Yugoslav Journal of Operations Research
21 (2011), Number 2, 307-324
DOI: 10.2298/YJOR1102307I

MODELS OF PRODUCTION RUNS FOR MULTIPLE PRODUCTS IN FLEXIBLE MANUFACTURING SYSTEM

Oliver ILIĆ, Milić RADOVIĆ

Faculty of Organizational Sciences,
University of Belgrade, Serbia
ioliver@fon.bg.ac.rs
radovicm@fon.bg.ac.rs

Received: June 2008 / Accepted: November 2011

Abstract: How to determine economic production runs (EPR) for multiple products in flexible manufacturing systems (FMS) is considered in this paper. Eight different although similar, models are developed and presented. The first four models are devoted to the cases when no shortage is allowed. The other four models are some kind of generalization of the previous ones when shortages may exist.The numerical examples are given as the illustration of the proposed models.

Keywords: Economic production runs, multiproduct case, deterministic inventory models.

MSC: 90B30.

1. INTRODUCTION

When a number of products share the use of the same equipment on a cyclic basis, the overall cycle length can be established in a way similar to the single case described in [9]. The more general problem, however, is not to determine the economical length of a production run for each product individually, but to determine jointly the runs for the entire group of products which share the use of the same facilities. If each part or product run is set independently, it is highly likely that some conflict of equipment needs would result unless the operating level is somewhat below capacity, where considerable idle equipment time is available [1]. The example presenting this situation are flexible manufacturing systems (FMS) that must be set up to produce different sizes and types of product [8], etc.

Conceptually, the problem to determine an economical cycle is the same as for the one-product case, that is, to determine the cycle length which will minimize the total of machine setup costs plus inventory holding costs jointly for the entire set of products [5], [6] and [7].

The models presented in this paper are the deterministic inventory models. In this paper, we present the procedure for determination of the number of *production runs*, *N*, for eight similar models. The eight models (see Table 1) are

Model I: gradual replenishment, with demand delivery during the production period, no shortages

Model II: instantaneous replenishment, with demand delivery during the production period, no shortages

Model III: gradual replenishment, no demand delivery during the production period, no shortages

Model IV: instantaneous replenishment, no demand delivery during the production period, no shortages

Model V: gradual replenishment, with demand delivery during the production period, with shortages

Model VI: instantaneous replenishment, with demand delivery during the production period, with shortages

Model VII: gradual replenishment, no demand delivery during the production period, with shortages

Model VIII: instantaneous replenishment, no demand delivery during the production period, with shortages.

The first four models and the seventh one, as will be seen later, are all special cases of the fifth, sixth, and the eighth. Our presentation of the eight models begins with model I, the basic economic production runs (EPR) model. Finally, models II, III, IV, V, VI, VII, and VIII are presented as the extensions to the basic model.

2. MODELS WITHOUT SHORTAGES

2.1. The basic economic production runs model

Our first model (model I) describes the case where no shortages are allowed, but the demand rate is greater than zero during the production period, and there is a finite replenishment rate. Figure 1 shows how the inventory levels for this model vary in time. Because the finite replenishment rate usually implies a production rate, model I is usually referred to as an EPR model. Within the context of this discussion, however, the EPR model is merely an extension of the basic economic production quantity (EPQ) or economic lot size (ELS) model [2], [3] and [4].

The total cost analysis for the EPR model is exactly the same as for the EPQ model. Inventory costs plus setup costs yield to total incremental cost. To develop the ERP model for several products, the following notations are used:

D_i = annual requirements for the individual products
d_i = equivalent requirements per production day for the individual products
p_i = daily production rates for the individual products - assuming, of course, that $p_i > d_i, i = 1, 2, ..., m$
H_i = holding cost per unit, per year for the individual products
S_i = setup costs per run for the individual products
m = number of products
q_i = production quantity for the individual products
y_i = peak inventory for the individual products
t_{pi} = production period for the individual products
t_{ci} = consumption period for the individual products
t = time between production runs
c = total incremental cost
N = number of production runs per year
$N*$ = number of production runs per year for an optimal solution
T = total time period

Inventory costs. The maximum inventory for a given product is $(p_i - d_i)t_{pi}$, and the average inventory is $(p_i - d_i)t_{pi} / 2$. However, $q_i = p_i t_{pi} = D_i / N$. Therefore, average inventory can be expressed as

$$\frac{(p_i - d_i)t_{pi}}{2} = (p_i - d_i)\frac{D_i}{2p_i N} = (\frac{p_i - d_i}{p_i})\frac{D_i}{2N} \quad (1)$$

The annual inventory cost for a given product is then the product of the average inventory, given by (1), and the cost to hold a unit in inventory per year, H_i, or

$$\frac{H_i T D_i}{2N}(\frac{p_i - d_i}{p_i}) \quad (2)$$

The annual inventory cost for the entire set of m products is, then, the sum of m expressions of the form of (2), or

$$\frac{T}{2N}\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})$$

Setup costs. The setup costs for a given product are given by S_i, in dollars per run. Therefore, the total setup cost per year for that product is NS_i. Finally, the total annual setup cost is the sum of NS_i for the entire set of m products, or

$$\sum_{i=1}^{m} NS_i$$

and since N is the same for all products, the total annual setup cost is,

$$N\sum_{i=1}^{m} S_i$$

Total incremental cost. The total incremental cost associated with the entire set of *m* products is then

$$C(N) = N\sum_{i=1}^{m} S_i + \frac{T}{2N}\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i}) \qquad (3)$$

Our objective is to determine the minimum of the *C* curve with respect to *N*, the number of production runs. Therefore, following the basic procedure for the derivation of the classical production quantity model, the first derivative of *C* with respect to *N* is

$$\frac{dC}{dN} = \sum_{i=1}^{m} S_i - \frac{T}{2N^2}\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i}) = 0$$

solving for *N*, we have

$$N^* = \sqrt{\frac{T\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})}{2\sum_{i=1}^{m} S_i}} \qquad (4)$$

The total cost of an optimal solution, C^*. The total cost of an optimal solution is found by substituting *N** for *N* in (3), or

$$C^* = N^*\sum_{i=1}^{m} S_i + \frac{T}{2N^*}\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})$$

Substituting and simplifying the expression for *N** shown in (4) leads to

$$C^* = \sqrt{2T\sum_{i=1}^{m} S_i \sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})}$$

2.2. Model II

Figure 2 presents inventory levels as a function of time for this model. No shortages are allowed, so each new run arrives at the moment when the production level with demand delivery during the production period reaches maximum inventory level.

The total incremental cost analysis for this model is exactly the same as for the basic EPR model. The maximum inventory level for a given product is the same as for the one previously defined. The cost that changes is the annual inventory holding cost for the entire set of m products because the parameter which is the annual inventory holding time changes. Therefore, the total incremental cost equation is

$$C(N) = N\sum_{i=1}^{m} S_i + \frac{T}{2N}\sum_{i=1}^{m} H_i D_i \left(\frac{p_i - d_i}{p_i}\right)^2$$

Then, the number of production runs per year for an optimal solution, N^*, satisfies

$$N^*\sum_{i=1}^{m} S_i = \frac{T}{2N^*}\sum_{i=1}^{m} H_i D_i \left(\frac{p_i - d_i}{p_i}\right)^2$$

or

$$N^* = \sqrt{\frac{T\sum_{i=1}^{m} H_i D_i \left(\frac{p_i - d_i}{p_i}\right)^2}{2\sum_{i=1}^{m} S_i}} \qquad (5)$$

The total incremental cost of an optimal solution is

$$C^* = \sqrt{2T\sum_{i=1}^{m} S_i \sum_{i=1}^{m} H_i D_i \left(\frac{p_i - d_i}{p_i}\right)^2} \qquad (6)$$

2.3. Model III

Our third model (model III) describes the case where no shortages and no demand delivery during the production period are allowed, but now there is a finite replenishment rate. Figure 3 shows how the inventory levels vary in time for this model. Now, $y_i = q_i, i = 1, 2, ..., m$. Therefore, the total incremental cost equation is

$$C(N) = N\sum_{i=1}^{m} S_i + \frac{T}{2N}\sum_{i=1}^{m} H_i D_i$$

and the optimal number of production runs, N^*, is

$$N^* = \sqrt{\frac{T\sum_{i=1}^{m} H_i D_i}{2\sum_{i=1}^{m} S_i}}$$

2.4. Model IV

Figure 4 presents inventory levels as a function of time for this model. No shortages are allowed, so each new run arrives the moment when the production level without demand delivery during the production period reaches maximum inventory level. For this model, $y_i = q_i, i = 1, 2, ..., m$, and the annual inventory holding time is the same as in model II. The total incremental cost for this model is the same as in equation (3). Also, the optimal number of production runs for this model is equal to the optimal number of production runs for model I.

3. MODEL WITH SHORTAGES

3.1. Model V

In terms of the replenishment rate and the demand rate during the production period, model V is the same as model I. A gradual replenishment is assumed. The difference is that, in model V, shortages are allowed, and a corresponding shortage cost is provided. In the shortage situation in this model, the demand that cannot be satisfied is backordered t and is to be met after the next shipment arrives. This is much different from the case of lost sales, where the customer does not return, thereby reducing the demand.

The inventory levels for model V are shown in Figure 5. Notice that the maximum shortage for a given product is b_i and the maximum inventory for a given product is y_i, which means that the figure is the same as Figure 1, but with all inventory levels reduced by the amount b_i. Again, common sense should tell us that, because inventory levels and the associated holding costs will be lower than in model I, the run quantity can be increased and runs can be placed less often.

To analyze this situation, let us define the cost of a backorder per unit per time (year) for a given product, G_i. That is, this cost is defined in terms of units (dollars per item per time), which is similar to the definition of the inventory holding cost. Also, the total incremental cost associated with the entire set of m products, for this model, is similar to the total cost for model I, with the addition of costs due to shortages.

C=annual setup costs + annual inventory holding costs + annual shortage costs

There is no change in the setup costs. However, the holding cost changes due to the difference in calculation of the average inventory level for this situation. The average inventory level is

$$\frac{\left[\frac{D_i}{N}\left(\frac{p_i - d_i}{p_i}\right) - b_i\right]^2}{2\frac{D_i}{N}\left(\frac{p_i - d_i}{p_i}\right)}$$

and the average backorder position is, similarly,

$$\frac{b_i^2}{2\frac{D_i}{N}(\frac{p_i - d_i}{p_i})}$$

Consequently, the total cost is

$$C(N,B) = \sum_{i=1}^{m}\left\{ NS_i + \frac{H_i T\left[\frac{D_i}{N}(\frac{p_i - d_i}{p_i})\right]^2}{2\frac{D_i}{N}(\frac{p_i - d_i}{p_i})} + \frac{G_i T b_i^2}{2\frac{D_i}{N}(\frac{p_i - d_i}{p_i})}\right\}$$

To obtain the EPR, we differentiate the total cost with respect to both *N* and *B* and solve two simultaneous equations, which yield to

$$N^* = \sqrt{\frac{T\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})\frac{G_i}{H_i + G_i}}{2\sum_{i=1}^{m} S_i}} \quad (7)$$

Because $H_i + G_i$ is more than G_i, the term $\frac{G_i}{H_i + G_i} < 1$, loading to the decreased *N*, which was expected.

The determination of the maximum number of demands outstanding, b_i, is

$$b_i^* = \frac{H_i \frac{D_i}{N}(\frac{p_i - d_i}{p_i})}{H_i + G_i} \quad (8)$$

The maximum inventory, then, is

$$y_i = \frac{D_i}{N}(\frac{p_i - d_i}{p_i}) - b_i \quad (9)$$

The length of the cycle, t, is T / N, as it has happened previously. The cycle, t, was broken down into t_{pi} and t_{ci} for model I, and into inventory and shortage time in this model. For this model, all the four time are important. As shown in Figure 5,

$$t = t_{pi} + t_{ci} = (t_{1i} + t_{2i}) + (t_{3i} - t_{4i})$$

where

t_{1i} = time of producing while there is a shortage situation for the individual products

$$t_{1i} = \frac{b_i}{p_i - d_i}$$

t_{2i} = time of producing, while there is inventory on hand for the individual products

$$t_{2i} = \frac{y_i}{p_i - d_i}$$

t_{3i} = time of pure consumption while there is inventory on hand for the individual products

$$t_{3i} = y_i / d_i$$

t_{4i} = time of pure consumption while there is a shortage situation for the individual products

$$t_{4i} = b_i / d_i$$

3.2. Model VI

Model VI allows shortages (finite shortage cost) and has an infinite rate of replenishment with demand delivery during the production period. The inventory levels over time for this model are shown in Figure 6. The total cost for this model is

$$C(N,B) = \sum_{i=1}^{m} \left\{ NS_i + \frac{H_i T \left[\frac{D_i}{N} (\frac{p_i - d_i}{p_i}) - b_i \right]^2}{2\frac{D_i}{N}} + \frac{G_i T b_i^2}{2\frac{D_i}{N}} \right\}$$

which yields to the following formulas:

$$N^* = \sqrt{\frac{T\sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})^2 \frac{G_i}{H_i + G_i}}{2\sum_{i=1}^{m} S_i}} \tag{10}$$

$$C^* = \sqrt{2T\sum_{i=1}^{m} S_i \sum_{i=1}^{m} H_i D_i (\frac{p_i - d_i}{p_i})^2 \frac{G_i}{H_i + G_i}} \tag{11}$$

and a maximal backorder position of the equation (8). The maximum inventory also is defined as an equation (9). For this model, the time of pure consumption is the same as in model V.

3.3. Model VII

Model VII is similar to model III. The difference is that, in model VII, shortages are allowed. The inventory levels for this model are shown in Figure 7. The total cost for this model is

$$C(N,B)=\sum_{i=1}^{m}\left[NS_i+\frac{H_iT(\frac{D_i}{N}-b_i)}{2\frac{D_i}{N}}+\frac{G_iTb_i^2}{2\frac{D_i}{N}}\right]$$

which yields to the EPR formula of

$$N^*=\sqrt{\frac{T\sum_{i=1}^{m}H_iD_i\frac{G_i}{H_i+G_i}}{2\sum_{i=1}^{m}S_i}}$$

and a maximal backorder position of

$$b_i^*=\frac{H_i\frac{D_i}{N}}{H_i+G_i} \qquad (12)$$

The maximum inventory level, then, is q_i-b_i. The cycle, *t*, was broken down into four times, where

$$t_{1i}=\frac{b_i}{q_i}t_{pi} \qquad t_{1i}=\frac{b_i}{p_i}$$

$$t_{2i}=\frac{q_i-b_i}{q_i}t_{pi} \qquad t_{2i}=\frac{y_i}{p_i}$$

or

$$t_{3i}=\frac{q_i-b_i}{q_i}t_{ci} \qquad t_{3i}=\frac{y_i}{d_i'}$$

$$t_{4i}=\frac{b_i}{q_i}t_{ci} \qquad t_{4i}=\frac{b_i}{d_i'}$$

3.4. Model VIII

Model VIII allows shortages (finite shortage cost) and has an infinite rate of replenishment and no demand delivery during the production period. The inventory levels over time are shown in Figure 8. The total cost is

$$C(N,B)=\sum_{i=1}^{m}\left[NS_i+\frac{H_iT(\frac{D_i}{N}-b_i)^2}{2\frac{D_i}{N}(\frac{p_i}{p_i-d_i})}+\frac{G_iTb_i^2}{2\frac{D_i}{N}(\frac{p_i}{p_i-d_i})}\right]$$

which yields to the EPR formula of equation (7), and a maximal backorder position of equation (12). The maximum inventory, then, is p_i-b_i. The time of pure consumption for model VIII is the same as in model VII.

3. NUMERICAL EXAMPLES

4.1. Example 1

Let us work out an example to determinate the cycle length by model II for the group of five products shown in Table 2, which shows the annual sales requirements, sales per production day (250 days per year), daily production rate, production days required, annual inventory holding cost, and setup costs. Table 3 shows the calculation of the number of runs per year calculated by formula 5. The minimum cost number of cycles which results in three per year, each cycle lasting approximately 78 days and producing one-third of the sales requirements during each run. The total incremental cost got by formula 6 is C*=$1361.

4.2. Example 2

What is the effect on N^* for Example 1 if shortage costs are G_1=$0.10, G_2=$0.10, G_3=$0.05, G_4=$0.04, and G_5=$0.70 per unit per year? What is the total incremental cost of this solution?

Table 4 shows the calculation of the number of runs per year calculated by formula 10. The minimum cost number of cycles which results is two per year, each cycle lasting approximately 117 days and producing a half of the sales requirements during each run. The total incremental cost got by formula 11 is C*=$913.

5. CONCLUSIONS

The eight similar models presented in this paper are the EPR models for several products. Although historically, these models follow in the line of approaches on inventory analysis, they have found their greatest application within the FMS environment.

Models V, VI, VII and VIII are seldom used in practice. The major reason is the difficulty to obtain an accurate estimate of the shortage cost. The models presented here are to emphasize some of the many assumptions that can be built into an EPR model and to show how these assumptions can be incorporated into the model. Table 5 summarizes the formulas for the eight models.

Model III is a special case of models I, II, and IV, with p_i=∞, $i=1,2,...,m$. It should be recognized that, in model VII, G_i=∞, $i=1,2,...,m$, leads to model III, where no shortages are allowed. Models V, VI, and VIII are the most general of all the eight models presented. In fact, models I, II, III, IV, and VII are all special cases of models V, VI, and VIII, which allow shortages (finite shortage cost). In short, models I, II, IV, and VII each presents generalization of one assumption from model III, but models V, VI, and VIII include both generalizations simultaneously.

REFERENCES

[1] Buffa, E. S., *Models for Production and Operations Management*, John Wiley & Sons, New York, 1963.

[2] Ilić, O., "Lot size models without shortages for a single product in MRP systems", *Proceedings of V SymOrg Conference*, Vrnjačka Banja, 1996, 413-418.

[3] Ilić, O., "Lot size models with shortages for a single sroduct in MRP systems", *Proceedings of XXIII SYM-OP-IS Conference*, Zlatibor, 1996, 852-855.

[4] Ilić, O., "Economic production quantity models for a single product in MRP systems", in: P. Jovanović and D. Petrović (eds.), *Contemporary Trends in the Development of Management*, FON, Belgrade, 2007, 203-219. (in Serbian)

[5] Ilić, O., and Radović, M., "Models of production runs without shortages for the multiproduct case in FMS", *Proceedings of I SIE Conference*, Belgrade, 1996, 454-456.

[6] Ilić, O., and Radović, M., "Models of production runs with shortages for the multiproduct case in FMS", *Proceedings of II SIE Conference*, Belgrade, 1998, 273-276.

[7] Radović, M., and Ilić, O., "Production runs for several parts or products", *Yugoslav Journal of Engineering*, 36 (3) (1986) 10-17. (in Serbian)

[8] Rankey, P. G., *Computer Integrated Manufacturing*, Prentice Hall, New Jersey, 1986.

[9] Weiss, H. J., and Gershon, M. E., *Production and Operations Management*, Allyn and Bacon, Boston, 1989.

APPENDIX

Nomenclature

$B=(b_i)$	Vector of maximum amount shortages
C	Total (annual) incremental cost (dollars/time)
C^*	Total incremental cost of an optimal solution
d_i	Daily demand rate of the *i*th product (units/day)
d_i'	Daily pure consumption rate of the *i*th product (units/day)
D_i	Demand rate of the *i*th product (units/time)
G_i	Shortage cost of the *i*th product (dollars/unit-time), where time must match demand
H_i	Holding cost of the *i*th product (dollars/unit-time), where time must match demand
m	Number of products
N	Number of production runs per time (year)
N^*	Number of production runs per time for an optimal solution
p_i	Daily production rate of the *i*th product (units/day)
q_i	Production run quantity of the *i*th product (units/run)
S_i	Setup or fixed cost of the *i*th product (dollars/run)
t	Length of the cycle
t_{ci}	Time of pure consumption of the *i*th product
t_{p_i}	Time of producing of the *i*th product
t_{1i}	Time of producing while there is a shortage situation of the *i*th product
t_{2i}	Time of producing, while there is inventory on hand of the *i*th product
t_{3i}	Time of pure consumption while there is inventory on hand of the *i*th product
t_{4i}	Time of pure consumption while there is a shortage situation of the *i*th product
T	Total time period (number of working days per year)
y_i	Maximum inventory level of the *i*th product

Table 1: Assumptions and models

Shortages	Demand delivery during the production period	Replenishment rate	
		Gradual	Instantaneous
No	Yes	Model I	Model II
	No	Model III	Model IV
Yes	Yes	Model V	Model VI
	No	Model VII	Model VIII

Table 2: Sales, production, and cost data for five products to be run on the same equipment

Product Number	D_i	d_i	p_i	Production Days Required	H_i	S_i
1	10,000	40	250	40	$0.05	$25
2	20,000	80	500	40	0.10	15
3	5,000	20	200	25	0.15	40
4	15,000	60	500	30	0.02	50
5	4,000	16	40	100	1.05	95
Total				235		$225

Table 3: Determination of the number of runs, jointly, for five products from formula 5

Product Number	d_i / p_i	$(1-d_i/p_i)$	$(1-d_i/p_i)^2$	$H_i D_i$	$H_i D_i (1-d_i/p_i)^2$
1	0.160	0.840	0.706	500	353
2	0.160	0.840	0.706	2000	1,412
3	0.100	0.900	0.810	750	607
4	0.120	0.880	0.774	300	232
5	0.400	0.600	0.360	4200	1,512
Total					4,116

$$N^* = \sqrt{\frac{4{,}116}{2x225}} \approx 3 \text{ cycles per year}$$

Table 4: Determination of the number of runs, jointly, for five products from formula 10

Product Number	$H_i + G_i$	$\frac{G_i}{H_i + G_i}$	$H_i D_i (1-d_i/p_i)^2$	$H_i D_i (1-d_i/p_i)^2 \frac{G_i}{H_i + G_i}$
1	0.15	0.667	353	235
2	0.20	0.500	1412	706
3	0.20	0.250	607	152
4	0.06	0.667	232	155
5	1.75	0.400	1512	605
Total				1,853

$$N^* = \sqrt{\frac{1{,}853}{2x225}} \approx 2 \text{ cycles per year}$$

Model	Shortage Cost (G_i)	Demand Delivery During the Production Period	Replenishment Rate	Optimal Number of Production Runs (N^*)	Maximum Inventory Level (y_i)	Maximum Backorders (b_i)	Cost (C)
III	Infinite	No	Gradual	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}$	none	$\sum_{i=1}^{n}\left(NS_i+\frac{H_i T D_i}{2N}\right)$
IV	Infinite	No	Instantaneous	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\left(\frac{p_i-d_i}{p_i}\right)}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}$	none	$\sum_{i=1}^{n}\left[NS_i+\frac{H_i T D_i}{2N}\left(\frac{p_i-d_i}{p_i}\right)\right]$
I	Infinite	Yes	Gradual	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\left(\frac{p_i-d_i}{p_i}\right)}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)$	none	$\sum_{i=1}^{n}\left[NS_i+\frac{H_i T D_i}{2N}\left(\frac{p_i-d_i}{p_i}\right)\right]$
II	Infinite	Yes	Instantaneous	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\left(\frac{p_i-d_i}{p_i}\right)^2}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)$	none	$\sum_{i=1}^{n}\left[NS_i+\frac{H_i T D_i}{2N}\left(\frac{p_i-d_i}{p_i}\right)^2\right]$
VII	Finite	No	Gradual	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\frac{G_i}{H_i+G_i}}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}\left(\frac{G_i}{H_i+G_i}\right)$	$\frac{D_i}{N}\left(\frac{H_i}{H_i+G_i}\right)$	$\sum_{i=1}^{n}\left[NS_i+\frac{H_i T\left(\frac{D_i}{N}-b_i\right)^2}{2\frac{D_i}{N}}+\frac{G_i T b_i^2}{2\frac{D_i}{N}}\right]$
VIII	Finite	No	Instantaneous	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\frac{G_i}{H_i+G_i}\left(\frac{p_i-d_i}{p_i}\right)}{2\sum_{i=1}^{n} S_i}}$	$\frac{D_i}{N}\left(\frac{G_i}{H_i+G_i}\right)$	$\frac{D_i}{N}\left(\frac{H_i}{H_i+G_i}\right)$	$\sum_{i=1}^{n}\left[NS_i+\frac{H_i T\left(\frac{D_i}{N}-b_i\right)^2}{2\frac{D_i}{N}\left(\frac{p_i}{p_i-d_i}\right)}+\frac{G_i T b_i^2}{2\frac{D_i}{N}\left(\frac{p_i}{p_i-d_i}\right)}\right]$
V	Finite	Yes	Gradual	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\frac{G_i}{H_i+G_i}\left(\frac{p_i-d_i}{p_i}\right)}{2\sum_{i=1}^{n} S_i}}$	$\frac{\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)G_i}{H_i+G_i}$	$\frac{\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)H_i}{H_i+G_i}$	$\sum_{i=1}^{n}\left\{NS_i+\frac{H_i T\left[\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)-b_i\right]^2}{2\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)}+\frac{G_i T b_i^2}{2\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)}\right\}$
VI	Finite	Yes	Instantaneous	$\sqrt{\frac{T\sum_{i=1}^{n} D_i H_i\frac{G_i}{H_i+G_i}\left(\frac{p_i-d_i}{p_i}\right)^2}{2\sum_{i=1}^{n} S_i}}$	$\frac{\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)G_i}{H_i+G_i}$	$\frac{\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)H_i}{H_i+G_i}$	$\sum_{i=1}^{n}\left\{NS_i+\frac{H_i T\left[\frac{D_i}{N}\left(\frac{p_i-d_i}{p_i}\right)-b_i\right]^2}{2\frac{D_i}{N}}+\frac{G_i T b_i^2}{2\frac{D_i}{N}}\right\}$

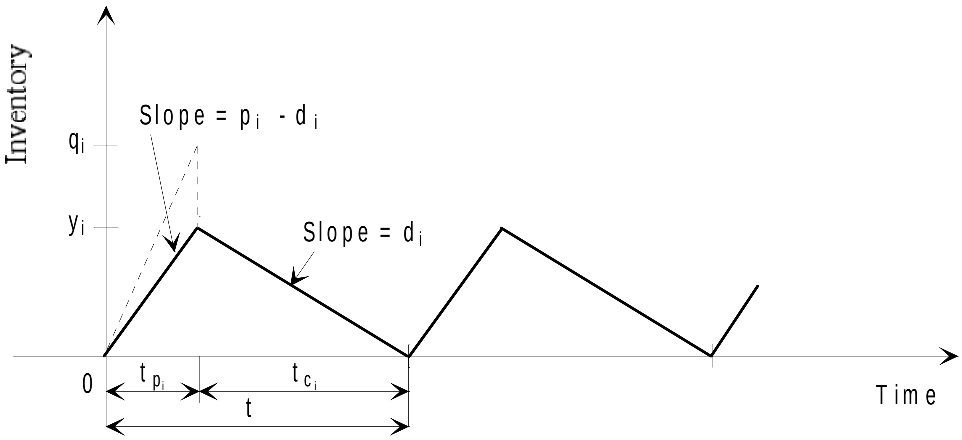

Figure 1: Inventory as a function of time-sawtooth curve, model I

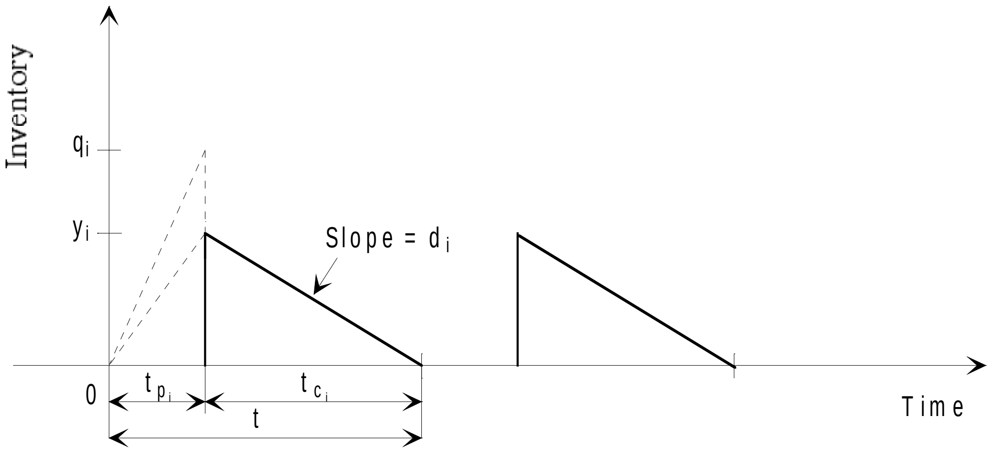

Figure 2: Sawtooth curve, model II

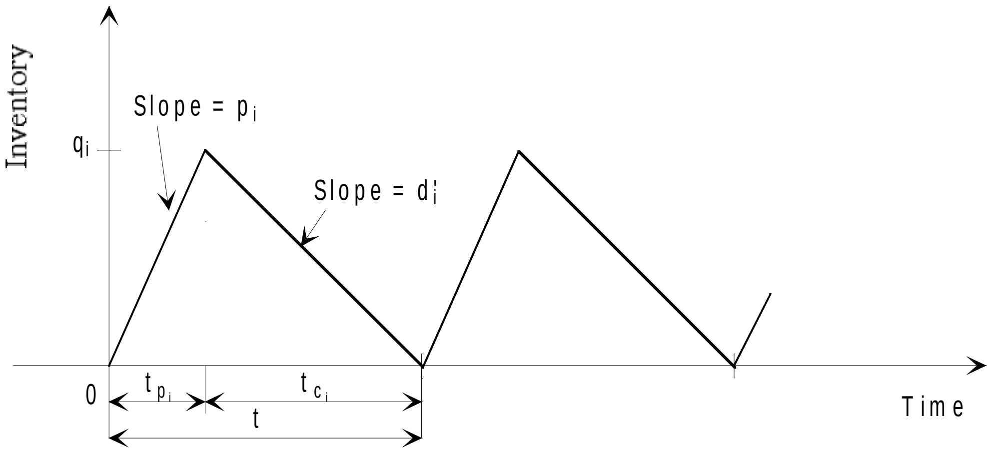

Figure 3: Sawtooth curve, model III

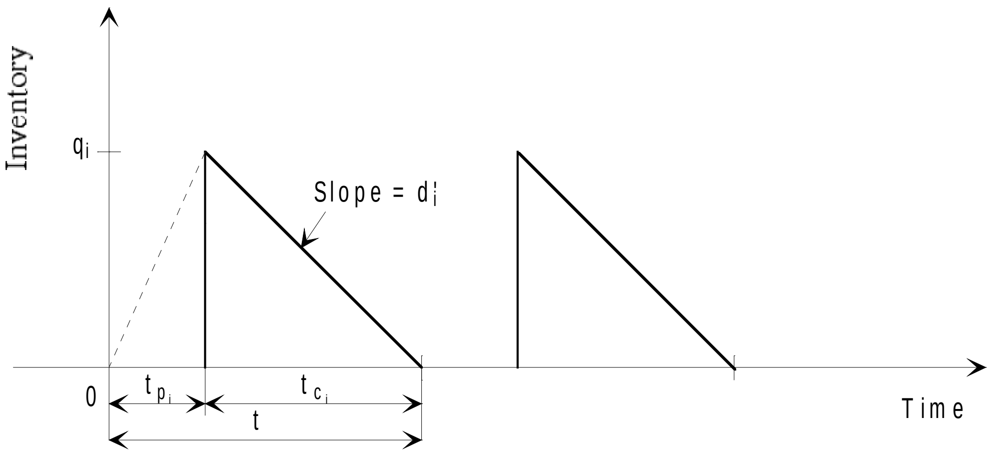

Figure 4: Sawtooth curve, model IV

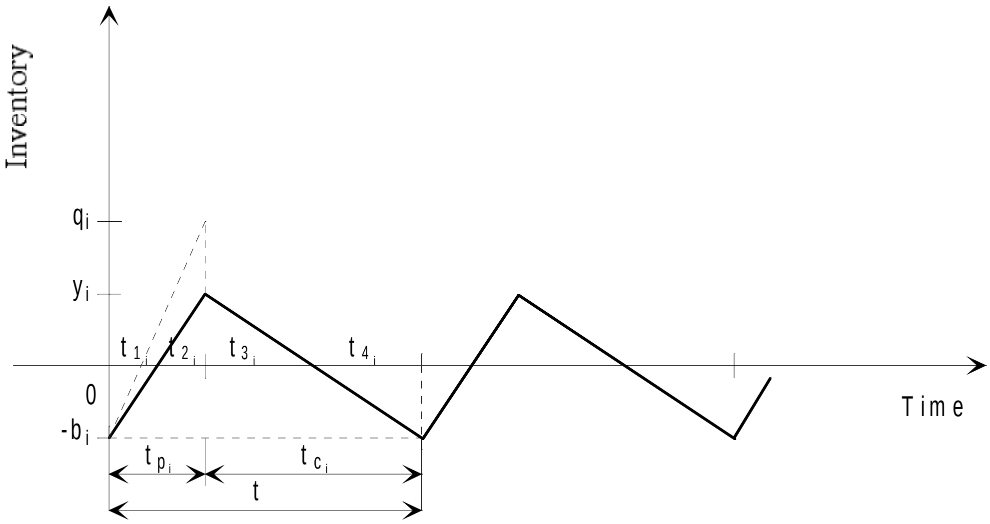

Figure 5: Sawtooth curve, model V

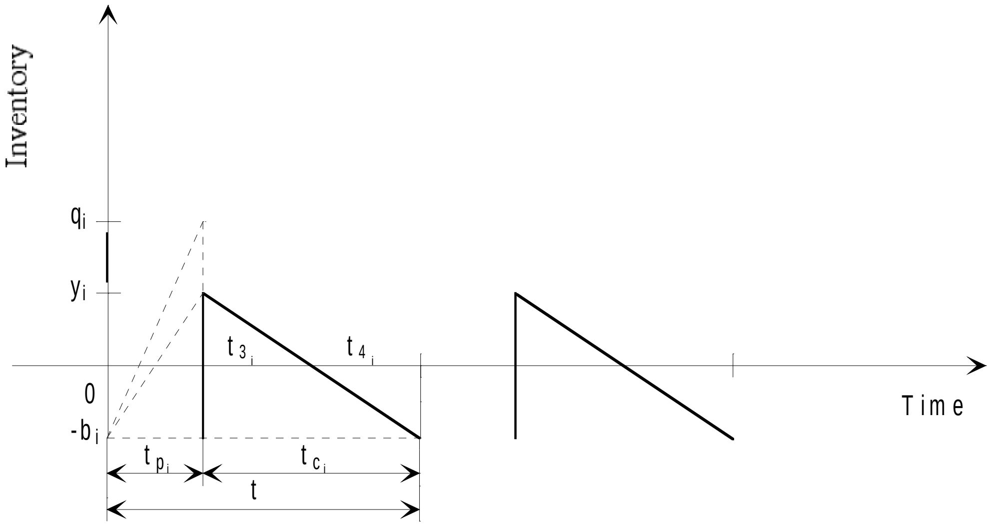

Figure 6: Sawtooth curve, model VI

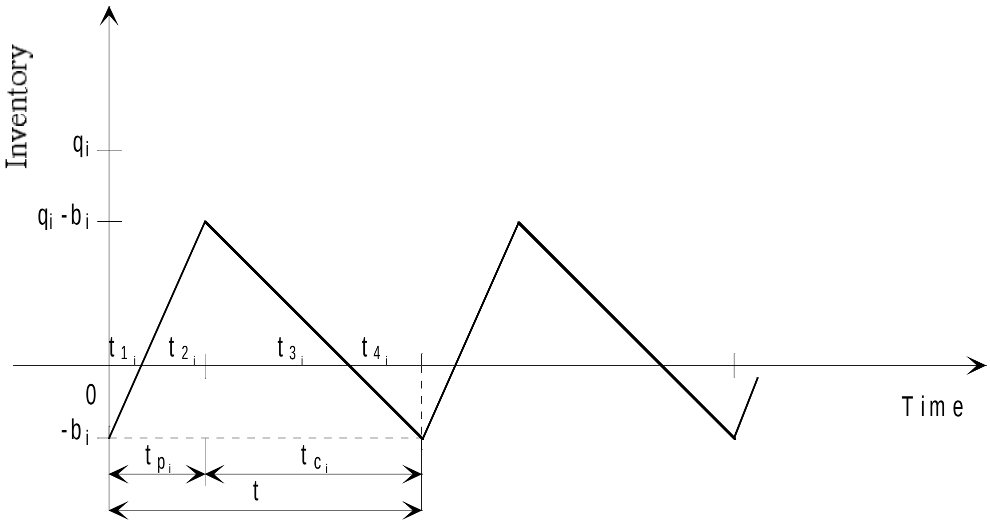

Figure 7: Sawtooth curve, model VII

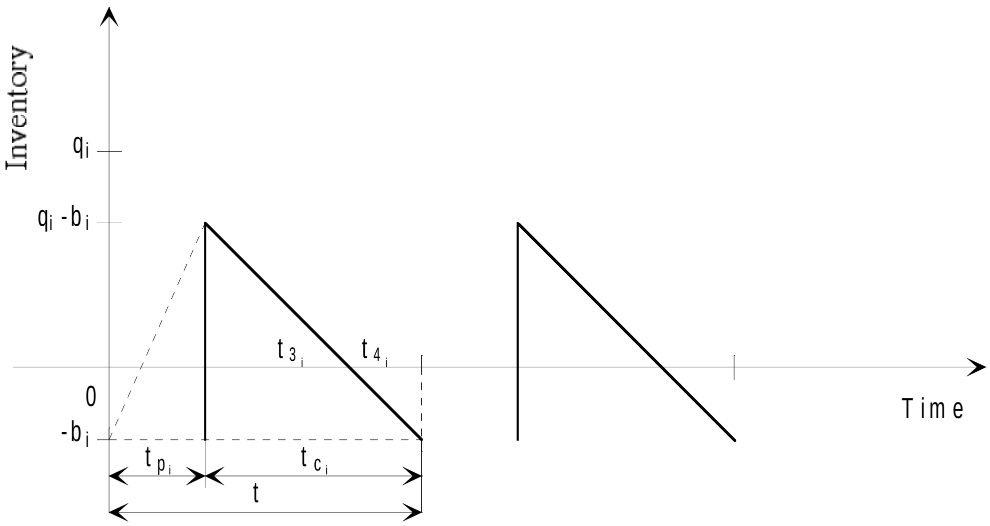

Figure 8: Sawtooth curve, model VIII