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VENDOR-BUYERS RELATIONSHIP MODEL FOR DETERIORATING ITEMS WITH SHORTAGES, FUZZY TRAPEZOIDAL COSTS AND INFLATION

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Abstract: In this paper, an integrated inventory model is developed from the perspective of a single vendor and multi-buyers for deteriorating items under fuzzy environment and inflation. In the development of the model, it is assumed that all costs parameters, demand and the production rates are imprecise in nature; they are represented by the trapezoidal fuzzy numbers, as these parameters are not constant and can be disturbed due to daily market changes. We use function principle as arithmetic operations to find the total inventory cost in fuzzy sense and Graded Mean – Integration Representation Method to defuzzify the fuzzy total inventory cost. Inflation is used to find the present worth of total cost. Since the optimal policy of buyers may not be the most economical for a vendor, thus to deal with this situation, integrated cost policy is used to reach the optimal policy. Finally, a numerical example is given to illustrate the model.

Keywords: Fuzzy, trapezoidal fuzzy number, inflation.

MSC: 90B05.

1. INTRODUCTION

The fuzzy set concept has been used to deal with the modern business problems and the day-by-day changing market scenario. Now it is the need of the hour that a real supply chain must be operated in an uncertain environment and the omission of any effects of uncertainty leads to inferior supply chain designs. These uncertainties are usually associated with the product supply, the customer demand, the inventory cost, the manufacturing cost, and so on. Typically, stochastic techniques, modeling uncertainties by probability distributions derived from the history data, have been adopted to cope with these problems. However, the history data of the inventory cost and the manufacturing cost are not always available (e.g. due to the innovation of products) or reliable (e.g. due to market turbulence). Moreover, a growing quota of manufacturing cost depends on factors such as the weather conditions, workers' attention and the aging of machines that can hardly be expressed in quantitative terms. Hence, quantitative demand forecasts and cost estimates based on decision maker's judgments', intuitions and experience seem to be more appropriate. Therefore, possibility theory rather than probability theory should be applied to deal with such kind of uncertainties. Zahed (1965) first introduced the fuzzy set theory. Kao and Hsu (2002) studied a lot size-reorder point inventory model with fuzzy demand. Chang et al. (2004) presented a lead–time production model based on continuous review inventory system in which the uncertainty of demand during lead-time was dealt with probabilistic fuzzy set and the annual average demand by a fuzzy number. Maiti and Maiti (2007) developed multi-item inventory models with stock dependent demand and two storage facilities in a fuzzy environment, where processing time of each unit is fuzzy and the processing time of a lot is correlated with its size. Singh and Singh (2008) discussed an EPQ model with imprecise costs. Singh and Singh (2010) developed a supply chain model with imprecise partial backlogging and fuzzy ramp-Type demand. Yadav et al. (2012) developed an inventory model for two warehouses with stock dependent demand using genetic algorithm in fuzzy environment.

Better coordination among the vendor and buyers is one of the key factors of a successful supply chain. The integration approach to supply chain management has been studied for years. Lu and Posner (1994) introduced two heuristic procedures for the one-warehouse, multi-retailer system. Thomas and Griffin (1996) reviewed the model for the coordination of supply and manufacturing as well as manufacturing and distribution. Ha and Kim (1997) made the analysis of integration between a buyer and a supplier by setting up the mathematical model in which the inventory cost of a vendor is derived through a discontinuous saw-tooth inventory-level function. Wee and Yang (2004) derived a heuristic solution model for producer–distributors–retailers inventory system using the principle of strategic partnership. Lee and Wu (2006) proposed a study on inventory replenishment policies in a two-echelon supply chain system. Chen and Kang (2007) derived an integrated vendor-buyer cooperative inventory models with variant permissible delay in payments. Kim and Park (2008) developed a three-echelon SC model to optimize coordination costs. In this study, the factors of deteriorating and integration of vendor and buyers are considered simultaneously. Most of the inventory models unrealistically ignore the influence of inflation. This was due to the belief that inflation would not influence the inventory policy to any significant degree. This belief is unrealistic since the resource of an enterprise is highly correlated to the return on investment. The concept of the inflation should be considered especially for long-term investment and forecasting. Buzacott (1975), Misra (1979), Chandra and Bahner (1985) were the first few who studied the effect of inflation with regard to inventory. Lieo et al. (2000) studied the effect of inflation on a permissible delay model. Mehta and Shah (2003) derived a lot-size inventory model for deteriorating items with exponentially increasing demand by allowing complete backlogging under inflation. Singh et al. (2007) discussed optimal policy for decaying items with stock-dependent demand under inflation in a supply chain. Singh and Singh (2010) developed two echelon supply chain model with imperfect production under imprecise and inflationary environment. Singh and Singh (2011) considered the production rate and the demand rate as fuzzy in nature to develop an integrated supply chain model for the perishable items.

In the present study, we have strived to develop a vendor-buyer relationship model for deteriorating items with shortages under imprecise and inflationary environment. It is assumed that all cost parameters involved in the total cost, demand rate and production rate are imprecise in nature. In order to express the fuzziness of these parameters, they are represented by the trapezoidal fuzzy numbers. Expressions for the average inventory cost are obtained in fuzzy sense. Later on, fuzzy total cost is defuzzified using the Graded Mean Integration Representation method. Thereafter, it is optimized with respect to the decision variables. A numerical example is given to illustrate that the integrated policy results in an impressive cost reduction when compared with the independent decisions made by the vendor and the buyers.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model in this study is developed under the following assumptions:

All cost parameters involved in the total cost are imprecise in nature.

A single item with constant deterioration rate of the on–hand inventory is considered.

Single-vendor multi-buyers with one item is assumed.

Shortages are allowed at the buyers' part only.

There is no replacement or repair of deteriorated units.

The production rate is finite and is greater than the sum of all buyers' demand.

Following notations are used to develop the model

θ	the deterioration rate
N	the number of buyers
n_i	the number of deliveries supply to the i^{th} byer, i = 1,2,3,4,…….,N
$\tilde{d}_i$	Fuzzy demand rate per year for i^{th} buyer, i = 1,2,3,4,…….,N
$\tilde{p}$	Fuzzy production rate per year
r	inflation rate
T	time length of each cycle, where $T = T_1 + T_2$
T_1	the length of production time in each production cycle T
T_2	the length of non-production time in each production cycle T

T_3 Shortage time period for the each buyer
T_4 the time at which buyer's inventory level reaches the zero level.
$I_{v1}(t_1)$ production inventory level of a vendor
$I_{v2}(t_2)$ non production inventory level of a vendor
$\tilde{c}_v$ Fuzzy setup cost of each production cycle for a vendor
$\tilde{c}_b$ Fuzzy setup or ordering cost per order for a vendor
$\tilde{h}_v$ Fuzzy holding cost per dollar per year for a vendor
$\tilde{h}_b$ Fuzzy holding cost per dollar per year for a buyer
$\tilde{d}_v$ Fuzzy unit deterioration cost for a vendor
$\tilde{d}_b$ Fuzzy unit deterioration cost for a buyer
$\tilde{s}_b$ Shortage cost per unit time per unit for each buyer
$\tilde{VC}$ Total cost of a vendor per unit time in fuzzy sense
$\tilde{BC}$ Total cost of all buyers per unit time in fuzzy sense
$\tilde{TC}$ Integrated total cost of the vendor and all buyers per unit time in fuzzy sense

3. MATHEMATICAL MODELING

3.1. Graded Mean Integration Representation Method

In 1998, Chen and Hsieh proposed the graded mean integration representation method based on the integral value of graded mean *h*-level of fuzzy number. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number with the membership function $\mu_{\tilde{A}}(x)$, defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for -\infty < x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & for\ a_1 \le x < a_2 \\ 1 & for\ a_2 \le x < a_3 \\ \dfrac{a_4-x}{a_4-a_3} & for\ a_3 \le x < a_4 \\ 0 & for\ a_4 < x < \infty \end{cases}$$

$$\begin{aligned} &L_A = (x) = \frac{x-a_1}{a_2-a_1},\ a_1 \le x < a_2 \\ &L_A^{-1}(h) = a_1 + h(a_2 - a_1), 0 \le h \le 1 \\ &R_A(x) = \frac{a_4 - x}{a_4 - a_3}, a_3 \le x < a_4 \\ &R_A^{-1}(h) = a_4 - h(a_4 - a_3), 0 \le h \le 1 \end{aligned} \tag{1}$$

L_A and R_A are the functions L and R of the trapezoidal fuzzy number A, respectively. $L^{-1}{}_A(h)$ and $R^{-1}{}_A(h)$ are inverse functions of the function $L_A(x)$ and $R_A(x)$ at h-level, respectively. Then, the graded mean h-level value of fuzzy number A is $\frac{h(L^{-1}{}_A(h)+R^{-1}{}_A(h))}{2}h$ (L-1(h) + R-1(h))/2. Then, the graded mean integration representation of A is denoted by $P(\tilde{A})$ and defined as

$$P(\tilde{A}) = \frac{\int_0^1 (h/2)\left\{L^{-1}(h) + R^{-1}(h)\right\} dh}{\int_0^1 h dh} \tag{2}$$

Here we suppose that

$$\tilde{c}_v = (c_{v1}, c_{v2}, c_{v3}, c_{v4}),\ \tilde{c}_b = (c_{b1}, c_{b2}, c_{b3}, c_{b4}),$$
$$\tilde{h}_v = (h_{v1}, h_{v2}, h_{v3}, h_{v4}),\ \tilde{h}_b = (h_{b1}, h_{b2}, h_{b3}, h_{b4}),\ \tilde{d}_v = (d_{v1}, d_{v2}, d_{v3}, d_{v4}),$$
$$\tilde{d}_b = (d_{b1}, d_{b2}, d_{b3}, d_{b4}),\ \tilde{s}_b = (s_{b1}, s_{b2}, s_{b3}, s_{b4}),\ \tilde{p} = (p_1, p_2, p_3, p_4),$$

and $\tilde{d}_i = (d_{i1}, d_{i2}, d_{i3}, d_{i4})$, are nonnegative trapezoidal fuzzy numbers.

3.2. Vendor's Inventory Model

A vendor starts the production at the time 0, and initially, vendors inventory levels increases up to time T_1, when the production is stopped; after that, the vendor's inventory level decreases due to the combined effect of demand and the deterioration, and reaches the zero level at the time T_2, where the cycle completes. The vendor's inventory system depicted in Fig.1 is represented by the following differential equations:

$$I'_{v1}(t_1)+\theta I_{v1}(t_1)=\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i \qquad 0\le t_1\le T_1 \tag{3}$$

$$I'_{v2}(t_2)+\theta I_{v2}(t_2)=-\sum_{i=1}^{N}\tilde{d}_i \qquad 0\le t_2\le T_2 \tag{4}$$

Using the boundary conditions $I_{v1}(0) = 0$ and $I_{v2}(T_2) = 0$ the solutions of the above differential equations are

$$I_{v1}(t_1)=\frac{\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i}{\theta}[1-e^{-\theta t_1}], \qquad 0\le t_1\le T_1, \tag{5}$$

$$I_{v2}(t_2)=\frac{\sum_{i=1}^{N}\tilde{d}_i}{\theta}[e^{\theta(T_2-t_2)}-1], \qquad 0\le t_2\le T_2, \tag{6}$$

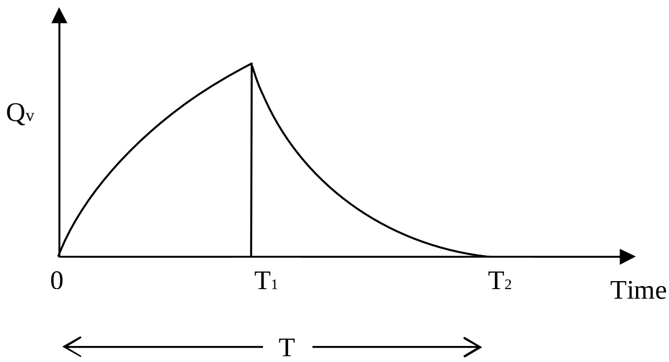

Figure 1: Vendor's inventory

Using the boundary condition $I_{v1}(T_1) = I_{v2}(0)$, we have

$$\left(\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i\right)[1-e^{-\theta T_1}]=\sum_{i=1}^{N}\tilde{d}_i[e^{\theta T_2}-1] \tag{6}$$

$$T_2\approx\frac{\left(\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i\right)}{\sum_{i=1}^{N}\tilde{d}_i}\frac{(1-e^{-\theta T_1})}{\theta} \tag{7}$$

We know that $T=T_1+T_2$, thus

$$T \approx \frac{\tilde{p}(1-e^{-\theta T_1}) + \sum_{i=1}^{N}\tilde{d}_i(e^{-\theta T_1} + \theta T_1 - 1)}{\sum_{i=1}^{N}\tilde{d}_i\theta} \tag{8}$$

Present worth of holding cost for the vendor is

$$HC_v = \frac{\tilde{h}_v}{T}\left[\int_0^{T_1} I_{v1}(t_1)e^{-rt}dt_1 + \int_0^{T_2} I_{v2}(t_2)e^{-r(T_1+t)}dt_2\right]$$

$$HC_v = \frac{\tilde{h}_v}{T}\left[\frac{\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i}{\theta}\int_0^{T_1}(1-e^{-\theta t_1})e^{-rt_1}dt_1 + \frac{\sum_{i=1}^{N}\tilde{d}_i}{\theta}\int_0^{T_2}(e^{\theta(T_2-t_2)}-1)e^{-r(T_1+t_2)}dt_2\right]$$

$$= \frac{\tilde{h}_v}{T}\left[\frac{\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i}{\theta}\left\{\frac{1}{r}(1-e^{-rT_1}) + \frac{1}{(\theta+r)}(e^{-(\theta+r)T_1}-1)\right\} + \frac{\sum_{i=1}^{N}\tilde{d}_i}{\theta}e^{-rT_1}\right.$$
$$\left.\left\{e^{\theta T_2}\frac{1}{(\theta+r)}(1-e^{-(\theta+r)T_2}) + \frac{1}{r}(e^{-rT_2}-1)\right\}\right] \tag{9}$$

Present worth of deterioration cost for the vendor is

$$\tilde{DC}_v = \frac{\tilde{d}_v\theta}{T}\left[\frac{\tilde{p}-\sum_{i=1}^{N}\tilde{d}_i}{\theta}\left\{\frac{1}{r}(1-e^{-rT_1}) + \frac{1}{(\theta+r)}(e^{-(\theta+r)T_1}-1)\right\} + \frac{\sum_{i=1}^{N}\tilde{d}_i}{\theta}e^{-rT_1}\right.$$
$$\left.\left\{e^{\theta T_2}\frac{1}{(\theta+r)}(1-e^{-(\theta+r)T_2}) + \frac{1}{r}(e^{-rT_2}-1)\right\}\right] \tag{10}$$

Present worth of setup cost per year for the vendor: since the setup is made at the start of the cycle so the inflation wouldn't affect the setup cost of the vendor i.e.

$$\tilde{SC}_v = \frac{\tilde{c}_v}{T} \tag{11}$$

The vendor's present worth of total cost is the sum of the present worth of holding cost, deteriorated cost and the setup cost as

$$\tilde{VC} = \tilde{HC}_v + \tilde{DC}_v + \tilde{SC}_v \tag{12}$$

3.3. Each buyer's inventory model:

The i^{th} buyer's inventory system for the j^{th} cycle is depicted in Fig.2 and represented by the following differential equations:

$$I'_{bi}(t) = -\tilde{d}_i \qquad 0 \le t \le T_3 \tag{13}$$

$$I'_{bi}(t) + \theta I_{bi}(t) = -\tilde{d}_i \qquad 0 \le t \le T_4\,, \tag{14}$$

$$where T_3 + T_4 = \frac{T}{n_i},\ i = 1,2,3,........,N \tag{15}$$

On using the boundary condition $I_{bi}(0) = 0$ *and* $I_{bi}(T_3) = 0$, solutions of the above differential equations are

$$I_{bi}(t) = -\tilde{d}_i t \qquad 0 \le t \le T_3 \tag{16}$$

$$I_{bi}(t) = \frac{\tilde{d}_i}{\theta}\left[e^{\theta(T_4 - t)} - 1\right] \qquad 0 < t < T_4 \tag{17}$$

Maximum ordered inventory of each buyer is

$$I_{mi}(b_i) = \frac{\tilde{d}_i}{\theta}\left[e^{\theta T_4} - 1\right] + \tilde{d}_i T_3 \tag{18}$$

Present worth of holding cost for i[th] buyer is

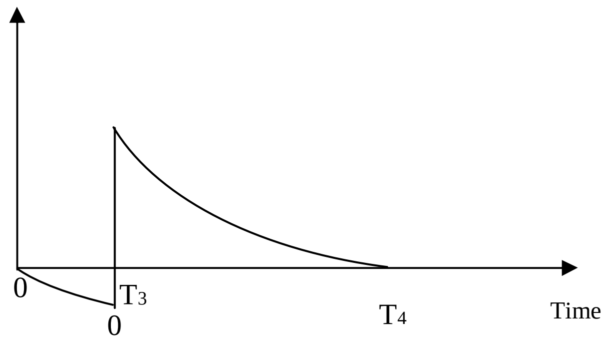

Figure 2: Each Buyer's inventory level for one cycle

$$\begin{aligned}\tilde{H}C_{bi} &= \frac{\tilde{h}_b}{T}\int_o^{T_4} I_{bi}(t)e^{-r(T_3+t)}dt\sum_{j=0}^{n_i-1} e^{-\frac{jrT}{n_i}}\\ &= \frac{\tilde{h}_b}{T}e^{-rT_3}\int_o^{T_4}\frac{\tilde{d}_i}{\theta}\left(e^{\theta(T_4-t)} - 1\right)e^{-rt}dt\sum_{j=0}^{n_i-1} e^{-\frac{jrT}{n_i}}\\ \tilde{H}C_{bi} &= \frac{\tilde{h}_b\tilde{d}_i}{T\theta}\left\{\frac{e^{\theta T_4}}{(\theta+r)}\left(1 - e^{-(\theta+r)T_4}\right) + \frac{1}{r}\left(e^{-rT_4} - 1\right)\right\}\sum_{i=1}^{n_i-1} e^{-\frac{irT}{n_i}}\end{aligned} \tag{19}$$

Present worth of deterioration costs for i[th] buyer is

$$\tilde{D}C_{bi} = \frac{\tilde{d}_b \tilde{d}_i e^{-rT_3}}{T} \left\{ \frac{e^{\theta T_4}}{(\theta + r)} \left(1 - e^{-(\theta+r)T_4}\right) + \frac{1}{r}\left(e^{-rT_4} - 1\right) \right\} \sum_{j=0}^{n_i - 1} e^{-\frac{jrT}{n_i}} \tag{20}$$

Present worth of shortages cost for the i[th] buyer is

$$\tilde{S}H_{bi} = \frac{\tilde{s}_b}{T} \int_0^{T_3} -I_{bi}(t)dt \sum_{j=0}^{n_i - 1} e^{-\frac{jrT}{n_i} - rT_3}$$

$$\tilde{S}H_{bi} = \frac{\tilde{s}_b}{T} \int_0^{T_3} \tilde{d}_i t dt \sum_{j=0}^{n_i - 1} e^{-\frac{jrT}{n_i} - rT_3} \tag{21}$$

$$\tilde{S}H_{bi} = \frac{\tilde{s}_b \tilde{d}_i}{2T} T_3^2 \sum_{j=0}^{n_i - 1} e^{-\frac{jrT}{n_i} - rT_3}$$

The setup cost for i[th] buyer is

$$\tilde{S}C_{bi} = \frac{\tilde{c}_b}{T} \sum_{j=0}^{n_i - 1} e^{-\frac{jrT}{n_i}} \tag{22}$$

Present worth of each buyer's total cost is the sum of the present worth of holding cost, deteriorated cost, and the setup cost of the buyer.

$$\tilde{B}C_i = \tilde{H}C_{bi} + \tilde{D}C_{bi} + \tilde{S}H_{bi} + \tilde{S}C_{bi} \tag{23}$$

Present worth of the entire buyer's total cost is the sum of present worth of total cost of each buyer:

$$\tilde{B}C = \sum_{i}^{N} \tilde{B}C_i \tag{24}$$

The integrated total cost of the vendor and the buyers, TC, is the sum of (12) and (24). $\tilde{T}C = \tilde{B}C + \tilde{V}C$

$$\begin{aligned} \tilde{T}C = \frac{\tilde{c}_v}{T} + \frac{\tilde{h}_v + \theta \tilde{d}_v}{T} \Bigg[& \frac{\tilde{p} - \sum_{i=1}^{N} \tilde{d}_i}{\theta} \left\{ \frac{1}{r}(1 - e^{-rT_1}) + \frac{1}{(\theta + r)}(e^{-(\theta+r)T_1} - 1) \right\} + \frac{\sum_{i=1}^{N} \tilde{d}_i}{\theta} e^{-rT_1} \\ & \left\{ e^{\theta T_2} \frac{1}{(\theta+r)}(1 - e^{-(\theta+r)T_2}) + \frac{1}{r}(e^{-rT_2} - 1) \right\} \Bigg] + \sum_{i=1}^{N} \Bigg[\frac{\tilde{c}_b}{T} \sum_{j=0}^{n_i-1} e^{-\frac{jrT}{n_i}} + \frac{(\tilde{h}_b + \theta \tilde{d}_b)}{T} \\ & \left\{ \frac{e^{\theta T_4}}{(\theta+r)}(1 - e^{-(\theta+r)T_4}) + \frac{1}{r}(e^{-rT_4} - 1) \right\} e^{-rT_3} \sum_{j=0}^{n_i-1} e^{-\frac{jrT}{n_i}} + \frac{\tilde{s}_b \tilde{d}_i}{2T} T_3^2 \sum_{j=0}^{n_i-1} e^{-\frac{jrT}{n_i} - rT_3} \Bigg] \end{aligned} \tag{25}$$

Firstly, we get the fuzzy present worth of integrated total cost in the form of trapezoidal fuzzy number as below

$$\begin{aligned}\tilde{T}C=&\frac{1}{T}\Big[(c_{v1},c_{v2},c_{v3},c_{v4})+\big((h_{v1},h_{v2},h_{v3},h_{v4})+\theta(d_{v1},d_{v2},d_{v3},d_{v4})\big)\\&\left[\frac{(p_1,p_2,p_3,p_4)-\sum_{i=1}^{N}(d_{i1},d_{i2},d_{i3},d_{i4})}{\theta}\left\{\frac{1}{r}(1-e^{-rT_1})+\frac{1}{(\theta+r)}(e^{-(\theta+r)T_1}-1)\right\}\right.\\&\left.+\frac{\sum_{i=1}^{N}(d_{i1},d_{i2},d_{i3},d_{i4})}{\theta}e^{-rT_1}\left\{e^{\theta T_2}\frac{1}{(\theta+r)}(1-e^{-(\theta+r)T_2})+\frac{1}{r}(e^{-rT_2}-1)\right\}\right]\\&+\sum_{i=1}^{N}i\Bigg[(c_{b1},c_{b2},c_{b3},c_{b4})\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}}+\big((h_{b1},h_{b2},h_{b3},h_{b4})+\theta(d_{b1},d_{b2},d_{b3},d_{b4})\big)\\&\left\{\frac{e^{\theta T_4}}{(\theta+r)}(1-e^{-(\theta+r)T_4})+\frac{1}{r}(e^{-rT_4}-1)\right\}e^{-rT_3}\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}}+\\&\frac{1}{2}(s_{b1},s_{b2},s_{b3},s_{b4})(d_{i1},d_{i2},d_{i3},d_{i4})T_3^2\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}-rT_3}\Bigg]\Big]\end{aligned}\tag{26}$$

Defuzzifying the fuzzy integrated total cost using graded mean integration representation method, we have

$$\begin{aligned}F(\tilde{T}C)=&\frac{1}{6T}\Big[\big(c_{v1}+2c_{v2}+2c_{v3}+c_{v4}\big)+\big\{(h_{v1}+\theta d_{v1})+2(h_{v2}+\theta d_{v2})+2\\&(h_{v3}+\theta d_{v3})+(h_{v4}+\theta d_{v4})\big\}\frac{1}{\theta}\Bigg[\Bigg\{\left(p_1-\sum_{i=1}^{N}d_{i4}\right)+2\left(p_2-\sum_{i=1}^{N}d_{i3}\right)+2\\&\left(p_3-\sum_{i=1}^{N}d_{i2}\right)+\left(p_4-\sum_{i=1}^{N}d_{i1}\right)\Bigg\}\left\{\frac{1}{r}(1-e^{-rT_1})+\frac{1}{(\theta+r)}(e^{-(\theta+r)T_1}-1)\right\}+\tfrac{1}{\theta}\\&\left(\sum_{i=1}^{N}(d_{i1}+2d_{i2}+2d_{i3}+d_{i4})\right)e^{-rT_1}\left\{e^{\theta T_2}\frac{1}{(\theta+r)}(1-e^{-(\theta+r)T_2})+\frac{1}{r}(e^{-rT_2}-1)\right\}\Bigg]\\&+\sum_{i=1}^{N}\Bigg[(c_{b1}+2c_{b2}+2c_{b3}+c_{b4})\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}}+\big((h_{b1}+\theta d_{b1})+2(h_{b2}+\theta d_{b2})+\\&2(h_{b3}+\theta d_{b3})+(h_{b4}+\theta d_{b4})\big)\left\{\frac{e^{\theta T_4}}{(\theta+r)}(1-e^{-(\theta+r)T_4})+\frac{1}{r}(e^{-rT_4}-1)\right\}e^{-rT_3}\\&\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}}+\frac{1}{2}\big((s_{b1}d_{i1})+2(s_{b2}d_{i2})+2(s_{b3}d_{i3})+(s_{b4}d_{i4})\big)T_3^2\sum_{j=0}^{n_i-1}e^{-\frac{jrT}{n_i}-rT_3}\Bigg]\Big]\end{aligned}$$

Using equations (7), (8) and (15), for a fixed value of n_i, $F(\tilde{T}C)$ is a function of T_1 *and* T_4 only, thus the optimal policies are obtained if $F(\tilde{T}C)$ is minimized.

4. NUMERICAL EXAMPLE

The preceding theory can be illustrated by considering two buyers, i.e. N=2. The capacity of production is (150000, 180000, 220000, 250000) units per year: the annual demand rate of the first and the second buyers are (3500, 3800, 4200, 4500) and (5500, 6000, 6500, 7000) units, respectively; the yearly percentage of holding cost per dollar for the vendor and the buyers are (0.10, 0.13, 0.17, 0.20) and (0.14, 0.17, 0.19, 0.22), respectively. The other related factors are: the ordering cost is $(350, 420, 480, 550) for the buyers, the production setup cost is $(4500, 4800, 5200, 5500). The unit deterioration cost for the vendor is $(5, 8, 12, 15), the unit deterioration cost for the buyer is $(7, 10, 14, 17). Shortage cost for the buyer is $ (10, 12, 16, 18), deterioration rate is 0.1 per year, and the inflation rate is 0.06 per year. The results obtained by applying the above solution procedure are presented in Tables 1 and 2.

4.1. Table 1: Optimal values of n_1 and n_2

n_1	n_2	T_1	T_2	T	T_4	BC	VC	TC
2	2	0.0552	0.9275	0.9716	0.5020	6005.97	11859.75	17865.72
2	3	0.0559	0.9304	0.9852	0.4154	6814.64	11849.48	18664.12
2	4	0.0548	0.9101	0.9638	0.3522	8256.11	11866.91	20123.02
3*	3*	0.0601	1.0348	1.0847	0.4063	6453.41	11392.11	17845.52*
$3^{\#}$	$4^{\#}$	0.0623	1.0589	1.1101	0.3630	$5992.42^{\#}$	12334.21	18326.63
4	3	0.0609	1.0505	1.1123	0.3770	6602.56	11866.27	18468.83
4	4	0.0657	1.1010	1.1745	0.3530	6220.16	11953.31	18173.47

4.2. Table 2: Buyers' costs

Cost items	($n_1^{\#}$ = 3, $n_2^{\#}$ = 4)	(n_1^{*} = 3, n_2^{*} = 3)	Cost
Buyers' ordering cost	4166.69	3971.56	-195.13
Buyers' holding cost	324.04	361.25	37.21
Buyer's deterioration cost	1211.69	1457.11	245.42
Buyers' shortage cost	290.00	664.05	374.05
Buyer's total cost	5992.42	6453.41	460.99

4.3. Table 3: Vendor's costs

Cost items	($n_1^{\#}$ = 3, $n_2^{\#}$ = 4)	(n_1^{*} = 3, n_2^{*} = 3)	Cost
Vendor's set up cost	6115.89	6246.38	130.49
Vendor's holding cost	861.05	642.42	-218.63
Vendor's deterioration cost	5357.27	4503.31	-853.96
Vendor's total cost	12334.21	11392.11	-942.10

5. RESULTS FROM THE NUMERICAL EXAMPLE

If the buyers follow the integrated policy and agree on the integrated deliveries of n_1^{*} = 3, n_2^{*} = 3, instead of their original policy ($n_1^{\#}$ = 3, $n_2^{\#}$ = 4), they will incur an increased cost of $460.99. On the other hand, the vendor will have a cost saving of $942.10. The percentage of the overall total integrated cost reduction is 2.69%. Since the vendor is the winner in the integrated policy, it is logical for him to offer some incentive for the buyers to accept the integrated policy.

6. CONCLUSION AND FUTURE ASPECT

In this paper, it is considered that all the cost parameters are trapezoidal fuzzy numbers and Graded Mean Integration Representation Method is used to defuzzify the total fuzzy cost. It has been observed that the integrated policy results in an impressive cost reduction if compared with the independent decisions made by the vendor and the buyers. However, the buyers' cost has been increased in the integrated approach when compared with the independent decision without considering the vendor's perspective. Thus, to develop a win-win situation for both the buyers and the vendor, the vendor should offer some quantity discount, or cost reduction to a certain percentage of his extra benefit due to the integrated approach (this will be discussed in a further research). This is worthwhile long-range strategy for both the vendor and the buyers. Therefore, it is concluded that integrated approach is much more practical in the daily life, and the assumption of fuzzy environment is nothing but adapting the model to suite more realistic situations. Most of the research papers in the existing literature have paid no or little attention towards the coordination of vendor and buyers in the imprecise and inflationary environment. In this paper, a vendor-buyer relationship model is developed for deteriorating items with shortages under fuzzy and inflationary environment. Thus, this study is a unique in its category, and deals the best with the competitive market situations. This inventory model can be extended by incorporating trade credit policy, two ware-house storage, etc.

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