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ANNUAL PREVENTIVE MAINTENANCE SCHEDULING FOR THERMAL UNITS IN AN ELECTRIC POWER SYSTEM

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Abstract: The system approach to the problem of preventive maintenance scheduling for thermal units in a large scale electric power system is considered in this paper. The maintenance scheduling program determines a set of thermal units maintenance switch off for a time period of one year. This paper considers the application of dynamic programming and successive approximations method in determination of annual thermal unit maintenance schedules. The objective function is multiple component and consists of system operation costs and system reliability indices (loss-of-load-probability and expected unserved energy). The evaluation of these costs is performed through a simulation method which uses a cumulant load model. The software package, developed in FORTRAN and integrated with an ORACLE data base, produces many useful outputs.

Keywords: Preventive maintenance, maintenance scheduling, thermal units, dynamic programming method, successive approximations.

AMS Subject Classification: 90C39,90B25.

1. INTRODUCTION

The complexity, continuous growth and request for increase of reliability of operation of electric power systems require the introduction of a systems approach to thermal generation units and transmission lines maintenance scheduling. Maintenance costs reach considerable amounts and record annual increases of 15-20%. The electric power system maintenance scheduling problem is also very important from resource utilization standpoint, because an average generator maintenance switch off lasts about 45 days. This amounts to about 14% of the total possible annual operation capacity. In addition, direct maintenance costs are accompanied by other "hidden" maintenance related costs, such as losses due to unserved energy, the cost of buying energy from some other sources, etc. On the other hand, the savings achievable by timely maintenance may "defer" the need for investment into the instruction of new power generating units systems, as the available capacities are used in an optimal way.

The existence of high-capacity thermal generating units (over 600 MW) in the electric power system requires precise scheduling of system reserve and fast rescheduling in case of failures and emergency conditions. In addition, various events, such as unpredictable outages, connection of a new generating plant to the system or the termination of a power plant operation, etc. also require fast changes in maintenance schedules.

2. PROBLEM STATEMENT

An optimal maintenance schedule for thermal generator units is obtained by solving a large-scale optimization problem with stochastically time-varying components.

The general task of the electric power system maintenance scheduling consists of determining the duration and sequence of the switch-offs of generator units and transmission lines over a given time period (usually a year) to permit maintenance to be performed. Most commonly, this task is formulated as the problem of finding the optimal maintenance schedule for a given criterion function, with all local and system constraints being satisfied.

System constraints refer to the maintenance process (maintenance duration and continuity), the permissible interval time (window) during which maintenance may be performed, the number of units under maintenance and the total capacity of thermal generator units under maintenance. The two last mentioned constraints may be specified for the whole system or for certain parts.

The criterion function may take the following forms:

- System operation costs including fuel cost, energy exchange cost and emergency power cost,
- System reliability, expressed and measured in terms of expected unserved energy (EUE), and/or loss of load probability (LOLP),
- A linear combination of system operation costs and system reliability parameters.

System operation costs and system reliability indices are calculated for each week by using a probabilistic simulation method that takes into account system load, the availability and characteristics of thermal generator units, hydro power plant generation and energy exchange contracts.

The problem of optimal annual maintenance scheduling for thermal units was treated by applying modern theoretical achievements and techniques, such as:

- Dynamic programming and successive approximations [1,2,8],
- Incorporation of system uncertainty into problem solving (load uncertainty, generator failures) [3,5],
- Simulation of the procedure of thermal units and hydro power plants merit order in loading and energy production [4],

- The cumulant method (in Gram-Charlier expansion) for solving the convolution problem [4,5].

Notation used in this paper:
j - thermal unit variable index
J - the total number of thermal units in the system
i – the time unit interval index (week)
I - the total number of intervals for which maintenance is scheduled
M_j - duration of maintenance for unit *j*
C_j - capacity of thermal unit *j*
r_j - forced outage rate of thermal unit *j*

d(*i*) – system demand forecast for time interval *i* presented in the form of a load duration curve – LDC,

$u_j(i)$ - control variable for thermal unit *j* in interval *i*, namely:

$$u_j(i) = \begin{cases} 1 & \text{if thermal unit j is under maintenance in interval i} \\ 0 & \text{otherwise} \end{cases}$$

$x_j(i)$ - state variable denoting the degree of maintenance completion for the thermal unit *j* in the interval *i*:

$$x_j(i) = \begin{cases} 0 & \text{maintenance not started} \\ m & \text{maintenance in progress, } 0 < m < M_j \\ M_j & \text{maintenance completed} \end{cases}$$

f^c (.) - expected production costs for the time interval *i*,
f^r (.) - expected cost of unreliability for the time interval *i*,
α_1 - proportionality factor for generation costs,
α_2 - proportionality factor for unreliability costs,
$\underline{v}$ - a vector
[M]‘ - a transposed vector or matri

The total costs $f(\underline{x}(i), \underline{u}(i), d(i))$ are represented as a linear convex combination of the expected production costs with the expected cost of unreliability in week *i* is:

$$f(\underline{x}(i), \underline{u}(i), d(i)) = \alpha_1 f^c(\underline{x}(i), \underline{u}(i), d(i)) + \alpha_2 f^r(\underline{x}(i), \underline{u}(i), d(i)) \tag{1}$$

$$\alpha_1 + \alpha_2 = 1, \quad \alpha_1 \geq 0, \alpha_2 \geq 0 \tag{2}$$

2. DYNAMIC PROGRAMMING AND SUCCESSIVE APPROXIMATIONS

The first paper on maintenance scheduling for thermal units using dynamic programming was published by Zürn and Quintana [1]. A more detailed analysis of the method proposed was given by Yamayee and Sidenblad [5]. Zurn and Quintana used dynamic programming and successive approximations (DPSA), which consists of sequential applications of standard dynamic programming to suitably chosen groups of thermal units. The grouping of thermal units reduces the state space, and thus the problem dimensionality as well. However, this method can yield a local minimum as the solution.

An optimal schedule requires the selection of an optimal control sequence $\{\underline{u}^*(i)\}, \underline{u}^*(i) \in U_i$.

Here U_i is a set of admissible controls for the time interval i. Forward dynamic programming (FDP) has been chosen for calculating the optimal control sequence, because it allows a user to search easily the large number of alternatives, i.e. sub optimal control sequences.

The maintenance scheduling problem may be formulated as:

$$\min_{\{\underline{u}(i)\}} F = \sum_{i=1}^{I} f(\underline{x}(i), \underline{u}(i), d(i)) \tag{3}$$

in accordance with state equation:

$$x_j(i) = x_j(i-1) + u_j(i), \quad \forall i \in \{1,2,...,I\}, \forall j \in \{1,2,...,J\} \tag{4}$$

the initial and terminal states:

$$\underline{x}_j(0) = \underline{0}, \quad \underline{x}_j(i) = M_j, \tag{5}$$

and other system constraints.

If $f_i(\underline{x}(i), i)$ are the minimum total costs (of generation and unreliability) of the feasible state $\underline{x}(i)$ at the end of the interval i, and starting from the initial interval $\underline{x}(0)$,the functional equation of dynamic programming is given by:

$$f_i(\underline{x}(i), i) = \min_{\{\underline{u}(i)\}} [f(\underline{x}(i), \underline{u}(i), d(i)) + f_{i-1}(\underline{x}(i) - \underline{u}(i))], \quad with \quad f_0(\underline{x}^0) = 0 \tag{6}$$

Successive approximations

The dynamic programming and successive approximations (DPSA) approach is used for solving such a problem of a large dimension. The problem is solved by an iterative procedure. One subset of control variables $\underline{u}_j(i)$ of thermal units is chosen and adequate cost function is optimized in each iteration, while the remaining control variables and associated states are unchanged.

Formally speaking, in maintenance scheduling the DPSA method is defined for solving N subsets of thermal units S_n , n=1,...,N. These are separate subsets, and their union consists of all thermal units.

The state equations for a subset n in iteration (h+1) are:

$$x_m^{(h+1)}(i) = x_m^{(h+1)}(i-1) + u_m^{(h+1)}(i); \quad m \in S_n \tag{7}$$

where S_n is a subset of set S.

After minimizing the sub processes described according to the above equation, the completely updated state and control vectors are:

$$\begin{aligned} \underline{x} &\equiv [\underline{x}_{S_1}^{(h+1)},...,\underline{x}_{S_n}^{(h+1)},\underline{x}_{S_{n+1}}^{(h)},...,\underline{x}_{S_N}^{(h)}]' \\ \underline{u} &\equiv [\underline{u}_{S_1}^{(h+1)},...,\underline{u}_{S_n}^{(h+1)},\underline{u}_{S_{n+1}}^{(h)},...,\underline{u}_{S_N}^{(h)}]' \end{aligned} \tag{8}$$

The group objective function used in (h+1)-st iteration for subset S_n and for the time interval i is:

$$f_{S_n,i}^{(h+1)}\left[\underline{x}_{S_n}^{(h+1)}(i)\right]=\min_{\underline{u}_{S_n}(i)}\left\{f\left[\underline{x}_{S_n}^{(h+1)}(i),\underline{u}_{S_n}^{(h+1)}(i),d(i)\right]+f_{S_n,i-1}^{(h+1)}\left[\underline{x}_{S_n}^{(h+1)}(i)-\underline{u}_{S_n}^{(h+1)}(i)\right]\right\} \tag{9}$$

Solution convergence has been achieved when successive iterations produce identical plans, i.e. when there is no improvement in criterion function value.

The successive approximations algorithm consists of the following steps:

1. Find an initial feasible solution of $\underline{x}$ and $\underline{u}$.
2. Form groups of units $S_n \in \{S_1,...,S_N\}$
   - 2.1. Set the index of the first group that is optimized $n=1$.
     - 2.1.1. Find the optimal solution and criterion function values for a specified group of units.
     - 2.1.2. Update $\underline{x}$ and $\underline{u}$.
     - 2.1.3. Set the index $n=n+1$. If $n \le N$, go to 2.1.1.; otherwise, go to 2.2.
   - 2.2. If the convergence is not achieved, go to 2.1.; otherwise, go to 2.3.
   - 2.3. If regrouping of units is required, regroup and go to 2.1.; otherwise, terminate successive approximation.

Finding an initial feasible solution

An initial feasible solution must be available for the successive approximations method to start. In addition, to reduce the number of iterations in problem solving, a good initial solution should be available at the very beginning of the procedure. The initial solution is used as the initial upper bound of the solution.

A good initial solution is found by the method of maximal element. The problem solving procedure is based on increasing the values of the components of vector $\underline{x}$ successively by one. In each iteration that component $x_j(i)$ of vector $\underline{x}(i)$ is increased by one for which the following holds:

$$\Delta_j = \max_k \Delta_k \tag{10}$$

where:

$$\Delta_k = f(x_k(i)) - f(x_k(i+1)), k=1,...,n \tag{11}$$

(the components of vector $\underline{x}$ that remain unchanged have been omitted from this relation).

To increase the speed of calculating the initial solution, a simplified form of criterion function is used. Namely, the least square of the difference between the generation and forecasted load should be found, what corresponds largely to the criterion function. The solution found is used for calculating the real value of criterion function.

If no feasible initial solution is found, the status of constraints should be changed (from hard to soft) and the region of bounds on constraints expanded. If necessary, some constraints may be relaxed completely.

Grouping of units

Model complexity and the large number of variables make it impossible to solve the whole problem simultaneously. The number of functions solved per i interval increases exponentially:

$$\prod_{j=1}^{J}(M_j + 1) \tag{12}$$

where:

M_j- maintenance duration for unit *j*.

In solving real models this leads to a too long program running time. This is why the system is decomposed into groups consisting of a few thermal units and optimization is performed successively for each group; during each optimization step units belonging to one group are optimized, the schedules of thermal units operation of the remaining groups and are considered as fixed.

In the model we have developed it is suggested that each group consists of 1 to 6 thermal units. The criterion for grouping is maximum overlapping of maintenance intervals. If groups are formed of units whose permissible maintenance intervals do not overlap, optimization is performed unit by unit.

General program algorithm

The general algorithm of the program for maintenance scheduling for thermal units is described in the sequel.

1. Select input parameters. (Enter data.)
2. Check data for completeness and consistency.
3. Select operation mode (Automatically or interactive correction of constraints).
   - 3.1. Evaluate a pre specified solution, if one is given.
     - 3.1.1. Calculate criterion function value and output results for the pre specified solution.
     - 3.1.2. Go to selected operation mode (3.2. or 3.3.).
   - 3.2. Automatically correction of constraints.
     - 3.2.1. Call the algorithm for finding the initial solution.
     - 3.2.2. A feasible solution found? If yes, go to 3.2.4.
     - 3.2.3. Constraint status change permitted? If yes, perform the change automatically and go to 3.2.1. Otherwise, a message is printed and the program terminated.

3.2.4. Call the optimization algorithm.
3.2.5. Go to 4.
3.3. Interactive correction of constraints.
3.3.1. Call the algorithm for finding the initial solution.
3.3.2. A feasible solution found? If yes, go to 3.3.4.
3.3.3. Change interactively the status of constraints and go to 3.3.1.
3.3.4. Call the optimization algorithm.
4. Calculate monthly results (if required).
5. Print output reports.
6. Save the results (if required).
7. Terminate.

4. PROBABILISTIC SIMULATION OF GENERATION

The criterion function consists of two components: expected power generation costs and system unavailability costs for interval i. The calculation of both components requires the knowledge of the expected energy generation by each thermal unit not under maintenance, the loss of load probability (LOLP) and the expected unserved energy (EUE) for a given interval. These values are calculated using the model of system demand and the models of thermal units, as well as the equivalent load duration curve (ELDC).

The criterion function is calculated by probabilistic simulation of power system operation. Probabilistic simulation was introduced by Baleriaux and Booth [3,4] and later improved by many authors. At present, the method permits the required results to be obtained in a reasonable time using standard computer.

Probabilistic simulation takes into account:

- Consumption represented by the load duration curve (LDC)
- Expected energy generation by thermal units
- Energy exchange with other systems
- Thermal units availability
- The curve of specific heat consumption by thermal units.

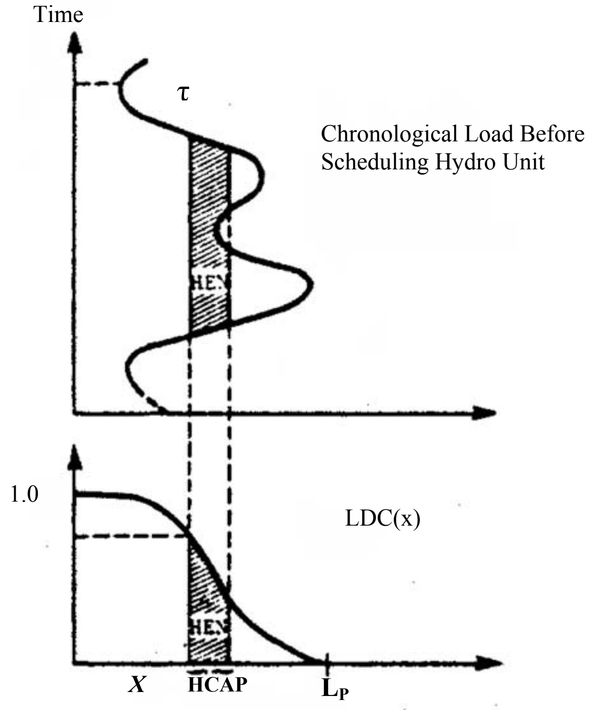

Figure 1.

The load duration curve (LDC) is used to represent the load at each level. The hourly load over a week is used for making the weekly LDC, which represents the number of hours during which load is equal to or larger than a certain value (see Figure 1.). Load uncertainty is included by modifying the LDC. A normal load distribution has been accepted (this is a generally accepted distribution). This means that a mean value and a peak load variance are determined for each week.

The cost function consists of two components: the expected energy generation costs and system unavailability costs in interval i. The calculation of both components requires the knowledge of the expected energy produced by each thermal unit not under maintenance, the LOLP for the whole system and the expected unserved energy for a given interval.

The problem of uncertainty in the operation of thermal units arises because of the possibility of forced outages. By definition, a forced outage is the outage caused by some unexpected event related directly to a thermal unit, which requires that thermal unit to be disconnected immediately, automatically or manually, for as long as the switch-off process is performed. In other words, a forced outage is the outage due to improper equipment functioning or a human error. A r_j - forced outage rate (FOR) is the measure of *j*-th thermal unit unavailability. FOR is defined as the ratio between the hours of forced outage and the sum of the hours of operation and forced outage.

Probabilistic load simulation by load duration curve has been widely used for the modeling of merit order commitment and energy production.

The combined effects of system load and the generation by units with which outages are possible may be represented as equivalent system load. In fact, demand for energy from generator units consists of two components: system load (demand by consumers) which is served by the generation of a unit, assuming there are no unit outages, and load to be served because of the outages of other units. The equivalent system load, which should be served, is the convolution of these two components.

The convolution of probability density functions associated to the system load and capacity in failure of a random variable leads to the equivalent probability density function of a random variable of load.

PROBABILISTIC STIMULATION OF UNIT GENERATION BALLERIAUX METHOD

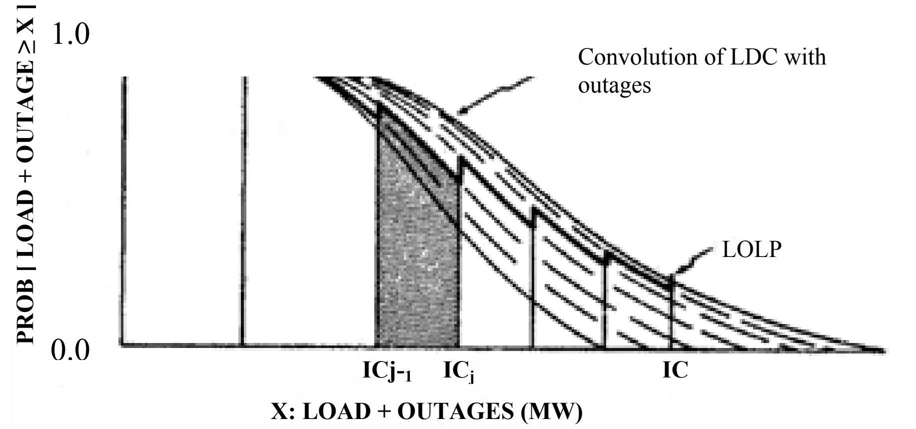

Figure 2.

The Baleriaux's procedure is illustrated in Fig. 2. Each unit is loaded, according to merit order, under a corresponding ELDC. The point of the beginning of loading unit j is the end of the sum of the first j-1 units (IC_{j-1}). The equivalent load curve obtained with unit j is the convolution of the failures of curves of j-1 units with the initial LDC. The shaded area under the curve reduced to unit availability is the expected generation. The failure of unit j is convolved with the equivalent curve, a new area is obtained and the next unit in the merit order is considered.

Cumulant method, developed by Gram and Charlier, allows the development of analytical representation of the area under the ELDC. In this way convolution is transformed into the addition of the cumulants of thermal units. In the cumulant method, the load distribution probability and unit failure are represented by a set of numbers, i.e., cumulants. These cumulants are functions of the moments (a mean value, variance, etc.) of probability functions.

Convolution in time is determined by the cumulant method, i. e., the moments method, where the LDC is represented using the Gram-Charlier's expansion. After calculating the ELDC, one can see that the convolutions included are simplified and replaced by multiplications and additions, what reduces the calculation time required for the Booth-Baleriaux's method. This simplification is very important for maintenance scheduling, because every time some thermal unit is out of the system (under maintenance) or is being connected to the system after maintenance completion, so convolutions and deconvolutions have to be performed and they require a long computer time. If equivalent thermal units are treated as having several segments, loading of higher segments requires the deconvolution of previously loaded segments.

Load cumulants:

$$ML_k = \frac{1}{i}\int_0^L l^{(k)} * LDC(1)dl \tag{13}$$

$$\begin{aligned}
LK_1 &= ML_1 \\
LK_2 &= ML_2 - ML_{1^2} \\
LK_3 &= ML_3 - 2ML_{1^3} - 3ML_1 ML_2 \\
LK_4 &= ML_4 - 3ML_{1^4} + 6ML_{1^2}\, ML_2 - 4ML_1 ML_3 \\
LK_5 &= ML_5 - 5ML_4 ML_1 + 10.\, ML_3 ML_{1^2} - 10ML_2 ML_{1^3} + 4ML_{1^5} \\
LK_6 &= ML_6 - 6ML_5 ML_1 + 15ML_4 ML_{1^2} - 20ML_3 ML_{1^3} + 15ML_2 ML_{1^4} - 5ML_{1^6}
\end{aligned} \tag{14}$$

In a similar way, generation cumulants $gK_j(k)$, k=1,..,6 are determined from the moments of failure distribution for unit j:

$$M_j(k) = r_j \times (C_j)^k \tag{15}$$

It should be noted that the moments, i. e., cumulants of thermal units are calculated once and these values are used throughout the calculation of maintenance period, whereas load moments have to be calculated once for each interval (52 load cumulants are calculated for a time horizon of one year and interval length of 7 days - 52 weeks).

As n thermal units are under consideration, cumulant k of ELDC is given by:

$$ELK_n(k) = LK_k + \sum_{j=1}^{n} gK_j(k) \quad for \quad k = 1,... \tag{16}$$

For n thermal units, the equivalent load duration curve is given by the Gram-Charlier's expansion:

$$\mathrm{ELDC}_n\,(z_l) = 1 - \int_{-\infty}^{z} N(l)dl + \frac{G_1}{3!}N^{(2)}(z_l) - \frac{G_2}{4!}N^{(3)}(z_l) - \frac{10G_1^2}{6!}N^{(5)}(z_l) \quad (17)$$

and

$$z_l = \frac{y-\mu}{\sigma}, \quad (18)$$

where:

$\mu = ELK_n(l)$ - mean value

$\sigma^2 = ELK_n(l)$ - variance

y - equivalent load level for which ELDC value is calculated (MW)

$G_1 = \frac{1}{\sigma^3} ELK_n(3)$

$G_2 = \frac{1}{\sigma^4} ELK_n(4)$

$N^{(i)}$ - i-th derivation of N(z)

$N(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$

Using linear approximation of ELDC_n, the expected unserved energy can be approximated by:

$$EUE = \frac{ELDC_J(C_j)}{2} \times \bar{i} \quad (19)$$

and loss of load probability by:

$$LOLP = ELDC_J(C_J) \quad (20)$$

where is:

C_J - sum of all thermal units capacities in the system.

The simulator includes: a demand model, hydro and thermal unit models, an exchange model and the model of pumped-storage units. System load is represented by a forecasted load duration curve (LDC). Hydro-power plants are represented by the expected weekly production of each hydro unit according to the result of long-term hydro production program. The availability of thermal units is calculated using their failure probability distribution function. These units are represented by a number of segments and are loaded according to the merit order in loading. The pumped-storage units are either in pumping or production mode of operation. Exchange contracts with other systems are specified through the agreed quantities of power and energy exchange (import/export).

The expected energy generation by thermal units is determined by their convolution/deconvolution under a LDC, depending on the merit order in loading. Hydro-power plants are loaded under the steep portion of LDC using the peak shaving technique.

The use of cumulants reduces the process of convolution and deconvolution with the load duration curve to additions and subtractions of load cumulants and the cumulants of generating units, i.e., exchange. The cumulant method also facilitates the calculation of reliability indices.

Simulation of thermal and hydro unit loading is performed during calculating the criterion function. The run-of-river hydro-power plants are loaded first, and then the thermal units according to the growth of their marginal costs. It must be checked continuously whether the available energy from hydro-power plants can satisfy the remaining demand. If the available hydro-energy is larger than required, unloading of the already loaded thermal units is performed. Operation costs are calculated in loading and unloading the thermal units, whereas EUE and LOLP are calculated in calculating the remaining unserved energy.

5. CONCLUSION

The mathematical model described in this paper has been developed so as to simulate as well as possible the electric power system operation, generation and loading of thermal and hydro units, respecting the existing operation mode and taking all uncertainties arising in planning into account. The software package is a powerful tool for maintenance scheduling and, perhaps even more, for rapidly testing various ideas and effects occurring when some parameters change. Maintenance scheduling is a discrete problem with a nonlinear criterion function and is very suitable for solving by dynamic programming. To overcome the dimensionality issue, the problem has been reduced to several smaller ones which are solved iteratively, by a monotone converging technique (DPSA). Another important problem is the calculation of criterion function. System operation costs and unreliability costs are calculated by a special simulator and cumulant method.

The software package has been developed for JP Elektroprivreda Srbije and tested for various characteristics and requirements. The required performances concerning both operation speed and the fidelity of simulation to real-life processes have been achieved. This software package, implemented in FORTRAN and ORACLE DBMS, provides all data and results required for maintenance scheduling purposes.

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