Yugoslav Journal of Operations Research
21 (2011), Number 2, 253-273
DOI: 10.2298/YJOR1102253A

AN INTERACTIVE ALGORITHM FOR LARGE SCALE MULTIPLE OBJECTIVE PROGRAMMING PROBLEMS WITH FUZZY PARAMETERS THROUGH TOPSIS APPROACH

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Received:June 2006 / Accepted: August 2011

Abstract: In this paper, we extend TOPSIS (Technique for Order Preference by Similarity Ideal Solution) for solving Large Scale Multiple Objective Programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the α-Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the α-Level sets of fuzzy numbers. An interactive fuzzy decision making algorithm for generating α-Pareto optimal solution through TOPSIS approach is provided, where a decision maker (DM) is asked to specify the degree α and the relative importance of objectives. Finally, a numerical example is given to clarify the main results developed in the paper.

Keywords: Interactive decision making, multiple objective programming problems, fuzzy parameters, TOPSIS, block angular structure.

MSC: 90C29.

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1. INTRODUCTION

Commonly, when formulating a large scale multiobjective programming model which closely describes and represents the real world decision situations, various factors of the real system should be reflected in the description of the objective functions and constraints. Naturally, these objective functions and constraints involve many parameters and the experts may assign them different values. In the traditional approaches, such parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters. In practice, however, it is natural to consider that the possible values of these parameters are often only ambiguously known to experts' understanding of the parameters as fuzzy numerical data, which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers ([25], [26]).

After the publication of the Dantzig-Wolfe decomposition method [5], there have been numerous subsequent works on large scale linear and nonlinear programming problems with block angular structure (see f. i. [11,12,17, 24, 26]).

M Sakawa and K Kato [21] formulated a large scale multiobjective linear programming problems involving fuzzy numbers. It is shown that the corresponding α-Pareto optimal solution can be easily obtained by solving the minimax problems for which the Dantzig-Wolfe decomposition method is applicable.

M. S. Osman, O. M. Saad and A. G. Hasan [20] presented a method for solving a special class of large scale fuzzy multiobjective integer problems depending on the decomposition algorithm.

A. A. El-Sawy, N. A. El-Khouly and T. H. M. Abou-El-Enien [9] introduced an algorihm for decomposing the parametric space in large scale linear vector optimization problems under fuzzy environment.

S. Opricovic and G. H. Tzeng [19] presented a comparative analysis of VIKOR and TOPSIS methods. The two methods are illustrated with a numerical example, showing their similarity and some differences. The multiple criteria decision making (MCDM) methods VIKOR and TOPSIS are based on an aggregating function representing "closeness to the ideal", which originated in the compromise programming method. In contrast, VIKOR linear normalization and TOPSIS vector normalization are used to eliminate the units of criterion functions. The VIKOR method of compromise ranking determines a compromise solution, providing a maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent". The TOPSIS method determines a solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances.

M. A. Abo-Sinna [1] extends TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. He shows that using the fuzzy max-min operator with nonlinear membership functions, leads to the solutions that are always nondominated solutions of the original MODP problems.

M. A. Abo-Sinna [2] presents an interactive fuzzy decision making for generating α-Pareto optimal solution to multiobjective dynamic programming problems with fuzzy parameters through the decomposition method

H. Deng et. al. [7] formulated the inter-company comparison process as a multi-criteria analysis model, and presented an effective approach by modifying TOPSIS for solving such a problem.

C. T. Chen [4] extends the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in fuzzy environment.

In this paper, we extend TOPSIS [15] for solving large scale multiple objective programming (LSMOP) problems with fuzzy parameters in the objective functions and the right-hand side of the independent constraints (LSFMOP). TOPSIS was first developed by C. L. Hwang and K. Yoon [14] for solving a multiple attribute decision making problem. It is based upon the principle that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). The single criterion for the shortest distance from the given goal or the PIS may be not enough to decision makers. In practice, we might like to have a decision which does not only make as much profit as possible, but which also avoids as much risk as possible. A similar concept has also been pointed out by M. Zeleny [26] (see Y. J. Lai et. al. [15]).

In the following section, we will give the formulation of the large scale multiple objective programming problem with fuzzy parameters in the objective functions and the right-hand side of the independent constraints (LSFMOP), which have block angular structure on which the Dantzig-Wolfe decomposition method was successfully applied. The family of d_p-distance and its normalization is discussed in subsection 3.1. The TOPSIS approach is presented in subsection 3.2. By the use of TOPSIS, we propose an interactive algorithm for solving LSFMOP problems in section 4, where the DM is asked to specify the degree α and the relative importance of objectives. The satisfying solution for the DM can be derived efficiently form among all α-Pareto optimal solutions. We also give a numerical example in section 5 for the sake of illustration. Finally, concluding remarks and future works are given in section 6.

2. FORMULATION OF THE PROBLEM

In practice, it would certainly be more appropriate to consider that the possible values of parameters in the description of the objective functions and the constraints, usually involve the experts' ambiguous understanding of the real systems. Thus, in this paper, we consider a LSFMOP problem of the following block angular structure [13, 16, 26]:

(LSFMOP):

$$Maximize \quad \left[f_1(X,\tilde{U}_2), f_2(X,\tilde{U}_2), ..., f_1(X,\tilde{U}_k) \right] \tag{1-a}$$

subject to

$$X \in M = \left\{ \begin{array}{l} X \in R^n : \sum_{j=1}^{q} A_j X_j \le b_0, D_j X_j \le \tilde{\tilde{Y}}j, \\ X_j \ge d_j > 0, j = 1, 2, ..., q, q > 1 \end{array} \right\} \tag{1-b}$$

If the objective functions are linear, then the objective function can be written as follows:

$$f_i(X,\tilde{U}_i)=\tilde{U}_iC_iX=\sum_{j=1}^{q}f_{ij}(X_j,\tilde{U}_{ij})=\sum_{j=1}^{q}\tilde{U}_iC_{ij}X_j, i=1,2,\ldots,k \tag{2}$$

where

k : the number of objective functions,
q : the number of subproblems,
m : the number of constraints,
n : the number of variables,
n_j : the number of variables of the j^{th} subproblem, $j=1,2,\ldots,q$,
m_0 : the number of the common constraints represented by

$A_jX_j \le b_0$, $\sum_{j=1}^{q}$

m_j : the number of independent constraints of the j^{th} subproblem represented by $D_jX_j \le \tilde{Y}_j, j=1,2,\ldots,q$
R : the set of all real numbers,
X : an n - dimensional column vector of variables,
X_j : an n_j - dimensional column vector of variables for the j^{th} subproblem, $j=1,2,\ldots,q$,
A_j : an $(m_0 \times n_j)$ coefficient matrix,
D_j : an $(m_j \times n_j)$ coefficient matrix,
b_0 : an m_0 -dimensional column vector of right-hand sides of the common constraints whose elements are constants,
$\tilde{Y}_j$: an m_j - dimensional column vector of independent constraints right-hand sides whose elements are fuzzy parameters for the j^{th} subproblem, $j=1,2,\ldots,q$,
d_j : is a certain lower bound for the variables X_j for all j,
$\tilde{U}_i$: an n - dimensional row vector of fuzzy parameters for the i^{th} objective function,
$\tilde{U}_{ij}$: an n_j - dimensional row vector of fuzzy parameters for the j^{th} subproblem in the i^{th} objective function,
C_i : is an $(n \times n)$ diagonal matrix for the i^{th} function,
C_{ij} : is an $(n_j \times n_j)$ diagonal matrix for the j^{th} subproblem in the i^{th} function,
$K=\{1,2,\ldots,k\}$
$N=\{1,2,\ldots,n\}$,

$$R^n = \left\{X = (x_1, x_2, ..., x_n\right\}^T : x_i \in R, i \in N\right\}$$

Throughout this paper, we assume that the column vectors of fuzzy parameters $\tilde{Y}_j, j = 1,2,...,q,$ and $\tilde{U}_i, i = 1,...,k$, the row vectors of fuzzy parameters are characterized as the column vectors of fuzzy numbers and row vectors of fuzzy numbers respectively [8,22,26].

These fuzzy numbers, reflecting the experts' ambiguous understanding of the nature of the parameters in the problem-formulation process, are assumed to be characterized as the fuzzy numbers introduced by D. Dubois and A. Prade [8]. In this paper, we deal with a real fuzzy number ã whose membership function μ_a (a) is defined as [8,22,26]:

(1) A continuous mapping from R^1 to the closed interval [0,1],

(2) $\mu_{\tilde{a}}(a) = 0$ for all a $\in (-\infty, a_1]$,

(3) Strictly increasing on $[a_1, a_2]$,

(4) $\mu_{\tilde{a}}(a) = 1$ for all a $\in (a_2, a_3]$,

(5) Strictly decreasing on $[a_3, a_4]$,

(6) $\mu_{\tilde{a}}(a) = 0$ for all a $[a_4, +\infty]$.

A possible shape of the fuzzy number ã is illustrated in figure 1.

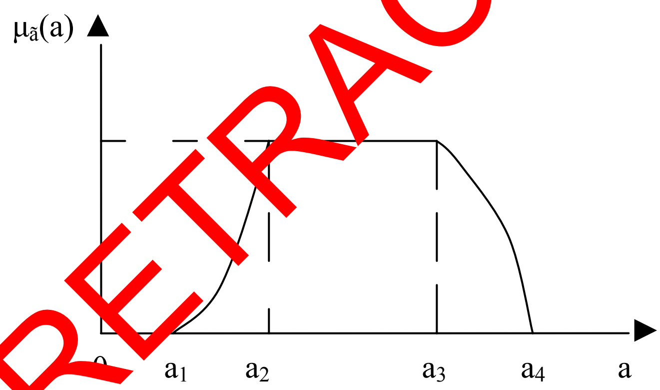

Figure1. Membership function of the fuzzy number ã

Observing the LSFMOP problem, it is evident that the notion of Pareto optimality [13] defined for the LSMOP problem cannot be applied directly. Thus, it seems essential to extend the notion of usual Pareto optimality. For that purpose, we first introduce the concept of the α-level set or α-cut of all the vectors whose elements are the fuzzy numbers as follows [8,22,26]:

Definition1. (α-level set). The α-level set of $(\tilde{U},\tilde{Y})$ *is defined as the ordinary set* $(\tilde{U},\tilde{Y})_a$ *for which the degree of its membership function exceeds the level*

$$\alpha \in [0,1] : (\tilde{U},\tilde{Y})_a = \{(\tilde{U},\tilde{Y}) : \mu_{\tilde{U}_{v,tj}}(U_{v,tj}) \geq \alpha, \mu_{\tilde{y}_{s,tj}}(y_{s,tj}) \geq \alpha, v = 1,\ldots,n_j, \\ j = 1,\ldots,q, t = 1,\ldots,k, s = 1,\ldots,m_j\} \tag{3}$$

For a certain degree of α, the LSFMOP problem (1) can be understood as the following nonfuzzy α -large scale multiple objective decision making α - LSMOP problem [13,16, 26]:

$$(\alpha - LSMOP):$$

$$Maximize[f_1(X,U_1), f_2(X,U_2),\ldots, f_k(X,U_k)] \tag{4}$$

$$subject\ to\ X \in M' = \left\{ X \in R_n : \sum_{j=1}^{q} A_j X_j \leq b_0, \right. \tag{4-a}$$

$$D_j X_j \leq Y_j, \tag{4-b}$$

$$D_j X_j \leq Y_j, \tag{4-c}$$

$$X_j \geq d_j > 0, j = 1,2,\ldots,q, q > 1, \tag{4-d}$$

$$\left. (U,Y) \in (\tilde{U},\tilde{Y})_\alpha \right\} \tag{4-e}$$

In the α -LSMOP problem (4), the parameters $\tilde{Y}_j, j = 1,\ldots,q, and\, U_i, i = 1,1,\ldots,k,$ are treated as variables rather than constants.

Based on the definition of α-level sets of the fuzzy numbers, we characterize α – Pareto optimal solution of α-LSMOP problem (4) [8,22,26]:

Definition2. (α *-Pareto optimal solution). A solution* X_j^* *є* M'*, j=1,2,…,q, is said to be an α - Pareto optimal solution to the α - LSMOP problem (4), if and only if there does not exist another* $X_j, j = 1,2,\ldots,q, (U,Y) \in (\tilde{U},\tilde{Y})_\alpha$, *such that* $\sum_{j=1}^{q} f_{ij}(X_j,U_{ij}) \geq \sum_{j=1}^{q} f_{ij}(X_j^*,U_{ij}^*), i = 1,2,\ldots,k,$ *with strictly inequality holding for at least one i, where the corresponding value of the parameter (*U^* *,* Y^* *) is called* α *-level optimal parameter.*

Thus, the α -LSMOP problem (4) can be written as follows [10,13,18]:

$$Maximize[f_1(X,U_1), f_2(X,U_2),\ldots, f_k(X,U_k)] \tag{5-a}$$

subject to

$$\sum_{j=1}^{q} A_j X_j \leq b_0, \tag{5-b}$$

$$D_j X_j \le Y_j, \tag{5-c}$$

$$X_j \ge d_j > 0, j = 1,2,...,q, q > 1, \tag{5-d}$$

$$G_{ij}^T \le U_{ij}^T \le H_{ij}^T, i = 1,2,...,k \tag{5-e}$$

$$\eta_j \le Y_j \le \beta_j, j = 0,1,...,q, q > 1 \tag{5-f}$$

It should be noted that the constraints (4-e) are replaced by the equivalent constraints (5-e) and (5-f), where G_{ij}, η_j and β_j, H_{ij} are lower and upper bounds of U_{ij} and y_j, respectively.

If the objective function is linear as in the equation (2), it will become

$$f_i(X,U_i) = U_i C_i X = \sum_{j=1}^{q} f_{ij}(X_j, U_{ij}) = \sum_{j=1}^{q} U_{ij} C_{ij} X_j, ...i = 1,2,...,k \tag{6}$$

If the equation (6) is used in problem (5), the nonlinearity of the objective functions can be treated by using the following transformations:

$$z_{i'js} = u_{i'js}\, x_j, i = 1,2,...k, j = 1,2,...,q, s = 1,2,...,n_j \tag{7}$$

Thus, equation (6) can be written as follows:

$$f_i(Z_i) = C_i' \; Z_i = \sum_{j=1}^{q} f_{ij}(Z_{ij}) = \sum_{j=1}^{q} C_{ij}' \; Z_{ij}, i = 1,2,...,k. \tag{8}$$

Consequently, problem (5) becomes:

$$Maximize \left[f_1(Z_1), f_2(Z_2), ..., f_k(Z_k) \right] \tag{9-a}$$

subject to

$$\sum_{j=1}^{q} A_j X_j \le b_0, \tag{9-b}$$

$$D_j X_j \le Y_j, \tag{9-c}$$

$$X_j \ge d_j > 0, j = 1,2,...,q, q > 1, \tag{9-d}$$

$$G_{ij}'(X) \le Z_{ij} \le H_{ij}'(X), i = 1,2,...,k \tag{9-e}$$

$$\eta_j \le Y_j \le \beta_j, j = 0,1,...,q, 1 > 1 \tag{9-f}$$

where

$$g_{i'js}'(x) = g_{i'js}\, x_j, \; and \; h_{i'js}'(x) = h_{i'js}'(x) = h_{i'js}\, x_j, i = 1,2,...k, \\ j = 1,2,...,q, s = 1,2,...,n_j \tag{10}$$

Problem (9) can be written as follows:

$$Maximize\left[f_1(Z_1), f_2(Z_2), \ldots, f_k(Z_k)\right] \tag{10-a}$$

subject to

$$X' \in M'' = \left\{X' \in R^{n+(m-m_0)}\right\} : \sum_{j=1}^{q} A_j' X_j' \le b_0, \tag{10-b}$$

$$D_j' X_j' \le b_j', \tag{10-c}$$

$$X_j' \ge d_j > 0, j = 1,2,\ldots,q, q > 1\Big\} \tag{10-d}$$

where $X_j' = (X_j, Z_{ij}, Y_j)^T$ is an $(n_j + kn_j + m_j)$ - dimensional column vector of variables for the j^{th} subproblem, $j = 1,2,\ldots,q, A_j' = (A_j\,|0)$ is an $(m_0 \times (n_j + kn_j + m_j))$ matrix , where 0 is an $(m_0 \times (kn_j + m_j)$ zero matrix, D_j' is a $(3m_j + 2kn_j) \times (n_i + kn_j + m_i))$ matrix which is the coefficient of the left-hand side of the following system:

$$D_j X_j - y_j \le 0, \tag{11-a}$$

$$y_j \le U_j \tag{11-b}$$

$$y_j \ge L_j \tag{11-c}$$

$$Z_{ij} - H_{ij}'(X) \le 0 \tag{11-d}$$

$$G_{ij}'(X) - Z_{ij} \ge 0 \tag{11-e}$$

$$Y_j \ge \eta_j, \tag{11-f}$$

$$Y_j \le \beta_j, j = 1,2,\ldots,q, q > 1, i = 1,2,\ldots,k, \tag{11-g}$$

and b_j' is a $(3m_j + 2kn_j)$ - dimensional column vector of independent constraints right - hand side of the system (7) for the j^{th} subproblem, $j = 1,2,\ldots,q$,

Now, if $(X_j^*, Z_{ij}^*, Y_j^*)^T, j = 1,2,\ldots,k,$ is the optimal solution for the problem (11), then $(X_j^*, U_{ij}^*, Y_j^*)^T$ becomes the optimal solution for the problem (5) by using

$$u_{i,js}^{\ *} = z_{i,js}^{\ *} / x_s^* \; i = 1,2,\ldots k, j = 1,2,\ldots,q, s = 1,2,\ldots,n_j \tag{12}$$

3. TOPSIS METHOD

This section consists of two subsections. In subsection (3.1), we will redefine some basic concepts of distance measures for problem (10). In subsection (3.1), we will extend the concept of TOPSIS to obtain a compromise (nondominated) solution for α - LSMOP problems.

3.1. Some Basic Concepts of distance Measures

The compromise programming approach (see [7, 28]) was developed to solve multiple objective programming problem, by reducing the set of nondominated solutions. The compromise solutions are the closest, by some distance measure, to the ideal ones. In this section, we redefine some concepts for the problem (10).

The point $f_i(Z_i^*) f_i(Z_i^*) = \sum_{j=1}^{q} f_{ij}(Z_{ij}^*)$ in the criteria space is called the ideal point (reference point). As the measure of "closeness", d_p - metric is used. The d_p - metric defines the distance between two points, $f_i(Z_i) = \sum_{j=1}^{q} f_{ij}(Z_{ij})$ and $f_i(Z_i^*) = \sum_{j=1}^{q} f_{ij}(Z_{ij}^*)$ (the reference point) in k-dimensional space [18] as:

$$d_p = (\sum_{i=1}^{k} (f_i^* - f_i^*)^p)^{1/p} = (\sum_{i=1}^{k} [\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}]^p)^{1/p} \tag{13}$$

where $p \geq 1$.

Unfortunately, because of the incommensurability among objectives, it is impossible to directly use the above distance family. To remove the effects of the incommensurability, we need to normalize the distance family of the equation (8) by using the reference point (see [27, 28]) as:

$$\mathrm{d}_p = (\sum_{i=1}^{k} ([\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}] / \sum_{j=1}^{q} f_{ij}^*)^p)^{1/p} \tag{14}$$

where $p \geq 1$.

To obtain a compromise solution for the problem (6), the global criteria method [13,16] for large scale problems uses the distance family of the equation (9) by the ideal solution being the reference point. The difficulties appear when solving the following auxiliary problem :

$$\underset{x' \in M^*}{Minimize}\ d_p = (\sum_{i=1}^{k} [\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}]^p)^{1/p} \tag{15}$$

where X^* is the PIS and $p = 1, 2, \ldots, \infty$.

Usually, the solutions based on PIS are different from the solutions based on NIS. Thus, both PIS . (f^*) and NIS (f^-) can be used to normalize the distance family and obtain [14] :

$$d_p = (\sum_{i=1}^{k} [(\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}) / (\sum_{j=1}^{q} f_{ij}^* - \sum_{j=1}^{q} f_{ij}^-)^p]^{1/p} \tag{16}$$

where $p \geq 1$.

In this study, we further extend the concept of TOPSIS to obtain a compromise (nondominated) solution for α - LSMOP problems.

3.2 TOPSIS for α -LSMOP Problems

Consider the following α -LSMOP Problem [13,16,23]:

$$Maximize\,/\,Minimize\left[f_1(Z_1), f_2(Z_2), \ldots, f_k(Z_k)\right] \tag{17-a}$$

subject to

$$X' \in M'' \tag{17-b}$$

where

$\sum_{j=1}^{q} f_{tj}(Z_t)$:

Objective Function for Maximization, $t \in K_1 \subset K$,

$\sum_{j=1}^{q} f_{vj}(Z_v)$:

Objective Function for Minimization, $v \in K_2 \subset K$

In order to use the distance family of the equation (16) to solve the problem (17), we must first find $PIS(f^*)$ and $NIS(f^-)$ which are [14]:

$$f^* = \underset{X' \in M''}{Maximize}(or\ Minimize)\sum_{j=1}^{q} f_{tj}(Z_t)\ (or \sum_{j=1}^{q} f_{vj}(Z_v)),\quad \forall t(and\ v) \tag{18-a}$$

$$f^- = \underset{X' \in M''}{Minimize}(or Maximize)\sum_{j=1}^{q} f_{tj}(Z_t)\ (or \sum_{j=1}^{q} f_{vj}(Z_v)),\quad \forall t(and\ \ v) \tag{18-b}$$

$K_2 f^* = (f_1^*, f_2^*, \ldots, f_k^*)$ and $f^- = (f_1^-, f_2^-, \ldots, f_k^-)$ are the individual $\cup$ where $K = K_1$

positive (negative) ideal solutions.

Using the PIS and the NIS, we obtain the following distance functions, respectively:

$$d_p^{PIS} = [\sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} f_{tj}^* - \sum_{j=1}^{q} f_{tj}(Z_t)}{\sum_{j=1}^{q} f_{tj}^* - \sum_{j=1}^{q} f_{tj}^-}\right)^p + \sum_{v\in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} f_{vj}(Z_v) - \sum_{j=1}^{q} f_{vj}^*}{\sum_{j=1}^{q} f_{vj}^- - \sum_{j=1}^{q} f_{vj}^*}\right)^p]^{1/p} \tag{19-a}$$

and

$$d_p^{NIS} = [\sum_{t\in K_1} w_t^p \left(\frac{\sum_{j=1}^{q} f_{tj}(Z_t) - \sum_{j=1}^{q} f_{tj}^-}{\sum_{j=1}^{q} f_{tj}^* - \sum_{j=1}^{q} f_{tj}^-}\right)^p + \sum_{t\in K_2} w_v^p \left(\frac{\sum_{j=1}^{q} f_{vj}^- - \sum_{j=1}^{q} f_{vj}(Z_v)}{\sum_{j=1}^{q} f_{vj}^- - \sum_{j=1}^{q} f_{vj}^*}\right)^p]^{1/p} \tag{19-b}$$

where $w_i = 1,2,...,k,$ are the relative importance (weighs) of the objectives, and $\infty p = 1,2,...,.$

In order to obtain a compromise solution, we transfer the problem (17) into the following bi-objective problem with two commensurable (but often conflicting) objectives [15]:

$$\begin{aligned} &Minimize\ d_p^{PIS}(Z) \\ &Maximize\ d_p^{NIS}(Z) \end{aligned} \tag{20}$$

subject to

$M'' \in X'$

where $p = 1,2,...,\infty$

Since these two objectives are usually conflicting to each other, we can simultaneously obtain their individual optima. Thus, we can use membership functions to represent these individual optima. Assume that the membership functions $(\mu_1(Z) and\, \mu_2(Z))$ of the two objective functions are linear. Then, based on the preference concept, we assign larger degree to the one with shorter distance from the PIS for $\eta_1(Z)$, and assign larger degree to the one with farther distance from NIS for $\eta_2(Z)$. Therefore, as shown in figure 2, $\eta_1(Z)$ and $\eta_2(Z)$ can be obtained as (see [3, 30]):

$$\mu_1(Z) = \begin{cases} 1, & if\ d_p^{PIS}(Z) < (d_p^{PIS})^*, \\ 1 - \dfrac{d_p^{PIS}(Z) - (d_p^{PIS})^*}{(d_p^{PIS})' - (d_p^{PIS})^*} & if\ (d_p^{PIS})' \geq d_p^{PIS}(Z) \geq (d_p^{PIS})^*, \\ 0, & if\ d_p^{PIS}(Z) > (d_p^{PIS})', \end{cases} \tag{21-a}$$

and

$$\mu_2(Z) = \begin{cases} 1, & if\ d_p^{NIS}(Z) < (d_p^{NIS})^*, \\ 1 - \dfrac{d_p^{NIS}(Z) - d_p^{PIS}(Z)}{(d_p^{NIS})^* - (d_p^{PIS})'} & if\ (d_p^{NIS})' \leq d_p^{NIS}(Z) \leq (d_p^{NIS})^*, \\ 0, & if\ d_p^{NIS}(Z) < (d_p^{NIS})', \end{cases} \tag{21-b}$$

where

$$\begin{aligned} &(d_p^{PIS})^* = \underset{X' \in M''}{Minimize}\ d_p^{PIS}(Z)\ and\ the\ solution\ is\ Z^{PIS}, \\ &(d_p^{PIS})^* = \underset{X' \in M''}{Maximize}\ d_p^{NIS}(Z)\ and\ the\ solution\ is\ Z^{NIS}, \\ &(d_p^{PIS})' = d_p^{PIS}(Z^{NIS})\ and\ (d_p^{NIS})' = d_p^{NIS}(Z^{PIS}). \end{aligned}$$

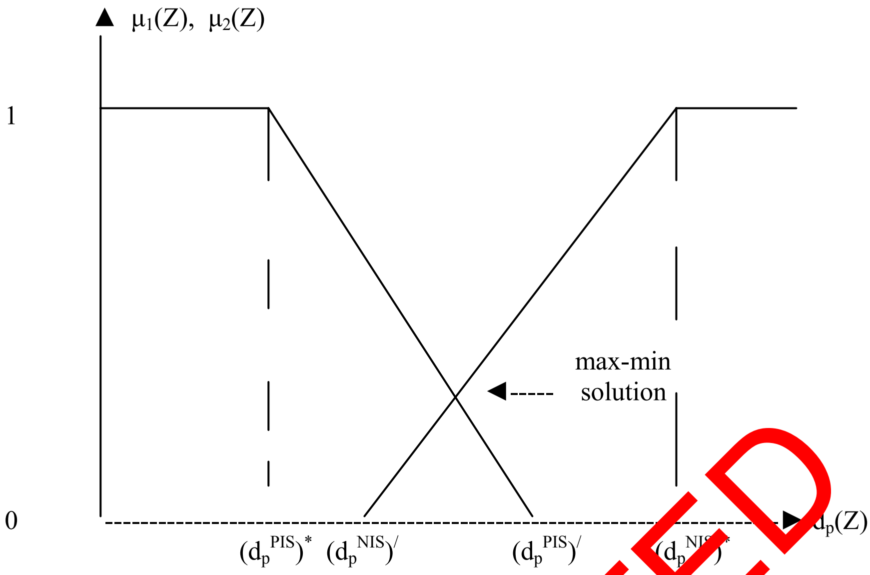

Figure 2. The membership functions of $\mu_1(Z)$ and $\mu_2(Z)$

Now, by applying the max-min decision model proposed by Bellman and Zadeh [3], and extended by Zimmermann [30], we can resolve problem (15). The satisfying decision, X^*, can be obtained by using the following model [3, 26]:

$$\mu_D(Z^*) = \underset{X \in M''}{Maximize}\{Minimize(\mu_1(Z), \mu_2(Z))\} \tag{22}$$

Finally, if $\delta = Minimize(\mu_1(Z), \mu_2(Z))$, the model (22) is equivalent to the form of Tchebycheff model (see [illegible]), which is equivalent to the following model:

$$Maximize\ \delta \tag{23-a}$$

subject to

$$\mu_1(Z) \geq \delta, \tag{23-b}$$

$$\mu_2(Z) \geq \delta, \tag{23-c}$$

$$[0,1], \in M'', \delta \in X' \tag{23-d}$$

where δ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

For finite value of p, problem (23) can be written as follows:

$$Maximize\ \delta \tag{24-a}$$

subject to

$$(\left[d_p^{PIS}(Z)-(d_p^{PIS})^*\right]/\left[(d_p^{PIS})'-(d_p^{PIS})^*\right])\geq\delta, \tag{24-b}$$

$$(\left[(d_p^{NIS})^*-d_p^{NIS}(Z)\right]/\left[(d_p^{NIS})^*-(d_p^{NIS})'\right])\geq\delta, \tag{24-c}$$

$$\left[0,1\right],\in M'',\delta\in X' \tag{24-d}$$

we have the following problem (instead ∞ For the special case of p = of the problem (20)) [15]:

$$\begin{aligned}&Minimize\ d_p^{PIS}(Z)\\&Maximize\ d_p^{NIS}(Z)\end{aligned} \tag{25-a}$$

subject to

$$X'\in M'' \tag{25-b}$$

$$\sum_{j=1}^{q}f_{tj}^{*}-\sum_{j=1}^{q}f_{tj}(Z_t))/(\sum_{j=1}^{q}f_{tj}^{*}-\sum_{j=1}^{q}w_t\left[(f_{tj}^{-})\right]\leq d_\infty^{PIS} \tag{25-c}$$

$$\sum_{j=1}^{q}f_{vj}^{-}-\sum_{j=1}^{q}f_{vj}^{*})/(\sum_{j=1}^{q}f_{vj}(Z_v)-\sum_{j=1}^{q}w_v\left[(f_{vj}^{*})\right]\leq d_\infty^{NIS} \tag{25-d}$$

$$\sum_{j=1}^{q}f_{tj}^{*}-\sum_{j=1}^{q}f_{tj}^{-})/(\sum_{j=1}^{q}f_{tj}(Z_t)-\sum_{j=1}^{q}w_t\left[(f_{tj}^{-})\right]\geq d_\infty^{PIS} \tag{25-e}$$

$$\sum_{j=1}^{q}f_{vj}^{-}-\sum_{j=1}^{q}f_{vj}(Z_v))/(\sum_{j=1}^{q}f_{tj}^{-}-\sum_{j=1}^{q}w_v\left[(f_{tj}^{*})\right]\geq d_\infty^{NIS} \tag{25-f}$$

where $p=1,2,\ldots,\infty$

4. THE ALGORITHM OF TOPSIS FOR SOLVING LSFMOP PROBLEMS

Thus, we can introduce the following algorithm to generate a set of α -nondominated solutions for the α-LSMOP problem:

<u>The algorithm (Alg-I):</u>

<u>Step 1. (i)</u> Formulate LSFMOP problem (1) which has linear objective functions as in equation (2).

<u>(ii)</u> Ask the DM to select $\alpha=\alpha^*\in\left[0,1\right]$.

<u>(iii)</u> Elicit a membership function from the DM for each fuzzy number in LSFMOP problem (1). For example, the fuzzy parameter $\tilde{a}=(a_1,a_2,a_3,a_4)$ can have a membership function of the following form [8,22,26]:

$$\mu_{\tilde{a}}(a)=\begin{cases}0, & a\le a_1,\\ 1-\left[(a-a_2)/(a_1-a_2)\right]^2, & a_1\le a\le a_2,\\ 1, & a_2\le a\le a_3,\\ 1-\left[(a-a_3)/(a_4-a_3)\right]^2, & a_3\le a\le a_4,\\ 0, & a\le a_4.\end{cases} \tag{26}$$

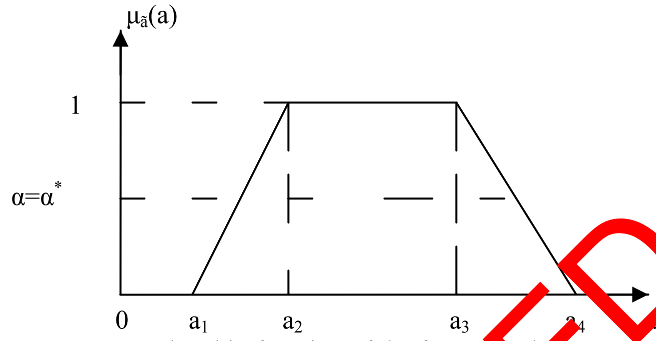

Figure 3. Membership function of the fuzzy number ã at $\alpha=\alpha^*$

Figure 3. illustrates the graph of a possible shape of a fuzzy number ã at $\alpha=\alpha^*$.

(iv) Transform LSMOP problem (1) into the form of the problem (5) by using (i), (ii), (iii) and equation (6).

Step 2. Transform problem (5) into the form of the problem (10).

Step 3. Construct the PIS payoff table of FOR?? the problem (10) by using the decomposition algorithm [5], and obtain $f^*=(f_1^*,f_2^*,\ldots,f_k^*)$, the individual positive ideal solutions.

Step 4. Construct the NIS payoff table of FOR??the problem (10) by using the decomposition algorithm [5] and obtain $f^-=(f_1^-,f_2^-,\ldots,f_k^-)$, the individual negative ideal solutions.

Step 5. Use the equations (18) & (19) and the steps 4 & 5 to construct d_p^{PIS} and d_p^{NIS}.

Step 6. Transform problem (17) to the form of problem (20).

Step 7. (i) Ask the DM to select $p=p^*\in\{1,2,\ldots,\infty\}$, (ii) Ask the DM to select $w_i=w_i^*, i=1,2,\ldots,k,$ where $\sum_{i=1}^{k} w_i=1,$

(iii) Use (i), (ii) and step 5 to compute $d_p^{PIS}(X)$ and $d_p^{NIS}(X)$.

Step 8. Construct the payoff table of FOR?? the problem (20):
At $p=1$, use the decomposition algorithm [5].
At $p\ge 2$, use the generalized reduced gradient method [17],
and obtain:

$d_p^* = ((d_p^{PIS})^*, (d_p^{NIS})^*), d_p^- = ((d_p^{PIS})', (d_p^{NIS})')$,

Step 9. (i) Transform problem (20) into the form of problem (23) by using the membership functions (21)..

(ii) Transform problem (23) into the form of problem (24).

Step 10. Solve problem (24).

Step 11. If the DM is satisfied with the current solution, go to step 11. Otherwise, go to step (1-ii).

Step 11. Stop.

6. A NUMERICAL EXAMPLE

Consider the following LSFMOP problem which has the angular structure:

$$Maximize\ f_1(X) = 3u''_{11}x_1 + 4u''_{12}x_2 \qquad (27\text{-a})$$

$$Minimize\ f_2(X) = -5u''_{21}x_1 - 6u''_{11}x_2 \qquad (27\text{-b})$$

subject to

$$x_1 + x_2 \geq 6\,, \qquad (27\text{-c})$$

$$3x_1 \leq \tilde{y}_1, \qquad (27\text{-d})$$

$$x_1 \geq 1\,, \qquad (27\text{-e})$$

$$4x_2 \leq \tilde{y}_2, \qquad (27\text{-f})$$

$$x_2 \geq 1\,, \qquad (27\text{-g})$$

where

$\tilde{y}_1 = (3,4,10,11)$, $\tilde{y}_2 = (5,6,12,22)$, $u''_{11} = (0,1,3,5)$, $u''_{12} = (1,6,7,8)$, $u''_{21} = (0,5,7,10)$, and $u''_{12} = (0,2,4,6)$.

We use (Alg-I) to solve the above problem.

Let $\alpha = \alpha^* = 0.36$ and by using the membership function (20), thus

$$Maximize\ f_1(X) = 3u''_{11}x_1 + 4u''_{12}x_2 \qquad (28\text{-a})$$

$$Minimize\ f_2(X) = -5u''_{21}x_1 - 6u''_{22}x_2 \qquad (28\text{-b})$$

subject to

$$x_1 + x_2 \geq 6\,, \qquad (28\text{-c})$$

$$3x_1 \leq y_1, \qquad (28\text{-d})$$

$$x_1 \geq 1\,, \qquad (28\text{-e})$$

$$4x_2 \leq y_2, \qquad (28\text{-f})$$

$x_2 \geq 1$, (28-g)

$0.2 \leq u_{11} \leq 4.6$, (28-h)

$2 \leq u_{21} \leq 7.8$, (28-i)

$1 \leq u_{12} \leq 9.4$, (28-j)

$0.4 \leq u_{22} \leq 5.6$ (28-k)

$3.2 \leq y_1 \leq 10.8$ (28-l)

$5.2 \leq y_2 \leq 19.2$ (28-m)

which can be written as follows :

$Maximize\ f_1(X) = 3z_{11} + 4z_{12}$ (29-a)

$Minimize\ f_2(X) = -5z_{21} - 6z_{22}$ (29-b)

subject to

$x_1 + x_2 \geq 6$, (29-c)

$3x_1 \leq y_1$, (29-d)

$x_1 \geq 1$, (29-e)

$4x_2 \leq y_2$, (29-f)

$x_2 \geq 1$ (29-g)

$0.2x_1 \leq z_{11} \leq 4.6x_1$ (29-h)

$2x_1 \leq z_{21} \leq 7.8x_1$ (29-i)

$x_2 \leq z_{12} \leq 9.4x_2$ (29-j)

$0.4x_2 \leq z_{22} \leq 5.6x_2$ (29-k)

$3.2 \leq y_1 \leq 10.8$ (29-l)

$5.2 \leq y_2 \leq 19.2$ (29-m)

We first obtain PIS and NIS for the problem (29) and use the transformation (12) to get :

Table1 : PIS payoff table

	f_1	f_2	x_1	x_2	y_1	y_2	u_{11}	u_{12}	u_{21}	u_{22}
$Maximize\ f_1(X)$	320.16*	-47.52	3.6	3.8	10.8	19.2	4.6	11.87	2	0.505
$Minimize\ f_2(X)$	21.36	-301.68*	3.6	4.8	10.8	19.2	0.2	1	7.8	5.6

PIS: f^{*}=(320.16 , 301.68)

Table2 : NIS payoff table

	f_1	f_2	x_1	x_2	y_1	y_2	u_{11}	u_{12}	u_{21}	u_{22}
Maximize $f_1(X)$	11.76	-41.76	3.6	2.4	10.8	9.6	0.2	1	2	0.4
Minimize $f_2(X)$	19.92	-23.52^{-}	1.2	4.8	10.8	19.2	0.2	1	2	0.4

NIS: f^{-}= (11.76 , -23.52)

Next, we use the equations (18 and 19) to construct the following ones:

$$d_p^{PIS} = \left[w_1^p (\left[320.16 - f_1(Z)\right] / \left[320-16-11.76\right])^p + w_2^p (\left[f_2(X) - (-301.68)\right] / \left[-23.52-(-301.68)\right])^{\,p}\right]^{1/p},$$

$$d_p^{NIS} = \left[w_1^p (\left[f_1(X) - (11.76)\right] / \left[320.16-11.76\right])^p + w_2^p (\left[-23.52 - f_2(X)\right] / \left[-23.52-(-301.68)\right])^{\,p}\right]^{1/p}$$

Thus, problem (20) is obtained.

In order to get numerical solutions, let us assume that $w_1^p = w_2^p = 0.5$ and at $p = 1$,

$$d_1^{PIS} = 0.00864x_3 + 0.00649x_4 + 0.00899x_5 + 0.11x_6 - 1.0623$$

$$d_1^{NIS} = 0.00864x_3 + 0.00649x_4 + 0.00899x_5 + 0.11x_6 - 0.0614$$

Table 3: PIS payoff table of the problem (20) at $p = 1$ and $w_1^p = w_2^p = 0.5$

	Minimize d_1^{PIS}	*Maximize* d_1^{NIS}
d_1^{PIS}	-0.921*	0.1408
d_1^{NIS}	0.014	-0.075*
f_1	230.16	19.92
f_2	301.68	-23.52
x_1	2.6	1.2
x_2	4.8	4.8
y_1	10.8	10.8
y_2	19.2	19.2
u_{11}	4.6	0.2
u_{12}	9.4	1
u_{21}	7.8	2
u_{22}	5.6	0.4

$d_1^{*} = (-0.921, -0.075), d_1^{-} = (0.1408, 0.014)$.

Now, it is easy to use problem (24) to formulate the following problem:

Maximize δ

subject to

$X \in M'', \delta \in [0,1]$,

$(d_1^{PIS}(X) - (-0.9211)) / 1.06193 \geq \delta,$

$(-0.075 - d_1^{NIS}(X)) / (-0.2162) \geq \delta$

The maximum "satisfactory level" $(\delta = 1)$ is achieved for the solution $x_1 = 3.6, x_2 = 2.4, y_1 = 10.8, y_2 = 9.6, u_{11} = 0.2, u_{12} = 1, u_{21} = 2, u_{22} = 0.4$.

Also, at $p = 2$, we have :

$$d_2^{PIS} = 0.000024x_3^2 + 0.000042x_4^2 - 0.005046x_3 + 0.006738x_4$$
$$-0.000063x_3x_4 + 0.000081x_5^2 + 0.000116x_6^2 + 0.000194x_5x_6$$
$$-0.00585x_5 - 0.00585x_6 + 0.56358)^{0.5}$$
$$d_2^{NIS} = (0.000024x_3^2 + 0.000042x_4^2 - 0.0001853x_3 - 0.000247x_4$$
$$-0.000063x_3x_4 + 0.000081x_5^2 + 0.000116x_6^2 + 0.000194x_5x_6$$
$$-0.00076x_5 - 0.000912x_6 + 0.000543)^{0.5}$$

Table 4 : PIS payoff table of the problem (20) at p=2 and $w_1^p = w_2^p = 0.5$

	Minimize d_1^{PIS}	*Maximize* d_1^{NIS}
d_1^{PIS}	0.5040*	0.5182
d_1^{NIS}	0.3557	0.4593*
f_1	181.863	186.544
f_2	-150.485	-225.797
x_1	2.2191	2.[illegible]172
x_2	4.0252	4.1490
y_1	7.2915	7.2915
y_2	16.5995	16.5995
u_{11}	10.[illegible]005	4.5921
u_{12}	9.394[illegible]	9.3998
u_{21}	7.778	7.796
u_{22}	2.658	5.598

$d_2^* = (0.5040, 0.4593), d_2' = (0.5182, 0.3557)$

Now, it is easy to use problem (24) to formulate the following problem:

Maximize δ

subject to

$X \in M'', \delta \in [0,1]$,

$(d_2^{PIS}(X) - 0.504) / 0.0142 \geq \delta$,

$(0.4593 - d_2^{NIS}(X)) / 0.1036 \geq \delta$

The maximum "satisfactory level" $(\delta = 0.9999)$ is achieved for the solution

$$X_1 = 1.5852, X_2 = 0.4.4231, y_1 = 8.6894, y_2 = 17.7168, u_{11} = 2.1633,$$
$$u_{12} = 2.0296, u_{21} = 3.4997, u_{22} = 3.4997$$

7. CONCLUDING REMARKS

In this paper, a TOPSIS approach is extended to solve Interactive Large Scale Multiple Objective Programming problems involving fuzzy parameters (LSFMOP). The LSFMOP problems using TOPSIS approach provides an effective way to find the compromise (satisfactory) solution of such problems. Generally, TOPSIS provides a broader principle of compromise for solving multiple criteria decision making problems. It transfers k-objectives (criteria), which are conflicting and non-commensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS), which are commensurable but most of the time conflicting. Then, the bi-objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS, compromise solution by a second–order compromise. The max-min operator is then considered as a suitable one to resolve the conflict between the new criteria (the shortest distance from the PIS and the longest distance from the NIS).

Also, in this paper, an interactive algorithm for generating an α-Pareto Optimal (compromise) solution of LSFMOP problems is presented. It is based on the decomposition algorithm of LSFMOP problems with block angular structure via TOPSIS approach for p=1, and Generalized reduced gradient method for p≥2. This algorithm has few features, (i) it combines both LSFMOP and TOPSIS approach to obtain TOPSIS's compromise solution for the problem, (ii) it can be efficiently coded, (iii) it is found that the decomposition based methods generally leads better results than the traditional simplex-based methods. Especially, the efficiency of the decomposition-based method increased sharply with the scale of the problem. Finally, an illustrative numerical example clarified the various aspects of both the solution concept and the proposed algorithm. Also, applications of the proposed algorithm will be required.

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