structures of oriented graph.

Unique solution for critical activity flow and the resulting time flow based on the expected times for specific activities represents one of the most problematic effects of the network planning implementation according to stochastic methods. Both the stochastic and the deterministic activities networks based on ADM (Arrow Diagram Method) structures can be very complex. On the basis of the critical flow analysis, a specific problem represent parallel critical flows, which are either dependent (correlation $corr_{1,2} > 0$) or independent (correlation $corr_{1,2} = 0$).

Common elements for these two parallel flows are: the start and the end event and the same, nearly the same, or very different time intervals of their realization. In that case we will take *k*-event, the moment when the both activities have been done. In case of more parallel flows we will take the longest one

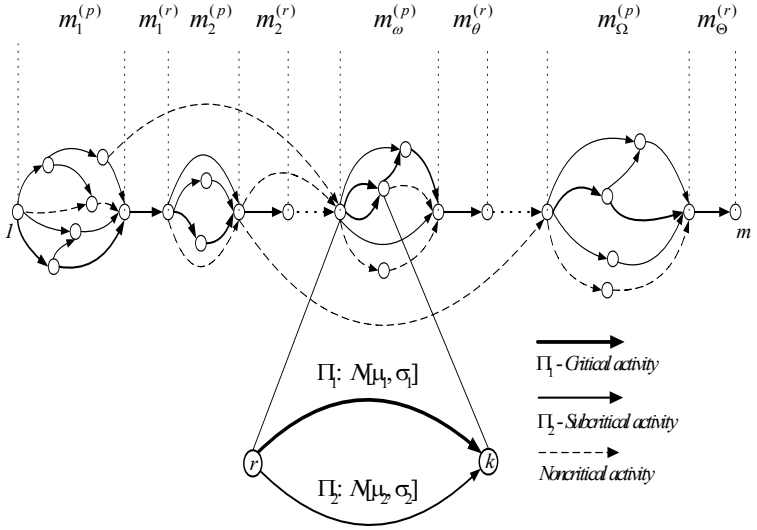

Figure 1.1: Sub network with two sub critical activities

Sub network with pure ordinal activities is much simpler to solve, because in such a case we can use the results of Central Limit Theorem (CLT) [3]. If we use them for solving the PERT critical flows we will not get a precisely defined problem and a solution.

We cannot define these with the classical PERT technique, because the realization time of critical flows in parallel sub networks $\left(m_{\omega}^{(p)}\right)$ is superponed, and that phenomenon is underestimated by using the standard PERT methodology. In the end we have a question: what is the probability that the resulting flow time *T* will be in the planned *Tp* period, taking into account that this kind of activity graph can contain only one, two or more parallel and ordinal flows.

2. THE AIM OF THE PAPER

In order to give the right answers to the previous questions, it is necessary to define the exact criterion and algorithm for the influence quantification, first of all, the influence of the critical and sub critical flows on forming of the results, e.g. superponed activity flow time. Noncritical flows are excluded from the analysis due to the fact that the theoretical side has little influence on superponed time value.

The basic aim of this paper is the impact quantification of critical and subcritical flows on the formation of the results, that is, the superponed flow time. Complementary goal relates to the criteria development for equivalency defining ($\equiv$) of those parallel flows.

Together with its solving, the fundamental base for the defining of the probability distribution function has been created here, and the relativity of those flows has been noticed by applying the frames method.

Therefore, it is assumed that individual activities are independent and normally distributed, with the characteristics of mean value and the standard deviation of their realization.

3. SOLVING PROBLEM METHODS

According to the researches [1], [2] the superponing intervals of the critical and subcritical flow time and their deviations as well as their reducing onto an equivalent flow can be deduced by:

- Analytical methods: with Clark's equations for the parallel flows solving, on the basis of the central limit theorem, for ordinal flows solving and
- Numerical method: Monte Carlo – frame simulation for the parallel-ordinal flows. To illustrate the application of the above-mentioned fundamental algorithms, we shall take the AON – network with two parallel flows Π_1 and Π_2 (Figure 3.1).

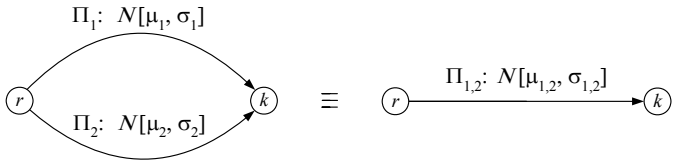

Figure 3.1: Sub network with two sub critical activities

4. THE DEFINING OF THE BASIC TIME PARAMETERS FOR THE AUTONOMOUS CRITICAL FLOWS

4.1. The Superponed Time and the Flow Variant

In the algorithm structuring for the analytical solving of this critical flows variant, we shall start from Clark's authentic equations. With these equations the flows parameters have been solved as follows: the superponed flow time t_{12} and its variants σ^2 (t_{12}).

For the basic oriented graph with two parallel flows, from the initial (r) to the terminal (k) event (Figure 3.1), the following solutions are known [1]:

- The mean superponed flow time represented with Clark's equation

$$\bar{t}_{1,2} = \dot{t}_1 \cdot \Phi(\xi_{1,2}) + \dot{t}_2 \cdot \Phi(-\xi_{1,2}) + \lambda_{1,2} \cdot \Psi(\xi_{1,2}) \tag{4.1.1}$$

Where: $\Phi(\xi) = (2\pi)^{-\frac{1}{2}} \cdot \int_{-\infty}^{\xi} \exp(-\frac{1}{2}z^2)dz$ –represents Laplace integral, $\Psi(\xi) = (2\pi)^{-\frac{1}{2}} \cdot \exp(-\frac{1}{2}z^2)$ – the density function of the centered normal distribution and $\lambda_{1,2} = \sqrt{\sigma^2(t_1) + \sigma^2(t_2)}$, that is $\xi_{1,2} = \frac{1}{\lambda_{1,2}} \cdot (\bar{t}_1 - \bar{t}_2)$ – the parameter of Clark's functions. In addition to this, the predicted or mean values of time intervals $\overline{t_1} = \mu_1$, $\overline{t_2} = \mu_2$ and standard deviation: $\sigma_1 = \sigma(t_1)$ and are usually taken $\sigma_2 = \sigma(t_2)$, thus we have that

- The mean superponed flow time is

$$\mu_{1,2} = \mu_1 \cdot \Phi(\xi_{1,2}) + \mu_2 \cdot \Phi(-\xi_{1,2}) + \lambda_{1,2} \cdot \Psi(\xi_{1,2}) \tag{4.1.2}$$

- The superponed dispersion is presented by the second Clark's equation

$$\sigma^2_{1,2} = (\mu_1^2 + \sigma^2(P_1)) \cdot \Phi(\xi_{1,2}) + (\mu_2^2 + \sigma^2(P_2)) \cdot \Phi(-\xi_{1,2}) + (\mu_1 + \mu_2) \cdot \lambda_{1,2} \cdot \varphi(\xi_{1,2}) - \mu^2_{1,2} \tag{4.1.3}$$

With these equations we can describe the characteristics of one equivalent flow instead of the previous two (Figure 3.1), which is the key moment in forming and solving a more complex model.

4.2. The Growth of the Superponed Flow Time in Relation to the Critical Flow

On the basis of the formulae 4.2 and 4.3, the new superponed of the time distribution t_{12} with the characteristics $\mathcal{N} \sim [m_{12}, s_{12}]$ has been defined. Depending on the fact which single time μ_1 or μ_2 has the critical feature $\mu_{1,2}$, the growth of the expected time can be quantified. For the elementary network with the autonomous flows Π_1 and Π_2, that growth or "the superponed extract", after more straightforward deriving results in:

$$\Delta\mu_{1,2} = \lambda_{1,2} \cdot \varphi(\xi_{1,2}) + (\mu_2 - \mu_1) \cdot \Phi(-\xi_{1,2}) \tag{4.2.1}$$

Meanwhile, in case of reversed choice, it follows

$$\Delta\mu_{2,1} = \lambda_{2,1} \cdot \varphi(\xi_{2,1}) + (\mu_1 - \mu_2) \cdot \Phi(-\xi_{2,1}) \tag{4.2.2}$$

In addition, these values are always nonnegative, i.e.: $\Delta\mu_{1,2} \geq 0$ and $\Delta\mu_{2,1} \geq 0$. With classical PERT they are taken as a zero value.

4.3. The Invariability Testing of the Flow Model

By invariability testing in both cases, 4.2.1 and 4.2.2, we should prove that those values remain unchanged and uniquely determined during the schedule altering of flows in the calculation process.

It is already known that with only two flows, Π_1 and Π_2, and with two parameters (μ_1, σ_1) and (μ_2, σ_2), we can have nine relations. In other words, analyzing the next possible relations between the expected time and the deviations of single flows, as in

$$\mu_1 \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \mu_2 \text{ and } \sigma_1 \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \sigma_2 \tag{4.3.1}$$

We can form only nine different combinations Table 4.3.1.

$\mu_1 \rho \mu_2$	<	<	<	=	=	=	>	>	>
$\sigma_1 \rho \sigma_2$	<	=	>	<	=	>	<	=	>

Table 4.3.1: The combinations

Where $\rho = \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\}$.

The probability theory recognizes the following invariants, and they can be easily proved:

$$\lambda_{1,2} = \lambda_{2,1}\,,\ \xi_{1,2} = -\xi_{2,1}\ \ \varphi(-\xi_{1,2}) = \varphi(\xi_{2,1}) \text{ and } \Phi(-\xi_{1,2}) = \Phi(\xi_{2,1}) \tag{4.3.2}$$

We get the invariant relations of the basic tested values that are connected to the superponed flow, i.e.

$$\mu_{1,2} = \mu_{2,1};\ \ \Delta\mu_{1,2} = \Delta\mu_{2,1} \text{ and } \sigma^2_{1,2} = \sigma^2_{2,1} \tag{4.3.3}$$

We can conclude that it is irrelevant which of the two observed flows we shall proclaim critical, and which one subcritical. This characteristic of the models invariability [4] is essential for the criteria equivalency development for the $w > 2$ flows case.

4.4. Parallel flows defining by the criteria for equivalency

As for network (sub) models with parallel and independent flows where $w \geq 2$, there must be some criteria defined to be implemented in the analytical solving of the superponed problem. We have the following variations:

Case $w = 2$: The equivalency condition of the two parallel flows can be described on the basis of two parameters and three relations. (Figure 4.4.1).

$$(\mu_1 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \mu_2 \wedge \sigma_1 = \sigma_2) \vee (\mu_1 = \mu_2 \wedge \sigma_1 \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \sigma_2) \Rightarrow (\sigma_{1,2} \equiv \sigma_{2,1}) \wedge (\mu_{1,2} \equiv \mu_{2,1}) \tag{4.4.1}$$

This criterion is based on the relations 4.3.2 about the two flows invariability Π_1 and Π_2.

Case $w = 3$: Neither of the three parallel flows is equivalent in the following cases (Figure 4.4.1):

$$\begin{aligned} &[(\mu_1 \neq \mu_2 \neq \mu_3) \wedge (\sigma_1 \begin{Bmatrix} = \\ \neq \end{Bmatrix} \sigma_2 \begin{Bmatrix} = \\ \neq \end{Bmatrix} \sigma_3)] \vee \\ &[(\mu_1 \begin{Bmatrix} = \\ \neq \end{Bmatrix} \mu_2 \begin{Bmatrix} = \\ \neq \end{Bmatrix} \mu_3) \wedge (\sigma_1 \neq \sigma_2 \neq \sigma_3)] \Rightarrow \\ &(\sigma_{1,2} \neq \sigma_{1,3} \neq \sigma_{2,3}) \wedge (\mu_{1,2} \neq \mu_{1,3} \neq \mu_{2,3}) \end{aligned} \tag{4.4.2}$$

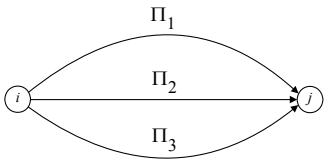

Figure 4.4.1: Sub network of three parallel flows with similar characteristics

Table of relations operators for the three parallel flows with all the combinations of expected values μ_ν $(\nu = 1,2,3)$ and the coresponding standard deviations σ_ν is given in Table 4.4.1. The number of these combinations at $w = 2$ is nine. With the three flows it is 81. In general case, the number of combinations (u) is exponential and it is:

$$u = 3^{w+1} \quad (w \geq 2,\ w \in N) \tag{4.4.3}$$

Equivalency w - parallel flows has been fulfilled only in the following case:

$$\begin{aligned} &\left[\forall\, \mu_\nu = \mu_{\nu+1}, \nu = \overline{1, w-1}\right] \wedge \left[\forall\, \sigma_\nu = \sigma_{\nu+1}\right] \Rightarrow \\ &(\mu_{1,2\ldots w} \equiv \mu_{2,1\ldots w} \equiv \ldots \equiv \mu_{w,w-1\ldots 1}) \wedge \\ &(\sigma_{1,2\ldots w} \equiv \sigma_{2,1\ldots w} \equiv \ldots \equiv \sigma_{w,w-1\ldots 1}) \end{aligned} \tag{4.4.4}$$

In that sense (4.4.4) these flows can be considered as one equivalent flow with:

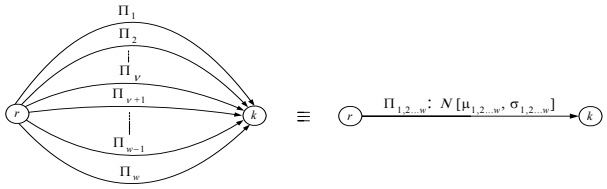

Figure 4.4.2: Sub network with w-parallel flows and their equivalent flow (on the right)

$\mu_1 \rho \mu_2 \rho \mu_3$	$\sigma_1 \rho \sigma_2 \rho \sigma_3$	$\mu_1 \rho \mu_2 \rho \mu_3$	$\sigma_1 \rho \sigma_2 \rho \sigma_3$	$\mu_1 \rho \mu_2 \rho \mu_3$	$\sigma_1 \rho \sigma_2 \rho \sigma_3$
< <	< <	= <	< <	> <	< <
< <	< =	= <	< =	> <	< =
< <	< >	= <	< >	> <	< >
< <	= <	= <	= <	> <	= <
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Table 4.4.1: Sub Table of relations operators for three parallel flows

According to the prior criteria, equivalency condition for three flows (Figure 4.4.1) fulfills only one case (bolded in table 4.4.1). This superponed model of parallel flows can be further integrated with ordinal flows model. Other variations must be simultaneously solved.

5. THE APPLICATION OF THE SIMULATION MODELS

Since the elementary activities of the flow time have the normal distribution with the parameters $N\left[\mu_\nu, \sigma_\nu\right]$, $(\nu = 1,2)$, simulation algorithm is simple and it does not require special criteria, apart from logical dependency as follows:

Two parallel flows algorithm:

$$T_{1,2} = if\,(T_1 > T_2, T_1, T_2) \quad \Rightarrow \; N[m_{1,2},\; s_{1,2}] \tag{5.1}$$

Three parallel flows algorithm:

$$T_{1,2,3} = if[T_1 > T_2 \wedge T_1 > T_3, T_1, if(T_2 > T_3, T_2, T_3)] \Rightarrow N[m = m_{1,2,3}, s = s_{1,2,3}] \quad (5.2)$$

Or *Mathcad* notation [5]:

$$t123_i : if(t1_i > t2_i \wedge t1_i > t3_i, t1_i, if(t2_i > t3_i, t2_i, t3_i))$$

$t1^T =$

	0	1	2	3	4	5	6	7	8	9
0	9.379	10.615	9.515	9.198	9.153	9.461	10.891	8.76	7.882	9.591

$t2^T =$

	0	1	2	3	4	5	6	7	8	9
0	10.611	11.317	10.654	9.857	9.91	10.731	10.351	8.912	9.417	10.029

$t3^T =$

	0	1	2	3	4	5	6	7	8	9
0	10.364	9.737	9.591	9.727	8.628	11.603	9.405	10.226	10.308	9.661

$t123^T =$

	0	1	2	3	4	5	6	7	8	9
0	10.611	11.317	10.654	9.857	9.91	11.603	10.891	10.226	10.308	10.029

Figure 5.1: Mathcad presentation of the t1, t2, t3 and t123 vectors

In the vectors t1, t2, t3 and t123 only the first 10 values are visible, as shown in Figure 5.1.

Example: For the two flows, Figure 4.4.1 represents the results of numerical simulations of $n = 10^5$ replications for the chosen characteristics of normal distribution: $N[\mu_1 = 10, \sigma_1 = 1]$, $N[\mu_2 = 10, \sigma_2 = 2]$ and the final simulations result, in the form of the mean value $m_{1,2}$ and the standard deviation $s_{1,2}$, which are needed for superponed distribution $N[m_{1,2}, s_{1,2}]$.

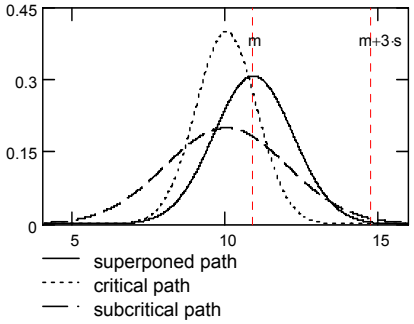

Figure 5.1: Probability distribution of critical, subcritical and superponed flow time

Here are also obtained:

1. theoretical values (Clark's equations):N~[μ_{12}=103,678066; σ_{12}=5,382548],
2. simulation values (Monte Carlo): N ~ [m_{12}=103,703373; s_{12}=5,35445] and
3. differences between theoretic and simulation values: $\Delta\mu_{12}$= 5,069x10^{-3} and $\Delta\sigma_{12}$ =2,809x10^{-2}

Meanwhile, as this algorithm is computer fixed, the main point of the problem is now within the purview of simulation. Namely, in this case, in one simulation session with n=10^5 replications, the testing of only one chosen variant was done, with μ_1=μ_2 and σ_1< σ_2 of the possible nine ones.

5.1. The Application of the Monte Carlo - Frame Methods in Solving the Clark's Flow Model

The extension of the Monte Carlo method is possible to do by using frames (4.3.3). With computer frames and by means of changing the fixed parameter, through the vector values (μ_1, σ_1) and (μ_2, σ_2) in the set Monte Carlo algorithm the convenience for understanding and visualization of a broader class of appearances is formed, more than it was in the previous stereotype – static view on the process and the simulation results. In that sense the supposition (4.3.2) can be solved up to three variants in one simulation session:

$$\mu_1 \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \mu_2 \text{ and } \sigma_1 < \sigma_2 \tag{5.1.1}$$

The frame number depends on the problem complexity, i.e. the studied process. One should not neglect the esthetic moment of the frames presentation, so the integrated Monte Carlo method-frame has also a significant educational role. In this case, the frames have been spontaneously connected for the simulation process, and so the Monte Carlo simulation for "a new dimension" has been broadened, which can be partially presented by a series of selected frames (Figures 5.1.1 - 3).

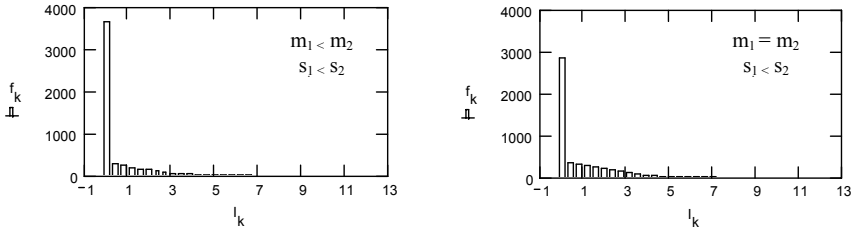

Fig. 5.1.1: The frames for the values:

a) $[\mu_1 = 9, \sigma_1 = 1]$ and $[\mu_2 = 10, \sigma_2 = 2]$ *b)* $[\mu_1 = 10, \sigma_1 = 1]$ and $[\mu_2 = 10, \sigma_2 = 2]$

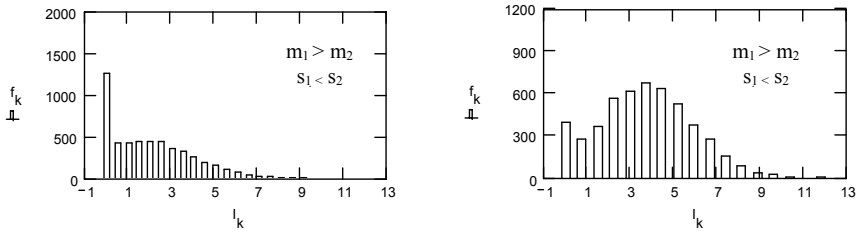

Figure 5.1.2: The frames for the values:

c) $[\mu_1 = 12, \sigma_1 = 1]$ and $[\mu_2 = 10, \sigma_2 = 2]$ *d)* $[\mu_1 = 14, \sigma_1 = 1]$ and $[\mu_2 = 10, \sigma_2 = 2]$

Figure 5.1.3: The frames for the values*: e)* $[\mu_1 = 16, \sigma_1 = 1]$ and $[\mu_2 = 10, \sigma_2 = 2]$

The most important advantage of the Monte Carlo simulation method is that the solving of the flow problem represents an opportunity for modeling distribution function of superponed flow time of basic network model given in Figure 1.1. However, the advantages of the Monte Carlo are significantly increased due to the dynamic network flow modeling. Frames provide a reliable basis for knowledge expansion, especially in terms of the critical flow relativity. Apart from that, the domination of the critical flow is significantly lower in favor of the subcritical one, in the sense that the mean value of the subcritical flow is increased in relation to the critical one (from frame to frame). Thus, it is possible to explore other cases by analytical and/or numerical methods, i.e.

$$\mu_\nu \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \mu_{\nu+1} \text{ and } \sigma_\nu \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} \sigma_{\nu+1} \text{ for } \nu = \overline{1, w-1} \tag{5.1.3}$$

These influences with the simulations at complex ADM networks can be easily noted [2], [4]. Complex flow calculations, with developed criteria, result in interesting values. The consequences of the lack of knowledge about the essence of the given results can be negative, especially in planing and controling of the complex stochastic activities flows. Clark's equations for three or more parallel flows have not been developed. If we had them we would get complex results. However, the existing equations for two parallel flows can be simplified in order to solve more complex flow cases $w \geq 3$, with the criteria (4.4.1), (4.4.2) and (4.4.4). Then they can be used as recurent ones, i.e. with form given for the last iteration:

w – iteration: superponing of flows $\Pi_{12\ldots w-1}$ and Π_w into the $\Pi_{12\ldots w-1,w}$.

Superponed flow time $\overline{t_{12\ldots w-1,w}}$ is:

- $$\overline{t_{12\ldots w-1,w}} = \overline{t_{12\ldots w-1}} \cdot \Phi(\xi_{12\ldots w-1,w}) + \overline{t_w} \cdot \Phi(-\xi_{12\ldots w-1,w}) + \lambda_{12\ldots w-1,w} \cdot \varphi(\xi_{12\ldots w-1,w}) \quad (5.1.4)$$
- Superponed disperzion $\sigma^2_{12\ldots w-1,w}$ is:

$$\sigma^2_{12\ldots w-1,w} = (\overline{t_{12\ldots w-1}}^2 + \sigma^2_{12\ldots w-1}) \cdot \Phi(\xi_{12\ldots w-1,w}) + (\overline{t_w}^2 + \sigma^2_w) \cdot \Phi(-\xi_{12\ldots w-1,w}) + \\ + (\overline{t_{12\ldots w-1}} + \overline{P_w}) \cdot \lambda_{12\ldots w-1,w} \cdot \Phi(\xi_{12\ldots w-1,w}) - \overline{t_{12\ldots w-1,w}}^2 \quad (5.1.5)$$

With the increasing number of flows we have an exponentially increased number of expected combinations time/deviation. I.e. for $w = 2, 3, 4, 5$ and 6 this array of numbers is $u = 9,\ 27,\ 81,\ 243,\ 729$ and 2187 etc.

6. THE CONCLUSION

Using the Clark's, CLT and Monte Carlo methods, flow times can be clarified while planning, or doing other stochastic processes which employ standard network planning methods, such as PERT. Standard PERT for flow time planning is based on the expected values of elementary flow times. By using the ordinal critical flows only, we make a sigificant mistake in planning, because we dimish the influence of the subcritical flows, and therefore the superponed phenomena. The previous example shows that the project risk is $\alpha \approx 13,5\,\%$. The value of the standard procedure is $\beta \approx 7,8\,\%$. The difference is obvious and this planning mistake can be very negative, especially in cases of project management in stochastic activities flows, that exist in traffic, civil engineering, machinery, shipyard, physical processes, telecomunication, air industry, cosmic researches, etc.

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