Yugoslav Journal of Operations Research
22 (2012), Number 1, 51-78
DOI:10.2298/YJOR100219005Y

INVENTORY MODEL OF DETERIORATING ITEMS WITH TWO-WAREHOUSE AND STOCK DEPENDENT DEMAND USING GENETIC ALGORITHM IN FUZZY ENVIRONMENT

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Received: February 2010 / Accepted: February 2012

Abstract: Multi-item inventory model for deteriorating items with stock dependent demand under two-warehouse system is developed in fuzzy environment (purchase cost, investment amount and storehouse capacity are imprecise) under inflation and time value of money. For display and storage, the retailers hire one warehouse of finite capacity at market place, treated as their own warehouse (OW), and another warehouse of imprecise capacity which may be required at some place distant from the market, treated as a rented warehouse (RW). Joint replenishment and simultaneous transfer of items from one warehouse to another is proposed using basic period (BP) policy. As some parameters are fuzzy in nature, objective (average profit) functions as well as some constraints are imprecise in nature, too. The model is formulated so to optimize the possibility/necessity measure of the fuzzy goal of the objective functions, and the constraints satisfy some pre-defined necessity. A genetic algorithm (GA) is used to solve the model, which is illustrated on a numerical example.

Keywords: Possibility/necessity measures, inflation, time value of money, deterioration, genetic algorithm.

MSC: 90B05

1. INTRODUCTION

The classical inventory models are mainly developed for the single storage facility. But, in the field of inventory management, when a purchase (or production) of large amount of units of items that can not be stored in the existing storage (viz., own warehouse-OW) at the market place due to its limited capacity, then excess units are stocked in a rented warehouse (RW) located at some distance from OW. In a real life situation, management goes for large purchase at a time, when either an attractive price discounts can be got or the acquisition cost is higher than the holding cost in RW. That's why, it is assumed that the capacity of a rented warehouse is imprecise in nature i.e., the capacity of a rented warehouse can be adjusted according to the requirement. The actual service to the customer is done at OW only. Items are transferred from RW to OW using basic period (BP) policy.

In the present competitive market, the inventory/stock is decoratively displayed through electronic media to attract the customer and to push the sale. Levin *et al.* [1972] established the impact of product availability for stimulating demand. Mandal and Maiti [1989] consider linear form of stock-dependent demand, i.e., $D = c + dq$, where D, q represent demand and stock level, respectively. Two constant c, d are chosen so to fit the demand function the best, whereas Urban [1992], Giri *et al.* [1996], Mandal and Maiti [2000], Maiti and Maiti [2005, 2006] and others consider the demand of the form $D = dq^r$ where d, r are constant, chosen so to fit the demand function the best. Goyal and Chang [2009] obtained the optimal ordering and transfer policy with stock dependent demand.

In general, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or loss of original usefulness. It is reasonable to note that a product may be understood as to have a lifetime which ends when its utility reaches zero. IC chip, blood, fish, strawberries, alcohol, gasoline, radioactive chemicals and grain products are the examples of deteriorating item. Several researchers have studied deteriorating inventory in the past. Ghare and Schrader [1963] were the first to develop an EOQ model for an item with exponential decay and constant demand. Covert and Philip [1973] extended the model to consider Weibull distribution deterioration. Mishra [1975] formulated an inventory model with a variable rate of deterioration with a finite rate of production. Several researchers like Goyal and Gunasekaran [1995], Benkherouf [1997], Giri and Chaudhuri [1998] have developed the inventory models of deteriorating items in different aspects. Kar *et al.* [2001] developed a two-shop inventory model for two levels of deterioration. A comprehensive survey on continuous deterioration of the on-hand inventory has been done by Goyal and Giri [2001]. Several researchers such as Yang [2004], Roy *et al.* [2007] analyze the effect of deterioration on the optimal strategy. Mandal *et al.* [2010] and Yadav *et al.* [2011] obtained the optimal ordering policy for deteriorating items.

It has been recognized that one's ability to make precise statement concerning different parameters of inventory model diminishes with the increase of the environment complexity. As a result, it may not be possible to define the different inventory parameters and the constraints precisely. During the controlling period of inventory, the resources constraints may be possible in nature, and it may happen that the constraints on resources satisfy, in almost all cases, except in a very few where they may be allowed to violate. In a fuzzy environment, it is assumed that some constraints may be satisfied using some predefined necessity, η_2 (cf. Dubois and Prade [1983, 1997]). Zadeh [1978] first introduced the necessity and possibility constraints, which are very relevant to the real life decision problems, and presented the process of defuzzification for these constraints. After this, several authors have extended the ideas and applied them to different areas such as linear programming, inventory model, etc. The purpose of the present paper is to use necessity and possibility constraints and their combination for a real-life two warehouse inventory model. These possibility and necessity resources constraints may be imposed as per the demand of the situation.

From financial standpoint, an inventory represents a capital investment and must compete with other assets within the firm's limited capital funds. Most of the classical inventory models did not take into account the effects of inflation and time value of money. This was mostly based on the belief that inflation and time value of money do not influence the cost and price components (i.e., the inventory policy) to any significant degree. But, during the last few decades, due to high inflation and consequent sharp decline in the purchasing power of money in the developing countries like Brazil, Argentina, India, Bangladesh, etc., the financial situation has been changed, and so it is not possible to ignore the effects of inflation and time value of money. Following Buzacott [1975], Mishra [1979] has extended the approach to different inventory models with finite replenishment and shortages by considering the time value of money and different inflation rates for the costs. Hariga [1995] further extends the concept of inflation. Liao *et al.* [2000] studied the effects of inflation on a deteriorating inventory. Chung and Lin [2001] studied an EOQ model for a deteriorating inventory subjected to inflation. Yang [2004]develop a model for deteriorating inventory stored at two warehouses, and extended inflation to the idea of deterioration as amelioration when the environment is inflationary. Several related articles were presented dealing with such inventory problems (Chung and Liao [2006], Maiti and Maiti [2007], Rang *et al.* [2008], Chen *et al.* [2008], Ouyang *et al.* [2009]).

Here, a deteriorating multi-item inventory model is developed considering inflation and time value of money. Analysis of inventory of goods whose utility does not remain constant over time has involved a number of different concepts of deterioration. Maintenance of such inventory is of a major concern for a manager in a modern business organization. The quality of stocks maintained by an organization depends very heavily on the facility of its preserving. Keeping all this in mind, it is considered that items deteriorate with constant rate. Two rented warehouses are used for storage, one (own warehouse) is located at the heart of the market place and the other (rented warehouse) is located at a short distance from the market place. The items are jointly replenished and transferred from RW using basic period (BP) policy. Under BP, a replenishment and transfer of items from RW to OW are made at regular time intervals. Each item has a replenishment quantity sufficient to last for exactly an integer multiple of T. Similarly, each item has a transferred quantity sufficient to last for exactly an integer multiple of L_t.

Demand rate of an item is assumed to be stock dependent and shortages are not allowed. Here, the size of OW is finite and deterministic, but that of RW is imprecise. Although business starts with two rented warehouses of fixed capacity, in some extra temporary arrangement, it may be run near RW as it is away from the heart of the market place. This temporary arrangement capacity is fuzzy in nature. Therefore, the capacity of

RW may be taken as fuzzy in nature, too. Unit costs of the items and the capital for investment are also fuzzy in nature. Hence, there are two constraints-one is on the storage space and the other on the investment amount, and these constraints will hold good to at least some necessity α. Since purchase cost is fuzzy in nature, the average profit is fuzzy in nature, too. As optimization of a fuzzy objective is not well defined, a fuzzy goal for average profit is set and possibility/necessity of the fuzzy objective (i.e., average profit) with respect to fuzzy goal is optimized under the above mentioned necessity constraints in optimistic/pessimistic sense.

2. OPTIMIZATION USING POSSIBILITY/NECESSITY MEASURE

A general single-objective mathematical programming problem should have the following form:

$$\begin{array}{ll} \max & f(x,\xi) \\ \text{subject to} & g_i(x,\xi) \le 0, \qquad i = 1,2,3,........,n \end{array} \tag{1}$$

where x is a decision vector, ξ is a crisp parameter, $f(x,\xi)$ is the return function, $g_i(x,\xi)$ are continuous functions, $i = 1,2,3,...,n$. In the above problem, when ξ is a fuzzy vector $\tilde{\xi}$ (i.e., a vector of fuzzy numbers), then return function $f(x,\tilde{\xi})$ and constraints functions $g_i(x,\tilde{\xi})$ are imprecise in nature and can be represented by a fuzzy number whose membership function involve the decision vector x as the parameter, and which can be obtained when membership functions of the fuzzy numbers in $\tilde{\xi}$ are known (since f and g_i are functions of decision vector x and the fuzzy number $\tilde{\xi}$). In that case, the statements maximize $f(x,\tilde{\xi})$ as well as $g_i(x,\tilde{\xi}) \le 0$ are not defined. Since $g(x,\tilde{\xi})$ represents a fuzzy number whose membership function involves decision vector x, and for a particular value of x, the necessity of $g(x,\tilde{\xi})$ can be measured by using formula (58) (see Appendix 1), therefore a value x_o of the decisions vector x is said to be feasible if necessity measure of the event $\{\xi : g_i(x,\xi) \le 0\}$ exceeds some pre-defined level α_i in pessimistic sense, i.e., if $nes\{g_i(x,\xi) \le 0\} \ge \alpha_i$,which may also be written as $nes\{\tilde{\xi} : g_i(x,\tilde{\xi}) \le 0\} \ge \alpha_i$. If an analytical form of the membership function of $g_i(x,\tilde{\xi})$ is available, then this constraint can be transformed to an equivalent crisp constraint (cf. Lemmas 1 and 2 of Appendix 1).

Again, as maximize $f(x,\tilde{\xi})$ is not well defined, a fuzzy goal of the objective function may be as proposed by Katagiri *et al.* [2004], Mandal *et al.* [2005]. To make optimal decision, DM can maximize the degree of possibility/necessity that the objective function value satisfies the fuzzy goal in optimistic/pessimistic sense as proposed by Katagiri *et al.* [2004]. When ξ is a fuzzy vector $\tilde{\xi}$ and $\tilde{G}(= an\ LFN(G_1, G_2))$ is the goal of the objective function, then according to the above discussion, the problem (1) is reduced to the following chance constrained programming in optimistic and pessimistic sense, respectively

$$\begin{aligned} &\max \quad Z_P = \pi_{f(x,\tilde{\xi})}(\tilde{G}),\\ &\text{subject to} \quad nes\left\{\tilde{\xi} : g_i(x,\tilde{\xi}) \le 0\right\} \ge \alpha_i,\\ &\qquad i = 1,2,3,\ldots\ldots..,n \end{aligned} \tag{2}$$

$$\begin{aligned} &\max \quad Z_N = N_{f(x,\tilde{\xi})}(\tilde{G}),\\ &\text{subject to} \quad nes\left\{\tilde{\xi} : g_i(x,\tilde{\xi}) \le 0\right\} \ge \alpha_i,\\ &\qquad i = 1,2,3,\ldots\ldots..,n \end{aligned} \tag{3}$$

If the analytical form of membership function of $f(x,\tilde{\xi})$ (obtained using formula (58) of Appendix 1) is a $TFN(F_1(x),F_2(x),F_3(x))$, then Lemma 3 of Appendix 1 gives $Z_p = \frac{F_3(x)-G_1}{F_3(x)-F_2(x)+G_2-G_1}$. Hence maximization of Z_p implies maximization of $F_2(x)$ (most feasible equivalent of $f(x,\tilde{\xi})$) and $F_3(x)$ (least feasible equivalent of $f(x,\tilde{\xi})$) together and $Z_p = 1$ implies $F_2(x) \ge G_2$, i.e., most feasible profit function achieves the highest level of profit goal (G_2). Therefore, if DM is optimistic and allows some risk, then she/he will take decision depending on possibility measure. On the other hand, in this case Lemma 4 of Appendix 1 gives $Z_p = \frac{F_3(x)-G_1}{F_3(x)-F_2(x)+G_2-G_1}$. In this case maximization of Z_N implies maximization of $F_1(x)$ (worst possible equivalent of $f(x,\tilde{\xi})$ and $F_2(x)$ (most feasible equivalent of $f(x,\tilde{\xi})$) together and $Z_N = 1$ implies $F_1(x) \ge G_2$, i.e., worst possible profit function achieves the highest level of profit goal (G_2). Thus, if any risk highly affects the company, the DM will go for optimization of Z_N to get optimal decision. However, one can optimize the weighted average of possibility and necessity measures. In that case, the problem is reduced to

$$\begin{aligned} &\max Z = \beta Z_p + (1-\beta) Z_N\\ &\textit{subject to nes}\left\{\xi : g_i(x,\xi) \le 0\right\} \ge \alpha_i,\\ &i = 1,2,3,\ldots,n. \end{aligned} \tag{4}$$

where Z_p and Z_N are given by equation (2) and (3), respectively, and β is the managerial attitude factor. Here, $\beta = 1$ represents the most optimistic attitude, and $\beta = 0$ represents the most pessimistic attitude.

3. DETERMINATION OF FUZZY GOAL

Fuzzy goal $\tilde{G}$ of the fuzzy objective function $\tilde{f}(\tilde{x},\tilde{\xi})$ is considered as a LFN (G_1, G_2) and the values of G_1, G_2 can be determined in different ways. Here, the following formulae are proposed and used in numerical illustrations for the fuzzy models. In the formulae, X denotes the feasible search of the problem:

$$G_1 = \inf_{\xi_0 \in \tilde{\xi}}(\inf_{x_0 \in X} f(\xi_0, x_0)),$$

$$G_2 = \sup_{\xi_0 \in \tilde{\xi}}(\sup_{x_0 \in X} f(\xi_0, x_0)),$$

4. GENETIC ALGORITHM

Genetic Algorithm is exhaustive search algorithm based on the mechanics of natural selection and genesis (crossover, mutation, etc.). It was developed by Holland, his colleagues and students at the University of Michigan. Because of its generality and other advantages over conventional optimization methods, it has been successfully allied to different decision making problems.

In natural genesis, we know that chromosomes are the main carriers of hereditary factors. At the time of reproduction, crossover and mutation take place among the chromosomes of parents. In this way, hereditary factors of parents are mixed-up and carried over to their offspring. Again, Darwinian principle states that only the fittest animals can survive in nature. So, a pair of parents normally reproduces a better offspring.

The above-mentioned phenomenon is followed to create a genetic algorithm for an optimization problem. Here, potential solutions of the problem are analogous with the chromosomes, and the chromosome of better offspring with the better solution of the problem. Crossover and mutation among a set of potential solutions to get a new set of solutions are made, and it continues until terminating conditions are encountered. Michalewich proposed a genetic algorithm named Contractive Mapping Genetic Algorithm (CMGA) and proved the asymptotic convergence of the algorithm by Banach fixed point theorem. In CMGA, a movement from the old population to a new one takes place only if an average fitness of the new population is better than the fitness of the old one. In the algorithm, p_c, p_m are probability of crossover and probability of mutation respectively, T is the generation counter and $P(T)$ is the population of potential solutions for the generation T. M is an iteration counter in each generation to improve $P(T)$ and M_0 is the upper limit of M. Initialize $(P(1))$ function generate the initial population $P(1)$ (initial guess of solution set) at the time of initialization. Objective function value due to each solution is taken as fitness of the solution. Evaluate $(P(T))$ function evaluates fitness of each member of $P(T)$. Even though when fuzzy model can be transformed into equivalent crisp model, only ordinary GA is used for a solution.

GA Algorithm:

1. Set generation counter $T = 1$, iteration counter in each generation $M = 0$.
2. Initialize probability of crossover p_c, probability of mutation p_m, upper limit of iteration counter M_0, population size N.
3. Initialize $(P(T))$.
4. Evaluate $(P(T))$.
5. While $(M < M_0)$.
6. Select N solutions from $P(T)$ for mating pool using Roulette-Wheel process.
7. Select solutions from $P(T)$, for crossover depending on p_c.
8. Make crossover on selected solutions.
9. Select solutions from $P(T)$, for mutation depending on p_m.
10. Make mutation on selected solutions for mutation to get population $P_1(T)$.
11. Evaluate $(P_1(T))$
12. Set $M = M + 1$
13. If average fitness of $P_1(T) >$ average fitness of $P(T)$ then
14. Set $P(T+1) = P(T_1)$
15. Set $T = T + 1$
16. Set $M = 0$
17. End if
18. End While
19. Output: Best solution of $P(T)$
20. End algorithm.

5. ASSUMPTIONS AND NOTATIONS FOR THE PROPOSED MODEL

The following notations and assumptions are used in developing the model.

Inventory system involves N items and two warehouse, one is Own warehouse situated in the main market, and the other is a rented warehouse situated away from the market place. They are respectively represented by OW and RW. The holding cost of OW warehouse is higher than the one of RW.

1. Storage area of OW and RW are AR_1 and AR_2 units, respectively.
2. T is planning horizon.
3. N_M orders are done during T.
4. T_o is the basic time interval between orders, i.e., $T_0 = T / N_M$.
5. M is the number of times items are transferred from RW to OW during T_0.

6. L_t basic time interval between transferred of items from RW to OW . So, $L_t = T_o / M$.
7. INV is the total investment.
8. Z is the profit per unit time.
9. G is the goal of Z (for fuzzy model).
10. β_1 and β_2 denote the confidence levels for investment and space constraint, respectively.
11. Z_p and Z_N represent degree of possibility, necessity that the average profit satisfies the fuzzy goal (for fuzzy model). F is the weighted average of Z_p and Z_N , i.e., $F = \beta Z_p + (1-\beta) Z_N$ and β is the managerial attitude factor.
12. I is the inflation rate.
13. d is the discount rate.
14. $R = d - I$.
15. c_{om} is the major ordering cost.
16. c_{tm} is the major transportation cost.

For i^{th} item following notations are used.

17. n_i the number of integer multiple of T_o when the replenishment of i^{th} item is part of group replenishment.
18. L_i is the cycle length, i.e., $L_i = n_i T_0$.
19. m_i the number of integer multiple of L_t when the transfer of i^{th} item is a part of group transfer from RW to OW .
20. T_{ti} is duration between two consecutive shipments of the item from RW to OW . So, $T_{ti} = m_i L_t$.
21. Item is transferred from RW to OW in N_i shipments. So

$$N_i = \begin{cases} \left[\dfrac{L_i}{T_{ti}}\right] & \text{if } L_i \text{ is an integer multiple of } T_{ti} \\ \left[\dfrac{L_i}{T_{ti}}\right] + 1 & \text{otherwise,} \end{cases}$$

where $\left[\dfrac{L_i}{T_{ti}}\right]$ represents integral part of $\dfrac{L_i}{T_{ti}}$

22. θ deterioration rate in OW and RW .

23. $c_{hOW(i)}$ and $c_{hRW(i)}$ are holding costs per unit quantity per unit time at OW and RW, respectively, so $c_{hOW(i)} = h_{OW(i)}c_{pi}$ and $c_{hOW(i)} = h_{RW(i)}c_{pi}$
24. Total cycles for i^{th} item

$$\mathrm{M_i} = \begin{cases} \left[\dfrac{\mathrm{H}}{\mathrm{L_i}}\right] & \text{if H is an integer multiple of } \mathrm{L_i} \\ \left[\dfrac{\mathrm{H}}{\mathrm{L_i}}\right] + 1 & \text{otherwise,} \end{cases}$$

$$\text{where} \left[\frac{\mathrm{H}}{\mathrm{L_i}}\right] \text{represents integral part of } \frac{\mathrm{H}}{\mathrm{L_i}}$$

25. In the j^{th} cycle item is transferred at $t = T_{ij1}, T_{ij2}, \ldots\ldots, T_{ijNi}$, where $T_{ij1} = (j-1), T_{ijk} = T_{ij1} + (k-1)T_{ti}$.
26. A_i be the area required to store one unit.
27. A fraction λ_i of AR_1 is allocated for i^{th} item. So, maximum displayed inventory level $Q_{di} = \lambda_i AR_1 / A_i \; and \; \sum_{i-1}^{N} \lambda_1 = 1$.
28. Q_{ij} is the order quantity at the beginning of j^{th} cycle, which is the same in all cycles except for the last.
29. Q_{i1} is the order quantity at the beginning of the last cycle.
30. Q_{OWijk} is the stock level at OW at the beginning of k^{-th} sub-cycle in j^{th} cycle, when items are transferred from OW to RW which is the same for all sub-cycles except for the first sub cycle where $Q_{OWsij1} = 0$.
31. Fractions $h_{OW(i)}$ and $h_{RW(i)}$ of purchase cost are assumed as holding costs per unit quantity per unit time at OW and RW, respectively, where $h_{OW(i)} > h_{RW(i)}$.
32. c_{oi} is the minor ordering cost of the item in \$, which is partly constant and partly order quantity dependent and of the form: $c_{oi} = c_{o1i} + c_{o2i}Q_i$.
33. $q_{OW(i)}$ is the inventory level at OW at any time t.
34. Demand of the item D_i is linearly dependent on the inventory level at OW and is of the form: $D_i(q_{OW(i)}) = x_i + y_i q_{OW(i)}$.
35. c_{ti} represents minor transportation cost in \$ per unit item from RW to OW.
36. c_{pi} represents minor transportation cost in \$, and η_i is the mark-up of selling price $c_{si} = \eta_i c_{pi}$.

6. MODEL DEVELOPMENT AND ANALYSIS

Rented Warehouse (RW):

In the development of the model, it is assumed that the items are jointly replenished using BP policy. Under BP, the replenishment is made at regular time intervals (every T_o unit of time) and each item (i^{th} item) has a replenishment quantity (Q_{ijk}) sufficient to last for exactly an integer multiple (n_i) of T_o, i.e., i^{th} item is ordered at regular time intervals $n_i T_o$. The inventory level at RW goes down discretely at a fixed time interval during which the stock at RW is depleted continuously only due to deterioration of the units. Hence, the inventory level $q_{RW(i)}(t)$ at RW at any instant t during $T_{ijk} \le t < T_{ijk+1}$, satisfies the differential equation

$$\frac{dq_{RW(i)}(t)}{dt} = -\theta_1 \mathrm{q}_{\mathrm{RW(i)}}(\mathrm{t}) \quad for\, T_{ijk} \le t < T_{ijk+1} \tag{5}$$

Therefore, in each time interval $T_{ijk} \le t < T_{ijk+1}$, $q_{RW(i)}(t)$ continuously decreases from the level q_{ijk} but it has left-hand discontinuity at T_{ijk+1}, because from the model description it is clear that $\underset{t \to (T_{ijk+1})^{-1}}{Lim} q_{RW(i)}(t) = Q_{Tijk} + q_{ijk}$.

Using this condition, the solution of the differential equation is given by

$$q_{RW(i)}(t) = (Q_{Tijk} + q_{ijk+1}) e^{\theta_1 (T_{ijk+1} - t)} \tag{6}$$

for $T_{ijk} \le t < T_{ijk+1}$.

Moreover, $q_{RW(i)}(T_{ijk}) = q_{ijk}$, so we can deduce q_{ijk} from equation (5)

$$q_{ijk} = (Q_{Tijk} + q_{ijk+1}) e^{\theta_1 (T_{ijk+1} - T_{ijk})}$$

$$q_{ijk} = (Q_{Tijk} + q_{ijk+1}) e^{\theta_1 T_{ti}} \quad or$$

$$q_{ijk+1} = Q_{Tijk} e^{\theta_1 T_{ti}} + q_{ijk+2} e^{\theta_1 T_{ti}}$$

$$q_{ijk+1} = Q_{Tijk} e^{\theta_1 T_{ti}} + Q_{Tijk} e^{2\theta_1 T_{ti}} + q_{ijk+3} e^{\theta_1 T_{ti}}$$

Continuing in this way, we get $q_{ijk+1} = Q_{Tijk} \sum_{s=1}^{N_i - k - 1} e^{s\theta_1 T_{ti}} + q_{ij\, N_i - k - 1} e^{\theta_1 T_{ti}}$

$$q_{ijk+1} = Q_{Tijk} e^{\theta_1 T_{ti}} \left[\frac{e^{(N_i - k - 1)\theta_1 T_{ti-1}}}{e^{\theta_1 T_{ti}} - 1} \right]$$

Since, $q_{ijN_i - k - 1} = 0$

Evaluation of Holding Cost at RW:

Stock at RW during $T_{ijk} \le t < T_{ijk+1}, Q_{2ijk}$ is given by

$$Q_{2ijk} = Q_{ij} - \sum_{l=1}^{k} Q_{Tijl} - \sum_{l=1}^{k} \theta_i T_{ti}$$

$$Q_{2ijk} = Q_{ij} - kQ_{di} + \frac{k-1}{y_i+\theta_i}\left[-x_i + \left\{ x_i + (y_i+\theta_i)Q_{di} \right\} e^{-(y_i+\theta_i)T_{ti}} \right]$$

Present value of holding cost at RW during $T_{ijk} \le t < T_{ijk+1}$ is $c_{h2i}H_{2ijk}$, where

$$\mathrm{H}_{2\mathrm{ijk}} = \int_{T_{ijk}}^{T_{ijk+1}} Q_{2ijk} e^{-Rt} dt = \frac{Q_{2ijk}}{R}(1-e^{-RT_{ti}})e^{-RT_{ijk}}$$

Present value of holding cost at RW in first M_{i-1} cycles is $c_{h2i}H_{2G}$, where

$$\mathrm{H}_{2\mathrm{G}} = \sum_{j=1}^{M_i-1}\sum_{k=1}^{N_i-1} H_{2ijk}$$

$$= \left\{\frac{1-\mathrm{e}^{-\mathrm{RT_{ti}}}}{\mathrm{R}}\right\}\left[\left[Q_{ij}\left\{\frac{1-e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}}\right\} - Q_{di}S_1 + \frac{1}{y_i+\theta_i}\left\{ -x_i + (x_i+(y_i+\theta_i)Q_{di})e^{-(y_i+\theta_i)T_{ti}} \right\}S_2\right]\left[\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}}\right]\right] \quad (7)$$

where

$$S_1 = \left\{\frac{1-e^{-RT_{ti}(N_i-1)}}{\left(1-e^{-RT_{ti}}\right)^2}\right\} - \left\{\frac{(N_i-1)e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}}\right\} \quad (8)$$

$$S_2 = \left\{\frac{e^{-RT_{ti}}\left(1-e^{-RT_{ti}(N_i-2)}\right)}{\left(1-e^{-RT_{ti}}\right)^2}\right\} - \left\{\frac{(N_i-2)e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}}\right\} \quad (9)$$

Own Warehouse:

On the other hand, the stock depletion at OW is due to demand and deterioration of the items. Instantaneous state $q_{OW(i)}(t)$ of i^{th} item at OW is given by

$$\frac{dq_{OW(i)}(t)}{dt} = -(x_i + y_i q_{OW(i)}(t)) - \theta_i q_{OW(i)}(t) \quad for \;\; T_{ijk} \le t \le T_{ijk+1} \quad (10)$$

With boundary conditions $q_{OW(i)}(T_{ijk}) = Q_{di}$ for $k = 1,2,3,...,N_i - 1$

From equation (10)

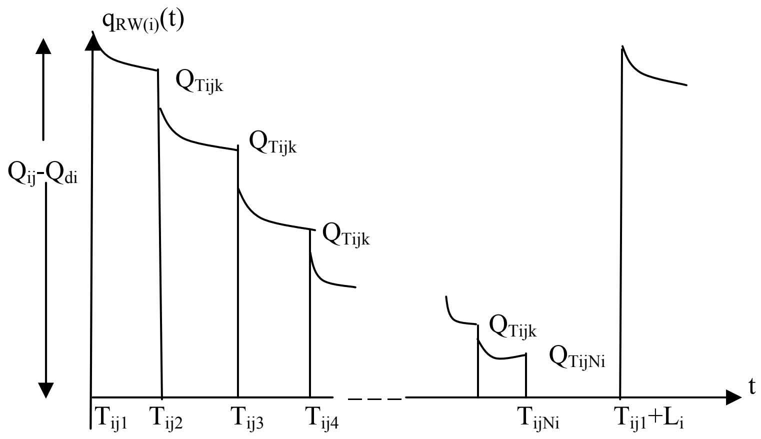
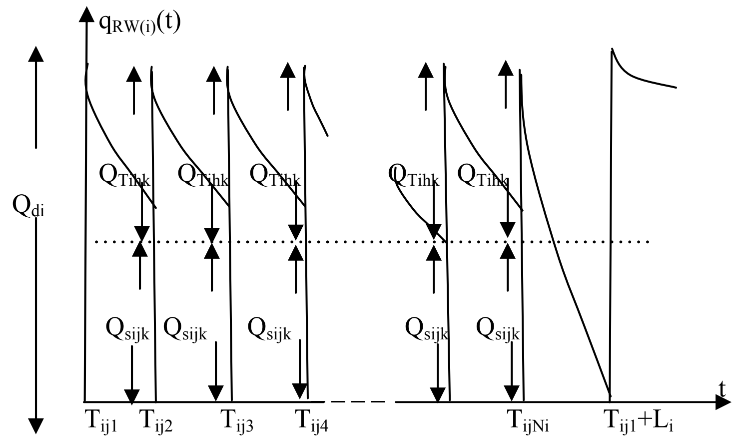


Figure 1.



Inventory Levels of i^th item in j^th cycle at RW and OW

Figure 2.

$$q_{OW(i)}(t) = \frac{1}{(\theta_i + y_i)}\left[-x_i + \left\{x_i + (\theta_i + y_i)\, Q_{di}\right\} e^{-(\theta_i + y_i)(t - T_{ijk})} \right] \tag{11}$$

$$Q_{sijk+1} = \frac{1}{(\theta_i + y_i)}\left[-x_i + \left\{x_i + (\theta_i + y_i)\, Q_{di}\right\} e^{-(\theta_i + y_i)T_{ti}} \right] \tag{12}$$

Amount transferred from RW to OW at $t = T_{ijk}, Q_{Tijk}$ is given by

$$Q_{Tijk} = Q_{di} - Q_{sijk} \quad \text{for } k = 1, 2, 3, ..., N_i - 1 \tag{13}$$

$$Q_{TijN_i} = Q_{ij} - \sum_{k=1}^{N_i-1} Q_{Tijk} \tag{14}$$

Instantaneous state $q_{OW(i)}(t)$ of i^{th} item at OW in the last sub-cycle is given by

$$\frac{dq_{OW(i)}(t)}{dt} = -(x_i + y_i q_i) - \theta_i q_i \qquad for\ T_{ijN_i} \le t \le T_{ij1} + L_i \tag{15}$$

With boundary conditions

$$Q_{OW(i)}(T_{ijNi}) = Q_{TijkNi} + Q_{sijNi}, q_i(T_{ij1} + L_i) = 0$$

From equation (15)

$$q_{OW(i)}(t) = \frac{1}{\theta_i + y_i}\left[-x_i + \left(x_i + (\mathrm{Q_{TijNi}} + \mathrm{Q_{sijNi}})(\theta_i + y_i)\right)e^{-(\theta_i + y_i)(t - \mathrm{TijNi})}\ \right]$$
$$for\ T_{ijN_i} \le t \le T_{ij1} + L_i \tag{16}$$

$$q_{OW(i)}(\mathrm{T_{ijNi}}) = \frac{x_i}{\theta_i + y_i}\left[e^{(\theta_i + y_i)(L_i - (N_i - 1)\mathrm{T_{ti}})} - 1\right] \tag{17}$$

$$Q_{ij} = (N_i - 1)\left[Q_{di} - \frac{1}{\theta_i + y_i}\left[-x_i + (x_i + y_i Q_{di})e^{-(\theta_i + y_i)\mathrm{T_{ti}}} - 1\right]\right] + \frac{x_i}{\theta_i + y_i}e^{(\theta_i + y_i)(L_i - (\mathrm{N_i} - 1)\mathrm{T_{ti}})} \tag{18}$$

Evaluation of holding cost at OW:

Present value of holding cost at OW in the k^{th} sub-cycles of the j^{th} cycle is

$$= c_{h1i} H_{1ijk}$$
$$= c_{h1i} \int_{T_{ijk}}^{T_{ijk+1}} q_{OW(i)^{(t)e^{-Rt}dt}} \tag{19}$$
$$= \frac{c_{h1i}^{\ e-RTijk}}{y_i + \theta_i}\left[\frac{x_i}{R}(e^{-RT_{ti}} - 1) + \frac{\{x_i + (y_i + \theta_i)Q_{di}\}(1 - e^{-(y_i + \theta_i + R)T_{ti}})}{(y_i + \theta_i + R)}\right]$$

Present value of holding cost at OW in the last-cycle of the j^{th} cycle is

$$= c_{h1i} H_{1ijNi} = c_{h1i} \int_{T_{ijNi}}^{T_{ijk+Li}} q_{OW(i)^{(t)e^{-Rt}dt}}$$
$$= \frac{c_{h1i}^{\ e-RTijN_i}}{y_i + \theta_i}\left[\frac{x_i}{R}(e^{-R(L_i - (N_i - 1)T_{ti})} - 1) + \{x_i + (y_i + \theta_i) q_{OW(i)}(T_{ij} N_i)\}\frac{(1 - e^{-(y_i + \theta_i + R)(L_i - (N_i - 1)T_{ti})})}{(y_i + \theta_i + R)}\right] \tag{20}$$

So, present value of holding cost at OW in the j^{th} cycles is

$$= c_{h1i} H_{1ij}$$

$$= c_{h1i} \sum_{k=1}^{N_i-1} H_{1ijk} + H_{1ijN_i}$$

So, present value of holding cost at OW in the first M_{i-1} cycles is

$$= c_{h1i} H_{1G} = c_{h1i} \sum_{j=1}^{M_i-1} \left[\sum_{k=1}^{N_i-1} H_{1ijk} + H_{1ijN_i} \right]$$

$$= c_{h1i} \left[\frac{1}{y_i+\theta_i} \left[\frac{x_i}{R}(e^{-RT_{ti-1}}-1) + \frac{(1-e^{-(y_i+\theta_i+R)T_{ti}})}{(y_i+\theta_i+R)} \left\{ x_i\,(y_i+\theta_i)Q_{di} \right\} \right] \right.$$
$$\left(\frac{1-e^{-R(N_i-1/T_{ti})}}{1-e^{-RT_{ti}}} \right) \left(\frac{1-e^{-R(M_i-1/T_{ti})}}{1-e^{-RT_{ti}}} \right) e^{-RT_{ti}} + \frac{1}{y_i+\theta_i}$$
$$\left[\frac{x_i}{R}(e^{-R(L_i-(N_i-1)T_{ti})}-1) + \left\{ x_i + (y_i+\theta_i) q_{OW(i)}(T_{ijNi}) \right\} \frac{(1-e^{-(y_i+\theta_i R)(L_i-(N_i-1)/T_{ti})})}{(y_i+\theta_i+R)} \right]\Bigg] \quad (21)$$
$$\left(\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RLi}} \right) e^{-RT_{i1Ni}}$$

Evaluation of Sell Revenue:

Present value of sell revenue during $T_{ijk} \le t \le T_{ijk+1}$ is

$$= c_{si} SP_{ijk} \quad k = 1,2,3,...,N_{i-1}$$

$$= c_{si} \left[\frac{x_i \theta_i e^{-RT_{ijk}}}{y_i+\theta_i} \left\{ \frac{1-e^{-RT_{ti}}}{R} \right\} + \left(Q_{di} + \frac{x_i}{y_i+\theta_i} \right) e^{-RT_{ijk}} \left\{ \frac{1-e^{-(y_i+\theta_i+R)T_{ti}}}{y_i+\theta_i+R} \right\} \right] \quad (22)$$

Present value of sell revenue during the last cycle is

$$= c_{si} SP_{ijNi} = c_{si} \int_{T_{ijN_i}}^{T_{ij1}+L_i} D_i(t) e^{-RT} dt$$

$$= c_{si} \left[\frac{x_i \theta_i e^{-RT_{ijN_i}}}{y_i+\theta_i} \left\{ \frac{1-e^{-R(T_{ij1}+L_i-T_{ijN_i})}}{R} \right\} + y_i (q_{OW(i)}(T_{ijN_i}) + \frac{x_i}{y_i+\theta_i} \right) e^{-(y_i+\theta_i+R)T_{ijN_i}} \quad (23)$$
$$\left\{ \frac{1-e^{-(y_i+\theta_i+R)(T_{ij1}+L_i-T_{ijN_i})}}{y_i+\theta_i+R} \right\} \Bigg]$$

So, present value of sell revenue in the first M_i-1 cycle is

$$c_{si} SP_G = c_{si} \left[\left(\frac{x_i + (y_i+\theta_i)Q_{di}}{y_i+\theta_i+R} \right) \left\{ 1-e^{-(y_i+\theta_i+R)T_{ti}} \right\} \left[\frac{1-e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}} \right] \left[\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right] \right.$$
$$\left. + \left(\frac{x_i + (y_i+\theta_i) q_i(T_{ijN_i})}{y_i+\theta_i+R} \right) \left\{ 1-e^{-(y_i+\theta_i+R)L_i} \right\} e^{-R(N_i-1)T_{ti}} \left[\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right] \right] \quad (24)$$

Evaluation of Transportation Cost:

Present value of transportation cost in the first M_i-1 cycles CT_G is given by

$$CT_G = c_{ti} \sum_{j=1}^{Mi-1} \left[\sum_{k=1}^{Ni-1} Q_{T_{ijk}} e^{-RT_{ijk}} + Q_{T_{ijN_i}} e^{-RT_{ijN_i}} \right]$$

$$= c_{ti} \left[Q_{di} \left\{ \frac{1-e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}} \right\} - \left\{ \frac{-x_i + (x_i + (y_i + \theta_i) Q_{di}) e^{-(y_i+\theta_i)T_{ti}}}{(y_i + \theta_i)} \right\} \left\{ \frac{1-e^{-RT_{ti}(N_i-2)}}{1-e^{-RT_{ti}}} \right\} \right] \left\{ \frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right\}$$

$$= c_{ti} \left[Q_{ij} - (N_i - 1) Q_{di} + (N_i - 2) \left\{ \frac{-x_i + (x_i + (y_i + \theta_i) Q_{di})}{(y_i + \theta_i)} \right\} \right] \left\{ \frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right\} e^{-RT_{ti}(N_i-1)} \quad (25)$$

Evaluation of replacement cost:

Ordering cost in the first M_i-1 cycles OC_G is given by

$$OC_G = \sum_{j=1}^{M_i-1} \left(c_{o1i} + c_{o2i} Q_{ij} \right) e^{-RT_{ij1}} = \left(c_{o1i} + c_{o2i} Q_{ij} \right) \left[\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right] \quad (26)$$

Evaluation of purchase cost:

Present value of purchase cost in the first M_i-1 cycles is $c_{pi}PC_G$ where

$$PC_G = \sum_{j=1}^{M_i-1} Q_{ij} e^{-RT_{ij1}} = Q_{ij} \left[\frac{1-e^{-RL_i(M_i-1)}}{1-e^{-RL_i}} \right] \quad (27)$$

Formulation for the i^{th} item in last cycle, i.e., for $T_{iM_i} \le t \le T$

The last cycle length $Ll_i = T - (M_i - 1)L_i$.

Items are transferred from RW to OW in Nl_i shipment where

$$Nl_i = \begin{cases} \left[\dfrac{Ll_i}{T_{ti}} \right] & \text{if } Ll_i \text{ is an integer multiple of } T_{ti} \\ \left[\dfrac{Ll_i}{T_{ti}} \right] + 1 & \text{otherwise} \end{cases} \quad (28)$$

Where $\left[\dfrac{Ll_i}{T_{ti}} \right]$ represent integral part of $\dfrac{Ll_i}{T_{ti}}$.

Items are transferred from RW to OW at t = $T_{iM_i1}, T_{iM_i2}, \ldots\ldots\ldots, T_{iM_iNl_i}$, where $T_{iM_ik} = (M_i - 1)L_i + (k-1)T_{ti}$, for $k = 1,2,3,...,N1_i$.

Own Warehouse:

On the other hand, the stock depletion at OW is due to demand and deterioration of the items. Instantaneous state $q_{OW(i)}(t)$ of i^{th} item at OW is given by

$$\frac{dq_{OW_{(i)}}(t)}{dt} = -(x_i + y_i q_{OW_{(i)}}(t)) - \theta_i q_{OW_{(i)}}(t) \qquad for \quad T_{iM_i k} \le t \le T_{iM_i k+1} \tag{29}$$

With boundary conditions $q_{OW(i)}(T_{iMik}) = Q_{di} \quad for \quad k = 1,2,3,...,N1_{i-1}$

From equation (29)

$$q_{OW(i)}(t) = \frac{1}{(y_i + \theta_i)}\left[-x_i + \left\{x_i + (y_i + \theta_i)Q_{di}\right\} e^{-(\theta_i + y_i)(t - T_{iM_i k})}\right] \tag{30}$$

$$Q_{siM_i k+1} = \frac{1}{(\theta_i + y_i)}\left[-x_i + \left\{ x_i + (\theta_i + y_i)Q_{di}\right\}\right] e^{-(\theta_i + y_i)T_{ti}} \qquad for \quad k = 1,2,...N1_{i-1} \tag{31}$$

$Q_{siMi1}=0$

Amount transferred from RW to OW at $t = T_{iM_i k}$, $Q_{TiM_i k}$ is given by

$Q_{TiM_i k} = Q_{di} - Q_{siMik} \quad for \quad k = 1,2,3,...N_{1i-1}$

$Q_{TiM_i Nl_i} = Q_{iM_i} - \sum_{k=1}^{N_i-1} Q_{TiM_i k}$

Instantaneous state $q_{OW(i)}(t)$ of i^{th} item at OW in the last sub-cycle is given by

$$\frac{dq_{OW(i)}(t)}{dt} = -(x_i + y_i q_i) - \theta_i q_i \quad for \ T_{iM_i Nl_i} \le t \le T_{iM_i 1} + L_i \tag{32}$$

With boundary conditions

$Q_{OW(i)}(T_{iMiNi})=Q_{TiMikNi}+Q_{siMiNi}$, $q_i(T_{iMi1}+L_i)=0$

From equation (32)

$$q_{OW(i)}(t) = \frac{1}{\theta_i + y_i}\left[-x_i + (x_i + (Q_{TiM_i Nli} + Q_{siM_i Nli}))(\theta_i + y_i)e^{-(\theta_i + y_i)(t - TiM_i Nli)}\right]$$

$$for \ T_{iM_i Nl_i} \le t \le T_{iM_i 1} + L_i \tag{33}$$

$$q_{OW(i)}(T_{iM_i Nli}) = \frac{x_i}{\theta_i + y_i}\left[e^{(\theta_i + y_i)(L_i - (Nl_i - 1)T_{ti})} - 1\right] \tag{34}$$

$$Q_{iM_i} = (Nl_i - 1)\left[Q_{di} - \frac{1}{\theta_i + y_i}\left[-x_i + (x_i + y_i Q_{di})e^{-(\theta_i + y_i)T_{ti}}\right]\right] + \frac{x_i}{\theta_i + y_i} e^{(\theta_i + y_i)(L_i - (Nl_i - 1)T_{ti})} \tag{35}$$

Evaluation of holding cost at OW:

Present value of holding cost at OW in the k^th^ sub-cycles of the last cycle is

$$= c_{h1i} H_{1iMik}$$

$$= c_{h1i} \int_{T_{iM_i k}}^{T_{iM_i k+1}} q_{OW(i)}(t) e^{-Rt} dt \tag{36}$$

$$= \frac{c_{h1i} e^{-RT_{iM_i k}}}{y_i + \theta_i} \left[\frac{x_i}{R}(e^{-RT_{ti}} - 1) + \left\{ x_i + (y_i + \theta_i) Q_{di} \right\} \frac{(1 - e^{-(y_i + \theta_i + R)T_{ti}})}{(y_i + \theta_i + R)} \right]$$

Present value of holding cost at OW in the last-cycle of j^th^ cycle is

$$= c_{h1i} H_{1iMiN1i}$$

$$= c_{h1i} \int_{T_{iM_i Nl_i}}^{T_{iM_i 1+L_i}} q_{OW(i)}(t) e^{-Rt} dt$$

$$= \frac{c_{h1i} e^{-RT_{iM_i Nl_i}}}{y_i + \theta_i} \left[\frac{x_i}{R}(e^{-R(L_i - (N_i - 1)T_{ti})} - 1) + \left\{ x_i + (y_i + \theta_i) q_{OW(i)}(T_{ijNl_i}) \right\} \frac{(1 - e^{-(y_i + \theta_i + R)(L_i - (Nl_i - 1)T_{ti})})}{(y_i + \theta_i + R)} \right]$$

So, present value of holding cost at OW in the j^th^ cycles is

$$= c_{h1i} H_{1iM_i}$$

$$= c_{h1i} \sum_{k=1}^{Nl_i - 1} H_{1iM_i k} + H_{1iM_i Nl_i}$$

So, present value of holding cost at OW in the last cycles is

$$= c_{h1i} H_{1L}$$

$$= c_{h1i} \left[\sum_{k=1}^{Nl_i - 1} H_{1iM_i k} + H_{1iM_i N_i} \right]$$

$$= c_{h1i} \left[\sum_{k=1}^{Nl_i - 1} \frac{e^{-RT_{iM_i k}}}{y_i + \theta_i} \left[\frac{x_i}{R}(e^{-RT_{ti}} - 1) + \left\{ x_i + (y_i + \theta_i) Q_{di} \right\} \right. \right. \tag{37}$$

$$\left. \frac{(1 - e^{-(y_i + \theta_i + R)T_{ti}})}{(y_i + \theta_i + R)} \right] + \frac{e^{-RT_{iM_i Nl_i}}}{y_i + \theta_i} \left[\frac{x_i}{R}(e^{-R(L_i - (Nl_i - 1)T_{ti})} - 1) \right.$$

$$\left. \left. + \left\{ x_i + (y_i + \theta_i) q_{OW(i)}(T_{iM_i Nl_i}) \right\} \frac{\left(1 - e^{-(y_i + \theta_i + R)(L_i - (Nl_i - 1)T_{ti})}\right)}{(y_i + \theta_i + R)} \right] \right]$$

Evaluation of Sell Revenue:

Present value of sell revenue during $T_{iM_i k} \le t \le T_{iM_i k+1}$ is $= c_{si} SP_{iMik}$ $k = 1, 2, 3, ..., N1_{i-1}$.

$$= c_{si}\int_{T_{iM_ik}}^{T_{iM_ik+1}}\left[x_i + y_i\left\{-\frac{x_i}{y_i+\theta_i}+\left(Q_{di}+\frac{x_i}{y_i+\theta_i}\right)e^{(y_i+\theta_i)(T_{iM_ik}-1)}\right\}\right]e^{-RT}dt$$

$$= c_{si}\left[\frac{x_i\theta_i e^{-RT_{iM_ik}}}{y_i+\theta_i}\left\{\frac{1-e^{-RT_{ti}}}{R}\right\}+\left(Q_{di}+\frac{x_i}{y_i+\theta_i}\right)e^{-RT_{iM_ik}}\left\{\frac{1-e^{-(y_i+\theta_i+R)T_{ti}}}{y_i+\theta_i+R}\right\}\right]$$

Present value of sell revenue during the last cycle is

$$= c_{si}SP_{iMiN1i}$$

$$= c_{si}\int_{T_{iM_iN_i}}^{T_{iM_i1+L_i}} D_i(t)e^{-RT}dt$$

$$= c_{si}\left[\frac{x_i\theta_i e^{-RT_{iM_iN_i}}}{y_i+\theta_i}\left\{\frac{1-e^{-R(T_{iM_i1}+L_i-T_{iM_iNl_i})}}{R}\right\}+\left(y_i(q_{OW(i)}(T_{iM_iNl_i})+\frac{x_i}{y_i+\theta_i}\right)e^{-(y_i+\theta_i+R)T_{iM_iNl_i}}\left\{\frac{1-e^{-(y_i+\theta_i+R)(T_{iM_i1}+L_i-T_{iM_iNl_i})}}{y_i+\theta_i+R}\right\}\right]$$

So, present value of sell revenue in the last cycle is

$$c_{si}SPL = c_{si}\left[\sum_{k=1}^{Nl_i-1}\left[\frac{x_i\theta_i e^{-RT_{iM_ik}}}{y_i+\theta_i}\left\{\frac{1-e^{-RT_{ti}}}{R}\right\}+\left(Q_{di}+\frac{x_i}{y_i+\theta_i}\right)e^{-RT_{iM_ik}}\left\{\frac{1-e^{-(y_i+\theta_i+R)T_{ti}}}{y_i+\theta_i+R}\right\}\right]+\left[\frac{x_i\theta_i e^{-RT_{iM_iNl_i}}}{y_i+\theta_i}e^{-(y_i+\theta_i+R)T_{iM_iNl_i}}\left\{\frac{1-e^{-(y_i+\theta_i+R)(T_{iM_i1}+L_i-T_{iM_iNl_i})}}{y_i+\theta_i+R}\right\}\right]\right.$$
$$\left.\left\{\frac{1-e^{-R(T_{iM_i1}+L_i-T_{iM_iNl_i})}}{R}\right\}+y_i\left(q_{OW(i)}(T_{iM_iNl_i})+\frac{x_i}{y_i+\theta_i}\right)\right] \tag{39}$$

Evaluation of the holding cost at RW:

Present Value of holding cost at RW in the last cycle is $c_{h2i}H_{2L}$ where

$$H_{2L} = \left\{\frac{1-e^{-RT_{ti}}}{R}\right\}\left[Q_{ij}\left\{\frac{1-e^{-RT_{ti}(Nl_i-1)}}{1-e^{-RT_{ti}}}\right\}-Q_{di}Sl_1+\frac{1}{y_i+\theta_i}\left\{-x_i+(x_i+(y_i+\theta_i)Q_{di})e^{-(y_i+\theta_i)T_{ti}}\right\}Sl_2\right]e^{-RL_i(M_i-1)} \tag{40}$$

where

$$Sl_1 = \left\{\frac{1-e^{-RT_{ti}(Nl_i-1)}}{(1-e^{-RT_{ti}})^2}\right\}-\left\{\frac{(Nl_i-1)e^{-RT_{ti}(Nl_i-1)}}{1-e^{-RT_{ti}}}\right\} \tag{41}$$

$$Sl_2 = \left\{\frac{e^{-RT_{ti}}(1-e^{-RT_{ti}(Nl_i-2)})}{(1-e^{-RT_{ti}})^2}\right\}-\left\{\frac{(Nl_i-2)e^{-RT_{ti}(N_i-1)}}{1-e^{-RT_{ti}}}\right\} \tag{42}$$

Evaluation of sell revenue in last cycle:

Sell revenue $=c_{si}SPL$

$$= c_{si}\left[\left(\frac{x_i+(y_i+\theta_i)Q_{di}}{y_i+\theta_i+R}\right)\left\{1-e^{-(y_i+\theta_i+R)T_{ti}}\right\}\left(\frac{1-e^{-RT_{ti}(Nl_i-1)}}{1-e^{-RT_{ti}}}\right)e^{-RL_i(M_i-1)} + \left(\frac{x_i+(y_i+\theta_i)q_i(T_{iM_iNl_i})}{y_i+\theta_i+R}\right)\left\{1-e^{-(y_i+\theta_i+R)L_i}\right\}e^{-R(Nl_i-1)}e^{-RL_i(M_i-1)} \right] \tag{43}$$

Present value of transportation cost in the last cycle CT_L is given by:

$$CT_L = c_{ti}\left[\sum_{k=1}^{Nl_i-1} Q_{TiM_ik}e^{-RT_{iM_ik}} + Q_{TiM_iNl_i}e^{-RT_{iM_iNl_i}}\right]$$

$$CTL = c_{ti}\left[Q_{di}\left(\frac{1-e^{-RT_{ti}(Nl_i-1)}}{1-e^{-RT_{ti}}}\right)-\left(\frac{-x_i+(x_i+(y_i+\theta_i)Q_{di})e^{-(y_i+\theta_i)T_{ti}}}{y_i+\theta_i}\right)\left(\frac{1-e^{-RT_{ti}(Nl_i-2)}}{1-e^{-RT_{ti}}}\right)e^{-RL_i(M_i-1)}\right]+c_{ti}\left[Q_{iMi}-(Nl_i-1)Q_{di}+(Nl_i-2)\left(\frac{-x_i+(x_i+(y_i+\theta_i)Q_{di})e^{-(y_i+\theta_i)T_{ti}}}{y_i+\theta_i}\right)\right]e^{-RT_{ti}(Nl_i-1)}e^{-RL_i(M_i-1)} \tag{44}$$

Evaluation of Ordering Cost in the last cycle:

Present value of ordering cost in the last cycle CT_L is given by

$$OC_L = (c_{o1i}+c_{o2i}Q_{iM_i})e^{-RL_i(M_i-1)} \tag{45}$$

Evaluation of purchase cost in the last cycle:

Present value of purchase cost in the last cycle is

$$c_{pi}PC_L = Q_{iM_i}e^{-RL_i(M_i-1)} \tag{46}$$

Formulation for major ordering and transportation costs:

Present value of major ordering cost during the entire planning horizon, MOC, is given by

$$MOC = \sum_{i=1}^{N_M} c_{om}e^{-R(i-1)T_o} = c_{om}\left[\frac{1-e^{-RN_MT_o}}{1-e^{-RT_o}}\right] \tag{47}$$

Present value of major transportation cost during the entire planning horizon, MTC, is given by

$$MTC = \sum_{i=1}^{MN_M} c_{tm}e^{-R(i-1)L_t} = c_{tm}\left[\frac{1-e^{-RMN_ML_t}}{1-e^{-RL_t}}\right] \tag{48}$$

Crisp Model:

Model 1:
Present value of an average profit during the planning horizon, Z, is given by

$$Z=\Big[\sum_{i=1}^{N}\{c_{si}(SP_G+SP_L)-c_{pi}(PC_G+PC_L)-c_{h1i}(H_{1G}+H_{1L})-$$
$$c_{h2i}(H_{2G}+H_{2L})-(OC_G+OC_L)-(CTG+CTL)\}\tag{49}$$
$$+MOC+MTC\Big]\Big/H$$

So the problem reduces to
Maximize Z,
Subject to

$$\left.\begin{aligned}&\sum_{i=1}^{N}Q_{i1}c_{pi}\le INV\\&\sum_{i=1}^{N}Q_{di}=AR_1\\&\sum_{i=1}^{N}Q_{i1}A_i\le AR_1+AR_2\end{aligned}\right\}\tag{50}$$

Fuzzy Model:
As discussed in section 1, it is very difficult to define different inventory parameters precisely, i.e., as crisp numbers. It is easy to define these parameters as fuzzy. For example, purchase cost of an item fluctuates throughout the year. Hence, purchase cost of an item can be taken as about c per unit, which can be represented as a TFN (c-a, c, c+b). This implies that normally, price is near c and lies in the interval [c-a, c+b]. The possibility of price to be within (c-a, c) and (c, c+b) lies in (0.0, 1.0). Again, at the beginning, a business normally starts with some capital and its upper limit is fixed. But in the course of time, advantage of bulk transport, sudden increase of demand, price discount so as the decision of acquiring more items force the investor to augment the previously fixed capital by some amount in some situations. This augmented amount is clearly fuzzy in nature, in the sense of degree of uncertainty, and hence the total invested capital becomes imprecise in nature. The point is that the acquisition of extra amount of items needs some extra storage space in addition to the initially arranged warehouse area. Since the location of the rented warehouse, RW, is away from the heart of the market, the use of a temporary extra storage space can be arranged there. Thus, storage space of the far-away rented go-down, RW, is fuzzy in nature. Therefore, imprecise i.e., vaguely defined in some situations. Hence we take c_{pi}, INV, AR_2 as fuzzy numbers, i.e., as $\tilde{c}_{pi}, \tilde{INV}, \tilde{A}R_2$, respectively. Then, due to this assumption, Z becomes fuzzy number $\tilde{Z}$, and constraints in equation (50) also become imprecise in nature. Therefore, if $\tilde{G}$ (= an LFN (G_1,G_2)) is the fuzzy goal of the objective $\tilde{Z}$, then according to the discussion in section 2, the problem is reduced to the following, in optimistic sense, pessimistic sense and weighted average of optimistic and pessimistic sense, respectively,

Model 2:

Maximize $Z_p = \pi_{\tilde{Z}}(\tilde{G})$,

Subject to

$$\left.\begin{aligned} & Nes\left\{\sum_{i=1}^{N} Q_{i1}\tilde{c}_{pi} \le IN\tilde{V}\right\} \ge \beta_1 \\ & \sum_{i=1}^{N} Q_{di} = AR_1 \\ & Nes\left\{\sum_{i=1}^{N} Q_{i1}A_i \le AR_1 + A\tilde{R}_2\right\} \ge \beta_2 \end{aligned}\right\} \quad (51)$$

Model 3: Maximize $Z_p = \pi_{\tilde{Z}}(\tilde{G})$,

Subject to the constraint of model 2 (52)

Model 4: Maximize $F = \rho Z_p + (1-\rho) Z_N$

Subject to the constraint of model 2 (53)

Here, if it is assumed that $\tilde{c}_{pi} = (c_{pi1}, c_{pi2}, c_{pi3})$, $IN\tilde{V} = (INV_1, INV_2, INV_2)$, $A\tilde{R}_2 = (AR_{21}, AR_{22}, AR_{23})$ as TFNs, then using definition (58) we have $\tilde{Z} = (Z_1, Z_2, Z_3)$ where for j=1,2,3.

$$Z_j = \left[\sum_{i=1}^{N}\left[\{\eta_i(SP_G + SP_L) - (PC_G + PC_L) - h_{1i}(H_{1G} + H_{1L})\right.\right.$$
$$\left.-h_{2i}(H_{2G} + H_{2\ L})\} c_{pij} - (OC_G + OC_L) - (CT_G + CT_L)\} + MOC + MTC\right] / H$$

and let $P_j = \sum_{i=1}^{N} Q_{i1} c_{pij}$ for j=1,2,3. Then following lemmas 1-4, problems (51)-(53) are reduced to respectively

Maximize $Z_p = \dfrac{Z_3 - G_1}{Z_3 - Z_2 + G_2 - G_1}$

Subject to

$$\left.\begin{aligned} & \frac{P_3 - INV_1}{INV_2 - INV_1 + P_3 - P_2} < 1 - \beta_1 \\ & \sum_{i=1}^{N} Q_{di} A_i = AR_1 \\ & \frac{\sum_{i=1}^{N} Q_{i1} A_i - AR_1 - AR_{21}}{AR_{22} - AR_{21}} < 1 - \beta_2 \end{aligned}\right\} \quad (54)$$

$$Maximize\ Z_N = \frac{Z_2 - G_1}{Z_2 - Z_1 + G_2 - G_1} \tag{55}$$

Subject to the constraint of (54)

$$\left.\begin{array}{l} Maximize\ F = \rho\left[\dfrac{Z_3 - G_1}{Z_3 - Z_2 + G_2 - G_1}\right] + \\ \qquad (1-\rho)\left[\dfrac{Z_2 - G_1}{Z_2 - Z_1 + G_2 - G_1}\right] \\ \text{subject to constraints of (54)} \end{array}\right\} \tag{56}$$

These crisp problems can easily be solved using any non-linear optimization technique in crisp environment. GA is used here for this purpose.

7. NUMERICAL ILLUSTRATION

The models are illustrated for three items (N=3). Common parametric values to illustrate the models are presented in Table-1. Other common parametric values are R=0.03, H=10, c_{0m}=20, c_{tm}=10, AR_1=60.

Table 1: Common input data for different examples

Item (i)	x_1	y_i	θ_i	h_{1i}	h_{2i}	c_{o1i}	c_{o2i}
1	10	3.20	.01	0.1	0.05	4	0.24
2	15	3.50	.01	0.1	0.05	6	0.18
3	12	3.42	.01	0.1	0.05	4	0.20

Item (i)	c_{ti}	η_i	A_i
1	0.20	1.4	0.45
2	0.22	1.4	0.50
3	0.20	1.4	0.52

Example-1: Along with the common parametric values, other assumed parametric values are c_{p1}=10, c_{p2}=12, c_{p3}=11, AR_2=150, INV=\$5000.

Table-2: Results for Model (1) by using GA

Item (i)	n_i	m_i	λ_i	N_M	M	Z
1	2	1	0.16	10	3	579.62
2	2	1	0.31			
3	2	1	0.25			

Example-2: Here, it is assumed that $\tilde{c}_{\text{pi}} = (c_{pi1,} c_{pi2,} c_{pi3,})$, $IN\tilde{V} = (INV_{1,} INV_{2,} INV_{3,})$, $A\tilde{R}_2 = (AR_{21,} AR_{22,} AR_{23,})$ as TFNs with c_{p11}=9, c_{p12}=10, c_{p13}=10.5,c_{p21}=11.5, c_{p22}=12, c_{p23}=13, c_{p31}=10, c_{p32}=11, c_{p33}=12, INV_1=\$4500, INV_2=\$5000, INV_3=\$5200, AR_{21}=140, AR_{22}=150, AR_{23}=160, G_1=580, G_2=690, $\alpha_1 = 0.1$, $\alpha_2 = 0.1, \rho = 0.5$.

Table 3: Result for Model-2 by using GA

Item (i)	n_i	m_i	λ_i	N_M	M	$Z_P/Z_N/F$
1	1	2	0.32	10	3	0.579
2	2	1	0.36			
3	3	1	0.25			

8. CONCLUSION

A two-storage inventory model with deterioration is developed incorporating simultaneous ordering and transfer of items from back-room inventory to show-room, following BP approach, in fuzzy environment. The proposed approach is such that instead of objective function, possibility/necessity measure of objective function with respect to fuzzy goal is optimized. The reasons for the adaptation of this model are as follows:

1. It is very difficult to define different parameters of an inventory problem precisely-specially the purchase cost, investments amount etc., which are normally fuzzy in nature and so render optimization of fuzzy objective under necessity based resources constraints. This phenomenon is incorporated in the model.
2. At present, there is a crisis of having larger space in the market places. In most of the literature, two-warehouse models with one own warehouse (OW) at the market place and another rented warehouse (RW) situated little farther from the centre of the city are dealt with. The holding cost at OW is assumed to be less than the one at RW. But in real life, now-a-days, it is the reverse as both warehouses are hired. Hence, the holding cost at the main market place is higher than that of the distant storage house. Such a realistic situation has been considered in this model.
3. Due to the preserving condition of warehouses, items gradually lose their utility, i.e., deterioration takes place. This realistic phenomenon is incorporated in this model.
4. The shortcoming of the existing two-storage multi-item inventory models have been taken into account. In the existing models, it is observed that items are ordered and transferred from back-room inventory to show-room individually, which incurred a large amount of ordering and transportation cost, too. In this model items are ordered and transferred from back-room to show-room simultaneously using BP policy.
5. The possibility/necessity measure on fuzzy goal as a decision making tool for inventory control problems has been used.

APPENDIX 1

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership function $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$, respectively.

$$pos(\tilde{a} * \tilde{b}) = \sup\left\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x)), x, y, x * y \in R\right\} \quad (57)$$

where the abbreviation *pos* represents possibility, and * is one of the relations $>,<,=,\leq,\geq$:

$$nes(\tilde{a} * \tilde{b}) = 1 - pos\overline{(\tilde{a} * \tilde{b})} \quad (58)$$

where the abbreviation *nes* represents necessity.
Similarly, possibility and necessity measures of $\tilde{a}$ with respect to $\tilde{b}$ are denoted by $\Pi_{\tilde{b}}(\tilde{a})$ and $N_{\tilde{b}}(\tilde{a})$, respectively and are defined as

$$\Pi_{\tilde{b}}(\tilde{a}) = \sup\left\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x)), x \in R\right\}, \quad (59)$$

$$N_{\tilde{b}}(\tilde{a}) = \min\left\{\sup(\mu_{\tilde{a}}(x), 1 - \mu_{\tilde{b}}(x)), x \in R\right\} \quad (60)$$

If $\tilde{a}, \tilde{b} \subseteq R$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f : R \times R \to R$ is a binary operation, then membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as for each $z \in R$,

$$\mu_{\tilde{c}}(z) = \sup\left\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x)), x, y \in R \text{ and } z = f(x, y)\right\} \quad (61)$$

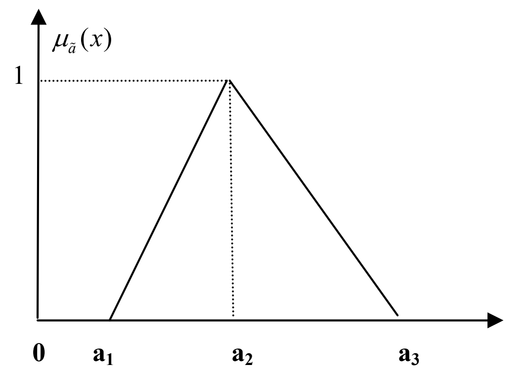


Figure 3: Membership function of triangular fuzzy number

Triangular fuzzy number (TFN): A TFN $\tilde{a}=(a_1,a_2,a_3)$ (Fig.3) has three parameters a_1, a_2, a_3 where $a_1<a_2<a_3$ and is characterized by the membership function $\mu_{\tilde{a}}(x)$, given by

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise.} \end{cases}$$

α-cut of fuzzy number: α-cut of a fuzzy number Ã in R with membership function $\mu_{\tilde{A}}(x)$ is denoted by A_α and defined as the following crisp set: $A_\alpha = \{x : \mu_{\tilde{A}}(x) \geq \alpha, x \in R\}$ *where* $\alpha \in [0;1]$.

A_α is a non-empty bounded closed interval contained in R which can be denoted by $A_\alpha = [A_L(\alpha), A_R(\alpha)]$.

Lemma 1. *If* $\tilde{a} = (a_1, a_2, a_3)$, $\tilde{b} = (b_1, b_2, b_3)$ *are TFNs with* $0<a_1$ *and* $0<b_1$, *then* $nes(\tilde{a} > \tilde{b}) \geq \alpha$ *iff* $\dfrac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} \leq 1 - \alpha$.

Proof: *We have* $nes(\tilde{a} > \tilde{b}) \geq \alpha \Rightarrow \{1 - Pos(\tilde{a} \leq \tilde{b})\} \geq \alpha \quad \Rightarrow Pos(\tilde{a} \leq \tilde{b}) \leq 1 - \alpha$ *It is clear that* $Pos(\tilde{a} \leq \tilde{b}) = \delta = \dfrac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2}$ *and hence, the result follows.*

Lemma 2. *If* $\tilde{a} = (a_1, a_2, a_3)$ *be a TFN with* $0<a_1$ *and b is a crisp number, then* $nes(\tilde{a} > \tilde{b}) \geq \alpha$ *iff* $\dfrac{b - a_1}{a_2 - a_1} \leq 1 - \alpha$.

Proof: *Proof follows from Lemma1.*

Lemma 3. *If* $\tilde{a} = (a_1, a_2, a_3)$ *be a TFN and* $\tilde{b} = (b_1, b_2)$ *be a LFN with* $0<a_1$ *and* $0<b_1$ *then*

$$\prod_{\tilde{a}}(\tilde{b}) = \begin{cases} 1 & \text{if } a_2 \geq b_2, \\ \dfrac{a_3 - b_1}{a_3 - a_2 + b_2 - b_1} & \text{if } a_2 < b_2 \text{ and } a_3 > b_1, \\ 0 & \text{otherwise.} \end{cases}$$

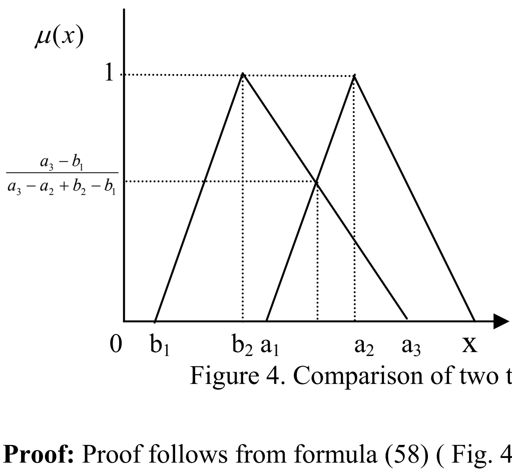


Figure 4. Comparison of two triangular fuzzy numbers

Proof: Proof follows from formula (58) (Fig. 4).

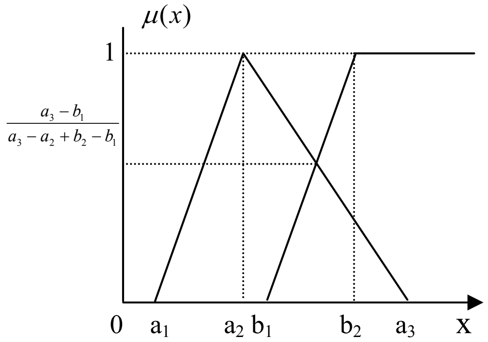


Figure 5. Pictorial representation of $\prod_{\tilde{a}}(\tilde{b})$

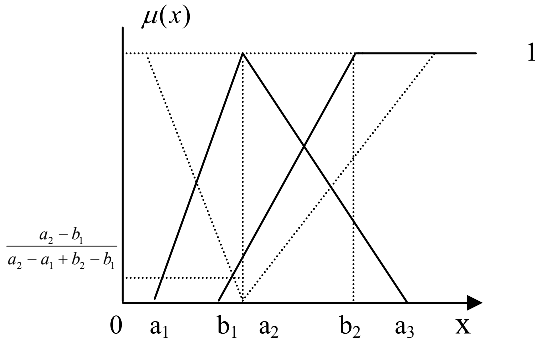


Figure 6. Pictorial representation of $N_{\tilde{a}}(\tilde{b})$

Lemma 4. *If* $\tilde{a} = (a_1, a_2, a_3)$ *be a TFN and* $\tilde{b} = (b_1, b_2)$ *be a LFN with* $0 < a_1$ *and* $0 < b_1$ *then*

$$N_{\tilde{a}}(\tilde{b}) = \begin{cases} 1 & \text{if } a_1 \geq b_2, \\ \dfrac{a_3 - b_1}{a_3 - a_2 + b_2 - b_1} & \text{if } a_2 < b_1 \text{ and } b_2 > a_1, \\ 0 & \text{otherwise.} \end{cases}$$

Proof: Proof follows from formula (59) (Fig. 6).

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