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MULTI-OBJECTIVE GEOMETRIC PROGRAMMING PROBLEM AND ITS APPLICATIONS

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Abstract: In this paper, we have discussed constrained posynomial Multi-Objective Geometric Programming Problem. Here we shall describe the fuzzy optimization technique (through Geometric Programming technique) In order to solve the above multi-objective problem. The solution procedure of the fuzzy technique is illustrated by a numerical example and real life applications.

Keywords: Posynomial, geometric programming, MOGPP, max-min operator, gravel box problem.

AMS Subject Classification: 90C29, 90C70.

1. INTRODUCTION

GP method is an effective method used to solve a non-linear programming problem. It has certain advantages over the other optimization methods. Here, the advantage is that it is usually much simpler to work with the dual than the primal one. Solving a non-linear programming problem by GP method with degree of difficulty (DD) plays a significant role. (It is defined as DD = total number of terms in objective function and constraints – total number of decision variables – 1).

Since late 1960’s, Geometric Programming (GP) has been known and used in various fields (like OR, Engineering sciences etc.). Duffin, Peterson and Zener [4] and Zener [11] discussed the basic theories on GP with engineering application in their books. Another famous book on GP and its application appeared in 1976 [2]. There are many references on applications and methods of GP in the survey paper by Ecker [5]. They described GP with positive or zero degree of difficulty.

Today, most of the real-world decision-making problems in economic, environmental, social, and technical areas are multi-dimensional and multi-objectives ones. Multi-objective optimization problems differ from single-objective optimization problem. It is significant to realize that multiple objectives are often non-commensurable and in conflict with each other in optimization problems. However, it is possible for him/her to state the desirability of achieving an aspiration level in an imprecise interval around it. An objective within exact target value is termed as a fuzzy goal. So, a multi-objective model with fuzzy objectives is more realistic than deterministic of it.

Zadeh [10] first gave the concept of fuzzy set theory. Later on, Bellman and Zadeh [2] used the fuzzy set theory to the decision-making problem. Tanaka [7] introduced the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmermann [12] gave the concept to solve multi-objective linear-programming problem. Fuzzy mathematical programming has been applied to several fields.

Geometric programming is a special method used to solve a class of nonlinear programming problems; mainly we use this problem to solve optimal design problems where we minimize cost and /or weight, maximize volume and/ or efficiency etc. Generally, an engineering design and management science problem has multi-objective functions. In this case it is not suitable to use any single objective programming to find an optimal compromise solution. We can use fuzzy programming to determine such a solution. Biswal [3], Verma [9] developed fuzzy geometric programming technique to solve Multi-Objective Geometric Programming (MOGP) problem. Here we have discussed another fuzzy geometric programming technique to solve MOGPP.

2. MULTI-OBJECTIVE OPTIMIZATION

In recent years there has been an increase in research on multi-objective optimization methods. Decisions with multi-objectives are quite successful in government, military and other organizations. Researchers from a wide variety of disciplines such as mathematics, management science, economics, engineering and others have contributed to the solution methods for multi-objective optimization problems. The situation is formulated as a multi-objective optimization problem in which the goal is to minimize (or maximize) not a single objective function but several objective functions simultaneously. The purpose of multi-objective problems in the mathematical programming framework is to optimize the different objective problems, (say 'k' in number) simultaneously subject to a set of system constraints. For example,

$$\text{Minimize } f(t) = \left[f_1(t), f_2(t), \ldots, f_k(t)\right]^T \tag{2.1}$$

$$\text{subject to } g_j(t) \le b_j \quad j=1,2,\ldots,m$$

$$t > 0.$$

Here now we shall describe the fuzzy optimization technique (through GP) to solve the above multi-objective problem.

2.1. Multi-Objective Geometric Programming Problem (MOGPP) using Fuzzy Technique

A multi-objective geometric programming problem can be stated as:

Find $t = (t_1, t_2, ..., t_n)^T$ so as to (2.1.1)

Minimize $f_1(t) = \sum_{i=1}^{T_1^0} c_{1i}^0 \prod_{r=1}^{n} t_r^{a_{1ir}^0}$

Minimize $f_2(t) = \sum_{i=1}^{T_2^0} c_{2i}^0 \prod_{r=1}^{n} t_r^{a_{2ir}^0}$

Minimize $f_k(t) = \sum_{i=1}^{T_k^0} c_{ki}^0 \prod_{r=1}^{n} t_r^{a_{kir}^0}$

Subject to

$$g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{mij}} \le 1 \qquad j = 1, 2......., m$$

t>0,

where $c_{ji}^0 (>0), c_{ks} (>0), a_{jir}^0, a_{jir}$ are all real numbers for j=1,2,..,m; i=1,2,.., T_j^0 ; k=1,2,…,m; s=1,2,…,. T_k

To solve this multi-objective geometric programming problem, we use the Zimmermann's (1978) solution procedure. This procedure consists of the following steps:

Step-1: Solve the MOGPP as a single objective GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solution.

Step-2: From the results of step-1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ t^1 \\ t^2 \\ ... \\ t^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(t) & f_2(t) & .... & f_k(t) \end{array} \\ \begin{bmatrix} f_1^*(t^1) & f_2(t^1) & .... & f_k(t^1) \\ f_1(t^2) & f_2^*(t^2) & ..... & f_k(t^2) \\ .... & .... & ..... & .... \\ f_1(t^k) & f_2(t^k) & ...... & f_k^*(t^k) \end{bmatrix} \end{array}$$

Here $t^1, t^2, ..., t^k$ are the ideal solutions of the objectives $f_1(t), f_2(t), ..., f_k(t)$ respectively.

So $U_r = \max\{f_r(t^1), f_r(t^2), ..., f_r(t^k)\}$ and $L_r = f_r^*(t^r)$ for $r = 1,2,...,k$

[L_r and U_r be lower and upper bounds of the r^{th} objective function $f_r(t)$ for $r = 1,...,k$].

Step 3: Using aspiration levels of each objective of the MOGPP (2.1.1) may be written as follows:

Find t so as to satisfy

$$f_r(t) \underset{\sim}{\le} L_r \, (r = 1,2,...,k) \tag{2.1.2}$$

subject to $g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{mij}} \le 1 \qquad j = 1,2.......,m$

t>0.

Here objective functions of the problem (2.1.2) are considered as fuzzy constraints. This type of fuzzy constraints can be quantified by eliciting a corresponding membership function

$$\left.\begin{aligned} \mu_r(\mathrm{f_r}(t)) &= 0 \text{ or } \to 0 \ \text{ if } \mathrm{f_r}(t) \ge U_r \\ &= \mathrm{u}_r(\mathrm{t}) \qquad \text{if } \mathrm{L_r} \le \mathrm{f_r}(\mathrm{t}) \le \mathrm{U_r} \\ &= 1 \text{ or } \to 1 \ \text{ if } \mathrm{f_r}(\mathrm{t}) \le \mathrm{L_r} \ . \end{aligned}\right\} \text{(r=1,2,...,k)} \tag{2.1.3}$$

Here $u_r(t)$ is a strictly monotonic decreasing function with respect to $f_r(t)$.

Following figure illustrates the graph of the membership function $\mu_r(f_r(t))$

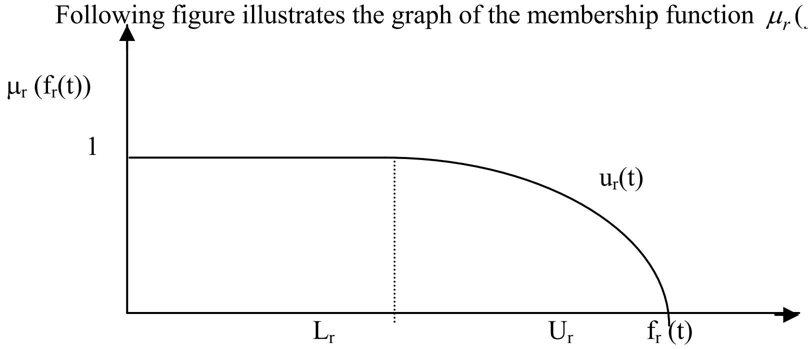


Figure-2.1: Membership function for minimization problem

Having elicited the membership functions $(as\ in\ Eqn.\,(2.1.3))$ $\mu_r(f_r(t))\ for\ r = 1,2,...,k,$ $\mu_r(f_r(t))$ a general aggregation function $\mu_{\underset{\sim}{D}}(t) = \mu_{\underset{\sim}{D}}\left(\mu_1(f_1(t)), \mu_2(f_2(t)),....., \mu_k(f_k(t))\right)$ is introduced.

So a fuzzy multi-objective decision making problem can be defined as

$$\text{Maximize } \mu_{\underset{\sim}{D}}(t) \tag{2.1.4}$$

subject to $g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{mij}} \le 1 \qquad j = 1,2.......,m$

t>0.

If we follow the fuzzy decision on fuzzy objective and constraint goals of Belman and Zadeh (1970) then using above said membership functions $\mu_r(f_r(t))$ (r=1,2,….,k), the problem of choosing the maximizing decision to find the optimal solution t (i.e. t*). There are two types of fuzzy decision and they are

(i) fuzzy decission based on minimum operator (like Zimmermann's approach (1978)).

(ii) convex-fuzzy decission based on addition operator (like Tewari et. al. (1987)).

Then the problem (2.1.4) is reduced to the following problems

(i) (according to **max-min operator**)

Maximize α (2.1.5)

subject to $\mu_r(f_r(t)) \ge \alpha \;\; for\; r = 1,2,...,k$

$$g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{mij}} \le 1 \qquad j = 1,2.......,m$$

$t > 0, 0 \le \alpha \le 1$.

and (ii) (according to **max-addition operator**)

$$Max\;\; \mu_D(t^*) = Max\left(\sum_{j=0}^{m} \lambda_j \mu_j(f_j(t))\right) \tag{2.1.6}$$

subject to $\mu_r(f_r(t)) = \dfrac{U_r - f_r(t)}{U_r - L_r}$ (r=1,2,….,k)

$$g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{jir}} \le 1 \qquad j = 1,2.......,m$$

$t0 \le \mu_r(f_r(x)) \le 1, t > 0 \;\; 0 \le \mu_r(f_r(x)) \le 1, t > 0$.

The above problem (2.1.6) reduces to

$$Max\, V(t) = \sum_{j=0}^{m} \lambda_j \frac{g_j^{'} - \sum_{i=1}^{T_j} c_{ji} \prod_{r=1}^{n} t_r^{a_{jir}}}{g_j^{'} - g_j^{0}} \tag{2.1.7}$$

subject to $g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{jir}} \le 1 \qquad j = 1,2.......,m$

t>0.

So optimal decision variable t^* with optimal objective value $V^*(t^*)$ can be obtained by $V^*(t^*) = \sum_{j=0}^{m} \frac{\lambda_j g_j'}{g_j' - g_j^0} - U^*(t)$ where t^* is optimal decision variable of the unconstrained geometric programming problem (for given λ_j j=1,2,…,k),

$$\text{Min } U(t) = \sum_{j=0}^{m} \frac{\lambda_j}{g_j' - g_j^0} \sum_{i=1}^{T_j} c_{ji} \prod_{r=1}^{n} t_r^{a_{jir}} \tag{2.1.8}$$

subject to $g_j(t) = \sum_{i=1}^{T_m} c_{ji} \prod_{r=1}^{n} t_r^{a_{jir}} \le 1 \quad j = 1, 2, \ldots, m$

t>0.

2.2. Example: Multi-Objective Primal Geometric Programming (MOPGP) Problem

$$\begin{aligned} &\textit{Minimize} \quad \{Z_1(X),\ Z_2(X)\} \\ &\textit{subject to} \quad Y_1(X) \le 1 \\ &x_1, x_2 > 0 \\ &\textit{where} \quad Z_1(X) = x_1^{-1} x_2^{-2}, \\ &Z_2(X) = 2 x_1^{-2} x_2^{-3} \ \textit{and} \ \ Y_1(X) = x_1 + x_2 \end{aligned} \tag{2.2.1}$$

In order to solve this MOGP problem, we shall first solve the two sub-problems (Sub-problem-1)

$$\begin{aligned} &\textit{Minimize } Z_1(X) \\ &\textit{subject to } Y_1(X) \le 1 \\ &x_1, x_2 > 0 \end{aligned} \tag{2.2.2}$$

and (Sub-problem-2)

$$\begin{aligned} &\textit{Minimize } Z_2(X) \\ &\textit{subject to } Y_1(X) \le 1 \\ &x_1, x_2 > 0 \end{aligned} \tag{2.2.3}$$

Solving the above problems by GP technique we have

For (Sub-problem-1) $x^1 = \left(\frac{1}{3}, \frac{2}{3}\right) \quad Z_1(X) = 6.75$

For (Sub-problem-1) $x^2 = \left(\frac{2}{5}, \frac{3}{5}\right) \quad Z_2(X) = 57.8703$

Now the pay-off matrix is given below

$$\begin{array}{c} \\ x^1 \\ x^2 \end{array}\begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{bmatrix} 6.75 & 60.75 \\ 6.94 & 57.87 \end{bmatrix} \end{array}$$

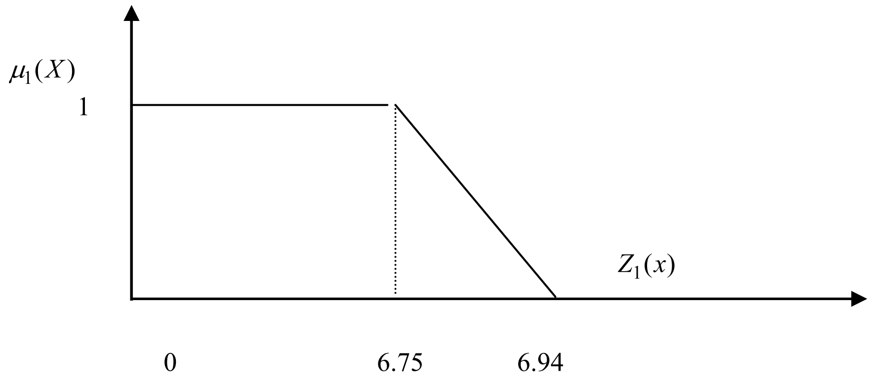
From the pay-off matrix the lower and upper bound of $Z_1(X)$ and $Z_2(X)$ be

$6.75 \le Z_1(X) \le 6.94$ *and* $57.87 \le Z_2(X) \le 60.75$

Let $\mu_1(X), \mu_2(X)$ be the fuzzy membership function of the objective function $Z_1(X)$ *and* $Z_2(X)$ respectively and they are defined as:

$$\mu_1(X) = \begin{cases} 1 & \text{if } Z_1(X) \le 6.75 \\ \dfrac{6.94 - Z_1(X)}{0.19} & \text{if } 6.75 \le Z_1(X) \le 6.94 \\ 0 & \text{if } Z_1(X) \ge 6.94 \end{cases}$$

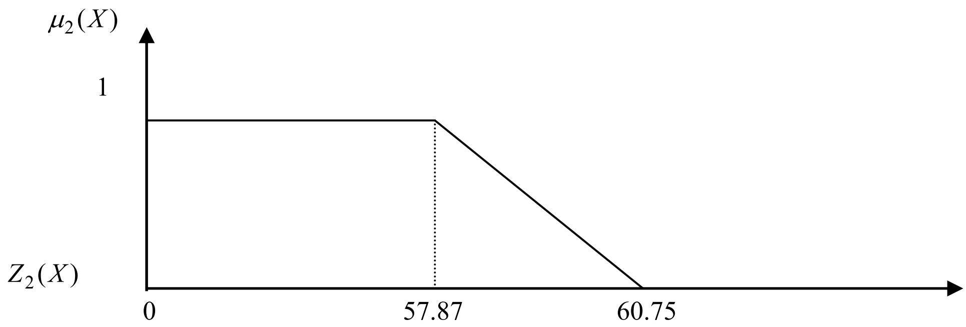
The following figure illustrated the graph of the fuzzy membership function $\mu_1(X)$



and

$$\mu_2(X) = \begin{cases} 1 & \text{if } Z_2(X) \le 57.87 \\ \dfrac{60.75 - Z_2(X)}{2.88} & \text{if } 57.87 \le Z_2(X) \le 60.75 \\ 0 & \text{if } Z_2(X) \ge 60.75 \end{cases}$$

Now the following figure illustrated the fuzzy membership function $\mu_2(X)$



According to **max-addition** operator, the MOGPP (2.2.1) reduces to the crisp problem

$$Maximize\,(\mu_1(X)+\mu_2(X))$$

$$i.e\quad Maximize\left(\frac{6.94-Z_1(X)}{0.19}+\frac{60.75-Z_2(X)}{2.88}\right)$$

$$i.e\quad Maximize\left\{57.61-\left(\frac{Z_1(X)}{0.19}+\frac{Z_2(X)}{2.88}\right)\right\} \tag{2.2.4}$$

$$subject\,to\;\; x_1+x_2\le 1$$

$$x_1, x_2 > 0.$$

[considering equal importance on both objective fuctions i.e. $\lambda_1=\lambda_2=1$]

For maximizing the above problem, we minimize $\frac{Z_1(X)}{0.19}+\frac{Z_2(X)}{2.88}$ *subject to* $x_1+x_2\le 1$.

So, our new problem is to solve

$$Minimize\, g(X)=\left(\frac{Z_1(X)}{0.19}+\frac{Z_2(X)}{2.88}\right)$$

$$i.e\;\; Mininimize\;\; g(X)=\;(5.269x_1^{-1}x_2^{-2}+0.699x_1^{-2}x_2^{-3}) \tag{2.2.5}$$

$$subject\,to\;\; x_1+x_2\le 1$$

$$x_1, x_2 > 0.$$

Degree of Difficulty of the problem (2.2.5) is =(4-(2+1))=1

The dual problem of the above problem (2.2.5) is

$$Maximize\ v(w) = \left(\frac{5.269}{w_{01}}\right)^{w_{01}} \left(\frac{0.699}{w_{02}}\right)^{w_{02}} \left(\frac{1}{w_{11}}\right)^{w_{11}} \left(\frac{1}{w_{12}}\right)^{w_{12}}$$

$$(w_{11}+w_{12})^{w_{11}+w_{12}}$$

$$\begin{aligned} subject\ to\ \ & w_{01}+w_{02}=1 \\ & -w_{01}-2w_{02}+w_{11}=0 \\ & -w_{01}-3w_{02}+w_{12}=0 \\ & w_{01},w_{02},w_{11},w_{12}>0. \end{aligned} \tag{2.2.6}$$

Solving the above equation by Newton Raphson method we ultimately get,

$w_{01}^* = 0.63745, w_{02}^* = 0.36254, w_{11}^* = 0.0065, w_{12}^* = 0.0113$

The value of the objective function of the problem (2.2.6) is $v(w^*) = 56.8389$.

Therefore, by using primal-dual variables relation-ship, the value of the objective function of the problem (2.2.5) is $g(X^*) = 56.8389$ and the values of decisions variables are $x_1^* = 0.36577$, $x_2^* = 0.63422$.

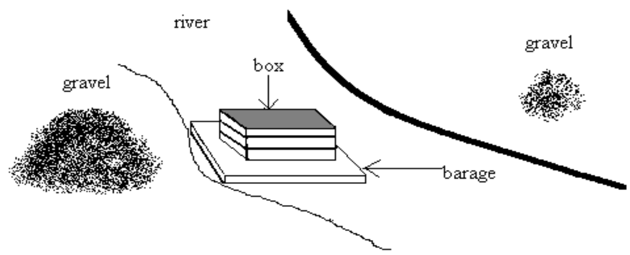
Thus, the values of the objective functions of the MOGPP (2.2.1) are $Z_1(X^*) = 6.796$ *and* $Z_2(X^*) = 58.599$.

2.3. Applications:

Problem-1: Gravel-Box problem

80 cubic-meter of gravel is to be ferried across a river on a barrage .A box (with open top) is to be built for this purpose. After the entire grave has been ferried, the box is to be discarded. The transport cost per round trip of barrage of box is Rs 1 and the cost of materials of sides and bottom of box are Rs 10/m^2 and Rs 80/m^2 and ends of box Rs 20/m^2. Find the dimension of the box that is to be building for this purpose and total optimal cost.

Gravel Box Design



Let us assume the gravel box has length = $t_1\ m$

width $= t_2\ m$

height $= t_3\ m$

$\therefore$ The area of the end of the gravel box $= t_2 t_3\ m^2$

The area of the side of the gravel box $= t_1 t_3\ m^2$

The area of the bottom of the gravel box $= t_1 t_2\ m^2$

$\therefore$ The volume of the gravel box $= t_1 t_2 t_3\ m^3$

Cost functions are:

Transport cost : $\left(Rs1/trip\right)\dfrac{80m^3}{t_1 t_2 t_3 m^3/trip} = Rs.\ 80 t_1^{-1} t_2^{-1} t_3^{-1}$,

Material cost: End of box: $2(Rs\,20/m^2)t_2 t_3 m^2 = Rs.\ 40 t_2 t_3$

Sides of box: $2(Rs10/m^2)t_1 t_3 m^2 = Rs.\ 20 t_1 t_3$

Bottom: $(Rs\,80/m^2)t_1 t_2 m^2 = Rs. t_1 t_2$

The total cost (Rupees)

$$g(t) = 80 t_1^{-1} t_2^{-1} t_3^{-1} + 40 t_2 t_3 + 20 t_1 t_3 + 80 t_1 t_2$$

It is a posynomial function.

As stated, this problem can be formulated as an unconstrained GP problem

$$Minimize\ g(t) = 80 t_1^{-1} t_2^{-1} t_3^{-1} + 40 t_2 t_3 + 20 t_1 t_3 + 80 t_1 t_2 \tag{2.3.1}$$

subject to $t_1, t_2, t_3 > 0$

Suppose that we now consider the following variant of the above problem (2.3.1) (similar discussion have done Duffin, Peterson and Zener(1967) in their book). It is required that the sides and bottom of the box should be made from scrap material but only 4 m^2 of this scrap material are available.

This variation of the problem leads us to the following constrained posynomial GP problem:

$$\begin{cases} Minimize\ g_0(t) = \dfrac{80}{t_1 t_2 t_3} + 40 t_2 t_3 \\ subject\ to\ g_1(t) \equiv 2 t_1 t_3 + t_1 t_2 \le 4, \\ \quad where\ t_1 > 0, t_2 > 0, t_3 > 0. \end{cases} \tag{2.3.2}$$

Not only minimizing total cost (= total transportation cost + material cost for two ends of the box) of the problem (2.3.2) but there is also another objective function which is to minimize the total number of trips.

Here no. of trips = $\frac{80}{t_1 t_2 t_3}$.

So, the problem is to determine dimensions of the box,

i.e. to find $t = (t_1, t_2, t_3)^T$ so as to satisfy

$$\begin{cases} Minimize\, g_0(t) = \dfrac{80}{t_1 t_2 t_3} + 40 t_2 t_3 \\ Minimize\, g_1(t) = \dfrac{80}{t_1 t_2 t_3} \\ subject\, to\;\; 2t_1 t_3 + t_1 t_2 \le 4, \\ where\; t_1, t_2, t_3 > 0. \end{cases} \tag{2.3.3}$$

It may be written as a Multi-Objective Geometric Programming Problem (MOGPP)

$$\begin{cases} Minimize\, g_0(t) = \dfrac{80}{t_1 t_2 t_3} + 40 t_2 t_3 \\ Minimize\, g_1(t) = \dfrac{80}{t_1 t_2 t_3} \\ subject\, to\;\; g_2(t) \equiv \dfrac{1}{2} t_1 t_3 + \dfrac{1}{4} t_1 t_2 \le 1, \\ where\; t_1, t_2, t_3 > 0. \end{cases} \tag{2.3.4}$$

Here two sub-problems are

$$(\text{Sub-problem-1}) \begin{cases} Minimize\;\; g_0(t) = \dfrac{80}{t_1 t_2 t_3} + 40 t_2 t_3 \\ subject\;\; to\;\; g_2(t) \equiv \dfrac{1}{2} t_1 t_3 + \dfrac{1}{4} t_1 t_2 \le 1, \\ where\;\; t_1, t_2, t_3 > 0. \end{cases} \tag{2.3.5}$$

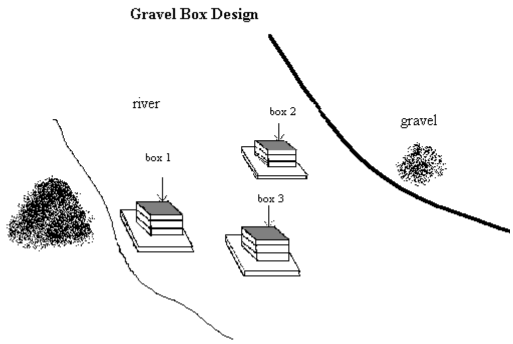
and

$$(\text{Sub-problem-2}) \begin{cases} Minimize\;\; g_1(t) = \dfrac{80}{t_1 t_2 t_3} \\ subject\;\; to\;\; g_2(t) \equiv \dfrac{1}{2} t_1 t_3 + \dfrac{1}{4} t_1 t_2 \le 1, \\ where\;\; t_1, t_2, t_3 > 0. \end{cases} \tag{2.3.6}$$

The above sub-problems (2.3.5) & (2.3.6) are two GP problem with DD = -1, 0 respectively. Solving this MOGPP (2.3.4) by using fuzzy techniques, we have t_1^* = 2.93, t_2^* = 1.17 and t_3^* = 0.43 and optimal objective goals $g_0^*(t^*) = 86.78$ and $g_1^*(t^*) = 3.3$.

Problem-2: Multi-Gravel box problem

Suppose that to shift gravel in a finite number (say n) of open rectangular boxes of lengths t_{1i} meters, widths t_{2i} meters, and heights t_{3i} meters (i=1,2,…,n). The bottom, sides and the ends of the each box cost Rs. a_i, Rs. b_i, and Rs. c_i/m^2. It costs Rs. 1 for each round trip of the boxes. Assuming that the boxes will have no salvage value, find the minimum cost of transporting d $(= \sum_{i=1}^{n} d_i)$ m^3 of gravels.



As stated, this problem can be formulated as an unconstrained modified geometric programming problem

$$\begin{cases} Minimize\, g(t) = \sum_{i=1}^{n} \left(\dfrac{d_i}{t_{1i}t_{2i}t_{3i}} + a_i t_{1i} t_{2i} + 2b_i t_{1i} t_{3i} + 2c_i t_{2i} t_{3i} \right) \\ where\;\; t_{1i} > 0, t_{2i} > 0,\; t_{3i} > 0 \qquad (i = 1,2,...,n). \end{cases} \tag{2.3.7}$$

Suppose that we know the following variant of the above problem. It is required that the sides and bottom of the boxes should be made from scrap material but only w m^2 of these scrap materials are available.

This variation of the problem leads us to the following constrained modified geometric programming problem:

$$\begin{cases} Minimize\;\; g(t) = \sum_{i=1}^{n} \left(\dfrac{d_i}{t_{1i}t_{2i}t_{3i}} + 2c_i t_{2i} t_{3i} \right) \\ subject\,to\;\; \sum_{i=1}^{n} \left(2t_{1i}t_{3i} + t_{1i}t_{2i} \right) \le w, \\ where\; t_{1i} > 0,\; t_{2i} > 0,\; t_{3i} > 0 \qquad (i = 1,2,....,n). \end{cases} \tag{2.3.8}$$

In particular, the problem is to minimize the 3 cost functions i.e.

$$\begin{cases} Minimize\ g(t) = (g_1(t), g_2(t), g_3(t)) \\ subject\,to\ \ \sum_{i=1}^{3}(2t_{1i}t_{3i} + t_{1i}t_{2i}) \le w, \\ where\ t_{1i} > 0,\ t_{2i} > 0,\ t_{3i} > 0 \qquad (i = 1,2,3) \\ g_1(t) = \dfrac{d_1}{t_{11}t_{21}t_{31}} + 2c_1t_{21}t_{31}, g_2(t) = \dfrac{d_2}{t_{12}t_{22}t_{32}} + 2c_2t_{22}t_{32}, g_3(t) \\ = \dfrac{d_3}{t_{13}t_{23}t_{33}} + 2c_3t_{23}t_{33} \end{cases} \tag{2.3.9}$$

It may be written as a Multi-Objective Geometric Programming Problem (MOGPP)

$$\begin{cases} Minimize\ g_1(t) = \dfrac{d_1}{t_{11}t_{21}t_{31}} + 2c_1t_{21}t_{31} \\ Minimize\ g_2(t) = \dfrac{d_2}{t_{12}t_{22}t_{32}} + 2c_2t_{22}t_{32} \\ Minimize\, g_3(t) = \dfrac{d_3}{t_{13}t_{23}t_{33}} + 2c_3t_{23}t_{33} \\ subject\,to\ \ \dfrac{1}{w}\sum_{i=1}^{3}\left(2t_{1i}t_{3i} + t_{1i}t_{2i}\right) \le 1 \\ t_{1i}, t_{2i}, t_{3i} > 0,\ \ (i = 1,2,3). \end{cases} \tag{2.3.10}$$

In particular here we assume transporting d m^3 of gravels by the three different open rectangular boxes. The final cost of each box is Rs. c_i /m^2 and the amount of the transporting gravels by three open rectangular boxes are $d\left(= \sum_{i=1}^{3} d_i \right)$ m^3. Input data of this MOGPP (2.3.10) is given in the table-1. It is a constrained posynomial MOGP problem. Solving this MOGPP (2.3.10) by the above specified fuzzy technique we get optimal solutions as shown in table-2.

Table-1
Input data for the MOGPP (2.3.10)

Boxes (i)	c_i (Rs. /m^2)	d_i (m^3)	w (m^2)
1	40	80	15
2	30	90	
3	20	70	

Table-2
Optimal solutions of the MOGPP (2.3.10)

Boxes (i)	t_{1i} (meter)	t_{2i} (meter)	t_{3i} (meter)	$g_1^*(t^*)$ (Rs.)	$g_2^*(t^*)$ (Rs.)	$g_3^*(t^*)$ (Rs.)
1	2.33	1.14	0.57	87.65	94.54	83.58
2	2.0	1.32	0.66			
3	1.49	1.47	0.74			

2.4. Conclusion

Here, we have discussed multi–objective geometric programming problem based on fuzzy programming technique through geometric programming. We have also formulated the multi–objective optimization model of gravel box design problem and solved by fuzzy programming technique. Geometric Programming technique is used to derive the optimal solutions for different preferences on objective functions. The multi-objective inventory models may also be solved by fuzzy geometric programming technique.

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