

A NEW GENETIC REPRESENTATION FOR QUADRATIC ASSIGNMENT PROBLEM¹

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Abstract: In this paper, we propose a new genetic encoding for well known Quadratic Assignment Problem (QAP). The new encoding schemes are implemented with appropriate objective function and modified genetic operators. The numerical experiments were carried out on the standard QAPLIB data sets known from the literature. The presented results show that in all cases proposed genetic algorithm reached known optimal solutions in reasonable time.

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1. INTRODUCTION

1.1. Quadratic assignment problem

The Quadratic Assignment Problem (QAP) is firstly proposed in [16] as a mathematical model related to economic activities. Since then, it has appeared in many practical applications as can be seen from [22]. We mention only several recent applications:

- facility layout design problem in order to minimize work-in-process [29];
- website structure improvement [27];
- placement of electronic components [9];
- index assignment problem related to error control in communications [3];
- memory layout optimization in signal processors [34].

Several NP-hard combinatorial optimization problems, such as the traveling salesman problem, the bin-packing problem and the max clique problem, also can be modeled as QAPs.

Since its first formulation, the QAP has been drawing researchers' attention worldwide, not only because of its practical and theoretical importance, but also because of its complexity. The QAP is one of the most difficult combinatorial optimization problems. In [28] was shown that QAP is NP-hard and that, unless $P = NP$, it is not possible to find an $1+\varepsilon$ - approximation algorithm, for a constant ε . Such results are valid even when flows and distances appear as symmetric coefficient matrices.

In general, the QAP instances of size greater than 30 cannot be solved exactly in a reasonable time. Although heuristic methods do not offer a guarantee for reaching the optimum, they give satisfactory results to a large range of various problems in a reasonable amount of time. Recently, so-called metaheuristics, or general frameworks for building heuristics, became popular for solving difficult combinatorial optimization problems. Metaheuristic approaches use different techniques in order to avoid entrapments in poor local minima and are based mainly on two principles: local search with globalization mechanisms and population search.

In local search methods, an intensive search of the solution space is performed by moving, at each step, from the current solution to another promising solution in its neighborhood. Globalization mechanisms are designed so to ensure diversification of the search. The population search consists of maintaining a pool of good solutions and combining them in order to produce hopefully better solutions.

Thus, a large number of metaheuristic methods have been used to solve the QAP and presentation of all such contributions is out of this paper's scope. We mention only several recent metaheuristic applications for QAP:

- genetic algorithms [6, 10, 33];
- tabu search [7, 14, 24];
- simulated annealing [23];
- ant colony optimization [27];
- particle swarm optimization [20];

- iterated local search [30]
- self-organizing migrating algorithm [4].

As it can be seen from the literature ([22]), hybrid approaches for solving QAP have some advantages compared to single metaheuristic approaches. Some of the recent hybrid approaches are:

- hybrid of genetic algorithm and several variants of tabu search [8];
- variable neighborhood particle swarm optimization [21];
- ant colony optimization approach coupled with a guided local search [12];
- ant colony optimization hybridized with the genetic algorithm and a local search method [32];
- GRASP with path-relinking [26].

1.2 Genetic algorithms

Genetic algorithms (GAs) represent a problem-solving metaheuristic method rooted in the mechanisms of evolution and natural genetics. The main idea was introduced by Holland [13]. In the last three decades GAs have emerged as effective, robust optimization and search methods.

GAs solve problems by creating a population of individuals (usually 10 - 200), represented by chromosomes, which are encoded solutions of the problem. The representation is the genetic code of an individual, and it is often a binary string, although other alphabets of higher cardinality can be used. A chromosome is composed of basic units named genes, which control the features of an individual. To each chromosome, a fitness value measuring its success is assigned. The initial population (the first generation of individuals) is usually randomly initialized. The individuals in the population then pass through a procedure of simulated "evolution" by means of randomized processes of selection, crossover, and mutation.

The selection operator favors individuals more capable to survive through the generations. The probability that a chromosome will be chosen depends on its fitness. The higher fitness value of a chromosome provides higher chances for its survival and reproduction. There are different ways of selecting the best-fitted individuals. One of the most often used is tournament selection (for more details see [1, 11, 25]). Crossover and mutation operators are also used during reproduction. The crossover operator provides a recombination of genetic material by exchanging portions between the parents with the chance that good solutions can generate even better ones.

Mutation causes sporadic and random changes by modifying individual's genetic material with some small probability. Its role is to regenerate the lost or unexplored genetic material into the population. Mutation has a direct analogy with nature, and it should prevent premature convergence of the GA to suboptimal solutions.

There are many different policies for generation replacement. Certain numbers of individuals (elite individuals) may skip selection (or even all genetic operators) going directly into the next generation. This approach is named the steady-state generation replacement policy with elitist strategy. It provides a smaller gradient in the genetic search, but preserves fitted individuals from the past generations.

There can be many modifications of the GA, but implementing the GA usually involves the following steps:

- evaluating the fitness of all individuals in a population;

- selecting the best-fitted individuals;
- creating a new population by performing crossover and mutation operators.

The process of reproduction and population replacement is repeated until a stopping criterion (fixed number of generations or satisfied quality of solutions obtained) is met. Detailed description of GAs is out of this paper's scope, and it can be found in [1,25].

GAs have a wide range of applications, growing rapidly, for example, from strong metric dimensions of graphs [19], through maximally balanced partition [5], spanning sets coverage [15], generalized Euclidean distances [2] to hub location [18] and modeling of chemical processes [31]. As it can be seen in previous section, GAs are frequently used for solving QAP in stand-alone or hybrid approaches ([6, 8, 10, 32]).

2. MATHEMATICAL FORMULATION OF THE QAP

The QAP can be described as the problem of assigning n facilities to n locations with given distances between the locations and given flows between the facilities. The goal is to place the facilities on locations in such a way that the sum of the product between flows and distances is minimized.

The QAP can be formulated in different ways. One of the most popular formulations is a permutation based formulation. Let S_n be the set of all permutations with n elements and $\pi \in S_n$. Consider f_{ij} the flows between facilities i and j and d_{kl} the distances between locations k and l . If each permutation π represents an allocation of facilities to locations, the problem expression becomes:

$$\min_{\pi \in S_n} \sum_{i=1}^n \sum_{j=1}^n f_{ij} \cdot d_{\pi(i)\pi(j)} \quad (1)$$

Another equivalent formulation is quadratic integer programming formulation.

If we define binary variables $x_{ik} = \begin{cases} 1, & k = \pi(i) \\ 0, & k \neq \pi(i) \end{cases}$ QAP can be formulated as:

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{k=1}^n \sum_{l=1}^n f_{ij} \cdot d_{kl} \cdot x_{ik} \cdot x_{jl} \quad (2)$$

s.t.

$$\sum_{i=1}^n x_{ij} = 1 \quad j = 1, 2, \dots, n \quad (3)$$

$$\sum_{j=1}^n x_{ij} = 1 \quad i = 1, 2, \dots, n \quad (4)$$

$$x_{ij} \in \{0, 1\} \quad i, j = 1, 2, \dots, n \quad (5)$$

In the literature there exist numerous formulations and presentation of the QAP but all of them are out of this paper's scope. Interested reader can seek for more information about different formulations of the QAP including the linear formulations in [22].

3. NEW GENETIC REPRESENTATION FOR THE QAP

3.1. Representation and objective function

Since the QAP is a minimization problem, it is obvious that, in the optimal permutation (solution) pairs of facilities with large flow usually corresponds to the pairs of locations with small distance between them. We introduce a new encoding scheme which forces frequent occurrence of this behavior, i.e. there are very small chances that the pair of facilities with the large flow corresponds to the pairs of locations with large distance. On that way, this encoding scheme push GA search towards promising search regions.

In this encoding scheme every individual consists of $n-1$ genes. Length of the individual is $n-1$ because it is not necessary to remember the last element in permutation when all other elements are set. Each gene is represented by an integer that corresponds to one element in permutation. In contrast to previous representations, that particular integer, let us say the i -th integer in permutation, which represents the gene, is not the index of a location assigned to facility i . Instead, it represents a "distance" of the corresponding partial assignment of locations to facilities $1, 2, \dots, i-1$ from a locally optimal solution of QAP restricted to these facilities. Therefore, the value of i -th gene belongs to $\{0, 1, \dots, n-i\}$.

For a given coded individual the corresponding permutation of locations assigned to facilities is obtained by an iterative procedure expressed by a pseudo-code in Figure 1. In each iteration of the procedure, for every non-assigned location its weight is calculated as the partial sum of the products between flows and distances from this location to all previously assigned locations. Then non-assigned locations are sorted in a sequence according to non-decreasing weights. The location from the sequence for which "distance" from its first member is equal to the value of the corresponding gene, is chosen as the next location in the permutation. In this way locations with lower weights are associated to facilities with smaller gene values.

```

S = []; gv := Take_Gene(1); Pi[1] := gv+1;
for i:=2 to n-1 do begin
  gv := Take_Gene(i);
  for j:=1 to n do
    if not (j in S) then begin
      w[j] = F(i,i) * D(j,j);
      for k:=1 to i-1 do begin
        w[j] := w[j] + F(k,i) * D(Pi[k],j);
        w[j] := w[j] + F(i,k) * D(j,Pi[k]);
      endfor
    end
  endfor
  Quick_Sort(n-i+1, w, w_index);
  Pi[i] := w_index[gv+1];
  S := S ∪ {Pi[i]};
endfor
Pi[n] := {1,2,...,n} \ S;

```

Figure 1: Pseudo-code for objective function.

Set S represents a set of currently assigned locations, Pi is the corresponding permutation of locations, array w stores calculated current weights, while w_index denotes indices of w arranged according to non-decreasing weights by function $Quick_Sort()$. Function $Take_Gene(i)$ returns the value of i -th gene.

Note that the previous procedure for calculating objective function gives the permutation so that the individuals are always feasible. In other words, if the initial order of locations $1, 2, \dots, n$ is respected whenever weights of some locations in sorted sequence are equal, the new encoding scheme is unique and complete, i.e. for each permutation of locations there exists a unique corresponding code and there are no two permutations with the same code.

Example 1. Suppose that GA is solving QAP with $n=4$ and with following flow and distance matrix:

$$F = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 2 & 0 & 0 & 7 \\ 3 & 4 & 0 & 0 \\ 5 & 2 & 5 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

In this case, each individual in the population has genetic representation with length 3. Suppose that the individual has following representation:

| | | |
|---|---|---|
| 2 | 0 | 1 |
|---|---|---|

Algorithm for calculating permutation represented by this individual is:

- At the beginning, $Pi[1] = gv+1=3$, i.e. location 3 is assigned to facility 1.

- In the next step, where $i=2$, non-assigned locations are $\{1,2,4\}$. For each of these locations, we are calculated their weights $w[1]=10$, $w[2]=w[4]=5$, so the sorted array w_index is 2,4,1. Since $gv=0$ then $Pi[2]=w_index[1]=2$.
- The same procedure is applied in the third step, so $Pi[3]=w_index[2]=1$.
- In the last step $Pi[4]=4$, which is only remaining location.

3.2. Improving heuristic

In order to improve individuals, we performed a local search on proposed genetic algorithms. The best results are obtained by applying best-known 2-opt heuristic with first improvement. This procedure is repeated until we are sure that swapping of each pair of the elements in permutation will not improve the quality of the permutation. The other strategy is to make only one local search for improving quality of obtained permutation. That approach is faster, but quality of obtained results is not as good as it is in the first case.

Local search procedure like 2-opt usually significantly decreases diversity of the GA population. In case when the solution is improved by heuristic corresponding GA code is not changed, so the diversity of the GA population is preserved. Therefore, 2-opt heuristic deals directly with permutations of locations and not with the new encoding, which imply that the algorithm for obtaining the genetic code from the improved solution is not needed at all.

3.3. Population initialization

The initial population of $N_{pop}=150$ individuals is randomly generated, allowing maximal diversity of genetic material, but initialization of the genetic code of all individuals in first generation should not be a pure random procedure. Natural model for this behavior is finite decreasing geometric progression with common ratio q .

Probability p_k that the i -th gene has value $k(k \in \{0,1,\dots,n-i\})$, decreases with increasing of k according to the geometric progression with a given ratio $q \in (0,1)$, i.e.

$$p_k = p_0 p^k. \text{ As } \sum_{k=0}^{n-i} p_k = p_0 \frac{1-q^{n-i+1}}{1-q} = 1, \text{ then } p_0 = \frac{1-q}{1-q^{n-i+1}}.$$

From the fixed value q , it is easy to calculate all probabilities $p_k, k=0,1,\dots,n-i$ and to generate i -th gene randomly with these probabilities. In algorithms that we propose, the value of q is equal to 0.5.

4. GENETIC OPERATORS AND OTHER GA ASPECTS

4.1. Fitness function and generation replacement policy

The number of elitist individuals passed directly to the next generation is $N_{elite}=100$. Non-elitist individuals N_{nonel} (the rest of the population) go through genetic

operators. This means that a lot of time is saved since the objective value is calculated only once for each elite individual.

To prevent undeserved domination of elite individuals over the population, their fitness is decreased by the next formula:

$$Ft_i^{new} = \begin{cases} Ft_i - \overline{Ft}, & Ft_i > \overline{Ft} \\ 0, & Ft_i \leq \overline{Ft} \end{cases}, \quad 1 \leq i \leq N_{elite}, \quad \overline{Ft} = \frac{1}{N_{pop}} \sum_{i=1}^{N_{pop}} Ft_i \quad (6)$$

In this way, even non-elite individuals preserve their chance to survive to the next generation.

As non-elitist individuals go through genetic operators, appearance of duplicated individuals is possible. Such individuals with the same genetic code are discarded - simply by setting fitness value of the duplicate to zero, so that selection operator allows them not to continue to the next generation. Furthermore, too many individuals with the same objective function, but different genetic codes, may predominate in population. This is why limiting the number of such individuals in population to some constant has been shown useful in [19, 5, 18]. Therefore, proposed algorithms prohibit an existence of more than 40 elite individuals with different genetic code and the same objection value, which prevent premature convergence of algorithms and increase diversity of genetic material.

4.2. Selection and crossover

A fine-grained tournament selection (FGTS) scheme has been used in the proposed GAs for deciding which individuals will produce the next generation. The average size of tournament, F_{tour} , is a real number, and is considered to be a constant in practice. We used the value of $F_{tour}=5.4$, because it gave good results in solving similar problems (for example, see [5,18,19]). Detailed information on FGTS scheme can be found in [11].

For recombination of individuals, we used the classical one-point crossover. The crossover rate is $p_{cross}=0.85$. This means that about 85% individuals participate in recombination of their genes.

4.3. Mutation

Finally, modified simple mutation operator changes randomly selected genes. During the GA execution, it is possible that all individuals in the population have the same gene in a certain position. These genes are called frozen. If the number of frozen genes is significant, the search space becomes much smaller, and the possibility of premature convergence rapidly increases. For that reason, the basic mutation rates are increased, but only for the frozen genes. The basic mutation rates are:

- $0.1/n$ for the bit on the first position.
- $0.05/n$ for the bit on the second position. Next bits in the gene have repeatedly two times smaller mutation rate ($0.025/n, 0.0125/n...$).

When compared with the basic mutation rates, frozen bits are mutated by 2.5 times higher rate:

- $0.25/n$ instead of $0.1/n$ if they are at the first position of the gene.
- $0.125/n$ for the bit on the second position. Next bits in the gene have repeatedly two times smaller mutation rate ($0.0625/n$, $0.03125/n\dots$).

4.4. Caching GA

Caching optimizes run-time of a genetic algorithm. The evaluated objective values are stored in a hash-queue structure using the Least Recently Used caching technique (LRU). Otherwise it would be necessary to calculate the same objective value each time genetic operators produce another individual with the same genetic code. With caching technique, when such individual appears, its objective value is taken from the caching table, and this saves a significant amount of time. Caching of GAs has no impact on results that are obtained by GAs - it only reduces execution time.

In proposed GA implementations, we limited the number of individuals stored in a caching table to $N_{cache}=5000$. Detailed information about caching GA can be found in [17].

Table 1: GA results on QAP instances

| Instance name | Opt | t (sec) | gen | $agap$ (%) | σ (%) | $eval$ | $cache$ (%) |
|---------------|----------|-----------|-------|------------|--------------|--------|-------------|
| bur26a | 5426670 | 77.422 | 2014 | 0.000 | 0.000 | 61114 | 60.6 |
| bur26b | 3817852 | 92.447 | 2020 | 0.000 | 0.000 | 63242 | 62.5 |
| bur26c | 5426795 | 81.654 | 2001 | 0.000 | 0.000 | 60001 | 59.9 |
| bur26d | 3821225 | 111.720 | 2299 | 0.000 | 0.000 | 71577 | 62.0 |
| bur26e | 5386879 | 108.612 | 2457 | 0.000 | 0.002 | 77288 | 62.5 |
| bur26f | 3782044 | 113.981 | 2001 | 0.000 | 0.000 | 58122 | 58.0 |
| bur26g | 10117172 | 84.253 | 2092 | 0.000 | 0.000 | 67248 | 64.2 |
| bur26h | 7098658 | 95.664 | 2178 | 0.000 | 0.000 | 67481 | 62.0 |
| chr12a | 9552 | 2.071 | 2001 | 0.000 | 0.000 | 73872 | 73.7 |
| chr12b | 9742 | 3.154 | 2001 | 0.000 | 0.000 | 68129 | 68.0 |
| chr12c | 11156 | 1.730 | 2014 | 0.000 | 0.000 | 77339 | 76.7 |
| chr15a | 9896 | 4.780 | 2262 | 0.061 | 0.148 | 86105 | 75.9 |
| chr15b | 7990 | 4.645 | 2001 | 0.000 | 0.000 | 73500 | 73.4 |
| chr15c | 9504 | 3.745 | 2274 | 1.306 | 2.229 | 88766 | 77.9 |
| chr18a | 11098 | 8.678 | 2261 | 0.299 | 1.026 | 84592 | 74.6 |
| chr18b | 1534 | 7.461 | 2001 | 0.000 | 0.000 | 67921 | 67.8 |
| chr20a | 2192 | 12.140 | 2265 | 0.771 | 1.377 | 82531 | 72.5 |
| chr20b | 2298 | 9.828 | 2777 | 5.013 | 1.725 | 102941 | 74.3 |
| chr20c | 14142 | 25.052 | 2356 | 0.472 | 1.454 | 82246 | 69.7 |
| chr22a | 6156 | 18.720 | 2465 | 0.369 | 0.291 | 93180 | 75.3 |
| chr22b | 6194 | 15.258 | 2914 | 1.088 | 0.481 | 110681 | 75.6 |
| chr25a | 3796 | 48.375 | 2946 | 2.903 | 1.719 | 107506 | 73.0 |
| els19 | 17212548 | 25.806 | 2194 | 0.177 | 0.364 | 74309 | 67.4 |
| esc16a | 68 | 3.353 | 2001 | 0.000 | 0.000 | 62204 | 62.1 |
| esc16b | 292 | 0.063 | 25 | 0.000 | 0.000 | 397 | 17.1 |
| esc16c | 160 | 4.802 | 2001 | 0.000 | 0.000 | 58745 | 58.6 |
| esc16d | 16 | 3.679 | 2001 | 0.000 | 0.000 | 59255 | 59.1 |
| esc16e | 28 | 2.729 | 2001 | 0.000 | 0.000 | 60625 | 60.5 |
| esc16f | 0 | 0.002 | 1 | 0.000 | 0.000 | 0 | 0.0 |
| esc16g | 26 | 2.994 | 2001 | 0.000 | 0.000 | 59974 | 59.9 |
| esc16h | 996 | 0.008 | 1 | 0.000 | 0.000 | 0 | 0.0 |
| esc16i | 14 | 0.492 | 315 | 0.000 | 0.000 | 8038 | 43.3 |
| esc16j | 8 | 2.716 | 2001 | 0.000 | 0.000 | 60994 | 60.9 |
| esc32e | 2 | 0.050 | 1 | 0.000 | 0.000 | 0 | 0.0 |
| esc32f | 2 | 0.050 | 1 | 0.000 | 0.000 | 0 | 0.0 |
| had12 | 1652 | 2.493 | 2001 | 0.000 | 0.000 | 77814 | 77.7 |
| had14 | 2724 | 7.797 | 2001 | 0.000 | 0.000 | 67434 | 67.3 |
| had16 | 3720 | 16.534 | 2001 | 0.000 | 0.000 | 61102 | 61.0 |
| had18 | 5358 | 22.627 | 2001 | 0.000 | 0.000 | 57944 | 57.8 |
| had20 | 6922 | 38.445 | 2001 | 0.000 | 0.000 | 57925 | 57.8 |

5. COMPUTATIONAL RESULTS

The tests were made on an Intel 2.5 GHz with 1GB memory, under Windows XP operating system. The algorithm was coded in C programming language. We tested our algorithm on QAPLIB instances (<http://www.seas.upenn.edu/qaplib/>) with known optimal solutions. The stopping criterion of GA was the maximum number of generations equal to 5000 or at most 2000 generations without the improvement of the objective value.

The GA was run 20 times for each instance, and the results are summarized in Table 1 and Table 2. For all the instances the algorithm reached the optimal solution. The tables are organized as follows:

- the first column contains the test instance's name;
- the second column contains the optimal solution for the particular instance;
- the average total running time (t) and the average number of generations for finishing GA (gen) are given in the third and the fourth columns;

the fifth and the sixth column ($agap$ and σ) contain information on the average solution quality: $agap$ is a percentage gap defined as $agap = \frac{1}{20} \sum_{i=1}^{20} gap_i$, where

$$gap_i = 100 \cdot \frac{GA_i - opt}{opt} \text{ and } GA_i \text{ represents the } GA \text{ solution obtained in the } i\text{-th}$$

run, while σ is the standard deviation of $gap_i, i=1,2,\dots,20$, obtained by formula

$$\sigma = \sqrt{\frac{1}{20} \sum_{i=1}^{20} (gap_i - agap)^2};$$

- in the last two columns $eval$ represents the average number of the objective function evaluations, while the cache displays savings (in percent) achieved by using the caching technique.

Table 2: GA results on QAP instances

| Instance name | Opt | t (sec) | gen | $agap$ (%) | σ (%) | $eval$ | $cache$ (%) |
|---------------|-----------|-----------|-------|------------|--------------|--------|-------------|
| kra30a | 88900 | 117.353 | 2078 | 0.067 | 0.302 | 69757 | 67.0 |
| kra30b | 91420 | 132.594 | 2269 | 0.009 | 0.029 | 77643 | 68.3 |
| lipa20a | 3683 | 11.044 | 2011 | 0.000 | 0.000 | 70296 | 69.8 |
| lipa20b | 27076 | 18.240 | 2001 | 0.000 | 0.000 | 55735 | 55.6 |
| lipa30a | 13178 | 57.660 | 2173 | 0.000 | 0.000 | 76183 | 70.0 |
| lipa30b | 151426 | 70.761 | 2001 | 0.000 | 0.000 | 62397 | 62.3 |
| nug12 | 578 | 1.589 | 2001 | 0.000 | 0.000 | 79805 | 79.6 |
| nug14 | 1014 | 3.341 | 2001 | 0.000 | 0.000 | 74519 | 74.4 |
| nug15 | 1150 | 5.779 | 2001 | 0.000 | 0.000 | 68544 | 68.4 |
| nug16a | 1610 | 6.454 | 2013 | 0.000 | 0.000 | 71358 | 70.8 |
| nug16b | 1240 | 9.817 | 2001 | 0.000 | 0.000 | 61327 | 61.2 |
| nug17 | 1732 | 9.315 | 2266 | 0.046 | 0.058 | 80968 | 71.2 |
| nug18 | 1930 | 10.218 | 2001 | 0.000 | 0.000 | 71870 | 71.7 |
| nug20 | 2570 | 20.740 | 2005 | 0.000 | 0.000 | 66932 | 66.6 |
| nug21 | 2438 | 27.497 | 2064 | 0.000 | 0.000 | 69749 | 67.3 |
| nug22 | 3596 | 35.819 | 2006 | 0.000 | 0.000 | 70199 | 69.9 |
| nug24 | 3488 | 48.132 | 2011 | 0.000 | 0.000 | 68349 | 67.8 |
| nug25 | 3744 | 65.388 | 2043 | 0.003 | 0.012 | 71173 | 69.5 |
| nug27 | 5234 | 57.737 | 2186 | 0.002 | 0.009 | 76857 | 70.0 |
| nug28 | 5166 | 71.577 | 2677 | 0.083 | 0.128 | 95321 | 71.1 |
| nug30 | 6124 | 160.993 | 2525 | 0.090 | 0.080 | 88657 | 69.9 |
| rou12 | 235528 | 1.376 | 2004 | 0.000 | 0.000 | 77921 | 77.6 |
| rou15 | 354210 | 3.582 | 2010 | 0.000 | 0.000 | 74054 | 73.6 |
| rou20 | 725522 | 12.864 | 2512 | 0.069 | 0.087 | 91333 | 72.6 |
| scr12 | 31410 | 1.573 | 2001 | 0.000 | 0.000 | 70681 | 70.5 |
| scr15 | 51140 | 4.467 | 2001 | 0.000 | 0.000 | 68398 | 68.3 |
| scr20 | 110030 | 11.825 | 2187 | 0.000 | 0.000 | 79151 | 72.3 |
| ste36a | 9526 | 484.717 | 2943 | 0.477 | 0.291 | 97933 | 66.3 |
| ste36b | 15852 | 418.567 | 2206 | 0.038 | 0.092 | 72428 | 65.4 |
| ste36c | 8239110 | 509.277 | 2941 | 0.181 | 0.122 | 99200 | 67.3 |
| tai10a | 135028 | 0.917 | 2001 | 0.000 | 0.000 | 76443 | 76.3 |
| tai10b | 1183760 | 1.103 | 2001 | 0.000 | 0.000 | 71168 | 71.0 |
| tai12a | 224416 | 1.871 | 2001 | 0.000 | 0.000 | 72101 | 72.0 |
| tai12b | 39464925 | 2.713 | 2002 | 0.000 | 0.000 | 73265 | 73.1 |
| tai15a | 388214 | 3.178 | 2001 | 0.000 | 0.000 | 75096 | 74.9 |
| tai15b | 51765268 | 7.523 | 2001 | 0.000 | 0.000 | 67744 | 67.6 |
| tai17a | 491812 | 6.716 | 2453 | 0.215 | 0.203 | 89057 | 72.5 |
| tai20a | 703482 | 10.470 | 2307 | 0.371 | 0.221 | 83099 | 71.9 |
| tai20b | 122455319 | 43.257 | 2001 | 0.000 | 0.000 | 65802 | 65.7 |
| tai25b | 344355646 | 84.672 | 2081 | 0.000 | 0.000 | 72678 | 69.6 |
| tho30 | 149936 | 171.737 | 2763 | 0.099 | 0.135 | 97441 | 70.1 |

Our approach reached optimal solutions within reasonable running time. If we compare these running time with the running time of other existing GA for QAP [6,10,33], applied to the same set of instances, we can see that our algorithm seems to perform slower. However, our main goals here are to develop a GA approach with completely new encoding and to generate high-quality solutions.

6. CONCLUSIONS

This paper is devoted to exploring the results of the new genetic encoding scheme to the quadratic assignment problem. Arranging possible locations of every facility in non-decreasing order of their weights directs GA to promising search regions. The proposed encoding scheme is performed with adequate objective function and appropriate modified genetic operators. The performance of the genetic algorithm is improved with a local search, the mutation with frozen genes, a limited number of different individuals with the same objective value and the caching GA technique. The experimental results are encouraging and show effectiveness of the new encoding scheme. The proposed GA obtains solutions that match all known optimal solutions from the literature.

Further research should be directed to testing large-scale instances on more powerful and/or parallel computers as well as to investigate the combination of presented GA approach with some other metaheuristic.

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