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CONTINUOUS REVIEW INVENTORY MODELS UNDER TIME VALUE OF MONEY AND CRASHABLE LEAD TIME CONSIDERATION

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Abstract: A stock is an asset if it can react to economic and seasonal influences in the management of the current assets. The financial manager must calculate the input of funds to the stock intelligently and the amount of money cycled through stocks, taking into account the time factors in the future. The purpose of this paper is to propose an inventory model considering issues of crash cost and current value. The sensitivity analysis of each parameter, in this research, differs from the traditional approach. We utilize a course of deduction with sound mathematics to develop several lemmas and one theorem to estimate optimal solutions. This study first tries to find the optimal order quantity at all lengths of lead time with components crashed at their minimum duration. Second, a simple method to locate the optimal solution unlike traditional sensitivity analysis is developed. Finally, some numerical examples are given to illustrate all lemmas and the theorem in the solution algorithm.

Keywords: Inventory model, crashable lead time, time value of money.

MSC: 90B05.

1. INTRODUCTION

From the perspective of financial management, stocks often comprise a very large proportion of a balance sheet. Funds invested in stock cannot be used elsewhere because they are not liquid assets. They become liquid only when the stocks are sold. Considering capital running factors, stocks must be turned over fast, so enterprises must determine appropriate inventory policies in order to reduce idleness of the stocks, and dead and scrap stocks in order to sell and produce effectively.

Studying inventory models and considering time and value, Moon and Yun [13] employed the discounted cash flow approach to fully recognize the time value of money and constructed a finite planning horizon EOQ model in which the planning horizon is a random variable. Jaggi and Aggarwal [8], in order to discuss an optimal replenishment policy with an infinite planning horizon, reported that a deteriorating product under the impact of a credit period did not allow shortages. Bose et al. [2] and Hariga [6] developed two inventory models, which incorporated the effects of inflation and time value of money with a constant rate of deterioration and time proportional demand. Moon and Lee [12] investigated the effect of inflation and time-value of money in an inventory model with a random product life cycle. Wee and Law [20] employed the concepts of inflation and the time value of money in a model where demand is price-dependent and shortages allowed. Chung and Tsai [3] derived an inventory model for deteriorating items with the demand of linear trends and shortages during the finite planning horizon, considering the time value of money. Sun and Queyranne [19] investigated general multi-product, multi-stage production and an inventory model using the net time value of money with its total cost as the objective function. Balkhi [1] considered a production lot size inventory model with deteriorated and imperfect products, taking into account inflation and the time value of money. Moon et al. [9] developed inventory models for ameliorating and deteriorating items with a time-variant demand pattern over a finite planning horizon, taking into account the effects of inflation and the time value of money. Shah [17] derived an inventory model by assuming a constant rate of deterioration of units in an inventory and the time value of money under the conditions of permissible delay in payments. Wee et al. [21] developed an optimal replenishment inventory strategy to consider both ameliorating and deteriorating effects, taking into account the time value of money and a finite planning horizon. Both the amelioration and deterioration rate were assumed to follow Weibull distribution. Dey et al. [5] considered an inventory model for a deteriorating item with time dependent demand and interval-valued lead-time over a finite time horizon. The inflation rate and time value of money are taken into account.

In addition, Ji [9] constructed a general framework of an inventory system for non-instantaneous deteriorating items with shortages, the time value of money, and inflation. Das et al. [4] developed a two-warehouse production-inventory model for deteriorating items considered under inflation and the time value of money over a random time horizon. Hou et al. [7] presented an inventory model for deteriorating items with a stock-dependent selling rate under inflation and the time value of money over a finite planning horizon. However, Kumar Maiti [10] also has developed an inventory model incorporating customers' credit-period dependent dynamic demand, inflation, and the time value of money, where the lifetime of the product is imprecise in nature.

In the recent studies, decomposing the lead time into several crashing periods is a controllable approach to lead time reduction. Ouyang et al. [15] constructed a variable lead time from a mixed inventory model with backorders and lost sales. In this article, we extend the inventory model of Ouyang et al. [15]. When the distribution of lead time demand is normal, we consider the time value of a continuous review inventory model with a mixture of backorders and lost sales.

This paper is organized as follows. In the next section, we define the notation of the inventory model and its assumptions. In section 3, first we construct the inventory model, taking into account the time value. Then we prove that the total expected annual cost is piece-wisely concave down with respect to lead time, and convex in order quantities. We apply a simple method to develop four lemmas and one theorem, and locate the optimal solution for constructing the procedure of solving a replenishment policy in section 4. This approach differs from the traditional methods. In section 5, numerical examples are offered to illustrate our algorithm. Section 6 summarizes the article and presents some conclusions.

2. NOTATION AND ASSUNPTIONS

We use the following notation and assumptions to develop inventory models with crashing component lead time and the time value of money.

A : Fixed ordering cost per order.

D : Average demand per year.

h : Inventory holding cost per item per year.

L : Lead time that has n mutually independent components. The i th component has a minimum duration a_i and normal duration b_i with a crashing cost c_i per unit time under the assumption $c_1 \le c_2 \le \cdots \le c_n$. The components of L are crashed one at a time, starting from the component of the least c_i and so forth. Hence, the range for L is from $\sum_{j=1}^{n} a_j$ to $\sum_{j=1}^{n} b_j$.

L_j : The length of lead time with components 1, 2,…, j are crashed to their minimum durations. We define $L_0 = \sum_{i=1}^{n} b_i$ and $L_n = \sum_{i=1}^{n} a_i$ and $L_j = L_n + \sum_{t=j+1}^{n} b_t - a_t$, for $j = j,...,n-1$. Since $b_j > a_j$, it follows that $L_{j-1} > L_j$, for $j = 1,...,n$.

$R(L)$: The lead time crashing cost per cycle for a given $L \in [L_i, L_{i-1}]$ is given by

$$R(L) = c_i(L_{i-1} - L) + \sum_{t=1}^{i-1} c_t(b_t - a_t) \ .$$

Q : Order quantity.

Q_j : The optimal order quantity when lead time is L_j .

X : Lead time demand that follows a normal distribution with mean μL and standard derivation $\sigma\sqrt{L}$.

r : Reorder point. Since $r =$ expected demand during lead time + safety stock, $r = \mu L + k\sigma\sqrt{L}$. Inventory is continuously reviewed. Replenishments are made whenever the inventory level falls to the reorder point r .

q : Allowable stockout probability during L .

k : Safety factor that satisfies $P(X > r) = P(Z > k) = q$, Z representing the standard normal random variable.

$B(r)$: Expected shortage at the end of the cycle. We quote the results of Ouyang et al. [15], $B(r) = \sigma\sqrt{L\Psi(k)}$ where $\Psi(k) = \varphi(k) - k\left[1 - \Phi(k)\right]$ as ϕ, where Φ denotes the standard normal probability density function and cumulative distribution.

β : The fraction of the demand during the stockout period that will be backordered.

π : Fixed penalty cost per unit short.

π_0 : Marginal profit per unit.

θ : The interest rate per year.

3. MATHEMATICAL FORMULATION

First, we study the total expected annual cost of the inventory model with backorders and lost sales for variable lead time. We quote the Equation (2) of Ouyang et al. [15], for $L \in \left[L_n, L_0\right]$, who derived the total expected annual cost, $EAC(Q, L)$, without considering the time value of money as follows:

$$EAC(Q, L) = EAC_j(Q, L) \tag{1}$$

for $L \in \left[L_j, L_{j-1}\right]$, with $j = 1, 2, ..., n$. We rewrite the total expected annual cost as

$$EAC_j(Q, L) = \frac{h}{2}Q + \frac{D}{Q}R_j(L) + \frac{D}{Q}p(L) + \Omega(L) \tag{2}$$

Where

$$\Omega(L) = h\sigma\left[k + (1-\beta)\Psi(k)\right]\sqrt{L} \ ,$$

$$p(L) = \sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)\sqrt{L} + A$$

and

$$R_j(L) = c_j(L_{j-1} - L) + \sum_{t=1}^{j-1} c_t(b_t - a_t)$$

for

$$L \in \left[L_j, L_{j-1}\right].$$

Secondly, we consider the inventory model, taking into account the time value. The expected net inventory level, just before the order arrives, is $k\sigma\sqrt{L} + (1-\beta)B(r)$, and the expected net inventory at the beginning of the cycle is $Q+k\sigma\sqrt{L}+(1-\beta)B(r)$. Therefore, the expected average inventory level is $Q+k\sigma\sqrt{L}+(1-\beta)B(r)-Dt$ for $t\in\left[0,\frac{Q}{D}\right]$. Hence, the inventory carrying cost for the first cycle equals

$$\int_{t=0}^{\frac{Q}{D}}\left[Q+k\sigma\sqrt{L}+(1-\beta)B(r)-Dt\right]he^{-\theta t}dt$$
$$=\frac{h}{\theta}\left[k\sigma\sqrt{L}+(1-\beta)B(r)\right]\left(1-e^{-\theta\frac{Q}{D}}\right)+h\frac{D}{\theta^2}\left(e^{-\theta\frac{Q}{D}}-1+\theta\frac{Q}{D}\right) \tag{3}$$

We adopt the discounted cash flow approach following Moon and Yun [14]. At the beginning of each cycle will be cash outflows for the ordering cost, stockout cost and lead time crashing cost. Therefore, the total relevant cost for the first cycle is

$$A+(\pi+\pi_0(1-\beta))\sigma\ \Psi(k)\sqrt{L}+R(L)+\frac{h}{\theta}\left[k\sigma\sqrt{L}+(1-\beta)B(r)\right]\left(1-e^{-\theta\frac{Q}{D}}\right)$$
$$+\frac{Dh}{\theta^2}\left(e^{-\theta\frac{Q}{D}}-1+\theta\frac{Q}{D}\right).$$

Referring to Silver and Peterson [18], we get that the time value of money of the expected total relevant cost over an infinite time horizon, $C(Q,L)$, is given by

$$\frac{1}{1-e^{-\theta\frac{Q}{D}}}\left\lfloor A+(\pi+\pi_0(1-\beta))\sigma\psi(k)\sqrt{L}+R(L)\right\rfloor$$
$$+\frac{1}{1-e^{-\theta\frac{Q}{D}}}\left\{\frac{h}{\theta}\left[r-\mu L+(1-\beta)B(r)\right](1-e^{-\theta\frac{Q}{D}})+\frac{Dh}{\theta^2}(e^{-\theta\frac{Q}{D}}-1+\theta\frac{Q}{D})\right\}$$

We can rewrite $C(Q,L)$ as follows:

$$C(Q,L)=\frac{f(L)}{1-e^{-\theta\frac{Q}{D}}}+g(L)+\frac{h}{\theta^2}\frac{Q\theta}{1-e^{-\theta\frac{Q}{D}}} \tag{4}$$

for $0<Q<\infty$ and $0\le L<\infty$,
where
$f(L)=p(L)+R(L), p(L)=\sigma\left[\pi+(1-\beta)\pi_0\right]\Psi(k)\sqrt{L}+A,$
for

$L\in\left[L_j,L_{j-1}\right], j=1,...,n, R(L)=R_j(L),$

$R_j(L)=c_j(L_{j-1}-L)+\sum_{t=1}^{j-1}c_t(b_t-a_t), g(L)=\frac{\Omega(L)}{\theta}-\frac{Dh}{\theta^2}$

and

$$\Omega(L) = h\sigma\left[k + (1-\beta)\Psi(k)\right]\sqrt{L} \ .$$

Third, we use $R(L)$ to denote the crashing cost. We have that

$$R(L) = R_j(L)$$

where

$$R_j(L) = c_j(L_{j-1} - L) + \sum_{t=1}^{j-1} c_t(b_t - a_t) \ \text{for} \ L \in \left[L_j, L_{j-1}\right],$$

with

$$j = 1, 2, ..., n \ .$$

Since $R_j(L)$ is a linear decreasing function on $\left[L_j, L_{j-1}\right]$, we get $L_{j-1} - L_j = b_j - a_j$

and $R_j(L_j) = c_j(L_{j-1} - L_j) + \sum_{t=1}^{j-1} c_t(b_t - a_t) = \sum_{t=1}^{j} c_t(b_t - a_t) = R_{j+1}(L_j)$,

it follows that $R(L)$ is a piece-wise linear decreasing and continuous function on $[L_n, L_0]$.

At the points $\{L_j : j = 1, 2, ..., n-1\}$, $R(L)$ has different slopes c_j and c_{j+1} of the tangent line from the right and left, respectively. Hence, $R(L)$ is not differentiable at those points, so we must divide the domain of L from $\left[L_n, L_0\right]$ into subintervals $\left[L_j, L_{j-1}\right]$, with $j = 1, 2, ..., n$.

According to Rachamadugu [16], in order to compare our results with the previous model of Ouyang et al. [15], we use $A(Q, L) = \theta \ C(Q, L)$, an alternate but equivalent measure. $A(Q, L)$ represents the equivalent uniform cash flow stream that generates the same $C(Q, L)$. From $\lim_{\theta \to 0} \theta \left(\frac{-Dh}{\theta^2} + \frac{h}{\theta^2} \frac{Q\theta}{1 - e^{-\theta \frac{Q}{D}}} \right) = \frac{h}{2} Q$, we have

$$\lim_{\theta \to 0} A(Q, L) = \frac{h}{2} Q + \frac{D}{Q} R_i(L) + \frac{D}{Q} p(L) + \Omega(L) \ .$$

That is equation (2) for the total expected annual cost of Ouyang et al. [15]. Hence, we extend their model. Now, we begin to find the minimum value of the total expected annual cost $C(Q, L)$ for $0 < Q < \infty$ and $0 \le L < \infty$. Taking the first and second partial derivatives of $C(Q, L)$ with respect to L gives

$$\frac{\partial C(Q,L)}{\partial L}=\frac{1}{1-e^{-\theta\frac{Q}{D}}}\left[(\pi+\pi_0(1-\beta))\frac{\sigma\ \Psi(k)}{2\sqrt{L}}-c_j\right]+ \frac{h\sigma}{2\theta\sqrt{L}}\left[k+(1-\beta)\Psi(k)\right] \tag{5}$$

and

$$\frac{\partial^2 C(Q,L)}{\partial L^2}=\frac{-\sigma(\pi+\pi_0(1-\beta))\psi(k)}{4\sqrt{L^3}\left(1-e^{-\theta\frac{Q}{D}}\right)}+\frac{-h\sigma}{4\theta\sqrt{L^3}}\left[k+(1-\beta)\psi(k)\right]. \tag{6}$$

From $\frac{\partial^2 C(Q,L)}{\partial L^2}<0$, $C(Q,L)$ is concave in $L\in\left[L_j,L_{j-1}\right]$. Hence, we can reduce the minimum problem from $\left\{C(Q,L):\sum_{i=1}^{n}a_i\le L\le\sum_{i=1}^{n}b_i, 0\le L<\infty\right\}$ to the boundary of each piece-wise defined domain as $\left\{C(Q,L):L=L_j,\ \text{for}\ j=0,1,\cdots,n,\ 0<Q<\infty\right\}$. Fixing $L=L_j$, with $j=0,1,...,n,$ taking the first and second partial derivative of $C(Q,L_j)$ with respect to Q, gives

$$\frac{\partial C(Q,L_j)}{\partial Q}=\frac{-\theta}{D}\frac{f(L_j)}{\left(1-e^{-\theta\frac{Q}{D}}\right)^2}e^{-\theta\frac{Q}{D}}+\frac{h}{\theta}\frac{1-e^{-\theta\frac{Q}{D}}-\theta\frac{Q}{D}e^{-\theta\frac{Q}{D}}}{\left(1-e^{-\theta\frac{Q}{D}}\right)^2}$$

and

$$\frac{\partial^2 C(Q,L_j)}{\partial Q^2}=f(L_j)\frac{\theta^2}{D^2}\frac{\left(1+e^{-\theta\frac{Q}{D}}\right)e^{-\theta\frac{Q}{D}}}{\left(1-e^{-\theta\frac{Q}{D}}\right)^3}+\frac{h}{D}e^{-\theta\frac{Q}{D}}\frac{\left(2+\theta\frac{Q}{D}\right)e^{-\theta\frac{Q}{D}}-2+\theta\frac{Q}{D}}{\left(1-e^{-\theta\frac{Q}{D}}\right)^3}.$$

Rachamadugu [10] derived that $e^{-x}>\frac{2-x}{2+x}$, for $x>0$. Hence, we know that the second term of the second partial derivative is positive, so $C(Q,L_j)$ is convex in $Q\in(0,\infty)$ with the minimum point at Q_j such that

$$e^{\frac{\theta}{D}Q_j}-1-\frac{\theta}{D}Q_j=\frac{\theta^2}{Dh}f(L_j) \tag{7}$$

Let $\phi(Q)=e^{\frac{\theta}{D}Q}-1-\frac{\theta}{D}Q$ for $0\le Q<\infty$. We know that $\phi(Q)$ is a strictly increasing function from $\phi(0)=0$ to $\lim_{Q\to\infty}\phi(Q)=\infty$. Therefore, given an L_j, there exists a unique point Q_j satisfying $e^{\frac{\theta}{D}Q_j}-1-\frac{\theta}{D}Q_j=\frac{\theta^2}{Dh}f(L_j)$.

We have shown that $C(Q,L)$ is concave down in $L\in\left[L_j,L_{j-1}\right]$. In addition, for $L=L_j$, with $j=0,1,...,n, C(Q_j,L_j)$ is concave up in Q. So the minimum problem is to consider the points (Q_j,L_j) for $j=0,1,...,n$ .We construct an algorithm as follows.

(i) Find the local minimum points (Q_j,L_j) for $j=0,1,...,n$ along the boundaries of each subinterval.

(ii) For each point (Q_j,L_j), evaluate the total expected annual cost $C(Q_j,L_j)$ for $j=0,1,...,n$.

(iii) Solve the minimum of $\left\{C(Q_j,L_j): j=0,1,\cdots,n\right\}$.

4. MONOTONIC PROPERTY AND PROPOSITIONS

We determine a criterion to reduce the computation of finding the local minimum for the inventory model. In addition, we construct a new function as the difference of the total expected annual cost function evaluated at two adjacent local minimum points. Then we verify if it is an increasing function of the fraction of backorders. Therefore, we can reduce the calculation for locating the optimal solution. Our purpose in this section is to develop a procedure that eliminates the need to compute the exact values of $\left\{Q_j: j=0,\cdots,n\right\}$ and $\left\{C(Q_j,L_j): j=0,\cdots,n\right\}$. We establish a criterion to compare Q_j and Q_{j-1} implicitly. Moreover, we change the value of β to investigate the sensitive analysis of backordered ratio per cycle. Our new method significantly reduces the amount of computation. First, we offer such a criterion that we can implicitly compare Q_j with Q_{j-1}. All the proofs for the Lemmas and the theorem are in the Appendix.

Lemma 1: Given a backordered fraction ratio β, then
$Q_j<Q_{j-1}\Leftrightarrow c_i(\sqrt{L_{i-1}}+\sqrt{L_i})<\sigma\left[\pi+(1-\beta)\pi_0\right]\Psi(k)$.

Secondly, we state the monotone property between $C(Q_i,L_i)$ and Q_j.

Lemma 2: For a given β , if $Q_j < Q_{j-1}$, then $C(Q_i, L_i) < C(Q_{i-1}, L_{i-1})$.

From the Table 2 of Ouyang et al. [15], if $Q_j > Q_{j-1}$, we know that there is no regulation between $C(Q_j, L_i)$ and $C(Q_{j-1}, L_{i-1})$. However, if we treat $Q_j(\beta)$ and $Q_{j-1}(\beta)$ as functions of β , then we can still measure the difference between $C(Q_j(\beta), L_i)$ and $C(Q_{j-1}(\beta), L_{i-1})$.

Lemma 3: *For a given interval* $\beta \in [\beta_0, \beta_1]$, *if* $Q_j(\beta) \geq Q_{j-1}(\beta)$, *then for* $\beta \in [\beta_0, \beta_1]$, $C(Q_j(\beta), L_i) - C(Q_{i-1}(\beta), L_{i-1})$ *is an increasing function of* β .

Here, we show the monotone property of $Q_j(\beta) \geq Q_{j-1}(\beta)$ with respect to β .

Lemma 4: *Given a fixed* β_0 , *if* $Q_j(\beta_0) \geq Q_{j-1}(\beta_0)$, *then* $Q_j(\beta) \geq Q_{j-1}(\beta)$ *for the interval* $\beta \in [\beta_0, 1]$.

Finally, we derive a criterion to compare $C(Q_{i-1}(\beta), L_{i-1})$ with $C(Q_i(\beta), L_i)$.

Theorem 1: *If* $C(Q_i(\beta_0), L_i) > C(Q_{i-1}(\beta_0), L_{i-1})$ *and* $Q_i(\beta_0) \geq Q_{i-1}(\beta_0)$ *for a fixed* β_0 , *then* $C(Q_i(\beta), L_i) > C(Q_{i-1}(\beta), L_{i-1})$ *for the interval* $\beta \in [\beta_0, 1]$.

5. NUMERICAL EXAMPLES

The following numerical examples explain how the above Lemmas and the theorem simplify the solution procedure. Using the numerical example from Ouyang et al. [15], we have the following data: $D = 600$ units/year, $k = 0.845$, $A = \$200$ /per order, $h = \$20$ /per item per year, $\pi = \$50$ /per unit short, $\pi_0 = \$150$ /per unit, $\sigma = 7$ units/per week, $q = 0.2$ (in this situation, from the normal distribution, we find $k = 0.845$ and $\psi(k) = 0.110$), and the lead time has three components with data shown in Table 1.

We assume that the interest rate $\theta = 0.1$. Following the solution algorithm, we obtain Table 2. When $\beta = 1$ in Table 2, we slightly change the decimal expression of Q_i , so apparently it implies $Q_2(1) < Q_1(1)$.

Table 1: Lead time data

Lead time component, i	0	1	2	3
L_i	8	6	4	3
$R(L_i)$	0	5.6	22.4	57.4
Normal duration, b_i (days)		20	20	16
Minimum duration, a_i (days)		6	6	9
$b_i - a_i$ (weeks)		2	2	1
Unit crashing cost, c_i ($/week)		2.8	8.4	35

Table 2: Summary of solutions (L_j in weeks)

	$\beta = 0$		$\beta = 0.5$		$\beta = 0.8$		$\beta = 1$	
j	Q_j	$C(Q_j, L_j)$	Q_j	$C(Q_j, L_j)$	Q_j	$C(Q_j, L_j)$	Q_j	$C(Q_j, L_j)$
0	239	52579.28	191	42293.07	161	36160.27	142.02	32088.04
1	223	48811.71	181	39921.80	156	34616.81	139.34	31092.47
2	208	44979.04	174	37734.19	153	33406.93	139.19	30530.02
3	206	44235.12	176	37960.58	158	34210.40	146.53	31716.41

Considering the cases for β = 0, 0.5, 0.8 and 1, we use Table 3 to evaluate $c_i(\sqrt{L_{i-1}} + \sqrt{L_i})$ along with $\sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)$.

Table 3: Data for comparison

j	$c_j(\sqrt{L_j} + \sqrt{L_{j-1}})$	β	$\sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)$
1	14.78	0	154.2
2	37.38	0.5	96.38
3	130.62	0.8	61.68
		1	38.55

When $\beta = 0$, we find $c_i(\sqrt{L_{i-1}} + \sqrt{L_i}) < \sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)$ for all $i = 1,2,3$. By Lemma 1, we get $Q_i(0) < Q_{i-1}(0)$ for all $i = 1,2,3$ For all $i = 1,2,3$, Lemma 2 implies $C(Q_i(0), L_i) < C(Q_{i-1}(0), L_{i-1})$, so the optimal solution is $(Q_3(0), L_3) = (206,3)$. When $\beta = 0.5, 0.8$ and 1, we find $c_i(\sqrt{L_{i-1}} + \sqrt{L_i})$ $< \sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)$ for all $i = 1,2$. Thus, by Lemma 2, we have $C(Q_i(\beta), L_i) < C(Q_{i-1}(\beta), L_{i-1})$ when $\beta = 0.5, 0.8$ and 1 with $i = 1,2$. Therefore, we need to calculate only $\min_{i=2,\,3} C(Q_i(\beta), L_i)$ instead of $\min_{i=0,1,2,3} C(Q_i(\beta), L_i)$ in order to get the optimal solution for $\beta = 0.5, 0.8$ and 1. Furthermore, we find $C(Q_3(0.5), L_3) = 34960 > 37734$ $= C(Q_2(0.5), L_2)$ and $Q_3(0.5) = 176 > 173 = Q_2(0.5)$. Using Theorem 1, we can conclude that $\min_{i=2,\,3} EAC(Q_i(\beta), L_i) = EAC(Q_2(\beta), L_2)$ for $\beta = 0.5, 0.8$ and 1. Consequently, Lemmas 1, 2, 3 and 4, and Theorem 1, can simplify the solution procedure. With our criterion, it is very easy to compare the local minimum points Q_i and Q_{i-1}. Using the monotone property between $C(Q_i, L_i)$ and Q_i, and the difference of the total expected annual cost function evaluated at two adjacent local minimum points as an increasing function of the fraction of backorders, our computation results become much simpler.

6. CONCLUSION

Usually, there are three kinds of stocks in a company: raw materials, work-in-process, and finished goods. These stocks all need funds to be managed. The current assets are the most difficult to be cashed. Good inventory management is often the mark of a well-run firm. This article considers the time value of money of a continuous review inventory model with a mixture of backorders and lost sales, where lead time demand has a normal distribution. We find the optimal order quantity and optimal lead time of the total expected annual costs at all lengths of lead times with components crashed to their minimum duration, and construct a process for an optimal solution. We develop a principle to compare the optimal order quantities $Q_i(\beta)$ at points $(Q_i(\beta),L_i)$ for all i =1, 2, …, n. Our approach, when solving most situations like this, differs from the traditional sensitivity analysis. We deduce the optimal values via complete procedures that are mathematically sound.

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APPENDIX

Proof of Lemma 1:

From $e^{\frac{\theta}{D}Q_j} - 1 - \frac{\theta}{D}Q_j = \frac{\theta^2}{Dh} f(L_j)$ and $\phi(Q) = e^{\frac{\theta}{D}Q} - 1 - \frac{\theta}{D}Q$, which is a strictly increasing function, we get the following criterion for comparing Q_j and Q_{j-1}:

$$Q_j < Q_{j-1} \Leftrightarrow f(L_j) < f(L_{j-1}) \Leftrightarrow p(L_i) + R(L_i) < p(L_{i-1}) + R(L_{i-1})$$

$$\Leftrightarrow c_i(\sqrt{L_{i-1}} + \sqrt{L_i}) < \sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k) .$$

Proof of Lemma 2: From Equations (4) and (7), we have

$$C(Q_j, L_i) = \frac{Dh}{\theta^2} e^{\frac{\theta}{D}Q_j} + \frac{h\sigma}{\theta}\left[k + (1-\beta)\Psi(k)\right]\sqrt{L_i} - \frac{Dh}{\theta^2} \tag{8}$$

Using Equation (8) and $L_{j-1} - L_j = b_j - a_j > 0$, we get Lemma 2.

Proof of Lemma 3: From equation (8), we obtain

$$C(Q_j(\beta), L_j) - C(Q_{j-1}(\beta), L_{j-1}) = \frac{Dh}{\theta^2}\left(e^{\frac{\theta}{D}Q_j(\beta)} - e^{\frac{\theta}{D}Q_{j-1}(\beta)}\right) + \frac{p(L_j) - p(L_{j-1})}{\theta}. \tag{9}$$

Since $p(L_j) - p(L_{j-1}) = h\sigma\left[k + (1-\beta)\psi(k)\right](\sqrt{L_j} - \sqrt{L_{j-1}})$ and $L_{j-1} > L_j$, we get that $p(L_i) - p(L_{i-1})$ is an increasing function of β .

From equation (7), $e^{\frac{\theta}{D}Q_j(\beta)} - 1 - \frac{\theta}{D}Q_j(\beta) = \frac{\theta^2}{Dh} f(L_j)$ and $f(L_j) = p(L_j) + R(L_j)$, with $p(L_j) = \sigma\left[\pi + (1-\beta)\pi_0\right]\Psi(k)\sqrt{L_j} + A$ and $R(L_j) = \sum_{k=1}^{j} c_k(b_k - a_k)$; then we take the derivative of Equation (7) with respect to β as follows:

$$\left(e^{\frac{\theta}{D}Q_j(\beta)} - 1\right)\frac{\theta}{D}\frac{dQ_j(\beta)}{d\beta} = \frac{-\sigma\theta^2\pi_0\psi(k)}{Dh}\sqrt{L_j} .$$

Hence, we get

$$\frac{dQ_j(\beta)}{d\beta} = \frac{-\sigma\theta\pi_0\psi(k)}{h\left(e^{\frac{\theta}{D}Q_j(\beta)} - 1\right)}\sqrt{L_j} \tag{10}$$

We assume $\mathrm{H}(x) = \frac{e^x}{e^x - 1}$, which shows that $\mathrm{H}'(x) = \frac{-e^x}{(e^x-1)^2} < 0$.

From $Q_j(\beta) \geq Q_{j-1}(\beta)$, we have

$$\frac{e^{\frac{\theta}{D}Q_{j-1}(\beta)}}{e^{\frac{\theta}{D}Q_{j-1}(\beta)} - 1} \geq \frac{e^{\frac{\theta}{D}Q_j(\beta)}}{e^{\frac{\theta}{D}Q_j(\beta)} - 1} \tag{11}$$

Combining Equation (11) with $L_{j-1} > L_j$ shows

$$\frac{e^{\frac{\theta}{D}Q_{j-1}(\beta)}}{e^{\frac{\theta}{D}Q_{j-1}(\beta)}-1}\sqrt{L_{j-1}} > \frac{e^{\frac{\theta}{D}Q_{j}(\beta)}}{e^{\frac{\theta}{D}Q_{j}(\beta)}-1}\sqrt{L_{j}} \tag{12}$$

Now, we compute the derivative of $e^{\frac{\theta}{D}Q_j(\beta)} - e^{\frac{\theta}{D}Q_{j-1}(\beta)}$ with respect to β as follows:

$$\frac{d}{d\beta}\left(e^{\frac{\theta}{D}Q_j(\beta)} - e^{\frac{\theta}{D}Q_{j-1}(\beta)}\right) = \frac{\theta}{D}e^{\frac{\theta}{D}Q_j(\beta)}\frac{dQ_j(\beta)}{d\beta} - \frac{\theta}{D}e^{\frac{\theta}{D}Q_{j-1}(\beta)}\frac{dQ_{j-1}(\beta)}{d\beta}.$$

Using Equations (10) and (12), we know

$$\frac{d}{d\beta}\left(e^{\frac{\theta}{D}Q_j(\beta)} - e^{\frac{\theta}{D}Q_{j-1}(\beta)}\right) = \frac{\sigma\theta^2\pi_0\psi(k)}{Dh}$$
$$\left(\frac{e^{\frac{\theta}{D}Q_{j-1}(\beta)}}{e^{\frac{\theta}{D}Q_{j-1}(\beta)}-1}\sqrt{L_{j-1}} - \frac{e^{\frac{\theta}{D}Q_{j}(\beta)}}{e^{\frac{\theta}{D}Q_{j}(\beta)}-1}\sqrt{L_{j}}\right) > 0.$$

Therefore, $e^{\frac{\theta}{D}Q_j(\beta)} - e^{\frac{\theta}{D}Q_{j-1}(\beta)}$ is an increasing function of β.

From equation (9), $C(Q_j(\beta), L_j) - C(Q_{j-1}(\beta), L_{j-1})$ is the sum of two increasing functions of β, so we finish the proof of Lemma 3.

Proof of Lemma 4:

Given a fixed β_0, if $Q_j(\beta_0) \geq Q_{j-1}(\beta_0)$, from the dual statement of Lemma 1, $c_i(\sqrt{L_{i-1}} + \sqrt{L_i}) \geq \sigma[\pi + (1-\beta_0)\pi_0]\Psi(k)$.

Therefore, $c_i(\sqrt{L_{i-1}} + \sqrt{L_i}) \geq \sigma[\pi + (1-\beta)\pi_0]\Psi(k)$, for $\beta \in [\beta_0, 1]$. Similarly, from the dual statement of Lemma 1, $Q_j(\beta) \geq Q_{j-1}(\beta)$, for $\beta \in [\beta_0, 1]$.

Proof of Theorem 1:

Using Lemma 4 induces that $Q_j(\beta) \geq Q_{j-1}(\beta)$, for $\beta \in [\beta_0, 1]$. Hence, from Lemma 3, we complete the proof of Theorem 1.