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# ON SUFFICIENCY FOR MATHEMATICAL PROGRAMMING PROBLEMS WITH EQUILIBRIUM CONSTRAINTS

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**Abstract:** In this paper, we consider a mathematical programming with equilibrium constraints (MPEC) where the objective and constraint functions are continuously differentiable. We establish the sufficient optimality condition for strict local minima of order m under the assumptions of generalized strong convexity of order m.

**Keywords:** Mathematical programming with equilibrium constraint, sufficient optimality condition, higher-order strong convexity.

**MSC:** 90C30, 90C33, 90C46.

## 1. INRODUCTION

Mathematical programming with equilibrium constraints (MPEC) is an optimization problem, where a parameter dependent variational inequality or, more specifically, a parameter dependent complementarity problem arises as a side constraint. MPEC is an extension of the class of bilevel programs, which was introduced in the operations research literature in the early 1970s by Bracken and McGill. So, MPECs are also called generalized bilevel programming problems. Book by Luo et al. [7] provides a solid foundation and an extensive study of MPEC. Many authors are developing interesting results on these topics, see for example [6, 9, 14, 15, 16]. The M-stationary condition in MPEC was first introduced in Ye and Ye [16] by using Mordukhovich coderivative of setvalued maps. Ye [13] established W-stationary, C-stationary, A-stationary and S-stationary conditions for MPEC; he showed that the M-stationary condition is the most appropriate stationary condition for MPEC, in the sense that it is the second strongest stationary condition (with the strongest one being the S-stationary condition) and it holds under almost all analogues of the constraint qualifications for nonlinear programming problems such as MPEC linear constraint qualification. Further, he showed that stationary conditions are sufficient for global or local optimality for MPEC under some generalized convexity assumptions. We refer to [3, 7, 9, 13, 14, 15, 16] and the references therein for a survey about recent developments and applications of mathematical programming with equilibrium constraints and bilevel programming.

The concept of local minimizer of higher order in nonlinear programming originated from the study of iterative numerical methods. Necessary and sufficient optimality conditions for local minima of order 1 and 2 were derived by Auslender [2]. Studniarski [11] extended the Auslender's [2] definition for local minima of order m as a positive integer. Further, Ward [12] renamed the minima of order m as the strict local minimizer of order m for scalar optimization problem.

Karmadian [5] gave the definition of strongly convex function of order 1 and 2 in ordinary sense. Further, Fukushima [6] gave the definition of strongly convex function of order m. Four new classes of strongly convex function, namely strongly pseudoconvex type I and type II of order m, and strongly quasiconvex type I and type II of order m have recently been introduced in [1].

We consider the following mathematical programming problem with equilibrium constraints (MPEC):

$(MPEC)$

$Minimize\ f(x)$

subject to:

$$g(z) \leq 0,\ h(z) = 0$$
$$G(z) \geq 0,\ H(z) \geq 0,\ \langle G(z), H(z) \rangle = 0,$$

where $f : R^n \to R, G : R^n \to R^l, H : R^n \to R^l, h : R^n \to R^p$ and $g : R^n \to R^k$.

Assume that $f, g, h$, G and $H$ are continuously differentiable on $R^n$.

In this paper, we show that M-stationary condition is sufficient for strict local minimizer of order m for (MPEC) under generalized strong convexity assumption. The content of this note is outlined as follows. Section 2 is devoted to the preliminaries. In Section 3, we derive sufficient optimality condition for (MPEC) to find minima of order m.

## 2. DEFINITION AND PRELIMINARIES

Given a feasible vector $z^*$ for (MPEC), we define the following index set:

$$I_g := \{i : g(z^*) = 0\},$$
$$\alpha := \alpha\{z^*\} = \{i : G_i(z^*) = 0, H_i(z^*) > 0\},$$
$$\beta := \beta\{z^*\} = \{i : G_i(z^*) = 0, H_i(z^*) = 0\},$$
$$\gamma := \gamma\{z^*\} = \{i : G_i(z^*) > 0, H_i(z^*) = 0\},$$

where $i \in \{1, 2, ..., l\}$. The set $\beta$ is known as a degenerate set, and if $\beta$ is empty, vector $z^*$ is said to satisfy the strict complementarity condition.

***Definition 2.1*** *(M-stationary point) [**13**]. A feasible point, $z^*$ of (MPEC) is called Mordukhovich stationary if there exists $\lambda = (\lambda^g, \lambda^h, \lambda^G, \lambda^H) \in R^{k+p+2l}$ such that following condition holds*

$$0 = \nabla f(z^*) + \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right]$$
$$\lambda_{I_g}^g \geq 0, \qquad \lambda_\gamma^G = 0, \qquad \lambda_\alpha^H = 0,$$

either

$$\lambda_i^G > 0, \qquad \lambda_i^H > 0, \quad or \quad \lambda_i^G \lambda_i^H = 0, \quad \forall i \in \beta.$$

***Definition 2.2*** *[**13**]. Let $z^*$ be a feasible point of (MPEC) where all the functions are continuously differentiable at $z^*$. We say that the No Nonzero Abnormal Multiplier Constraint Qualification (NNAMCQ) is satisfied at $z^*$ if there is no nonzero vector $\lambda = (\lambda^g, \lambda^h, \lambda^G, \lambda^H) \in R^{k+p+2l}$ such that*

$$0 = \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right]$$
$$\lambda_{I_g}^g \geq 0, \qquad \lambda_\gamma^G = 0, \qquad \lambda_\alpha^H = 0,$$

either

$$\lambda_i^G > 0, \qquad \lambda_i^H > 0, \quad or \quad \lambda_i^G \lambda_i^H = 0, \quad \forall i \in \beta.$$

***Theorem 2.1*** *[**1**]. Suppose $f : X \to R$ is continuously differentiable on $X$. Then, $f$ is a strongly convex function of order m on $X$ if and only if there exists a constant $c > 0$ such that*

$$f(z) \geq f(x) + \langle \nabla f(x), z - x \rangle + c\|z - x\|^m.$$

***Definition 2.3*** *[**1**]. A differentiable function $f : X \to R$ is said to be strongly pseudoconvex type $I$ of order m on $X$ if there exists a constant $c > 0$ such that for any $x, z \in X$*

$$\langle \nabla f(x), z - x \rangle \geq 0 \Rightarrow f(z) \geq f(x) + c\|z - x\|^m.$$

**Remark 2.1** Every strongly pseudoconvex type $I$ function of order $m$ is pseudoconvex. Converse is not true. For example $f(x)=x^2, x\le 0,\ f(x)=0, x>0$ is pseudoconvex but not strongly pseudoconvex type $I$ of any order, as for $x=0, z>0$ we have $\langle \nabla f(x), z-x\rangle = 0$, however, $f(z)\ge f(x)+c\|z-x\|^m$ is not true for any $c>0$.

***Definition 2.4*** *[**1**]. A differentiable function $f: X\to R$ is said to be strongly pseudoconvex type II of order m on X if there exists a constant $c>0$ such that for any $x, z\in X$*

$$\langle \nabla f(x), z-x\rangle + c\|z-x\|^m \ge 0 \Rightarrow f(z)\ge f(x).$$

***Definition 2.5*** *[**1**]. A differentiable function $f: X\to R$ is said to be strongly quasi-convex type I of order m on X if there exists a constant $c>0$ such that for $x, z\in X$*

$$f(z)\ge f(x)\Rightarrow \langle \nabla f(x), z-x\rangle + c\|z-x\|^m \le 0.$$

**Remark 2.2** Every strongly quasiconvex type *I* function of order m is quasiconvex. The converse of the above may not hold. The function $f(x)=\sqrt{4-x^2}, x\in[0,2]$ is quasiconvex but is not strongly quasiconvex type *I* of any order, take $z=1, x=0,$ we have $f(1)\le f(0)$ but $\langle \nabla f(x), z-x\rangle + c\|z-x\|^m \le 0$ does not hold for any $c>0$.

***Definition 2.6*** *[**1**]. A differentiable function $f: X\to R$ is said to be strongly quasiconvex type II of order m on X if there exists a constant $c>0$ such that for $x, z\in X$*

$$f(z)\le f(x)+c\|z-x\|^m \Rightarrow \langle \nabla f(x), z-x\rangle \le 0.$$

## 3. SUFFICIENT M-STATIONARY CONDITION

***Theorem 3.1****(Fritz John type M-stationary condition) [**14**]. Let $z^*$ be a local solution of (MPEC) where all functions are continuously differentiable at $z^*$. Then, there exists $r\ge 0, \lambda=(\lambda^g,\lambda^h,\lambda^G,\lambda^H)\in R^{p+q+2m}$ not all zero such that*

$$0 = r\nabla f(z^*) + \sum_{j=1}^{k}\lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p}\lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l}\left[\lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*)\right]$$

$$\lambda_{I_g}^g \ge 0, \qquad \lambda_\gamma^G = 0, \qquad \lambda_\alpha^H = 0,$$

either

$$\lambda_i^G > 0, \qquad \lambda_i^H > 0, \quad or \quad \lambda_i^G\lambda_i^H = 0, \quad \forall i\in\beta.$$

***Theorem 3.2****(Kuhn-Tucker type necessary M-stationary condition) [**13**]. Let $z^*$ be a local optimal solution for (MPEC) where all functions are continuously differentiable at $z^*$. Suppose (NNAMCQ) is satisfied at $z^*$ then, $z^*$ is M-stationary.*

Ye [15] has proved that the S-stationary conditions become sufficient or locally sufficient for optimality when the objective function is pseudoconvex and all constraint functions are affine. In Theorem 2.3 [13], we see that M-stationary condition also turns into a sufficient optimality condition for MPEC when the objective function is pseudoconvex and all constraint functions quasiconvex. Now, we prove that M-stationary condition is a sufficient condition for MPEC when the objective function is pseudoconvex type I of order m and all constraint functions are quasiconvex type I of order m.

***Theorem 3.3****(Sufficient M-stationary condition). Let* $z^*$ *be a feasible point of (MPEC) and M-stationary condition hold at* $z^*$*, i.e., there exists* $\lambda = (\lambda^g, \lambda^h, \lambda^G, \lambda^H) \in R^{k+p+2l}$

$$0 = \nabla f(z^*) + \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right] \quad (1)$$

$$\lambda_{I_g}^g \geq 0, \qquad \lambda_\gamma^G = 0, \qquad \lambda_\alpha^H = 0,$$

either

$$\lambda_i^G > 0, \qquad \lambda_i^H > 0, \quad or \quad \lambda_i^G \lambda_i^H = 0, \quad \forall i \in \beta .$$

Let

$$J^+ := \left\{ i : \lambda_i^h > 0 \right\}, \quad J^- := \left\{ i : \lambda_i^h < 0 \right\},$$

$$\beta^+ := \left\{ i \in \beta : \lambda_i^G > 0, \lambda_i^H > 0 \right\},$$

$$\beta_G^+ := \left\{ i \in \beta : \lambda_i^G = 0, \lambda_i^H > 0 \right\}, \beta_G^- := \left\{ i \in \beta : \lambda_i^G = 0, \lambda_i^H < 0 \right\},$$

$$\beta_H^+ := \left\{ i \in \beta : \lambda_i^H = 0, \lambda_i^G > 0 \right\}, \beta_H^- := \left\{ i \in \beta : \lambda_i^H = 0, \lambda_i^G < 0 \right\},$$

$$\alpha^+ := \left\{ i \in \alpha : \lambda_i^G > 0 \right\}, \quad \alpha^- := \left\{ i \in \alpha : \lambda_i^G < 0 \right\},$$

$$\gamma^+ := \left\{ i \in \gamma : \lambda_i^H > 0 \right\}, \quad \gamma^- := \left\{ i \in \gamma : \lambda_i^H < 0 \right\}.$$

Suppose that $f$ is strongly pseudoconvex type I of order m at $z^*$, $g_j(j \in I_g)$, $h_i(i \in J^+)$, $-h_i(i \in J^-), G_i(i \in \alpha^- \cup \beta_H^-), -G_i(i \in \alpha^+ + \cup \beta_H^+ \cup \beta_H^-)$,

$H_i(i \in \gamma^- \cup \beta_G^-), -H_i(i \in \gamma^+ \cup \beta_G^+ \cup \beta_H^-)$ are strongly quasiconvex type I of order m at $z^*$. Then, the following holds true

(1) If $\alpha^- \cup \gamma^- \cup \beta_G^- \cup \beta_H^- = \phi$, then, $z^*$ is a global optimal solution of order m for (MPEC).

(2) If $\beta_G^- \cup \beta_H^- = \phi$ or $z^*$ is an interior point relative to the set

$$S \cap \left\{ z : G_i(z) = 0, H_i(z) = 0, i \in \beta_G^- \cup \beta_H^+ \right\}$$

i.e., for every feasible point $z$ which is close to $z^*$, it holds that

$$G_i(z) = 0, \qquad H_i(z) = 0, \qquad \forall i \in \beta_G^- \cup \beta_H^+$$

then, $z*$ is a local optimal solution of order m for (MPEC), where $S$ denotes the set of feasible solutions of (MPEC).

**Proof: (1)** Let $z$ be any feasible point of (MPEC). Then, for any $j \in I_g$,

$$g_j(z) \le 0 = g_j(z^*).$$

Since $g_j(z)$ is strongly quasiconvex type I of order m, there exists $c_j > 0, \forall j \in I_g$ such that

$$\left\langle \nabla g_j(z^*), z - z^* \right\rangle + c_j \left\| z - z^* \right\|^m \le 0. \tag{2}$$

Similarly, we have

$$\left\langle \nabla h_j(z^*), z - z^* \right\rangle + c_i \left\| z - z^* \right\|^m \le 0, \quad \forall i \in J^+, \tag{3}$$

$$-\left\langle \nabla h_j(z^*), z - z^* \right\rangle + c_i \left\| z - z^* \right\|^m \le 0, \quad \forall i \in J^-. \tag{4}$$

Also, for any feasible $z, -G(z) \le 0, -H(z) \le$ and $G(z), H(z)$ are strongly quasiconvex type I of order m, and we get

$$-\left\langle \nabla G_i(z^*), z - z^* \right\rangle + c_i \left\| z - z^* \right\|^m \le 0, \quad \forall i \in \alpha^+ \cup \beta_H^+ \cup \beta^+, \tag{5}$$

$$-\left\langle \nabla H_i(z^*), z - z^* \right\rangle + c_i \left\| z - z^* \right\|^m \le 0, \quad \forall i \in \gamma^+ \cup \beta_G^+ \cup \beta^+. \tag{6}$$

Where $c_i > 0, \forall i.$ if $\alpha^- \cup \gamma^- \cup \beta_G^- \cup \beta_H^- = \phi$, multiplying (2) − (6) by

$\lambda_j^g \ge 0 (j \in I_g(z^*))$,
$\lambda_i^h > 0 (j \in J^+)$,
$-\lambda_i^h > 0 (j \in J^-)$,
$\lambda_i^G > 0 (j \in \alpha^+ \cup \beta_H^+ \cup \beta^+)$,
$\lambda_i^H > 0 (j \in \gamma^+ \cup \beta_G^+ \cup \beta^+)$,

respectively, and adding, we get

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle + \mu \left\| z - z^* \right\|^m \le 0,$$

where

$$\mu = (\sum_{j=1}^{k} \lambda_j^g c_j + \sum_{i \in J^+} \lambda_i^h c_i + \sum_{i \in J^-} (-\lambda_i^h) c_i + \sum_{i \in \alpha^+ \cup \beta_H^+ \cup \beta^+} \lambda_i^G c_i + \sum_{i \in \gamma^+ \cup \beta_G^+ \cup \beta^+} \lambda_i^H c_i) > 0.$$

Which implies

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle \leq 0.$$

Using (1), the above inequality implies that for $z$ sufficiently close to $z^*$,

$$\langle \nabla f(z^*), z - z^* \rangle \geq 0.$$

Since $f$ is strongly pseudoconvex type I of order m, there exists $c > 0$ such that

$$f(z) \geq f(z^*) + c\|z - z^*\|^m, \forall z \in S.$$

Hence, $z^*$ is strict minimal solution of order m for (MPEC).

(2) Now, we discuss the case when $\alpha^- \cup \gamma^- \neq \phi$ and $\beta_G^- \cup \beta_H^- = \phi$. For any $i \in \alpha$, since $H_i(z^*) > 0, H_i(z) > 0$ for $z$ sufficiently close to $z^*$ and hence, by the complementarity condition, $G_i = 0$ for such $z$. That is, for z sufficiently close to $z^*$, one has

$$G_i(z) = G_i(z^*), \quad i \in \alpha.$$

Since $G_i$ is strongly quasiconvex type I of order m at $z^*$, there exists $c_i > 0$ such that

$$\langle \nabla G_i(z^*), z - z^* \rangle + c_i \|z - z^*\|^m \leq 0, \quad \forall i \in \alpha^-. \tag{7}$$

Similarly, for any $i \in \gamma$, since $G_i(z^*) > 0, G_i(z) > 0$ for $z$ sufficiently close to $z^*$, and hence by the complementarity condition, $H_i = 0$ for such $z$. That is, for z sufficiently close to $z^*$, one has

$$H_i(z) = H_i(z^*), \quad i \in \gamma.$$

Since $H_i$ is strongly quasiconvex type I of order m at $z^*$, there exists $c_i > 0$ such that

$$\langle \nabla H_i(z^*), z - z^* \rangle + c_i \|z - z^*\|^m \leq 0, \quad \forall i \in \gamma^-. \tag{8}$$

Multiplying (2) − (8) by

$$\lambda_j^g \geq 0 (j \in I_g(z^*)),$$
$$\lambda_i^h > 0 (i \in J^+),$$
$$-\lambda_i^h > 0 (i \in J^-),$$
$$\lambda_i^G > 0 (i \in \alpha^+ \cup \beta_H^+ \cup \beta^+),$$
$$\lambda_i^H > 0 (i \in \gamma^+ \cup \beta_G^+ \cup \beta^+),$$
$$-\lambda_i^G > 0 (i \in \alpha^-), -\lambda_i^H > 0 (i \in \gamma^-),$$

respectively, and adding,we have

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle + \mu \|z - z^*\|^m \leq 0,$$

where

$$\mu = (\sum_{j=1}^{k} \lambda_j^g c_j + \sum_{i \in J^+} \lambda_i^h c_i + \sum_{i \in J^-} (-\lambda_i^h) c_i \sum_{i \in \alpha^+ \cup \beta_H^+ \cup \beta^+} \lambda_i^G c_i + \sum_{i \in \gamma^+ \cup \beta_G^+ \cup \beta^+} \lambda_i^H c_i$$
$$+ \sum_{i \in \alpha^-} (-\lambda_i^G) c_i + \sum_{i \in \gamma^-} (-\lambda_i^H) c_i) > 0.$$

Which implies

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle \leq 0.$$

Using (1), the above inequality implies that for $z$ sufficiently close to $z^*$

$$\langle \nabla f(z^*), z - z^* \rangle \geq 0.$$

Since *f* is strongly pseudoconvex type I of order m at $z^*$, we must have

$$f(z) \geq f(z^*) + c\|z - z^*\|^m, \forall z \in S.$$

That is, $z^*$ is local minimal solution of order m of (MPEC).
Now suppose that $z^*$ is an interior point relative to the set

$$S \cap \left\{ z : G_i(z) = 0, H_i(z) = 0, i \in \beta_G^- \cup \beta_H^+ \right\}.$$

Then, for any feasible point *z* sufficiently close to $z^*$, it holds that

$$G_i(z) = 0, \qquad H_i(z) = 0, \qquad \forall i \in \beta_G^- \cup \beta_H^+.$$

Since $G_i(i \in \beta_H^-)$ and $H_i(i \in \beta_G^-)$ are strongly quasiconvex type I of order m, there exists $c_i > 0$ such that

$$\langle \nabla G_i(z^*), z - z^* \rangle + c_i \|z - z^*\|^m \leq 0, \forall i \in \beta_H^-, \tag{9}$$

$$\langle \nabla H_i(z^*), z - z^* \rangle + c_i \|z - z^*\|^m \leq 0, \forall i \in \beta_G^-. \tag{10}$$

Multiplying (2) − (10) by

$\lambda_j^g \geq 0(j \in I_g(z^*)),$

$\lambda_i^h > 0(i \in J^+),$

$-\lambda_i^h > 0(i \in J^-),$

$\lambda_i^G > 0(i \in \alpha^+ \cup \beta_H^+ \cup \beta^+),$

$\lambda_i^H > 0(i \in \gamma^+ \cup \beta_G^+ \cup \beta^+),$

$-\lambda_i^G > 0(i \in \alpha^- \cap \beta_H^-),$

$-\lambda_I^H > 0(i \in \gamma^- \cap \beta_G^-),$

respectively, and adding, we have

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle + \mu \|z - z^*\|^m \leq 0,$$

$$\left\langle \sum_{j=1}^{k} \lambda_j^g \nabla g_j(z^*) + \sum_{i=1}^{p} \lambda_i^h \nabla h_i(z^*) - \sum_{i=1}^{l} \left[ \lambda_i^G \nabla G_i(z^*) + \lambda_i^H \nabla H_i(z^*) \right], z - z^* \right\rangle \leq 0.$$

Using (1), the above inequality implies that for $z$ sufficiently close to $z^*$,

$\langle \nabla f(z^*), z - z^* \rangle \geq 0.$

Since $f$ is strongly pseudoconvex type I of order m at $z^*$, there exists $c > 0$ such that

$$f(z) \geq f(z^*) + c\|z - z^*\|^m, \forall z \in S.$$

That is, $z^*$ is local minimal solution of (MPEC) if $z^*$ is an interior point relative to the set $S \cap \{z : G_i(z) = 0, H_i(z) = 0, i \in \beta_G^- \cup \beta_H^+\}$. This completes the proof.

## 4. CONCLUSION

In this paper, we considered a mathematical programming with equilibrium constraints (MPEC) where the objective and constraint functions are continuously differentiable. We established the sufficient optimality condition for strict local minima of order m under the assumptions of generalized strong convexity of order m. The result presented in this paper extends the result of Ye [13] regarding the generalized strong convex of order m case.

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