AN EPLS MODEL FOR A VARIABLE PRODUCTION RATE WITH STOCK-PRICE SENSITIVE DEMAND AND DETERIORATION

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Abstract: It is observed that large piles of consumer goods displayed in supermarkets lead consumers to buy more, which generates more profit to sellers. But a large number of on-hand display of stock leaves a negative impression on the buyer. Also, the amount of shelf or display space is limited. Due to this reason, we impose a restriction on the number of on-hand display of stock and also on initial and ending on-hand stock levels. We introduce an economic production lot size model, where production rate depends on stock and selling price per unit. A constant fraction deterioration rate is considered in this model. To illustrate the results of the model, four numerical examples are established. Sensitivity analysis of the changes of parameter values is also given.

Keywords: Inventory, lot size, deterioration, pricing, stock-dependent demand.

1. INTRODUCTION

We observe that the demand is usually influenced by the amount of stock displayed in the shelves in supermarkets. i.e., the demand rate increases or decreases according to the on-hand inventory level increase or decrease. An increase in shelf space for a particular item attracts the consumers to buy it more. On the other hand, low stock level of certain goods might raise the impression that they are not being fresh. It is a common belief that large piles of goods displayed in supermarkets tempt consumers to buy more. Therefore, the demand rate may be influenced by the stock level; such a phenomenon is termed as "stock-dependent demand rate". Many researchers have observed that inventory-level-dependent demand effects inventory control policies. Silver and Peterson (1985) noted that sales at the retail level tended to be proportional to the inventory displayed. Levin et al. (1972) also noted that "large piles of consumer goods displayed in a supermarket will lead consumers to buy more. Yet, too many goods piled up in everyone's way leave a negative impression on buyers and employees alike". Baker and Urban (1988) introduced an economic order quantity model for a demand rate $D(t) = \alpha \left[i(t) \right]^{\beta}$, where $i(t)$ is the inventory level, $\alpha > 0$ and $0 < \beta < 1$. Mondal and Phaujdar (1989) established an economic production quantity model for deteriorating items with constant production and linear stock-dependent demand $D(t) = \alpha + \beta i(t)$, where α and $\beta > 0$. Datta and Pal (1990) developed an economic order quantity model in which the demand rate was dependent on the instantaneous stock amount displayed until a given level of inventory L was achieved; otherwise, the demand rate was constant, i.e., $D(t) = \alpha \left[i(t) \right]^{\beta}$ if $i(t) > L$ and $D(t) = \alpha L^{\beta}$ if $0 \leq i(t) \leq L$.

Among the important papers published so far, considering the stock-dependent demand rate, the works of Gupta and Vrat (1986), Urban (1992), Giri et al.(1996), Bar-Lev et al. (1994), Padmanabhan and Vrat (1995), Pal et al. (1993), Ray and Chaudhuri (1997), Ray et al. (1998), etc., are worth mentioning. Urban and Backer (1997) also considered an EOQ (economic order quantity) model in which the demand was a multivariate function of price, time and inventory level. Datta and Paul (2001) developed a multivariate multi-period EOQ model with stock-dependent and price-sensitive demand rate. Teng and Chang (2005) analyzed an EPQ (economic production quantity) model for deteriorating items with constant production rate and price and stock-dependent demand given by $D(I(t), p) = \alpha(p) + \beta I(t)$, where $I(t)$ was on-hand display of stock at any time, $\beta > 0$ and $\alpha(p) > 0$. Roy and Chaudhuri (2009) discussed a production inventory model under stock dependent demand $D(t) = \alpha + \beta i(t)$ where $\alpha > 0$ and $0 < \beta < 1$, Weibull distribution deterioration and shortage. Sarkar et al. (2010) developed a stock-dependent inventory model in an imperfect production process. Roy and Chaudhuri (2006) analyzed a deterministic inventory model for deteriorating items with stock level-dependent demand, shortage, inflation and time discounting. Recently, Sarkar and I. K. Moon (2011) developed an EPQ model with inflation in an imperfect production system. We observe that too much stock in the supermarket leaves a negative impression on the buyer. If there is a large pile of stock in the supermarket, it is often associated with a price-cut to clear the stock and hence to induce more sales and profit as well. Only a few EPLS (economic production lot size) models with stock-dependent demand consider the

pricing policy. For that reason, we develop an EPLS model with demand rate depending on stock-level and selling price. Also, a ceiling is made on the number of on-hand display of stock and on initial and ending stock-levels of the item, due to limited shelf space and to avoid a negative impression on buyers. In this paper, we consider that the production rate is proportional to the stock-dependent demand. Most of the physical goods undergo decay or deterioration over time.

Thus decay or deterioration of physical goods in stock is a very realistic phenomenon, and researchers felt the need to take this into consideration. Whitin (1957) considered fashion goods deteriorating at the end of a prescribed shortage period. Ghare and Schrader (1963) established a model for an exponentially decaying inventory. Some of the recent works in this area are by Goswami and Chaudhuri (1991), Fujiwara (1993), Wee (1995), Giri and Chudhuri (1997), Yan and Chang (1998), Giri et al. (2003), Sana et al. (2004), Roy and Chaudhuri (2010), Roy and Chaudhuri (2011), etc.. In this paper, we consider deterioration of physical goods. Teng and Chang (2005) consider the demand given by $D(I(t), p) = \alpha(p) + \beta I(t)$, where $I(t)$ is on-hand display of stock at any time, $\beta > 0$ and $\alpha(p) > 0$, but in the present article, we consider the demand function $D(i(t), p) = r(p)[i(t)]^{\beta}$, where $0 < \beta < 1$ and $r(p)$ is a non-negative function such that $r'(p) < 0$, which is more applicable in the light of a practical scenario.

2. BASIC ASSUMPTIONS AND NOTATIONS OF THE MODEL

The mathematical model in this paper is developed on the basis of the following assumptions and notations:
Assumptions :

(a) $D(i(t), p)$ is the demand rate which is a function of the inventory level $i(t)$ and the selling price p. We consider $D(i(t), p) = r(p)[i(t)]^{\beta}$, where $0 < \beta < 1$ and $r(p)$ is a non-negative function such that $r'(p) < 0$.

(b) To avoid lost sales, shortages are not allowed.

(c) To avoid negative impression on consumers, and due to limited display space in the shelf,we consider the maximum allowable stock to be S.

(d) To avoid the negative impression on a buyer, we assume that the initial and the ending inventory level $Q \neq 0$ is the same, and the production cycle is repeatable.

(e) We introduce constant fraction deterioration per unit time. The value of a deteriorated item has two possible cases in this inventory problem that minimizes the cost (i) if there is a salvage value, which is either negative or zero and (ii) if there is a disposal value, which is positive.

(f) Without any loss of generality, we assume that the variable production rate is greater than the demand and deterioration rate during the production run, i.e., $\gamma r(p) > r(p) + \theta[i(t)]^{1-\beta}$, when $t \leq t_1$. Otherwise, the inventory level will reduce below Q during the entire inventory cycle, which contradicts our assumption that the ending inventory is Q.

Notations:

$P = $ the variable production rate per unit time $= \gamma D(i(t), p)$ where $\gamma(>1)$.

$i(t) = $ the inventory level at time t such that $i(t) \leq S$.

$T_1 = $ the production run time which is a decision variable.

$T = $ the production cycle duration where $T = T_1 + T_2$ and T_2 is the length of the time interval with production.

$\theta = $ the constant deterioration rate, where $O < \theta < 1$.

$Q = $ the initial and ending inventory level where $0 \leq Q \leq S$ and Q is a decision variable.

$C_0 = $ the set-up cost per cycle.

$C_h = $ the holding cost per unit per unit time.

$C_p = $ the production cost per unit.

$C_s = $ the salvage value (or disposal cost) per unit where $C_p > C_s (or - C_s)$.

$p = $ the constant selling price per unit, a decision variable.

3. BASIC EQUATIONS GOVERNING THE MODEL AND THEIR SOLUTIONS

Variable production rate starts after $t = 0$ when inventory-level is Q and stops at $t = T_1$. The stock reaches a level S at $t = T_1$. Just after $t = T_1$, the inventory-level gradually declines to $Q(\geq 0)$ at the end of the production cycle $t = T$, mainly due to demand and partly to deterioration up to $t = T_2$. Our problem is to determine the optimal values of p, T_1, T_2 and Q such that the total profit is maximized. The pictorial representation of the inventory cycle is given in Fig. 1.

The differential equations describing the instantaneous states of stock-level $i(t)$ at time t satisfy the following differential equations:

$$\frac{di(t)}{dt} + \theta i(t) = P - D(i(t), p), \quad 0 \leq t \leq T_1 \tag{1}$$

and

$$\frac{di(t)}{dt} + \theta i(t) = -D(i(t), p), \quad T_1 \leq t \leq T. \tag{2}$$

The initial and boundary conditions are

$$i(0) = Q, \; i(T_1) = S \; and \; i(T) = Q. \tag{3}$$

The solutions of the differential equations (1) and (2) respectively are

$$i(t) = [\frac{(1-\gamma)r(p)}{\theta}(1-e^{-\theta(1-\beta)t}) + Q^{1-\beta}e^{-\theta(1-\beta)t}]^{\frac{1}{1-\beta}}, \quad 0 \le t \le T_1 \tag{4}$$

and

$$i(t) = [\frac{r(p)}{\theta}(e^{\theta(1-\beta)(T_1+T_2-t)} - 1) + Q^{1-\beta}e^{\theta(1-\beta)(T_1+T_2-t)}]^{\frac{1}{1-\beta}}, \quad T_1 \le t \le T \tag{5}$$
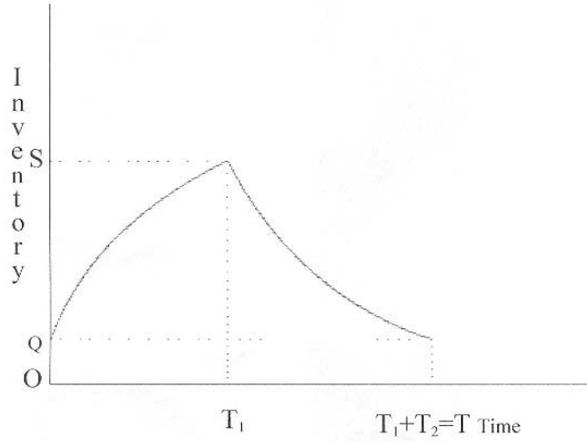

Figure 1: Pictorial representation of inventory cycle

Now, using the condition $i(T_1) = S$ in (3) and (4), we have

$$S^{1-\beta} = \frac{(1-\gamma)r(p)}{\theta}(1-e^{-\theta(1-\beta)T_1}) + Q^{1-\beta}e^{-\theta(1-\beta)T_1} \tag{6}$$

$$= \frac{r(p)}{\theta}(e^{\theta(1-\beta)T_2} - 1) + Q^{1-\beta}e^{\theta(1-\beta)T_2} \tag{7}$$

From equation (6), we get

$$T_1 = \frac{1}{\theta(1-\beta)} \ln[\frac{(1-\gamma)r(p) - \theta Q^{1-\beta}}{(1-\gamma)r(p) - \theta S^{1-\beta}}] \tag{8}$$

From equation (7), we get

$$T_1 = \frac{1}{\theta(1-\beta)} \ln[\frac{r(p) + \theta S^{1-\beta}}{r(p) + \theta Q^{1-\beta}}] \tag{9}$$

From (6) and (7), we have

$$T_1 = \frac{1}{\theta(1-\beta)} \ln\left[\frac{\gamma r(p)}{r(p)+\theta Q^{1-\beta}} - \frac{(\gamma-1)r(p)-\theta Q^{1-\beta}}{r(p)+\theta Q^{1-\beta}} e^{-\theta(1-\beta)T_1}\right] \qquad (10)$$

which indicates that T_2 is a function of T_1, p and Q.

Taking the partial derivative of T_2 with respect to T_1 in (6) and (7), we get

$$\frac{\partial T_2}{\partial T_1} = \frac{(\gamma-1)r(p)-\theta Q^{1-\beta}}{r(p)+\theta Q^{1-\beta}} e^{-\theta(1-\beta)(T_1+T_2)} > 0 \qquad (11)$$

Since we have from (1) that $\gamma r(p) > r(p) + \theta[i(t)]^{1-\beta}$ when $t \le t_1$, this implies that $(\gamma-1)r(p) > \theta[i(0)]^{1-\beta} = \theta Q^{1-\beta}$.

Now, using (6) and (7), the total profit TP during the interval [0, T] is given by [see Appendix A]

$$TP = (p-C_p)\int_0^T D(i(t),p)dt - C_0 - [C_h + \theta(C_p+C_s)]\int_0^T i(t)dt$$

$$= (p-C_p)\{(1+\frac{1}{\gamma-1})(S-Q) + \frac{\theta}{r(p)(2-\beta)}(\frac{1}{(\gamma-1)^2}-1)$$

$$\times (S^{2-\beta}-Q^{2-\beta})\} - C_0 - [C_h + \theta(C_p+C_s)]\{\frac{(\frac{1}{\gamma-1}+1)}{(2-\beta)r(p)}(S^{2-\beta}-Q^{2-\beta}) \qquad (12)$$

$$+ \frac{\theta(\frac{1}{(\gamma-1)^2}-1)}{(3-2\beta)\{r(p)\}^2}(S^{3-2\beta}-Q^{3-2\beta})\}$$

Our problem is to determine the optimal values of p and Q that maximize the total profit TP in (12). With these optimal values of p and Q, we obtain the optimal values of T_1 and T_2 from (9) and (10) respectively.

4. SOLUTION PROCEDURE: ALGORITHM

The above model can be solved by the following algorithm.

Step 1: Equation (12) gives the total profit TP which is a function of the decision variables p and Q. By MATHEMATICA 5.2 software, we obtain the corresponding optimal values of p and Q.

Step 2: Putting these optimal values of p and Q in equation (8), we get the optimal value of T_1.

Step 3: Similarly, we get optimal value of T_2 by putting optimal value of p and Q in equation (9).

5. NUMERICAL EXAMPLES

We assumed that the demand is a negative exponential function of the price in Examples 1 and 2. In Examples 3 and 4, we use demand as a constant elasticity of the price.

Example 1: Let us take $r(p) = \alpha e^{-Gp}$ and the parameter values of the inventory system

$$C_0 = 200, C_h = 0.5, C_p = 0.6, C_s = -0.03, \alpha = 200,$$
$$\beta = 0.02, \gamma = 5. \theta = 0.04, S = 250$$

and

$G = 1.2$ in appropriate units. Then the optimal solution is $p^* = 2.6162, Q^* = 35.8827,$

$T_1^* = 6.62492, T_2^* = 14.4506$ and maximum total profit $TP^* = 889.186$.

Example 2: Let us consider $r(p) = \alpha e^{-Gp}$ and the parameter values of the inventory system

$$C_0 = 200, C_h = 0.5, C_p = 0.6, C_s = -0.01, \alpha = 200,$$
$$\beta = 0.05, \gamma = 4, \theta = 0.04, S = 250$$

and $G = 2.2$ in appropriate units. We obtain the optimal solution $p^* = 1.54171,$

$Q^* = 13.8013, T_1^* = 11.7641, T_2^* = 18.0382$ and maximum total profit

$TP^* = 1263.14$.

Example 3: Let us take $r(p) = \alpha p^G$ and the parameter values of the inventory system

$$C_0 = 200, C_h = 0.5, C_p = 0.9, C_s = -0.03, \alpha = 200,$$
$$\beta = 0.04, \gamma = 4, \theta = 0.08, S = 250$$

and $G = -3.8$ in appropriate units. Then the optimal solution is $p^* = 2.01334, Q^* = 31.4103, T_1^* = 5.5649, T_2^* = 8.04764$ and maximum total profit $TP^* = 671.556$.

Example 4: Let us consider $r(p) = \alpha p^G$ and the parameter values of the inventory system

$$C_0 = 300, C_h = 0.5, C_p = 0.8, C_s = -0.01, \alpha = 200,$$
$$\beta = 0.03, \gamma = 3, 7, \theta = 0.09, S = 250$$

and $G = -3.7$ in appropriate units. We obtain the optimal solution $p^* = 1.99328, Q^* = 36.2593, T_1^* = 6.08721, T_2^* = 7.17976$ and maximum total profit $TP^* = 732.30$.

6. SENSITIVITY ANALYSIS

Using the numerical Example 3, the sensitivity of each of the decision variables p, Q, T_1, T_2 and the total profit TP to changes in each of the 10 parameters $C_0, C_h, C_p, C_s, \alpha, \beta, \gamma, \theta, S$ and G is examined in Table 1. The sensitivity analysis is performed by changing each of the parameters by -25%, -10%, +10% and +25%, taking one parameter at a time, and keeping the remaining parameters unchanged. It is observed that the constant price p is almost insensitive to changes in all the parameters.

Table 1: Sensitivity analysis

Parameter	%change in the parameter	p^*	Q^*	T_1^*	T_2^*	% change in TP^*
c_0	+25	2.01334	31.4114	5.56546	8.04758	7.44537
	+10	2.01334	31.4114	5.56546	8.04758	2.97815
	-10	2.01334	31.4114	5.56546	8.04758	-2.97815
	-25	2.01334	31.4114	8.32606	8.04758	-7.44537
c_h	+25	2.01081	25.6133	5.65672	8.33854	21.5005
	+10	2.01220	28.8084	5.60653	8.17722	8.57111
	-10	2.01472	34.5169	5.51639	7.8950	-8.51989
	-25	-	-	-	-	-
c_p	+25	2.01217	24.2482	5.56643	8.32424	8.77884
	+10	2.00890	28.5165	5.56719	8.15997	3.46061
	-10	2.01773	34.3488	5.56163	7.93265	-3.39152
	-25	2.02424	38.8390	5.55197	7.75557	-8.34701
c_s	+25	2.01335	31.4454	5.56487	8.04585	-0.102597
	+10	2.01334	31.4240	5.56517	8.04685	-0.0410985
	-10	2.01333	31.3978	5.56562	8.04822	0.0409496
	-25	2.02332	31.3774	5.56592	8.04922	0.102597
α	+25	-	-	-	-	-
	+10	2.06594	32.8663	5.55330	7.98340	-1.23995
	-10	2.95667	29.8410	5.57923	8.11698	1.35163
	-25	2.86267	27.2192	5.23328	8.23328	3.64287
β	+25	2.04291	31.5864	5.60636	8.08646	-0.366759
	+10	2.02511	31.4821	5.58167	8.06303	-1.146078
	-10	2.00163	31.3398	5.54929	8.03210	0.145185
	-25	2.98420	31.2307	5.52514	8.00903	0.361696
γ	+25	1.95175	33.4775	3.33372	7.34725	-13.1162
	+10	1.98323	32.4031	4.39004	7.70255	6.60166
	-10	2.05622	30.0577	7.89792	8.68824	10.0480
	-25	2.17148	26.7415	17.5475	9.92976	41.0483
θ	+25	1.89770	34.3201	4.39193	6.3072	-14.274
	+10	1.996319	32.6665	5.03094	7.25226	-6.54999
	-10	2.07013	30.0147	6.22059	9.02758	8.12320
	-25	2.17194	27.607	7.53605	11.0092	24.6869
S	+25	1.90002	35.3281	5.59076	8.18693	20.9633
	+10	1.96421	33.0506	5.57668	8.10791	8.31429
	-10	2.06921	29.6608	5.55259	7.97947	-8.20659
	-25	-	-	-	-	-
G	+25	1.74993	23.7419	5.72384	8.46238	6.59601-
	+10	1.88906	27.7790	5.64187	8.24281	3.06734
	-10	2.17619	37.2079	5.46102	7.79317	-3.90212
	-25	-	-	-	-	-

The parameter γ is highly sensitive and the parameters C_0, C_h, C_p, θ, S and G are slightly sensitive on total profit, whereas the other parameters C_s, α and β are practically insensitive on total profit. The following analysis can also be made from the Table 1:

(i) If the set-up cost per cycle increases (or decreases) by 25%, then the maximum total profit TP^* increases (or decreases) by 7% and the optimal value of decision variables p^*, Q^*, T_1^* and T_2^* remains the same.

(ii) We observe that if holding cost C_h increases by 25%, TP^* increases by 21%. If C_h decreases by 10%, TP^* decreases by 8%.

(iii) If the production cost increases (or decreases) by 25%, then TP^* increases (or decreases) by 8%.

(iv) When the parameter γ increases by 25%, then the maximum total profit TP^* decreases by 13%. If it decreases by 25%, then TP^* increases by 41%.

(v) If the constant deterioration rate θ increases by 25%, then the maximum total profit TP^* decreases by 14%. When θ decreases by 25%, then TP^* decreases by 24%.

(vi) When the parameters S and G increases (or decreases), then TP^* increases (or decreases).

7. CONCLUDING REMARKS

In this article, a single-item EPLS model was developed in which the demand was a function of the stock level and the selling price. Variable production rate was proportional to demand. We impose a ceiling on initial and ending on-hand inventory stock. Constant fraction deterioration is also considered in this model. Model is developed and solved allowing no shortages in inventory.

We then provided an algorithm of solution procedure for finding optimal solution to the problem. We solve this problem analytically and numerically. Four numerical examples are given to illustrate the model. Sensitivity analysis is also examined.

The proposed article can be extended in the following ways:

We could extend the proposed demand function to time-price or quadratic price dependent demand or stochastically fluctuating demand pattern.

We might generalize the model by taking into consideration Weibull deterioration, partial backlogging and other assumptions.

Appendix A

The total profit TP in the interval [0.T] is given by

$$TP = (p - C_p) \int_0^T D(i(t), p)dt - C_0 - [C_h + \theta(C_p + C_s) \int_0^T i(t)dt$$
$$= (p - C_p)r(p)[I_1 + I_2] - C_0 - [C_h + \theta(C_p + C_s)][I_3 + I_4]$$

where

$$I_1 = \int_0^T [\frac{(\gamma-1)r(p)}{\theta} + \{Q^{1-\beta} - \frac{(\gamma-1)r(p)}{\theta}\}e^{-\theta(1-\beta)t}]^{\frac{\beta}{1-\beta}} \, dt$$

$$= \frac{1}{(1-\beta)(1-\gamma)r(p)}[(1-\beta)[\{\frac{(\gamma-1)r(p)}{\theta} + (Q^{1-\beta} - \frac{(\gamma-1)r(p)}{\theta})$$

$$\times e^{-\theta(1-\beta)T_1}\}^{\frac{1}{1-\beta}} - Q] + \frac{\theta(1-\beta)}{(2-\beta)(\gamma-1)r(p)}[\{\frac{(\gamma-1)r(p)}{\theta}$$

$$+ (Q^{1-\beta} - \frac{(\gamma-1)r(p)}{\theta})e^{-\theta(1-\beta)T_1}\}^{\frac{2-\beta}{1-\beta}} - Q^{2-\beta}]],$$

taking $\theta \ll (\gamma-1)r(p)$ and neglecting small quantities above the first order

$$= \frac{1}{(\gamma-1)r(p)}(S-Q) + \frac{\theta}{(2-\beta)(\gamma-1)^2\{r(p)\}^2}(S^{2-\beta} - Q^{2-\beta}), \text{ using equation (6)},$$

$$I_2 = \int_{T_1}^{T_1+T_2} [(Q^{1-\beta} + \frac{r(p)}{\theta})e^{\theta(1-\beta)(T_1+T_2-t)} - \frac{r(p)}{\theta}]^{\frac{\beta}{1-\beta}} \, dt$$

$$= \frac{1}{-(1-\beta)r(p)}[(1-\beta)[Q - \{(Q^{1-\beta} + \frac{r(p)}{\theta})e^{\theta(1-\beta)T_2} - \frac{r(p)}{\theta}\}^{\frac{1}{1-\beta}}]$$

$$- \frac{\theta(1-\beta)}{(2-\beta)r(p)}[Q^{2-\beta} - \{(Q^{1-\beta} + \frac{r(p)}{\theta})e^{\theta(1-\beta)T_2} - \frac{r(p)}{\theta}\}^{\frac{2-\beta}{1-\beta}}]],$$

taking $\theta \ll r(p)$ and neglecting small quantities above the first order

$$= \frac{1}{r(p)}(S-Q) + \frac{\theta}{(2-\beta)\{r(p)\}^2}(Q^{2-\beta} - S^{2-\beta}), \text{ using equation (7)}.$$

Similarly

$$I_3 = \frac{1}{(\gamma-1)r(p)(2-\beta)}(S^{2-\beta} - Q^{2-\beta}) + \frac{\theta}{(\gamma-1)^2(3-2\beta)\{r(p)\}^2}(S^{3-2\beta} - Q^{3-2\beta}).$$

and

$$I_4 = \frac{1}{r(p)(2-\beta)}(S^{2-\beta} - Q^{2-\beta}) + \frac{\theta}{(3-2\beta)\{r(p)\}^2}(S^{3-2\beta} - Q^{3-2\beta}).$$

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