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A NEW EFFICIENT TRANSFORMATION OF THE GENERALIZED VEHICLE ROUTING PROBLEM INTO THE CLASSICAL VEHICLE ROUTING PROBLEM

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Abstract: Classical combinatorial optimization problems can be generalized in a natural way by considering a related problem relative to a given partition of the nodes of the graph into node sets. In the literature one can find generalized problems such as: generalized minimum spanning tree, generalized traveling salesman problem, generalized Steiner tree problem, generalized vehicle routing problem, etc. These generalized problems typically belong to the class of NP-complete problems; they are harder than the classical ones, and nowadays are intensively studied due to their interesting properties and applications in the real world. Because of the complexity of finding the optimal or near-optimal solution in case of the generalized combinatorial optimization problems, great effort has been made, by many researchers, to develop efficient ways of their transformation into classical corresponding variants. We present in this paper an efficient way of transforming the generalized vehicle routing problem into the vehicle routing problem, and a new integer programming formulation of the problem.

Keywords: Combinatorial optimization, efficient transformations, generalized combinatorial optimization problems, integer programming.

MCN: 90C27, 90C10, 68M10.

1. INTRODUCTION

Combinatorial optimization is a branch of optimization; its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution.

The challenge of combinatorial optimization is to develop the algorithms for which the number of elementary computational steps is acceptably small.

The study of combinatorial optimization owes its existence to the advent of modern digital computer. Most of the currently accepted methods for solving combinatorial optimization problems would have hardly been taken seriously 30 years ago, for no one could have carried out the computations involved. Moreover, the existence of digital computers created a multitude of technical problems with the combinatorial character.

A large number of combinatorial optimization problems were generated by research in computer design, the theory of computation, and by the application of computers to a myriad of numerical and non-numerical problems, which required new methods, new approaches and new mathematical insights.

Combinatorial optimization problems can be generalized in a natural way by considering a related problem relative to a given partition of the nodes of the graph into node sets, while the feasibility constraints are expressed in terms of clusters. In this way, the class of generalized combinatorial optimization problems is introduced. In the literature we can find several generalized problems such as: generalized minimum spanning tree, generalized traveling salesman problem, generalized vehicle routing problem, generalized (subset) assignment problem, etc. These generalized problems belong to the class of *NP*-complete problems; they are harder than the classical ones, and nowadays are intensively studied due to their interesting properties and applications in the real world; though, many practitioners are reluctant to use them for practical modeling problems because of their complexity of finding optimal or near-optimal solutions.

For this class of problems, several approaches were developed: exact algorithms [22], approximation algorithms [19] and relaxation methods [20], so as several models based on integer programming [18].

The complexity of obtaining optimum or even near-optimal solutions for the generalized combinatorial optimization problems led to the development of:

- several metaheuristics: tabu search [16], simulated annealing [23], genetic algorithms, variable neighborhood search [8], etc;
- efficient transformations of the generalized combinatorial optimization problems into classical combinatorial optimization problems [3,4,6,8,14].

Regarding the latest approach, there are at least two reasons for which it seems appropriate:

- first, the generalized combinatorial optimization problems are natural extensions of combinatorial optimization problems, and we may take advantage of the similarities between them;
- second, there are several efficient methods for solving classical combinatorial optimization problems.

In the present paper we confine ourselves to the generalized vehicle routing problem denoted by GVRP.

The aim of this paper is to describe a new integer programming formulation of the GVRP, and to present a new efficient transformation of the GVRP into the classical vehicle routing problem (VRP).

2. DEFINITION OF THE GENERALIZED VEHICLE ROUTING PROBLEM

The Vehicle Routing Problem (VRP) calls for determination of the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers, which is the subject to various constraints such as vehicle capacity, route length, time windows, etc. It is one of the most important and most studied combinatorial optimization problems.

The VRP has a significant economic importance due to its numerous practical applications in the field of distribution, collection, logistics, etc. A wide body of literature exists on the VRP problem (for an extensive bibliography, see Laporte and Osman [14], Laporte [12], etc).

The Generalized Vehicle Routing Problem (GVRP) is a generalization of the Vehicle Routing Problem (VRP) introduced by Ghiani and Improta [6]. Given is a directed graph whose nodes are partitioned into a given number of nodes sets (called clusters), the GVRP has to find the optimal routes from the given depot to the number of predefined clusters ,which include exactly one node from each cluster. They proposed, as well, a solution procedure by transforming the GVRP into a Capacitated Arc Routing problem, for which exact algorithms and several approximate procedures are reported in literature. Integer programming formulations for the GVRP were proposed by Kara and Bektas [10] and Kara and Pop [11]. In [21], Pop *et al.* proposed a metaheuristic algorithm for solving the GVRP based on ant colony optimization; it was tested on several benchmark problems drawn from *TSPLIB* library test problems. As far as we know, this is the only method proposed for solving the GVRP.

The GVRP is able to model the distribution of goods by sea to a number of customers situated in an archipelago as in Philippines, New Zealand, Indonesia, Italy, Greece and Croatia. In this application, a number of potential harbors are selected for every island, and a fleet of ships is required to visit exactly one harbor for every island.

Several applications of the GTSP (Laporte et al. [13]) may be extended naturally to GVRP. In addition, several other situations can be modeled as a GVRP, which include:

- the Traveling Salesman Problem (TSP) with profits (Feillet et al. [5]);
- a number of Vehicle Routing Problem (VRP) extensions: the VRP with selective backhauls, the covering VRP, the periodic VRP, the capacitated general windy routing problem, etc.;
- the design of tandem configurations for automated guided vehicles (Baldacci et al. [2]).

Let $G=(N,A)$ be a directed graph with $N=\{0,1,2,...,n\}$ as the set of nodes and $A=\{(i,j)\,|\,i,j\in N, i\neq j\}$. We assume that a nonnegative cost denoted by c_{ij} is associated with each arc $(i, j) \in N$. Node *0* represents the depot and remaining *n* nodes represent geographically dispersed customers. The node set N is partitioned into *k* mutually exclusive nonempty subsets V_i such that $N = V_0 \cup V_1 \cup ... \cup V_k$, where $V_0 = \{0\}$ is the depot (origin). Each customer has a certain amount of demands and the total demand of each cluster can be satisfied via any of its nodes. There exist *m* identical vehicles, each with a capacity *Q*.

Then the generalized vehicle routing problem (GVRP) consists of finding the minimum total cost tours of starting and ending at the depot, such that each cluster should be visited exactly once, the entering and leaving nodes of each cluster is the same, and the sum of all the demands of any tour (route) does not exceed the capacity of the vehicle *Q*. An illustrative scheme of the GVRP and a feasible tour is shown in the next figure.

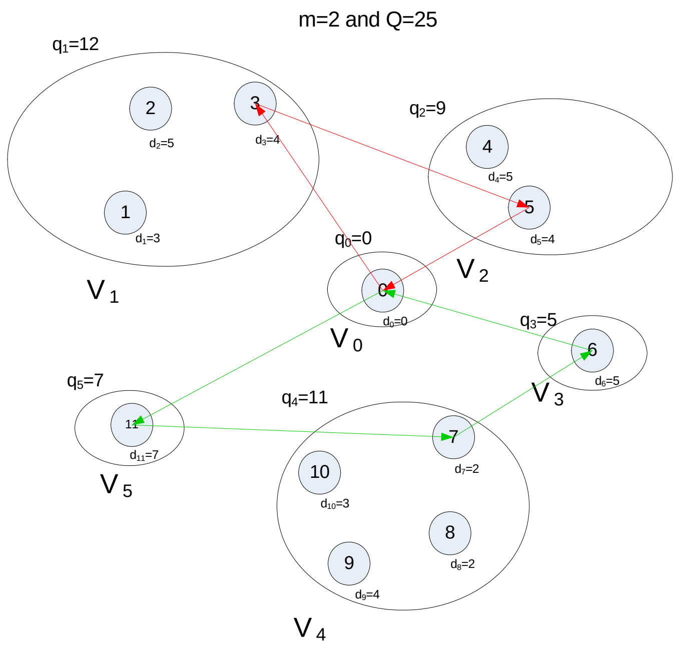

Figure 1: A feasible solution to the Generalized Vehicle Routing Problem

The GVRP is NP-hard because it includes the generalized traveling salesman problem (GTSP) as a special case when $m = 1$ and $Q = \infty$.

3. A NEW INTEGER PROGRAMMING FORMULATION OF THE GVRP

In 2003, Kara and Bektas [10] proposed an integer programming formulation for GVRP with a polynomially increasing number of binary variables and constraints; and in 2008 Kara and Pop [11] presented two integer linear programming formulations for GVRP with $O(n^2)$ binary variables and $O(n^2)$ constraints.

The model for the GVRP has the following parameters:

n is the number of customers which are partitioned into a given number of clusters;

m is the number of vehicles;

q_i denotes the demand of a customer *i* (in same units as vehicle capacity);

c_{ij} is the cost of traveling from a customer *i* to a customer *j*.

All the parameters are considered as non-negative. A homogeneous fleet of vehicles with a limited capacity *Q* and a central depot, with index *0*, makes deliveries to exactly one customer from each cluster. The problem is to determine the exact tour for each vehicle starting and ending at the depot and visiting exactly one customer from each cluster. The sum over the demands of the customers in every tour has to be within the limits of the vehicle capacity. The objective is to minimize the total travel cost. That could also be the distance between the nodes or other quantities on which the quality of the solution depends, based on the problem to be solved. Hereafter, it will be referred to as a cost.

The mathematical model is defined on a graph (N, A). The node set *N* corresponds to the set of customers from 1 to n and in addition to the depot number *0*. The arc set *A* consists of possible connections between the nodes. A connection between every two nodes in the graph will be included in *A* here. Each arc $(i, j) \in A$ has a travel cost c_{ij} associated to it. It is assumed that the cost is symmetric, i.e. $c_{ij} = c_{ij}$, and also that $c_{ij} = 0$. The set of uniform vehicles is *V*. The vehicles have a capacity *Q*, and all customers have a demand d_j.

In order to model the GVRP as an integer programming, we consider two binary variables: $x^v{}_{ij} = 1$, if the vehicle v drives from the customer i to the customer j, and $x^v{}_{ij} = 0$, otherwise; $z_i = 1$, if the customer i is selected and $z_i = 0$ otherwise.

The objective function and the constraints of the mathematical model of the GVRP can be described as follows:

minimize $\sum_{v \in V} \sum_{(i,j) \in A} c_{ij} x_{ij}^v$

subject to

$$\sum_{i \in V_l} z_i = 1, \quad for \quad 1 = 1, \dots, k \tag{1}$$

$$\sum_{v \in V} \sum_{j \in N} x_{ij}^v = z_i, \forall i \in \{1, \dots, n\}, \tag{2}$$

$$\sum_{i \in N \setminus \{0\}} d_i \sum_{j \in N} x_{ij}^v \le Q, \ \forall \text{ v} \in \text{V} \tag{3}$$

$$\sum_{i\in N\setminus\{0\}} x_{0j}^{v} = 1 \ , \forall \text{ v} \in \text{V} \tag{4}$$

$$\sum_{i\in N} x_{ik}^{v} - \sum_{j\in N} x_{kj}^{v} = 0, \forall k \in N\setminus\{0\}, \forall v \in V \tag{5}$$

$$x_{ij}^{v} \in \{0,1\}, z_i \in \{0,1\} \tag{6}$$

Constraints (1) are to make sure that exactly one customer is visited from each cluster, and the constraint (2) is to make sure that each visited customer is assigned to exactly one vehicle. In equation (3) the capacity constraints are stated: the sum over the demands of the customers within each vehicle v has to be less than or equal to the capacity of the vehicle. The flow constraints are shown in equations (4) and (5). Firstly, each vehicle can only leave the depot once. Secondly, the number of vehicles entering every customer *k* and the depot must be equal to the number of vehicles leaving. Finally, constraints (6) are the integrality constraints.

An even simpler version could have a constant number of vehicles, but here the number of vehicles can be modified in order to obtain the smallest possible cost. However, there is a lower bound of the number of vehicles, which is the smallest number of vehicles that can carry the total demand of the customers:

$$\lceil (\Sigma_{i\in N\setminus\{0\}} d_i \Sigma_{j\in N} x_{ij}^{v}) / \mathbb{Q} \rceil.$$

4. A NUMERICAL EXAMPLE

In this section we solve optimally the numerical example described by Ghiani and Improta [5] using our novel integer programming formulation of the GVRP. The example was derived from an VRP instance, namely test problem 7, introduced by Araque et al. [1] and has 50 vertices, 25 clusters and 4 vehicles. This instance is a randomly generated problem and was generated by placing the customers randomly on a square area. The vertex coordinates of the depot are (50,50) and of the customers are:

1. (10,42)	2. (23,6)	3. (8,46)	4. (51,29)	5. (64,24)
6. (28,6)	7. (30,69)	8. (87,3)	9. (13,56)	10.(80,87)
11.(95,41)	12. (92,21)	13. (86,10)	14. (39,45)	15. (76,44)
16.(30,34)	17. (47,3)	18. (84,70)	19. (65,5)	20. (98,18)
21. (3,13)	22. (52,98)	23. (73,17)	24. (48,82)	25. (28,32)
26. (16,86)	27. (50,56)	28. (53,72)	29. (75,89)	30. (41,38)
31. (11,28)	32. (76,57)	33. (86,18)	34. (34,19)	35. (70,25)
36. (79,50)	37. (25,13)	38. (55,94)	39. (4,33)	40. (8,66)
41. (51,70)	42. (3,22)	43. (93,17)	44. (96,45)	45. (71,85)
46. (39,32)	47. (37,71)	48. (79,45)	49. (96,66)	50. (69,92)

The distances between the customers are the Euclidean distances, and were rounded to obtain integer values. The set of vertices is partitioned into 25 clusters as follows:

$V_0 = \{0\}$ $V_1 = \{22, 38\}$ $V_2 = \{26\}$ $V_3 = \{24\}$ $V_5 = \{10, 29, 45, 50\}$
$V_5 = \{40\}$ $V_6 = \{7, 47\}$ $V_7 = \{28, 41\}$ $V_8 = \{18, 49\}$ $V_9 = \{1, 3, 9\}$
$V_{10} = \{14\}$ $V_{11} = \{32\}$ $V_{12} = \{21, 42, 39, 31\}$ $V_{13} = \{16, 25\}$ $V_{14} = \{30, 46\}$
$V_{15} = \{4\}$ $V_{16} = \{15, 36, 48\}$ $V_{17} = \{11, 44\}$ $V_{18} = \{2, 6, 37\}$ $V_{19} = \{34\}$
$V_{20} = \{5, 23, 35\}$ $V_{21} = \{12, 20, 33, 43\}$ $V_{22} = \{17\}$ $V_{23} = \{8, 13, 19\}$ $V_{24} = \{27\}$

In the next figure we present the vertices (customers) and their partitioned into clusters.

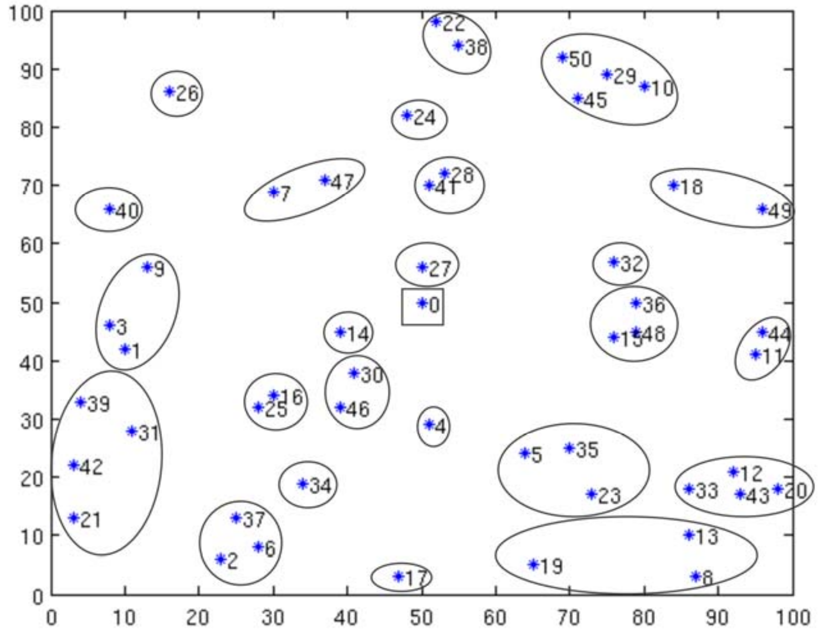

Figure 2: Representation of the 51 vertices and their partitioning into 25 clusters

Each customer has a unit demand, and the demand of a cluster is given by the cardinality of that cluster. The capacity of each vehicle is equal to 15.

The solution reported by Ghiani and Improta [6], was obtained by transforming the GVRP into a Capacitated Arc Routing Problem (CARP), which was then solved to yield the objective value 532.73. The same instance was solved to optimality by Kara and Bektas [10] using their proposed integer programming formulation by CPLEX 6.0 on a Pentium 1100 MHz PC with 1 GB RAM in 17600.85 CPU seconds.

Using our proposed formulation for the GVRP, we solved the same instance to optimality using CPLEX 12.2, getting the same value as reported by Kara and Bektas [10]. The required computational time was 1456.17 CPU seconds in the case of our novel integer programming formulation of the GVRP. The computation was performed on a Intel Core 2 Duo 2.00 GHz with 2 GB RAM.

In the next figure we point out the optimal solution of the GVRP obtained in the case of the instance described by Ghiani and Improta [6].

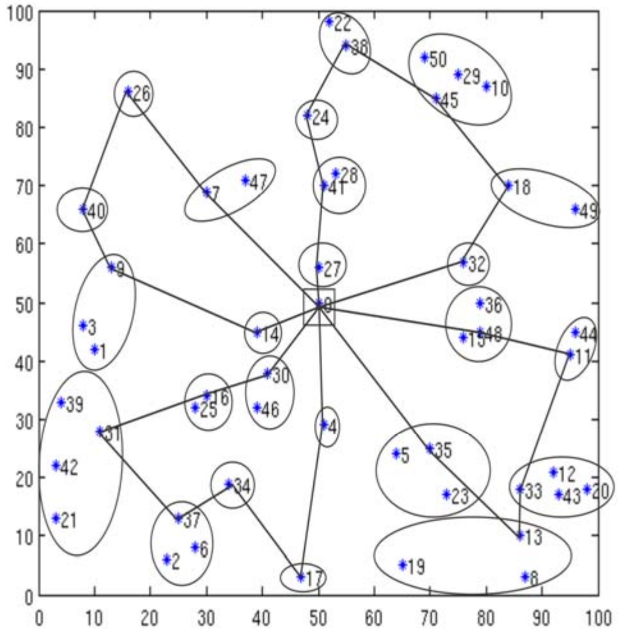

Figure 3: The optimal solution of the GVRP instance described by Ghiani and Improta

5. AN EFFICIENT TRANSFORMATION OF THE GVRP INTO THE VEHICLE ROUTING PROBLEM

Because of the complexity of the generalized combinatorial optimization problems, efficient transformations of these problems into classical combinatorial optimization problems seem to be an appropriate approach. In addition to the classical combinatorial optimization problems, there exist many methods for solving them.

Several efficient transformations of the generalized combinatorial optimization problems into classical combinatorial optimization problems were developed:

- in the case of the generalized traveling salesman problem (GTSP), the first transformation into the traveling salesman problem (TSP) was introduced by Lien, Ma and Wah [15], were the number of nodes of the transformed TSP was quite large, in fact more than three times larger than the number of nodes in the associated GTSP. Later, Dimitrijevic and Saric [4] developed another transformation that decreased the size of the corresponding TSP. In their method, the number of nodes of the TSP was twice the number of nodes of the original GTSP. Recently, Behzad and Modarres [3] provided an efficient transformation in which the number of nodes in the transformed TSP does not exceed the number of nodes in the original GTSP;

- in the case of the railway traveling salesman problem (RTSP) , which is a practical extension of the GTSP, considering a railway network and train schedules, and introduced by Hadjicaralambous *et al.*[7], Hu and Raidl [9] provided two transformation schemes to reformulated the RTSP as either a classical asymmetric or symmetric TSP;
- in the case of the GVRP, Ghiani and Improta [6] showed that the problem can be transformed into a capacitated arc routing problem (CARP), and Baldacci et al. [2] proved that the reverse transformation is valid.

Now we shall describe an efficient transformation of the GVRP into the VRP. The method that we propose is a modification of the transformation proposed by Behzad and Modarres [3] in the case of the GTSP.

For the sake of simplicity, we assume that the arcs are defined between nodes, which belong to different clusters (inter-cluster edges), but we do not restrict the GVRP to be symmetric or Euclidean. It is easy to show that the above mentioned assumption is not restrictive and every GVRP can be transformed into our problem.

Consider the GVRP with $k+1$ clusters represented by a directed graph $G=(N,A)$, where $N=V_0 \cup V_1 \cup ... \cup V_k, V_0=\{0\}$ and $V_1 \cap V_r = \varnothing$, for all $l,r \in \{0,1,...,k\}, l \neq r$ and the set of arcs $A=\{(i,j) | i \in V_l, j \in V_r \quad l \neq r\}$. The solution of the GVRP consists of a collection of routes. In the example presented Figure 1, the solution consists of two tours: $V_0 \to V_1 \to V_2 \to V_0$ and $V_0 \to V_5 \to V_4 \to V_3 \to V_0$.

It is important to mention that using the same approach as the one described by Pop et al. [21] in the case of the GTSP, called the local-global approach, given a sequence (V_{k1} , ..., V_{kp}) in which the clusters are visited in a tour, it is possible to find the best feasible tour that passes through exactly one node from each visited cluster (w.r.t cost minimization), by visiting the clusters according to the given sequence. This can be done in polynomial time, by solving $|V_{k1}|$ shortest path problems in a constructed layered network.

Let denote by v^r_i the i-th node of the cluster r.

Then we define the VRP on a directed graph G′ associated to G as follows:

1. The set of nodes of G and G′ are identical.
2. All nodes of each cluster are connected by arcs into a cycle in G′. We denote by $v^r_{i(s)}$ the node that succeeds v^r_i in the cycle.
3. The costs of the arcs of the transformed graph G′ are defined as:

$$c'(v_i^r, v_{i(s)}^r) = 0$$

and

$$c'(v_i^r, v_i^t) = c(v_{i(s)}^r, v_i^t) + M, r \neq t,$$

where M must be a sufficiently large number, for example $\Sigma_{(i,j)\in A} c(i,j)$.

We will call a tour that visits exactly one node from a given number of clusters a generalized tour.

We define a path in G′ as an intra-cluster path if it consists of all the nodes of only one cluster.

In addition, we define a tour in G′ containing the depot $\{0\}$ whose cost is less than $(p+1)M$ as an generalized tour visiting p clusters and containing as well the depot $V_0 = \{0\}$.

Lemma 1. *Every tour in G′, corresponding to a generalized tour visiting p clusters enters and leaves each cluster exactly once.*

Proof. From the definition of the costs in the directed graph G' we have that the costs of moving from one cluster to another is at least *M.* Since the tour in G' visits all the p clusters, its cost is at least *pM.* If the tour in G' does not traverse along the cluster paths, it will imply that it enters and leaves each of the *p* clusters more than once, and therefore the cost of the tour cannot be less than $(p+1)M$, which is a contradiction.

We can define now the one-to-one correspondence between tours in G' and generalized tours in *G:*

1. Consider a tour in G' and connect the first nodes of its clusters paths together in the order of their corresponding clusters, then the result is a generalized tour in *G.*
2. Consider a generalized tour in *G* that includes the following nodes $\cdots \to v^r_i \to v^t_j \cdots$, $r \neq t$. Replacing the node v^r_i with the *r-th* cluster path starting with v^r_i and then connecting the last node of this path to the next cluster path starting with v^t_j, we obtain a tour in G'.

It is obvious that the cost of a generalized tour in *G* visiting *p* clusters is equal to the cost of the corresponding tour in G' less *pM* , and the cost of the GVRP is the sum of the costs of the generalized tours.

In the next figure, we illustrate the described transformation procedure and show an example how the routes are adapted into the new graph G'.

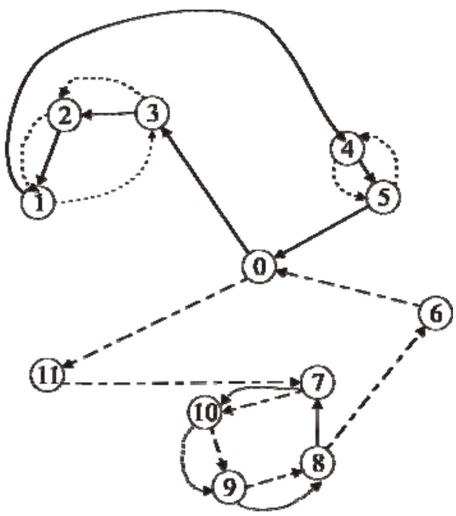

Figure 4: Transforming the graph *G* into G'

The optimal solution of the VRP consisting of two tours is illustrated by dash-dot arcs and bold arcs in figure 2. The nodes of each cluster are cycled in G' by pointed arcs as shown in the figure 2. The optimal cost of VRP is 22+7M and respectively the cost of the GVRP is 22.

6. CONCLUSIONS

In this paper we consider the generalized vehicle routing problem which looks for the optimal routes from a given depot to a number of predefined clusters which include exactly one node from each cluster. We present a new integer programming formulation of the GVRP as well as the description of an efficient transformation procedure of the GVRP into VRP.

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