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A NOTE ON MULTI-CRITERIA INVENTORY CLASSIFICATION USING WEIGHTED LINEAR OPTIMIZATION

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Abstract: Recently, Ramanathan (R., Ramanathan, ABC inventory classification with multiple-criteria using weighted linear optimization, Computer and Operations Research, 33(3) (2006) 695-700) introduced a simple DEA-like model to classify inventory items on the basis of multiple criteria. However, the classification results produced by Ramanathan are not consistent with the domination concept encouraged some researchers to extend his model. In this paper, we produce the correct results and compare them to the original results and those of the extended models. We also improve this model to rank items with an optimal score of 1 using a cross-efficiency technique. The classification results are considerably different from the original results. Despite the fact that the correct results are obtained in this paper, there is no significant difference between the original model and its extensions, while the original model is more simple and suitable for the situations in which decision-maker cannot assign specific weights to individual criteria.

Keywords: Inventory classification, weighted linear optimization, multi-criteria decision-making.

AMS Subject Classification: 90B05, 90B05.

1. INTRODUCTION

Most organizations classify their inventory items into three classes: A - very important, B - of average importance, and C - relatively unimportant. The more important the inventory item, the greater the level of attention and control it receives. While the traditional classification approach defines the importance of inventory items in terms of their ‘annual dollar usage’, the multi-criteria classification approach - introduced by

Flores and Whybark [4], [5] also includes other criteria, such as lead time, criticality, availability, commonality, inventory cost, demand distribution, stock ability, etc.

Multi-criteria inventory classification has received much attention in recent years. Various heuristic and multi-criteria decision-making (MCDM) methods have been applied, such as Analytic Hierarchy Process (AHP) [6], [8], fuzzy AHP [3], [11], Technique for Order Preferences by Similarity to the Ideal Solution (TOPSIS) [2] and fuzzy rule-based approach [10].

Ramanathan [9] considered the importance of inventory items in terms of their ‘performance’, proposing a linear programming model to obtain an item’s the so-called ‘optimal score’. Although this model was to a large extent able to rank inventory items, the results it produced were not correct. In section two, we provide a brief introduction of Ramanathan’s model. In section three, we present the correct results we obtained and compare them to the results obtained in [9] and the extended models [7], [13]. In addition we extend the original model in order to be able to rank items with an optimal score of 1 using a cross-efficiency technique. We discuss our conclusion in section four.

2. WEIGHTED LINEAR OPTIMIZATION

Let us propose that N is the number of inventory items that have to be classified to the three classes of A, B and C based on J criteria. If we translate the ‘importance’ of each inventory item into its ‘performance’, the result is a Data Envelopment Analysis (DEA)-like model in which we consider each item as a Decision-Making Unit (DMU). If we suppose that y_{mj} denotes the performance of mth item (DMU) in terms of jth criterion, the proposed model in [9] is as follows.

$$
\begin{aligned}
&\max \ \sum_{j=1}^{J} v_{mj} y_{mj}, \\
&s.t. \ \sum_{j=1}^{J} v_{mj} y_{nj} \le 1, \quad n = 1,2,\ldots,N, \\
&\qquad v_{mj} \ge 0, \quad j = 1,2,\ldots,J.
\end{aligned}
\tag{1}
$$

where v_{mj} indicates the relative importance of criterion j for item m.

The result of this model would be the aggregated importance (performance) of item m. Solving this model repeatedly for each item provides us with the aggregated performance of all N items, assuming that all the criteria are positively related to the importance level of the item. If there are inversely related criteria, reciprocals of the scores could be used to turn them into positive criteria. This is a simple model that is suitable for multi-criteria inventory classification in situations when to determine the relative importance of individual criteria is impossible or very difficult. In other words, it is suitable for situations in which decision-maker cannot assign specific weights to individual criteria. This model has been applied to inventory classification using data provided by Flores *et al*. [6] for 47 items based on four criteria, namely, average unit cost, annual dollar usage, critical factor and lead time. Unfortunately, however, the results were neither optimal scores nor even feasible. In the next section, we produce the correct results and compare them to the results obtained by the extended versions of the original model.

3. RESULTS, DISCUSSION AND COMPARISON

Table 1 shows four criteria measures (columns 2-5) for 47 items (S1-S47). Column 6 indicates the original so-called optimal score of each item obtained in [9]. All four criteria are positively related to the importance (performance) level of inventory items. In addition, while we are using the proposed model for item *m*, the same weights are applied to all the 47 items. We therefore expect item *Sn* with equal or greater measures for all *J* criteria than that of *Sk* to be assigned to a more important class, or at least in the same one. In other words, if $y_{n1} \geq y_{k1}, y_{n2} \geq y_{k2}, \ldots, y_{nJ} \geq y_{kJ}$, logic would dictate that *Sn* is more important than *Sk*. However, when we look at the original results, we see many contradictions in the classification, for instance when we consider two items S10 and S16. While S10 shows higher or equal scores compared to S16 with regard to the four criteria measures, S10 was assigned to class C and S16 to class A. We reproduced the results (using the four criteria) and found that the optimal score of most items is incorrect, which means that the classification of items and the comparison to the traditional approach and AHP method presented in [9] are invalid. Column 7 of Table 1 shows the optimal scores we obtained, which in most cases are different from the original results.

In our reproduced results, 15 items have an integrated score of 1. In terms of DEA, we have 15 efficient DMUs, which means we should rank these efficient items (DMUs) as well. Generally speaking, the two most popular techniques to rank efficient DMUs are: (1) supper-efficiency [1], and (2) cross-efficiency [12]. Because there are many zeros in these kinds of data, the former technique may result in infeasibility [14], which is why we adopt the cross-efficiency ratio matrix to rank items with an aggregated score of 1. As pointed by Ramanathan [9], the weighted linear optimization model is in fact an output-maximizing multiplier DEA model with many outputs and a constant input. Consequently, the modified cross-efficiency ratio is formulated as follows:

$$E_{kl} = \sum_{j=1}^{J} v_{kj} y_{lj} \tag{2}$$

where E_{kl} denotes the efficiency of item *k* calculated by using optimal weights of item *l*.

Subsequently, the modified optimal score of efficient item *k*, O_k can be calculated as follows;

$$O_k = \frac{\sum_{l=1, l \neq k}^{Ne} E_{kl}}{Ne - 1} \tag{3}$$

where *Ne* indicates the number of items with an optimal score of 1 (efficient items).

We apply equation (3) to rank efficient items. The results are presented in column 2 of Table 2. Finally, the optimal scores are sorted in a descending order and, following [7], [9] and [13] the first 10 items are assigned to class A, the next 14 items to class B and the remaining 23 items to class C (column 3 of Table 2). Column 4 of Table 2 contains the original results obtained in [9]. It is clear that the classification of items based on the correct optimal scores is considerably different from the original classification. In all, 27 of the 47 items are classified differently.

Table 1: Criteria measures, and original and reproduced optimal scores

Item #	Average unit cost $	Annual dollar usage$	Critical factor	Lead time	Ramanathan's optimal scores	Reproduced optimal scores
S1	49.92	5840.64	1	2	0.619	1
S2	210	5670	1	5	0.451	1
S3	23.76	5037.12	1	4	0.054	1
S4	27.73	4769.56	0.01	1	1	0.817
S5	57.98	3478.8	0.5	3	0.308	0.613
S6	31.24	2936.67	0.5	3	0.429	0.573
S7	28.2	2820	0.5	3	0.378	0.566
S8	55	2640	0.01	4	0.267	0.690
S9	73.44	2423.52	1	6	0.164	1
S10	160.5	2407.5	0.5	4	0.404	0.781
S11	5.12	1075.2	1	2	0.12	1
S12	20.87	1043.5	0.5	5	0.04	0.732
S13	86.5	1038	1	7	0.098	1
S14	110.4	883.2	0.5	5	1	0.782
S15	71.2	845.4	1	3	0.968	1
S16	45	810	0.5	3	0.917	0.500
S17	14.66	703.68	0.5	4	0.799	0.578
S18	49.5	594	0.5	6	1	0.857
S19	47.5	570	0.5	5	0.866	0.714
S20	58.45	467.6	0.5	4	0.699	0.581
S21	24.4	463.6	1	4	0.697	1
S22	65	445	0.5	4	0.694	0.593
S23	86.5	432.5	1	4	0.715	1
S24	33.2	398.4	1	3	0.544	1
S25	37.05	370.5	0.01	1	0.419	0.188
S26	33.84	338.4	0.01	3	0.518	0.429
S27	84.03	336.12	0.01	1	0.671	0.400
S28	78.4	313.6	0.01	6	0.89	0.857
S29	134.34	268.68	0.01	7	1	1
S30	56	224	0.01	1	0.447	0.267
S31	72	216	0.5	5	0.724	0.714
S32	53.02	212.08	1	2	0.424	1
S33	49.48	197.92	0.01	5	0.717	0.714
S34	7.07	190.89	0.01	7	1	1
S35	60.6	181.8	0.01	3	0.467	0.436
S36	40.82	163.28	1	3	0.449	1
S37	30	150	0.01	5	0.714	0.714
S38	67.4	134.8	0.5	3	0.502	0.500
S39	59.6	119.2	0.01	5	0.714	0.714
S40	51.68	103.36	0.01	6	0.857	0.857
S41	19.8	79.2	0.01	2	0.287	0.286
S42	37.7	75.4	0.01	2	0.286	0.286
S43	29.89	59.78	0.01	5	0.714	0.714
S44	48.3	48.3	0.01	3	0.429	0.429
S45	34.4	34.4	0.01	7	1	1
S46	28.8	28.8	0.01	3	0.429	0.429
S47	8.46	25.38	0.01	5	0.714	0.714

Based on the erroneous results reported in [9], Zhou and Fan [13], and Ng [7] implicitly assume that Ramanathan's model is unable to provide a logical classification of inventory items, which is why they proposed two extended versions of the original model. Although the extended versions may have some advantages, they complicate matters for the average inventory manager. Additionally, while the weights of criteria are determined endogenously in [9], this procedure is changed to some extent in [7] and [13], which means they make the weights assigned to the criteria somewhat subjective. Zhou and Fan [13] use two sets of weights that are most favorable and least favorable for each item, while in [7] it is the decision-maker (DM) who ranks the importance of the criteria. It is clear that the main advantage of the model proposed in [9] is that it can determine the weights without relying on a DM. If the DM is able or allowed to determine the weight or rank of the criteria, there are some powerful alternative methods to classify inventory items proposed in [2], [3], [6], [8], [10] and [11]. Consequently, we believe that the model proposed in [9] is more suitable than [7] and [13] with regard to situations in which the DM cannot assign weights to the criteria, because the original model does not need any information from DM with regard to the importance of criteria.

Both [7] and [13] have considered the same data set, using the criteria of average unit cost, annual dollar usage and lead time and excluding the element of critical factor. Therefore, in order to compare the correct results of Ramanathan's model, we also exclude the critical factor. To our surprise, the (invalid) results obtained in [9] were reported in [7] erroneously which means that the comparison results of [7] are also not valid. For example, while S16 was assigned to class A in [9], in the results of [9] as presented in [7], it is assigned to class C. Columns 6 and 7 of Table 2 indicate the optimal score and classification of inventory items respectively by using the model (1) and the three criteria of average unit cost, annual dollar usage and lead time. Columns 8 and 9 show the classification results obtained in [13] and [7] respectively. Compared to [13] and [7] respectively, there are two and four different classifications in class A; five and six different classifications in class B; three and three different classifications in class C. The differences are caused by the fact that, in model (1), the criteria weights are determined completely endogenously while, as mentioned earlier, they are not obtained dependently in [13] and [7].

3. CONCLUSION

In this paper, we have presented the correct optimal scores and classification of the model introduced in [9]. We compared the correct results to two extended versions of the original model and found that there is no significant difference between them. The correct results obtained in this paper highlight the robustness of the original weighted linear optimization model compared to its extended versions. We also applied a cross-efficiency technique to rank items with an optimal score of 1. These items are considered with the same importance in the previous models. Therefore if in a real-world problem we have a considerable proportion of items with optimal score of 1, we cannot determine a reasonable cut-off point between class A and B using the original model and its two extended versions. Our extension enables the decision-maker to rank these items as well.

Table 2: Left: The results based on 4 criteria; Right: The results based on 3 criteria

Item #	Reproduced scores	Correct classification	Ramanathan's classification	Item #	Reproduced scores	Correct classification	Zhou & Fan classification	Ng classification
S3	1(0.855)	A	C	S1	1	A	A	A
S9	1(0.819)	A	C	S2	1	A	A	A
S13	1(0.888)	A	C	S13	1	A	A	A
S1	1(0.887)	A	B	S29	1	A	A	A
S2	1(0.732)	A	C	S34	1	A	B	B
S23	1(0.884)	A	B	S45	1	A	B	B
S21	1(0.764)	A	B	S9	0.946	A	A	A
S15	1(0.776)	A	A	S3	0.884	A	A	A
S24	1(0.784)	A	C	S18	0.857	A	A	B
S36	1(0.748)	A	C	S28	0.857	A	A	B
S11	1(0.158)	B	C	S40	0.857	B	B	B
S32	1(0.718)	B	C	S4	0.817	B	C	A
S29	1(0.139)	B	A	S10	0.781	B	A	A
S34	1(0.743)	B	A	S14	0.750	B	A	B
S45	1(0.139)	B	A	S12	0.732	B	B	B
S18	0.857	B	A	S19	0.714	B	B	B
S28	0.857	B	A	S31	0.714	B	B	B
S40	0.857	B	B	S33	0.714	B	B	B
S4	0.817	B	A	S37	0.714	B	B	C
S14	0.782	B	A	S39	0.714	B	B	B
S10	0.781	B	C	S43	0.714	B	C	C
S12	0.732	B	C	S47	0.714	B	C	C
S19	0.714	B	A	S8	0.690	B	B	B
S31	0.714	B	B	S5	0.613	B	B	A
S33	0.714	C	B	S23	0.596	C	B	B
S37	0.714	C	B	S17	0.578	C	C	C
S39	0.714	C	B	S22	0.575	C	B	C
S43	0.714	C	B	S6	0.573	C	C	A
S47	0.714	C	B	S20	0.572	C	B	C
S8	0.690	C	C	S21	0.571	C	C	C
S5	0.613	C	C	S7	0.566	C	C	B
S22	0.593	C	B	S15	0.466	C	C	C
S20	0.581	C	B	S38	0.453	C	C	C
S17	0.578	C	B	S16	0.450	C	C	C
S6	0.573	C	C	S35	0.436	C	C	C
S7	0.566	C	C	S24	0.429	C	C	C
S16	0.500	C	A	S26	0.429	C	C	C
S38	0.500	C	C	S36	0.429	C	C	C
S35	0.436	C	C	S44	0.429	C	C	C
S26	0.429	C	C	S46	0.429	C	C	C
S44	0.429	C	C	S27	0.400	C	C	C
S46	0.429	C	C	S11	0.331	C	C	C
S27	0.400	C	B	S32	0.322	C	C	C
S41	0.286	C	C	S41	0.286	C	C	C
S42	0.286	C	C	S42	0.286	C	C	C
S30	0.267	C	C	S30	0.267	C	C	C
S25	0.188	C	C	S25	0.188	C	C	C

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