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AN EOQ MODEL FOR DETERIORATING ITEMS UNDER SUPPLIER CREDITS WHEN DEMAND IS STOCK DEPENDENT

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Abstract : In many circumstances retailer is not able to settle the account as soon as items are received. In that scenario supplier can offer two promotional schemes namely cash discount and /or a permissible delay to the customer. In this study, an EOQ model is developed when units in inventory deteriorate at a constant rate and demand is stock dependent. The salvage value is associated to deteriorated units. An algorithm is given to find the optimal solution. The sensitivity analysis is carried out to analyze the effect of critical parameters on optimal solution.

Keywords : Deterioration, salvage value, cash discount, trade credit, stock dependent demand.

1. INTRODUCTION

Classical inventory Economic Order Quantity (EOQ) model is based on the assumption that the retailer settles the accounts as soon as items are received in his inventory. Practically, this is not possible for retailer every time therefore supplier can offer a cash discount and / or a permissible delay to the retailer if the outstanding amount is paid within the allowable fixed settlement period. For example, the supplier offers 3 % discount off the unit purchase price if the payment is made within 15 days; otherwise full price of items is due within 30 days. This credit term is usually denoted as “3/15, net 30” (e.g, see Brigham (1995, p. 741)).

Goyal (1985) derived an EOQ model under the conditions of permissible delay in payments. Shah (1993) and Aggarwal and Jaggi (1995) extended Goyal’s model to allow for deteriorating items. Jamal et al. (1997) then further generalized the model to allow for shortages. Liao et al. (2000) developed an inventory model for stock-dependent consumption rate when a delay in payment is permissible. Some interesting articles are by Arcelus et al. (2001), Arcelus and Srinivasan (1993, 1995, 2001), Chu et al. (1998), Chung (1998), Liao et al. (2000), Shah (1993a, 1993b, 1997), Teng (2002) etc.

Soni and Shah (2005) developed a mathematical model when units in inventory are subject to constant deterioration under the scenario of progressive credit periods. Soni et. al. (2006) studied effect of inflation in above stated model. Levin et. al. (1972)'s quotation: "the presence of inventory has motivational effect on customer around it" is studied by Soni and Shah (2008). They developed a model in which demand is partially constant and partially dependent on the stock, and the supplier offers to retailer progressive credit period to settle the account.

The items like fruits and vegetables, radioactive chemicals, medicines, blood components etc deteriorate with time. The above stated models are derived under the assumption that deterioration of units in inventory is complete loss of retailer. In this research article, an optimal ordering policy is established when units in inventory are subject to constant deterioration when salvage value is associated to the deteriorated units, supplier offers a cash discount and/or a permissible delay to the retailer to settle the account when demand is stock dependent. An algorithm is given to find the optimal solution. Sensitivity analysis is carried out to study the variations in critical parameters on decision variable and objective function.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model is derived under following assumptions and notations :

2.1 Assumptions :

- The inventory system deals with single item.
- The demand for the item is stock dependent.
- Shortages are not allowed and lead time is zero.
- Replenishment rate is infinite. Replenishment is instantaneous.
- During the time account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of period the customer pays off all units sold, keep profits, and starts paying for the interest charges on the items on stocks.
- Time horizon is infinite.
- The following notations are used in the formulation of the model:
- $R(Q(t)) = \alpha + \beta Q(t)$: the demand rate per annum (stock dependent), where α (> 0) is fixed demand and β (> 0) denotes stock dependent parameter. Also $\alpha >> \beta$.
- C : the unit purchase cost.
- γ C : salvage value of deteriorated unit, $0 \le \gamma < 1$.
- h : the inventory holding cost per unit per year excluding interest charges.
- A : ordering cost per order.
- M : period of cash discount.
- N : period of permissible delay in setting the account with N > M

- Ic : the interest charged per \$ in stock per year by the supplier or a bank.
- Ie : the interest earned per \$ in stock per year
- r: cash discount ($0 < r < 1$).
- θ : the constant deterioration rate. $0 < \theta < 1$.
- Q : the procurement quantity (a decision variable)
- T : the replenishment cycle time (a decision variable).
- Q(t) : the on - hand inventory level at any instant of time t, $0 \le t \le T$.
- D(T) : the number of units deteriorated during the cycle time T.
- K(T) : the total inventory cost per time unit is the sum of : (a) ordering cost; OC, (b) inventory holding cost (excluding interest charges) IHC, (c) cost due to deterioration ; CD, (d) cash discount earned if payment is made at M; DS, (e) salvage value of deteriorated units; SV, and (f) cost of interest charges for unsold items after the permissible delay M or N, (g) interest earned from sales revenue during the permissible period [0, M] or [0, N].

3. MATHEMATICAL MODEL

The on – hand inventory depletes due to demand and deterioration of units. The instantaneous state of inventory at any instant of time t is governed by the differential equation.

$$\frac{dQ(t)}{dt} = -(\alpha + \beta Q(t) + \theta\, Q(t)) \tag{3.1}$$

with boundary condition Q(0) = Q and Q(T) = 0 , consequently solution of (3.1) can be given by

$$Q(t) = \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)(T-t)} - 1\right], 0 < t < T \tag{3.2}$$

and the order quantity is

$$Q = Q(0) = \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)T} - 1\right] \tag{3.3}$$

Total demand during one cycle is $R(Q(T))\,T = (\alpha + \beta\,(Q(T)))\,T = \alpha\, T$.
Therefore number of deteriorating items during the cycle is

$$Q - R(Q(T))\,T = \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)T} - 1\right] - \alpha T$$

$$= \frac{\alpha}{\theta + \beta}\left[e^{(\theta+\beta)T} - 1 - T(\theta + \beta)\right] \tag{3.4}$$

The total relevant cost per time unit consists of the following elements.

Ordering Cost; OC = $\frac{A}{T}$ (3.5)

Cost of deteriorating units; CD = $\frac{C\alpha}{(\theta+\beta)T}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]$ (3.6)

Salvage value of deteriorated units, SV = $\frac{\gamma C a}{(\theta+\beta)T}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]$ (3.7)

Inventory holding cost; IHC = $\frac{h\alpha}{(\theta+\beta)^2 T}\left[e^{(\theta+\beta)T}-(\theta+\beta)T-1\right]$ (3.8)

Due to cash discount, interest charged and earned, we have four cases based on retailer's two choices (i.e. pays at M and N) and length of T

- Case 1: The payment is made at M and N to get a cash discount and M $\leq$ T.
- Case 2: The retailer pays in full at M to get a cash discount but T < M.
- Case 3: The payment is made at N to get the permissible delay and N $\leq$ T.
- Case 4: The retailer pays in full at N but T < N.

Next, we discuss each case in detail.

Case 1 : M $\leq$ T

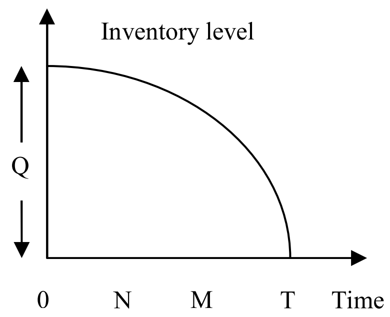

Figure 1: Inventory – time representation when M $\leq$ T

Here, the payment is made at M hence retailer saves rCQ per cycle due to price discount. The discount saving per year is given by

$$DS=\frac{rCQ}{T}=\frac{rC\alpha}{(\theta+\beta)T}\left[e^{(\theta+\beta)T}-1\right] \quad (3.9)$$

Next, as stated in assumption (3.4) the retailer sells R (Q(M)) M units in total at supplier. The items in stock have to be financed at interest rate Ic at time M to pay the supplier in full in order to get the cash discount. Thereafter, the retailer gradually reduced the amount of financed loan due to sales and revenue received. Therefore, the interest payable per year is

$$IC_1 = \frac{CIc}{T}\int_M^T Q(t)dt$$

$$= \frac{C\,Ic\,\alpha}{T(\theta+\beta)}\left[e^{(\theta+\beta)(T-M)} - (\theta+\beta)(T-M) - 1\right] \tag{3.10}$$

During [0, M], the retailer sells items and deposits the revenue into an account that earns Ie per $ per year.

Therefore interest earned per year is

$$IE_1 = \frac{PIe}{T}\int_0^M tR(I(t))dt = \frac{PIe\alpha\theta M^2}{2(\theta+\beta)T} + \frac{PIe\alpha\,\beta e^{(\theta+\beta)T}}{(\theta+\beta)^3 T}$$

$$\left[1 - (\theta+\beta)Me^{(\theta+\beta)M} - e^{(\theta+\beta)M}\right] \tag{3.11}$$

Therefore total cost per time unit is

$$K_1(T) = OC + CD + IHC - DS + + IC_1 - IE_1 - SV \tag{3.12}$$

The optimum value of T = T_1 is the solution of non- linear equation $\frac{\partial K_1(T)}{\partial T} = 0$.

The obtained T = T_1 minimizes the total cost provided $\frac{\partial^2 K_1(T)}{\partial T^2} > 0$ for all T.

Case 2 : T < M

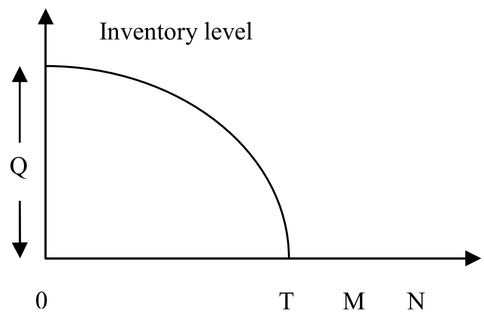

Figure 2: Inventory – time representation when M > T

In this case, the customer sells RT – units in total at time T and has CRT to pay the supplier in full at time M. Hence interest charges are zero, while the cash discount is same as that in case 1. Interest earned per time unit is

$$IE_2 = \frac{PIe}{T}\left\{\int_0^T tR(T)dt + \alpha T(M - T)\right\} =$$

$$\frac{PIe}{T}\left[\frac{\alpha\theta T^2}{2(\theta+\beta)} + \frac{\alpha\beta}{(\theta+\beta)^3}\left\{e^{(\theta+\beta)T} - (\theta+\beta)T - 1\right\}\right] \tag{3.13}$$

Therefore, total cost $K_2(T)$ per time unit is

$$K_2(T) = OC + CD + IHC - DS + + IC_2 - IE_2 - SV \tag{3.14}$$

The optimal value of $T = T_2$ is a solution of non – linear equation $\frac{\partial K_2(T)}{\partial T} = 0$ and $T = T_2$ minimizes the total cost $K_2(T)$ of an inventory system provided $\frac{\partial^2 K_2(T)}{\partial T^2} > 0$ for all T.

Case 3 : N ≤ T

Here, payment is made at time N, there is no cash discount. The interest charged per time unit is

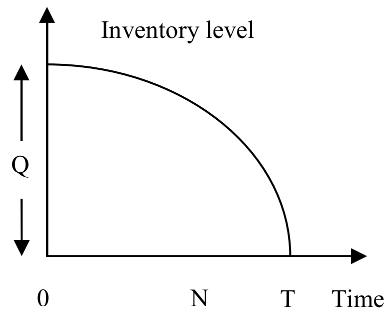

Figure 3: Inventory – time representation when N ≤ T

$$IC_3 = \frac{CIc}{T}\int_N^T Q(t)dt$$

$$= \frac{CIc\,\alpha}{(\theta+\beta)^2 T}\left[e^{(\theta+\beta)(T-N)} - (\theta+\beta)(T-N) - 1\right] \tag{3.15}$$

$$IE_3 = \frac{PIe}{T}\int_0^N tR(t)dt = \frac{PIe\ \alpha}{(\theta+\beta)T}$$

$$\left[\frac{\theta N^2}{2} + \frac{\beta}{(\theta+\beta)^2}\left\{- N(\theta+\beta)Te^{(\theta+\beta)(T-N)} - e^{(\theta+\beta)(T-N)} + e^{(\theta+\beta)T}\right\}\right] \tag{3.16}$$

Therefore, total cost K_3 (T) per time unit is

$$K_3(T) = OC + CD + IHC - DS + + IC_3 - IE_3 - SV \quad (3.17)$$

The optimal value of T = T_3 is a solution of non – linear equation $\frac{\partial K_3(T)}{\partial T} = 0$ and T = T_3 minimizes the total cost K_3(T) of an inventory system if and only if $\frac{\partial^2 K_3(T)}{\partial T^2} > 0$ for all T.

Case 4 : T < N

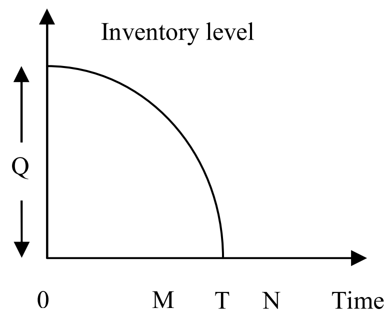

Figure 4: Inventory – time representation when T < N

Since, payment is made before 'N' there is no interest charged and interest earned per time unit is

$$IE_4 = \frac{PIe}{T}\left[\int_0^T tR(t)dt + \alpha T(N - T)\right] =$$

$$\frac{PIe}{T}\left[\frac{\alpha\theta T^2}{2(\theta+\beta)} + \frac{\alpha\beta}{(\theta+\beta)^3}(e^{(\theta+\beta)T} - (\theta+\beta)T - 1) + \alpha T(N - T)\right] \quad (3.19)$$

Therefore, total cost K_4(T) per time unit is

$$K_4(T) = OC + CD + IHC - DS + + IC_4 - IE_4 - SV \quad (3.20)$$

The optimal value of T = T_4 is a solution of non – linear equation $\frac{\partial K_4(T)}{\partial T} = 0$ and T = T_4 minimizes the total cost K_4 (T) of an inventory system unless $\frac{\partial^2 K_4(T)}{\partial T^2} > 0$ for all T.

4. THEORETICAL RESULTS

The first – order condition for $K_1(T)$ in (3.12) to be minimized is $\frac{\partial K_1(T)}{\partial T} = 0$

Now value for θ is sufficiently small. Therefore using exponential series

$e^{\theta T} \approx 1 + \theta T + \frac{(\theta T)^2}{2}$, as θT is small (ignoring θ^2 and powers), we get

$$T_1 = \sqrt{\frac{2A + \alpha M^2 [C\,Ic - P\,Ie]}{\alpha\left[h + C(1-\gamma - r)(\theta + \beta) + C\,Ic + P\,Ie\,\beta\,M\right]}} \tag{4.1}$$

For second – order condition, we obtain $\frac{\partial^2 K_1(T)}{\partial T^2} > 0$

To ensure $T_1 > M$, we substitute (4.1) into equality $T_1 > M$ to obtain

$$2A > M^2 \alpha \left[h + C(1 - \gamma - r)(\beta + \theta) + PI_e \beta M + PI_e\right] \tag{4.2}$$

Similarly, we get the first – order condition for (case 2). $K_2(T)$ in (3.14) to be minimized is $\frac{\partial K_2(T)}{\partial T} = 0$ which leads to

$$T_2 = \sqrt{\frac{2A}{\alpha\,[h + C\,(1-\gamma - r)\,(\theta + \beta) + P\,Ie]}} \tag{4.3}$$

For second – order condition, we obtain $\frac{\partial^2 K_2(T)}{\partial T^2} > 0$

Again substituting T_2 in the equality $T_2 < M$ we obtain

$$2A > M^2 \alpha \left[h + C(1 - \gamma - r)(\theta + \beta) + PI_e\right] \tag{4.4}$$

For case 3, first order condition $\frac{\partial K_3(T)}{\partial T} = 0$ in (3.20) gives optimal solution of

$$T_3 = \sqrt{\frac{2A + \alpha N^2 [C\,Ic - P\,Ie]}{\alpha\left[h + C(1-\gamma)(\theta + \beta) + C\,Ic + P\,Ie\,\beta\,N\right]}} \tag{4.5}$$

Similarly we can verify second order condition $\frac{\partial^2 K_3(T)}{\partial T^2} > 0$

Substituting (4.5) in inequality $N \le T_3$ we obtain that

$$2A > N^2 \alpha \left[h + C(1 - \gamma - r)(\beta + \theta) + PI_e \beta N + PI_e\right] \tag{4.6}$$

For case 4, first order condition $\frac{\partial K_4(T)}{\partial T} = 0$ in (3.20) gives optimal solution of

$$T_4 = \sqrt{\frac{2A}{\alpha\left[h + C(1-\gamma)(\theta+\beta) + PI_e\right]}} \tag{4.7}$$

Similarly we can verify second order condition $\frac{\partial^2 K_4(T)}{\partial T^2} > 0$

Substituting (4.7) in inequality $N > T_4$ we obtain that

$$2A > N^2\alpha\left[h + C(1-\gamma)(\theta+\beta) + PI_e\right] \tag{4.8}$$

Algorithm:

If $2A > M^2 \alpha [h + C(1 - \gamma - r)(\beta + \theta) + P\, Ie\, \beta M + P\, Ie]$, then $T^* = T_1$

If $2A = M^2 \alpha [h + C(1 - \gamma - r)(\beta + \theta) + P\, Ie\, \beta M + P\, Ie]$, then $T^* = M$

If $2A < M^2 \alpha [h + C(1 - \gamma - r)(\theta + \beta) + P I_e]$ then $T^* = T_2$

If $2A > N^2 \alpha [h + C(1 - \gamma)(\beta + \theta) + P\, Ie\, \beta N + P\, Ie]$ then $T^* = T_3$

If $2A = N^2 \alpha [h + C(1 - \gamma)(\beta + \theta) + P\, Ie\, \beta N + P\, Ie]$ then $T^* = N$

If $2A < N^2 \alpha [h + C(1 - \gamma)(\theta + \beta) + P I_e]$ then $T^* = T_4$

If $M^2 \alpha [h + C(1 - \gamma - r)(\beta + \theta) + P\, Ie\, \beta M + P\, Ie] < 2A < N^2 \alpha [h + C(1 - \gamma)(\theta + \beta) + P I_e]$ then (a) If $K_1(T_1) < K_4(T_4)$ then $T^* = T_1$ otherwise $T^* = T_4$

Special cases:

(1) If M = N and $r = \theta = 0$ in theorem 1 then same as result proved in Chung (1998) and Teng (2002) .

(2) If $\beta = 0$ *in* $R(Q(t)) = \alpha + \beta Q(t)$ then all results of theorem is same as already proved in Liang – Yuh Ouyang[1] , Chun – Tao Chang[2] and Jinn – Tsair Teng[3] (2003): An EOQ Model for deteriorating Item under Supplier Credits. Applied Mathematical Modeling vol 27, issue 12, 983 – 996.

5. NUMERICAL RESULTS

Consider the following parametric values in appropriate units :
[A, C, h, P, α, Ie, Ic] = [75, 40, 4, 80, 1000, 0.12, 0.15]

The effect of various parameters on decision variable and total cost of inventory system is exhibited in the following tables:

Table 1: Effect of deterioration on decision policy

θ	T	K(T)	K"(T)
0.02	0.1064	305.4669	1.1971 10^5
0.03	0.1048	325.9278	1.2522 10^5
0.04	0.1033	346.1211	1.3081 10^5
0.05	0.1019	366.0132	1.3648 10^5

As deterioration decreases optimal time decreases whereas total cost increases.

Table 2: Effect of stock dependent parameter on decision policy

β	T	K(T)	K"(T)
0.05	0.1064	305.4669	1.1971 10^5
0.10	0.0991	404.6970	1.4838 10^5
0.15	0.0931	497.695	1.7907 10^5
0.20	0.0881	585.5265	2.1165 10^5

Cycle time decreases as stock dependent parameter increases moreover total cost increases at the same time.

Table 3: Effect of cash discount on decision policy

r	T	K(T)	K"(T)
0.02	0.1064	305.4669	1.1971 10^5
0.03	0.1065	-96.0032	1.1932 10^5
0.04	0.1066	-497.5232	1.1892 10^5
0.05	0.1067	-898.993	1.8537 10^5

With increase in cash discount rate cycle time increases but total cost decreases.

Table 4: Effect of salvage parameter on decision policy

γ	T	K(T)	K"(T)
0.02	0.1064	305.4669	1.1971 10^5
0.03	0.1065	303..9867	1.1932 10^5
0.04	0.1066	302.4767	1.1892 10^5
0.05	0.1067	301.0067	1.8537 10^5

Cycle time increases as salvage parameter increases but total cost of inventory decreases.

Table 5: Effect of ordering cost on decision policy

A	T	K(T)	K"(T)
5	$T_4 = 0.0248$	- 416.47	$6.5722\ 10^5$
13	$T_4 = 0.0399$	-188.27	$4.0761\ 10^5$
13.7738	M = 0.0412	-524.6983	$3.0970\ 10^5$
15	$T_4 = 0.0429$	-143.72	$3.7948\ 10^5$
55.3502	N = 0.0824	731.030	$2.2867\ 10^5$
75	$T_1 = 0.1064$	305.9867	$1.1971\ 10^5$

(Negative sign comes only because purchase cost is not taken into account.)

CONCLUSION

In this study, an attempt is made to develop inventory model for deteriorating items when units in inventory deteriorate at a constant rate and demand is stock dependent. The optimal ordering policy is derived when the supplier offers a cash discount and/or a trade credit. The closed form solution is obtained using Taylor series expansion. An algorithm is provided to find the optimal policy. It suggests that instead of considering the deteriorated units to be a complete loss they can be available at reduced price and it has significant impact on total cost.

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