

AN INTEGRATED SUPPLY CHAIN MODEL FOR THE PERISHABLE ITEMS WITH FUZZY PRODUCTION RATE AND FUZZY DEMAND RATE

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Abstract: In the changing market scenario, supply chain management is getting phenomenal importance amongst researchers. Studies on supply chain management have emphasized the importance of a long-term strategic relationship between the manufacturer, distributor and retailer. In the present paper, a model has been developed by assuming that the demand rate and production rate as triangular fuzzy numbers and items deteriorate at a constant rate. The expressions for the average inventory cost are obtained both in crisp and fuzzy sense. The fuzzy model is defuzzified using the fuzzy extension principle, and its optimization with respect to the decision variable is also carried out. Finally, an example is given to illustrate the model and sensitivity analysis is performed to study the effect of parameters.

Keywords: Fuzzy numbers, fuzzy demand, fuzzy production, integrated supply chain.

MSC: 90B30

1. INTRODUCTION

Today, the study of the supply chain model in a fuzzy environment is gaining phenomenal importance around the globe. In such a scenario, it is the need of the hour that a real supply chain be operated in an uncertain environment and the omission of any effects of uncertainty leads to inferior supply chain designs. Indeed, attention has been focused on the randomness aspect of uncertainty. Due to the increased awareness and

more receptiveness to innovative ideas, organizations today are constantly looking for newer and better avenues to reduce their costs and increase revenues. This particular study shows how organizations in a supply chain can use their resources for the best possible outcome.

In the crisp environment, all parameters in the total cost such as holding cost, set-up cost, purchasing price, rate of deterioration, demand rate, production rate etc. are known and have definite value without ambiguity. Some of the business situations fit such conditions, but in most of the situations and in the day-by-day changing market scenario the parameters and variables are highly uncertain or imprecise. For any particular problem in the crisp scenario, the aim is to maximize or minimize the objective function under the given constraint. But in many practical situations, the decision maker may not be in the position to specify the objective or the constraints precisely, but rather specify them uncertainly or imprecisely. Under such circumstances, uncertainties are treated as randomness and handled by appealing to probability theory. Probability distributions are estimated based on historical data. However, shorter and shorter product life cycles as well as growing innovation rates make the parameters extremely variable, and the collection of statistical data less and less reliable. In many cases, especially for new products, the probability is not known due to lack of historical data and adequate information. In such situations, these parameters and variables are treated as fuzzy parameters. The fuzzification grants authenticity to the model in the sense that it allows vagueness in the whole setup which brings it closer to reality. The defuzzification is used to determine the equivalent crisp value dealing with all uncertainty in the fuzzy value of a parameter. The fuzzy set theory was first introduced by Zadeh in 1965. Afterwards, significant research work has been done on defuzzification techniques of fuzzy numbers. In all of these techniques the parameters are replaced by their nearest crisp number/interval, and the reduced crisp objective function is optimized. Chang et al. (2004) presented a lead-time production model based on continuous review inventory systems, where the uncertainty of demand during lead-time was dealt with probabilistic fuzzy set and the annual average demand by a fuzzy number only. Chang et al. (2006) presented a model in which they considered a lead-time demand as fuzzy random variable instead of a probabilistic fuzzy set. Dutta et al. (2007) considered a continuous review inventory system, where the annual average demand was treated as a fuzzy random variable. The lead-time demand was also assessed by a triangular fuzzy number. Maiti and Maiti (2007) developed multi-item inventory models with stock dependent demand, and two storage facilities were developed in a fuzzy environment where processing time of each unit is fuzzy and the processing time of a lot is correlated with its size.

Better coordination amongst the producer, distributors and retailers is the key to success for every supply chain. The integration approach to supply chain management has been studied for years. Wee (1998) developed a lot-for-lot discount pricing policy for deteriorating items with constant demand rate. Yang and Wee (2000) considered multiple lot size deliveries. Yang and Wee (2003) developed an optimal quantity-discount pricing strategy in a collaborative system for deteriorating items with instantaneous replenishment rate. Wu and Choi (2005) assumed supplier-supplier relationships in the buyer-supplier triad. Lee and Wu (2006) developed a study on inventory replenishment policies in a two-echelon supply chain system. Chen and Kang (2007) thought out integrated vendor-buyer cooperative inventory models with variant permissible delay in

payments. Singh et al. (2007) discussed optimal policy for decaying items with stock-dependent demand under inflation in a supply chain. Chung and Wee (2007) developed, optimizing the economic lot, size of a three-stage supply chain with backordering derived without derivatives. Rau and Ouyang (2008) have introduced an optimal batch size for integrated production-inventory policy in a supply chain. Kim and Park (2008) have assumed development of a three-echelon SC model to optimize coordination costs.

Most of the references cited above have considered single echelon or multi echelon inventory models with crisp parameters only, and some who develop the inventory model with fuzzy parameter consider only the single echelon inventory model. In the past, researchers paid no or little attention to the coordination of the producer, the distributor and the retailers in the fuzzy environment.

In the present study, we have strived to develop a supply chain model for the situations when items deteriorate at a constant rate, and demand and the production rates are imprecise in nature. It is assumed that the producer supply n_d delivery to distributor and distributor, in turns, supplies n_r deliveries to retailer in each of his replenishment. In order to express the fuzziness of the production and demand rates, these are expressed as triangular fuzzy numbers. Expressions for the average inventory cost are obtained both in crisp and fuzzy sense. Later on, the fuzzy total cost is defuzzified using the fuzzy extension principle. Thereafter, it is optimized with respect to the decision variables. Finally, the model is illustrated with some numerical data.

2. ASSUMPTIONS AND NOTATIONS

In this research, an integrated supply chain model for the perishable items with fuzzy production rate and fuzzy demand rate is developed from the perspective of a manufacturer, distributor and retailer. We assume that the demand and the production rates are imprecise in nature and they have been represented by the triangular fuzzy numbers. Mathematical model in this paper is developed under the following assumptions.

Assumptions:

1. Model assumes a single producer, single distributor and a single retailer.
2. The production rate is finite and greater than the demand rate.
3. The production and demand rates are fuzzy in nature.
4. Shortages are not allowed.
5. Deterioration rate is constant.
6. Lead time is Zero.

Notations: The following notations have been used throughout the paper to develop the model:

P	Production rate
\tilde{P}	Fuzzy production rate
d	Demand rate
\tilde{d}	Fuzzy demand rate
$I_{p1}(t)$	Single-echelon inventory level of producer during period T_1
$I_{p2}(t)$	Single-echelon inventory level of producer during period T_2
T	Cycle time
T_1	Time period of production cycle when there is positive inventory
T_2	Time period of non-production cycle when there is positive inventory
θ	Deterioration rate of on-hand inventory
n_d	Integer number of deliveries from the producer to the distributor during of inventory cycle when there is positive inventory
n_r	Integer number of deliveries from the distributor to his retailer during each delivery he got from the producer
$I_d(t)$	Single echelon inventory level of distributor
$I_r(t)$	Single echelon inventory level of retailer
Q_p	Producer's production lot size
Q_d	Distributor's lot size
Q_r	Retailer's lot size
C_{1p}	Setup cost of the producer per production cycle
C_{1d}	Ordering cost of distributor per order
C_{1r}	Ordering cost of retailer per order
C_{2p}	Inventory carrying cost for the producer per year per unit
C_{2d}	Inventory carrying cost for distributor per year per unit
C_p	Cost of deteriorated unit for the producer
C_d	Cost of deteriorated unit for the distributor
C_r	Cost of deteriorated unit for the retailer
TC_p	Total cost of the producer
TC_d	Total cost of the distributor
TC_r	Total cost of the retailer
TC	The integrated total annual cost
\tilde{TC}	Fuzzified integrated total annual cost
$M_{\tilde{TC}}$	Defuzzified integrated total annual cost

3. CRISP MODEL

3.1. Producer's Inventory Model

Based on our assumptions, the producer starts the production with zero inventory level. Initially, the inventory levels increases at a finite rate ($P-d$) units per unit time and decreases at a constant deterioration rate of (θ), up to a time period T_1 at which production is stopped. Thereafter, the inventory level decreases due to the constant demand rate (d) units per unit time and at a constant deterioration rate (θ) for a period of time T_2 at which the inventory level reaches zero level again, as shown in Figure 1 given below.

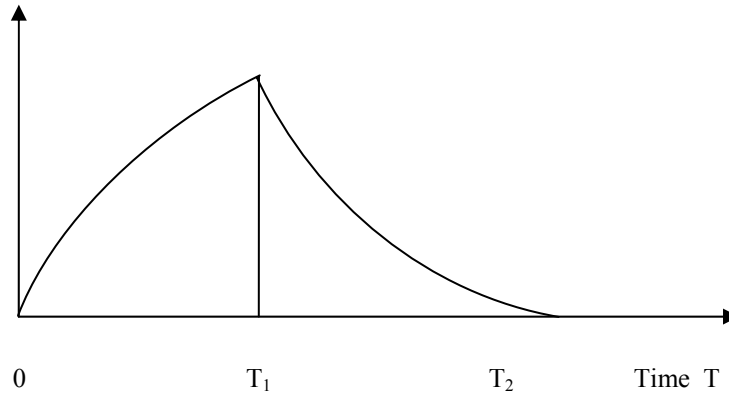


Figure 1: Producer's Inventory Level

The differential equations governing the single echelon producer model for different time durations are as follows:

$$I'_{p1}(t_1) = P - d - \theta I_{p1}(t_1), 0 \leq t_1 \leq T_1 \quad (1)$$

$$I'_{p2}(t_2) = -d - \theta I_{p2}(t_2), 0 \leq t_2 \leq T_2 \quad (2)$$

where $T = T_1 + T_2$ by solving the above equations with the boundary conditions

$$I_{p1}(0) = 0, I_{p2}(0) = Q_p \text{ and } I_{p2}(T_2) = 0$$

producer's inventory level $I_p(t)$ is given by

$$I_{p1}(t_1) = \frac{P-d}{\theta} [1 - e^{-\theta t_1}], 0 \leq t_1 \leq T_1 \quad (3)$$

$$I_{p2}(t_2) = \frac{d}{\theta} [e^{\theta(t_2 - T_2)} - 1], 0 \leq t_2 \leq T_2 \quad (4)$$

From the condition $I_{p1}(T_1) = Q_p = I_{p2}(0)$, we have

$$\begin{aligned} \frac{P-d}{\theta} [1 - e^{-\theta T_1}] &= Q_p = \frac{d}{\theta} [e^{\theta T_2} - 1] \\ T_2 &= \frac{1}{\theta} \ln \frac{[P - (P-d)e^{-\theta T_1}]}{d} \end{aligned} \quad (5)$$

Holding Cost of the Producer is

$$HC_p = C_{2p} \frac{P-d}{\theta^2} [e^{-\theta T_1} + \theta T_1 - 1] + C_{2p} \frac{d}{\theta^2} [e^{\theta(T-T_1)} - \theta(T-T_1) - 1]$$

Deterioration Cost of the Producer is

$$DC_p = C_p \frac{P-d}{\theta} [e^{-\theta T_1} + \theta T_1 - 1] + C_p \frac{d}{\theta} [e^{\theta(T-T_1)} - \theta(T-T_1) - 1]$$

The average total cost function TC_p for the producer is average of the sum of set-up cost, carrying cost and deterioration cost.

$$TC_p = \frac{C_{1p}}{T} + \frac{(C_{2p} + \theta C_p)(P-d)}{T} \frac{1}{\theta^2} \{e^{-\theta T_1} + \theta T_1 - 1\} + \frac{(C_{2p} + \theta C_p)d}{T} \frac{1}{\theta^2} \{e^{\theta(T-T_1)} - \theta(T-T_1) - 1\} \quad (6)$$

For the minimization of the total cost we have

$$\frac{d}{dT_1}(TC_p) = 0$$

This implies that $T_1 = \frac{1}{\theta} \ln \frac{[P-d + de^{\theta T}]}{P}$, putting this value in equation (5) we have T_2 , and then putting both of these values in the equation (6), we obtained the total cost for the producer.

3.2. Distributor's Inventory Model

Since the distributor receives a fixed quantity Q_d units in each of the replenishment, the distributor's cycle starts with the inventory levels Q_d units. Thereafter, inventory level decreases due to the constant demand rate of $(\frac{d}{n_d})$ units per unit time and at a constant deterioration rate (θ) , which reaches the zero level in the time period $\frac{T}{n_d}$, as shown in Figure 2 given below.

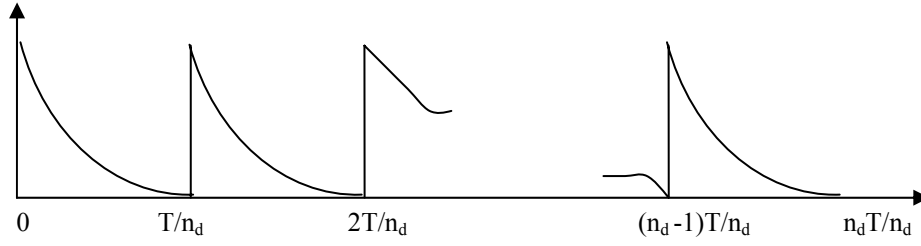


Figure 2 Distributor's Inventory level

Differential equations governing the distributor's inventory level are as follows

$$I_d'(t) = -\frac{d}{n_d} - \theta I_d(t), 0 \leq t \leq \frac{T}{n_d} \quad (7)$$

Solving the differential equation with boundary conditions $I_d(\frac{T}{n_d}) = 0$ gives

$$I_d(t) = \frac{d}{\theta n_d} \left[e^{\theta(\frac{T}{n_d}-t)} - 1 \right], 0 \leq t \leq \frac{T}{n_d} \quad (8)$$

Maximum Inventory of the distributor is

$$Q_d = \frac{d}{\theta n_d} \left[e^{\theta \frac{T}{n_d}} - 1 \right] \quad (9)$$

Holding cost of the distributor in each replenishment cycle is

$$HC_d = C_{2d} \frac{d}{\theta^2 n_d} \left[e^{\theta \frac{T}{n_d}} - \theta \frac{T}{n_d} - 1 \right]$$

Deterioration Cost of the distributor in each replenishment cycle is

$$DC_d = C_d \frac{d}{\theta n_d} \left[e^{\theta \frac{T}{n_d}} - \theta \frac{T}{n_d} - 1 \right]$$

Distributor's cost in each replenishment cycle is the sum of the ordering cost, carrying cost and deterioration cost.

Distributor's total cost function TC_d is the average of the sum of distributor's total annual ordering cost, carrying cost and deteriorating cost in n_d replenishments.

$$TC_d = \left[\frac{n_d C_{1d}}{T} + \frac{(C_{2d} + \theta C_d)}{T} \frac{d}{\theta^2} \left(e^{\theta \frac{T}{n_d}} - \theta \frac{T}{n_d} - 1 \right) \right] \quad (10)$$

3.3. The retailer's inventory model

Distributor, in turns, supplies n_r replenishments to the retailer in each of his replenishment cycles. In each replenishment, he supplies a fixed quantity Q_r to the retailer. Hence, retailer's inventory level starts with the quantity Q_r and then decreases due to the combined effect of both the constant demand and deterioration for a time period of $\frac{T}{n_d n_r}$ at which the inventory level reaches the zero level, as shown in Figure 3 given below.

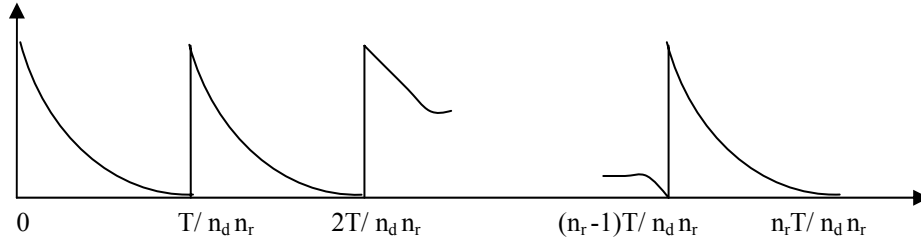


Figure 3 Retailer's Inventory level

Differential equations governing the retailer's inventory level are as follows

$$I_r'(t) = -\frac{d}{n_d n_r} - \theta I_r(t), 0 \leq t \leq \frac{T}{n_d n_r} \quad (11)$$

Solving the differential equation with boundary conditions $I_r(\frac{T}{n_d n_r}) = 0$ gives

$$I_r(t) = \frac{d}{\theta n_d n_r} \left[e^{\theta(\frac{T}{n_d n_r} - t)} - 1 \right], 0 \leq t \leq \frac{T}{n_d n_r} \quad (12)$$

Maximum Inventory of the retailer is

$$Q_r = \frac{d}{\theta n_d n_r} \left[e^{\theta \frac{T}{n_d n_r}} - 1 \right] \quad (13)$$

Retailer's holding cost in each replenishment he got is

$$HC_r = C_{2r} \frac{d}{\theta^2 n_d n_r} \left[e^{\theta \frac{T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1 \right]$$

Retailer's deterioration cost in each cycle is

$$HC_r = C_r \frac{d}{\theta n_d n_r} \left[e^{\frac{\theta T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1 \right]$$

Retailer's cost in each cycle is the sum of the ordering cost, holding cost and deterioration cost.

Retailer's average total cost function TC_r is the average of the sum of retailer's total annual ordering cost, carrying cost and deterioration cost in $n_d n_r$ replenishment cycles

$$TC_r = \left[\frac{n_d n_r C_{1r}}{T} + \frac{(C_{2r} + \theta C_r)}{T} \frac{d}{\theta^2} \left(e^{\frac{\theta T}{n_d n_r}} - \frac{\theta T}{n_d n_r} - 1 \right) \right] \quad (14)$$

The integrated joint total cost function TC for the producer, distributor and retailer is the sum of TC_p , TC_d , and TC_r .

$$TC = TC_p + TC_d + TC_r$$

$$TC = \frac{1}{T} \left[C_{1p} + n_d C_{1d} + n_d n_r C_{1r} + \frac{P}{\theta^2} (C_{2p} + \theta C_p) (e^{-\theta T_1} + \theta T_1 - 1) + \frac{d}{\theta^2} \left\{ (C_{2p} + \theta C_p) \left\{ e^{\theta(T-T_1)} - e^{-\theta T_1} - \theta T \right\} + (C_{2d} + \theta C_d) \left(e^{\frac{\theta T}{n_d}} - \theta \frac{T}{n_d} - 1 \right) + (C_{2r} + \theta C_r) \left(e^{\frac{\theta T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1 \right) \right\} \right] \quad (15)$$

$$TC = F_1(T) + PF_2(T) + dF_3(T) \quad (16)$$

where

$$F_1(T) = \frac{C_{1p} + n_d C_{1d} + n_d n_r C_{1r}}{T} \quad (17)$$

$$F_2(T) = \frac{(C_{2p} + \theta C_p)}{T \theta^2} (e^{-\theta T_1} + \theta T_1 - 1) \quad (18)$$

$$F_3(T) = \left[\frac{(C_{2p} + \theta C_p)}{T} \left\{ e^{\theta(T-T_1)} - e^{-\theta T_1} - \theta T \right\} + \frac{(C_{2d} + \theta C_d)}{T} \left(e^{\frac{\theta T}{n_d}} - \theta \frac{T}{n_d} - 1 \right) + \frac{(C_{2r} + \theta C_r)}{T} \left(e^{\frac{\theta T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1 \right) \right] \quad (19)$$

4. FUZZY MODEL BASED ON MODEL DEVELOPED IN SECTION 3

In a real situation and in a competitive market situation both the production rate and the demand rate are highly uncertain in nature. To deal with such a type of uncertainties in the super market, we consider these parameters to be fuzzy in nature.

In order to develop the model in a fuzzy environment, we consider the production rate p and the demand rate d as the triangular fuzzy numbers $\tilde{P} = (P_1, P_0, P_2)$ and $\tilde{d} = (d_1, d_0, d_2)$ respectively, where $P_1 = P - \Delta_1, P_0 = P, P_2 = P + \Delta_2$ and $d_1 = d - \Delta_3, d_0 = d$ and $d_2 = d + \Delta_4$, such that $0 < \Delta_1 < P, 0 < \Delta_2, 0 < \Delta_3 < d, 0 < \Delta_4$ and $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are determined by the decision maker based on the uncertainty of the problem. Thus, the production rate P and demand rate d are considered as the fuzzy numbers \tilde{P} and \tilde{d} with membership functions

$$\mu_{\tilde{P}}(P) = \begin{cases} \frac{P - P_1}{P_0 - P_1} & , P_1 \leq P \leq P_0 \\ \frac{P_2 - P}{P_2 - P_0} & , P_0 \leq P \leq P_2 \\ 0 & , otherwise \end{cases} \quad (20)$$

$$\mu_{\tilde{d}}(d) = \begin{cases} \frac{d - d_1}{d_0 - d_1} & , d_1 \leq d \leq d_0 \\ \frac{d_2 - d}{d_2 - d_0} & , d_0 \leq d \leq d_2 \\ 0 & , otherwise \end{cases} \quad (21)$$

Defuzzification of \tilde{P} and \tilde{d} by the centroid method is given by

$$M_p = \frac{P_1 + P_0 + P_2}{3} = P + \frac{1}{3}(\Delta_2 - \Delta_1)$$

$$M_d = \frac{d_1 + d_0 + d_2}{3} = d + \frac{1}{3}(\Delta_4 - \Delta_3), \text{ respectively}$$

For fixed value of T:

$$TC = \frac{1}{T} \left[C_{1p} + n_d C_{1d} + n_d n_r C_{1r} + \frac{P}{\theta^2} (C_{2p} + \theta C_p) (e^{-\theta T} + \theta T - 1) + \frac{d}{\theta^2} \{ (C_{2p} + \theta C_p) \{ e^{\theta(T-T_1)} - e^{-\theta T} - \theta T \} + (C_{2d} + \theta C_d) (e^{\frac{\theta T}{n_d}} - \theta \frac{T}{n_d} - 1) + (C_{2r} + \theta C_r) (e^{\frac{\theta T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1) \} \right]$$

$$TC = F_1(T) + PF_2(T) + dF_3(T)$$

where

$$F_1(T) = \frac{C_{1p} + n_d C_{1d} + n_d n_r C_{1r}}{T}$$

$$F_2(T) = \frac{(C_{2p} + \theta C_p)}{T\theta^2} (e^{-\theta T_1} + \theta T_1 - 1)$$

$$F_3(T) = \left[\frac{(C_{2p} + \theta C_p)}{T} \{e^{\theta(T-T_1)} - e^{-\theta T_1} - \theta T\} + \frac{(C_{2d} + \theta C_d)}{T} (e^{\theta \frac{T}{n_d}} - \theta \frac{T}{n_d} - 1) + \frac{(C_{2r} + \theta C_r)}{T} (e^{\theta \frac{T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1) \right]$$

Let $TC = y$, this implies that

$$p = \frac{y - F_1 - dF_3}{F_2}$$

$$\mu_{\bar{p}} \left(\frac{y - F_1 - dF_3}{F_2} \right)$$

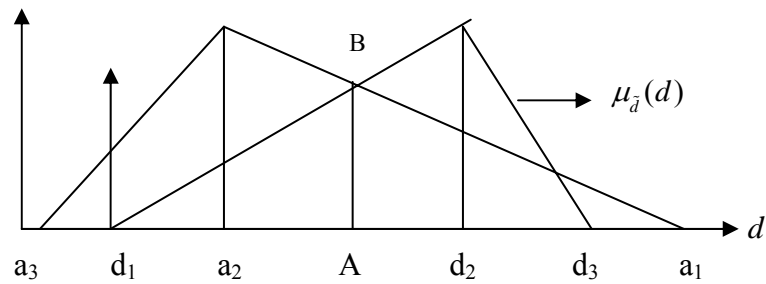


Figure 4 $\mu_{TC}(y) = AB$

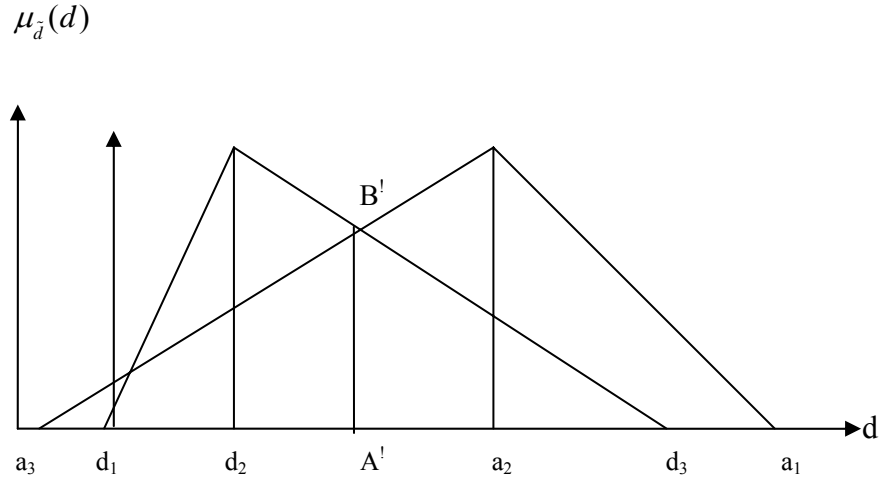


Figure 5 $\mu_{TC}(y) = A'B'$

The membership of the fuzzy cost function given by the extension principle is

$$\begin{aligned} \mu_{TC}(y) &= \text{Sup}_{(P,d) \in (TC)^{-1}(y)} [\mu_{\bar{P}}(P) \wedge \mu_{\bar{d}}(d)] \\ &= \text{Sup}_{d_1 \leq d \leq d_2} \left[\mu_{\bar{P}}\left(\frac{y-F_1-dF_3}{F_2}\right) \wedge \mu_{\bar{d}}(d) \right] \end{aligned} \quad (22)$$

Now

$$\mu_{\bar{P}}\left(\frac{y-F_1-dF_3}{F_2}\right) = \begin{cases} \frac{P_2F_2 + dF_3 + F_1 - y}{(P_2 - P_0)F_2} & , a_3 \leq d \leq a_2 \\ \frac{y - F_1 - dF_3 - P_1F_2}{(P_0 - P_1)F_2} & , a_2 \leq d \leq a_1 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Where

$$a_1 = \frac{y - F_1 - P_1F_2}{F_3}, a_2 = \frac{y - F_1 - P_0F_2}{F_3} \text{ and } a_3 = \frac{y - F_1 - P_2F_2}{F_3}$$

When $a_2 \leq d_0$ and $u_1 \geq d_1$, i.e. when $y \geq F_1 + P_1F_2 + d_1F_3$ and $y \leq F_1 + P_0F_2 + d_0F_3$, Figure 1 exhibits the Graphs of $\mu_{\bar{P}}\left(\frac{y-F_1-dF_3}{F_2}\right)$ and $\mu_{\bar{d}}(d)$.

It is clear that for every $y \in [F_1 + P_1F_2 + d_1F_3, F_1 + P_0F_2 + d_0F_3]$, $\mu_{\bar{y}}(y) = AB$. The value of AB is then calculated by solving the first equation of (21) and the second equation of (23), i.e.

$$\frac{d - d_1}{d_0 - d_1} = \frac{y - F_1 - dF_3 - P_1F_2}{(P_0 - P_1)F_2} \text{ or}$$

$$d = \frac{(y - F_1 - P_1F_2)(d_0 - d_1) + d_1(P_0 - P_1)F_2}{(P_0 - P_1)F_2 + (d_0 - d_1)F_3}$$

Therefore,

$$AB = \frac{d - d_1}{d_0 - d_1}$$

$$= \frac{y - F_1 - P_1F_2 - d_1F_3}{(P_0 - P_1)F_2 + (d_0 - d_1)F_3} = \mu_1(y)$$

When $a_3 \leq d_2$ and $u_2 \geq d_0$, i.e. when $y \geq F_1 + P_0F_2 + d_0F_3$

and $y \leq F_1 + P_2F_2 + d_2F_3$, Figure 2 exhibits the graph of $\mu_{\bar{p}}\left(\frac{y - F_1 - dF_3}{F_2}\right)$ and $\mu_{\bar{d}}(d)$.

The value of A^1B^1 is calculated by solving the second equation of (21) and the first equation of (23), i.e.

$$\frac{d_2 - d}{d_2 - d_0} = \frac{P_2F_2 + dF_3 + F_1 - y}{(P_2 - P_0)F_2} \text{ or } d = \frac{d_0(P_2 - P_0)F_2 - (P_2F_2 + F_1 - y)(d_2 - d_0)}{(P_2 - P_0)F_2 + (d_2 - d_0)F_3}$$

Therefore,

$$A^1B^1 = \frac{d_2 - d}{d_2 - d_0}$$

$$= \frac{P_2F_2 + d_2F_3 + F_1 - y}{(P_2 - P_0)F_2 + (d_2 - d_0)F_3} = \mu_2(y) \text{ (say)}$$
(25)

Membership function for the fuzzy total cost is given as below:

$$\mu_{\bar{TC}}(y) = \begin{cases} \mu_1(y) & , F_1 + P_1F_2 + d_1F_3 \leq y \leq F_1 + P_0F_2 + d_0F_3 \\ \mu_2(y) & , F_1 + P_0F_2 + d_0F_3 \leq y \leq F_1 + P_2F_2 + d_2F_3 \\ \text{otherwise} & \end{cases} \quad (26)$$

Now let

$$P_1 = \int_{-\infty}^{\infty} \mu_{\bar{TC}}(y) dy \text{ and } R_1 = \int_{-\infty}^{\infty} y \mu_{\bar{TC}}(y) dy$$

Defuzzification for the fuzzy total cost, given by the centroid method, is

$$M_{\bar{r}C}(T_1) = \frac{R_1}{P_1}$$

$$= F_1(T) + PF_2(T) + dF_3(T) + \frac{1}{3}\{(\Delta_2 - \Delta_1)F_2(T) + (\Delta_4 - \Delta_3)F_3(T)\}$$

Where $F_1(T)$, $F_2(T)$ and $F_3(T)$ are given by (17), (18) and (19) respectively.

$$M_{\bar{r}C}(T_1) = \frac{1}{T} \left[C_{1p} + n_d C_{1d} + n_d n_r C_{1r} + \frac{\left\{ P + \frac{1}{3}(\Delta_2 - \Delta_1) \right\}}{\theta^2} (C_{2p} + \theta C_p) \right.$$

$$\left. (e^{-\theta T_1} + \theta T_1 - 1) + \frac{\left\{ d + \frac{1}{3}(\Delta_4 - \Delta_3) \right\}}{\theta^2} \left\{ (C_{2p} + \theta C_p) \left\{ e^{\theta(T-T_1)} - e^{-\theta T_1} - \theta T \right\} + \right. \right.$$

$$\left. \left. (C_{2d} + \theta C_d) \left(e^{\frac{\theta T}{n_d}} - \theta \frac{T}{n_d} - 1 \right) + (C_{2r} + \theta C_r) \left(e^{\frac{\theta T}{n_d n_r}} - \theta \frac{T}{n_d n_r} - 1 \right) \right\} \right] \quad (27)$$

To minimize the total average cost per unit time, optimal value of T_1 (say T_1^*) is obtained by solving the following equation

$$\frac{d}{dT_1} M_{\bar{r}C}(T_1) = 0 \text{ which implies that}$$

$$T_1^* = \frac{1}{\theta} \ln \frac{\left[\left\{ P + \frac{1}{3}(\Delta_2 - \Delta_1) \right\} + \left\{ d + \frac{1}{3}(\Delta_4 - \Delta_3) \right\} (e^{\theta T} - 1) \right]}{\left\{ P + \frac{1}{3}(\Delta_2 - \Delta_1) \right\}} \quad (28)$$

$$\frac{d^2}{dT_1^2} M_{\bar{r}C}(T_1) = \left\{ P + \frac{1}{3}(\Delta_2 - \Delta_1) \right\} \theta^2 e^{-\theta T_1} +$$

$$\left\{ d + \frac{1}{3}(\Delta_4 - \Delta_3) \right\} \theta^2 \left(e^{\theta(T-T_1)} - e^{-\theta T_1} \right)$$

$$\text{and } \left[\frac{d^2}{dT_1^2} M_{\bar{r}C}(T_1) \right]_{T_1=T_1^*} > 0$$

Hence, the cost function is minimized at $T_1 = T_1^*$ and the minimum cost is given by

$$\left[M_{\bar{r}C}(T_1) \right]_{T_1=T_1^*}$$

5. NUMERICAL EXAMPLE

5.1. Crisp Model

To illustrate the proposed model, we consider that the producer supplies five deliveries to the distributor. Distributor in turn supplies six deliveries to the retailer in each of the replenishments he gets from the producer. We assume the production rate is $P = 20000$ units per year and the total demand is 12000 units per year while the rate of deterioration is 0.01 per year. In this sequence, we consider that the ordering cost is \$80, \$400 per order for retailer and distributor respectively and the production set-up cost is \$8000 per production. We also assume that the carrying costs per year for producer, distributor and retailer are \$20, \$35 and \$150 respectively. Similarly, the deterioration costs per unit for the producer, distributor and retailer are taken as \$100, \$150 and \$200 respectively. We also consider that the time horizon is finite, in particular – one year. Using the above data, the optimal values for the production time with minimum total cost have been calculated and the results are tabulated in Table 1.

Table 1: Results for the crisp model:

T_1	Q_p	Q_d	Q_r	TC_p	TC_d	TC_r	TC
0.60	4807	483	14	50900.62	47273.54	33176.00	131350.00

5.2. Fuzzy Model

In addition to the study on the model in fuzzy environment, the production and the demand rate are considered as the triangular fuzzy numbers (17000, 20000, 25000) and (10800, 12000, 14000) respectively, and all other data remain the same as in crisp model i.e. $\theta = 0.01$, $C_{1p} = \$ 8000$, $C_{1d} = \$ 400$, $C_{1r} = \$ 80$, $C_{2p} = \$ 20$, $C_{2d} = \$ 35$, $C_{2r} = \$ 150$, $C_p = \$ 100$, $C_d = \$ 150$, $C_r = \$ 200$, $\Delta_1 = 3000$, $\Delta_2 = 5000$, $\Delta_3 = 1200$, $\Delta_4 = 2000$. Using the above data, the optimal production time with various costs has been calculated and the results are displayed in Table 2.

5.3. Sensitivity Analysis

A sensitivity analysis is performed for the fuzzy model with respect to various parameters. Results are calculated and tabulated in the Table 3.

Table 3: Sensitivity analysis with respect to the various parameters for the fuzzy model:

Parameters	% Changes	Q_p^*	TC^*	T_1^*	T_2^*
Δ_1	-33.33	5247	136017	0.585	0.415
	-16.67	5147	135349	0.590	0.410
	+16.66	4947	134212	0.599	0.401
	+33.33	4847	133557	0.604	0.396
Δ_2	-30.00	4746	132907	0.609	0.391
	-16.00	4887	133634	0.603	0.397
	+16.00	5207	135749	0.587	0.413
	+33.00	5347	136500	0.581	0.419
Δ_3	-33.33	5276	135313	0601	0.399
	-16.67	5290	135061	0598	0.402
	+16.67	5316	134365	0.592	0.408
	+33.33	5319	134226	0.588	0.412
Δ_4	-25.00	5330	133968	0.587	0.413
	-10.00	5316	135303	0.592	0.408
	+10.00	5290	137081	0.598	0.402
	+25.00	5273	138416	0.603	0.397
P	-25.00	2667	110183	0.784	0216
	-10.00	4216	126536	0.658	0.342
	+10.00	5645	141608	0.542	0458
	+25.00	6431	149770	0.479	0.521
d	-50.00	4390	91496	0.304	0.696
	-25.00	4140	117614	0.450	0.550
	+25.00	3993	142736	0.739	0.261
	+50.00	2121	141421	0.884	0.116

6. OBSERVATIONS

Based on the sensitivity analysis, it is observed that the fuzzy expected cost is slightly higher than the crisp total cost, while the optimal production time in the fuzzy sense is decreased. As a result, the amount of economic production quantities decreased. The various observations are shown below.

The following observations have been made during the sensitivity analysis:

1. Total cost obtained in the fuzzy sense is slightly higher than the crisp total cost.
2. Optimal production length is slightly lower than the crisp cycle length.
3. It is observed that the optimal manufactured quantity obtained in the fuzzy sense is larger than the crisp optimal manufactured quantity.
4. As Δ_1 increases total cost TC^* increases and the optimal production quantity Q_p^* = decreases.
5. As Δ_2 increases both the total cost TC^* and the optimal production quantity Q_p^* increases. As Δ_3 increases, total cost TC^* decreases and the optimal production

quantity Q_p^* increases. As Δ_4 increases total cost TC^* increases and the optimal production quantity Q_p^* decreases. As P increases both the total cost TC^* and the optimal production quantity Q_p^* increases. As d increases total cost TC^* increases and the optimal production quantity Q_p^* decreases.

The overall observation from Table 3 is that in any case the total cost does not vary much from its original value. This is the most distinguished feature of the whole study. This finding is more than sufficient to justify the whole fuzzification process.

7. CONCLUSIONS

This study develops an integrated supply chain, multi-echelon deteriorating inventory model in the fuzzy environment. We have strived to develop a supply chain model for the situations when items deteriorate at a constant rate, the demand and production rates are imprecise in nature. It is assumed that the producer supplies n_d delivery to distributor and distributor, in turns, supplies n_r deliveries to retailer in each of his replenishment. In the development of inventory models, most of the previous researchers have considered the production rate and demand rate as constant quantity. Sometimes, a situation occurs when it is not possible to provide exact data, or if we consider realistic situations, these quantities are not exactly constant, but have little variations compared to the actual values. With fuzzy models, however, we have the advantage that, instead of providing the exact values for the variables, we are required to provide a range with the help of membership functions. This led us to developing a model with fuzzy production rate and fuzzy demand rate. Production and demand rates are taken as triangular fuzzy numbers and the membership function for the fuzzy total cost is obtained by using extension principle. The total cost, as suggested by the fuzzy approach, is far more practical and realistic than the crisp approach and provides a better chance for attainment. The sensitivity analysis shows in Table 3 that the total cost does not vary much from its original value in any case; therefore, the developed model is very stable and promises a better deal to the inventory manager.

Our analysis is the first step. In the next step, we will extend our approach and thoughts to the supply chain models with more innovative ideas, such as models with uncertain lead time problem, the model with shortages and partially backlogging and price discount with different demand and deterioration rates.

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