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BEHAVIORAL OPTIMIZATION MODELS FOR MULTICRITERIA PORTFOLIO SELECTION

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Abstract: In this paper, behavioral construct of suitability is used to develop a multi-criteria decision making framework for portfolio selection. To achieve this purpose, we rely on multiple methodologies. Analytical hierarchy process technique is used to model the suitability considerations with a view to obtaining the suitability performance score in respect of each asset. A fuzzy multiple criteria decision making method is used to obtain the financial quality score of each asset based upon investor's rating on the financial criteria. Two optimization models are developed for optimal asset allocation considering simultaneously financial and suitability criteria. An empirical study is conducted on randomly selected assets from National Stock Exchange, Mumbai, India to demonstrate the effectiveness of the proposed methodology.

Keywords: Portfolio selection; Behavioral optimization model; Fuzzy multiple criteria decision making; Analytical hierarchy process

MSC: 90C29; 91G10; 03E72

1. INTRODUCTION

Portfolio selection as a field of study began with the Markowitz model [20] in which return is quantified as the mean and risk as the variance. Traditionally, portfolio selection models have solely relied on financial criteria such as return, risk and liquidity as the determinants of asset quality [1, 8, 9, 12]. Of late, one witnesses some research effort toward incorporating suitability criteria as well. Suitability is a behavioral concept that refers to the propriety of the match between investor-preferences and portfolio characteristics. Financial experts and investment companies use various techniques to profile investors and then recommend a suitable asset allocation. In our view, portfolio selection models can be substantially improved by incorporating investor-preferences. In literature, we do not come across many studies to examine portfolio selection problem involving trade-off between financial and suitability criteria. Bolster and Warrick [2] developed a model of suitability for individual investors based on their personal attributes. Gupta, Mehlawat and Saxena [13] developed mathematical models for simultaneous consideration of suitability and optimality in asset allocation. Recently, Gupta, Inuiguchi and Mehlawat [14] developed a hybrid approach for asset allocation with simultaneous consideration of suitability and optimality. Other than these, to the best of our knowledge, there has not been much research on incorporating behavioral imperatives in portfolio selection. The present paper seeks to capture an important behavioral imperative of portfolio optimization, i.e. respect for differences in investor preferences by way of the construct of suitability.

This paper distinguishes itself in developing a multicriteria framework that consists of (a) survey of investor-preferences for investment alternatives; (b) measurement of asset quality on financial criteria using investor-preferences instead of historical data; and (c) hybrid optimization models for managing trade-off between financial and suitability criteria. For measuring suitability performance of the assets, we use a hierarchical basis of suitability evaluation of the assets using analytical hierarchy process (AHP). We measure asset suitability in respect of investor-preferences using an index called suitability performance (SP) score. We use a fuzzy multiple criteria decision making (Fuzzy-MCDM) method for calculating the financial performance (FP) score of the assets. The investor-ratings of the assets with respect to four key financial criteria, namely, short term return, long term return, risk and liquidity are used for calculating the FP scores. Two hybrid optimization models based upon SP and FP scores are developed to obtain portfolios that meet investor-preferences on both financial and suitability criteria as far as possible.

This paper is organized as follows. In Section 2, we present AHP model for determining SP scores of the assets, and present details of the computational procedure of AHP. In Section 3, we describe the Fuzzy-MCDM method to measure asset quality using financial criteria. In Section 4, we present hybrid optimization models of portfolio selection. The proposed models are test-run in Section 5. This section also pertains to a discussion of the results obtained. Finally in Section 6, we furnish our concluding observations.

2. SUITABILITY EVALUATION OF ASSETS

Suitability is a major concern for financial experts while recommending a suitable set of assets to an individual investor. According to them, once a investor's personal and financial situation is evaluated, a suitable asset allocation for an individual investor can be determined. A suitable portfolio is one in which the assets held are appropriate to the investment objectives, financial needs and level of sophistication of the individual investor. However, there is no guarantee that the recommended asset allocation is also optimal in a return-risk sense. Even if we fulfill a prescribed asset allocation with the best category specific assets (or combinations of assets), there is no guarantee that the resulting portfolio will yield the highest return at the given level of expected risk. Likewise, a return-risk efficient portfolio, with a reasonable level of risk may not be suitable for a particular investor. Ideally, the investors may have a portfolio that is based not only on financial considerations but also incorporates suitability. Whereas, the existing optimization models of portfolio selection adequately address to the consideration of the financial measures of asset performance, incorporation of suitability measures necessitates use of alternative frameworks.

2.1 AHP model of suitability performance score

For measuring suitability performance of an asset, we propose a measure called SP score which can be used as an input along with its financial performance. The SP scores allow us to profile investor-preferences for suitability considerations of the assets in portfolio selection.

2.1.1 The hierarchical basis

We follow the hierarchical basis of suitability evaluation of assets considered in Gupta, Mehlawat and Saxena [13]. The SP index is broken into three main criteria of suitability, namely, income and savings (IS), investment objectives (IO) and investing experience (IE). Each of these criteria is further decomposed into various sub-criteria apiece illustrative of the factors that weigh in investors' minds while making investment decisions. The resultant hierarchy is shown in Fig. 1. Level 1 represents the goal, i.e. SP score; level 2 represents the three main criteria: IS, IO and IE. At level 3, these criteria are decomposed into various sub-criteria, i.e. IS is decomposed into income (IN), source (SO), savings (SA) and saving rate (SR); IO is decomposed into age (AG), dependents (DE), time horizon (TH) and risk/loss (R/L); IE is decomposed into length of prior experience (LE), equity holding (EH) and education (ED); and finally, the bottom level of the hierarchy, i.e. level 4, represents the alternatives (assets). For detailed discussion on the variables considered here for AHP modeling of the suitability performance, we refer the reader to [13].

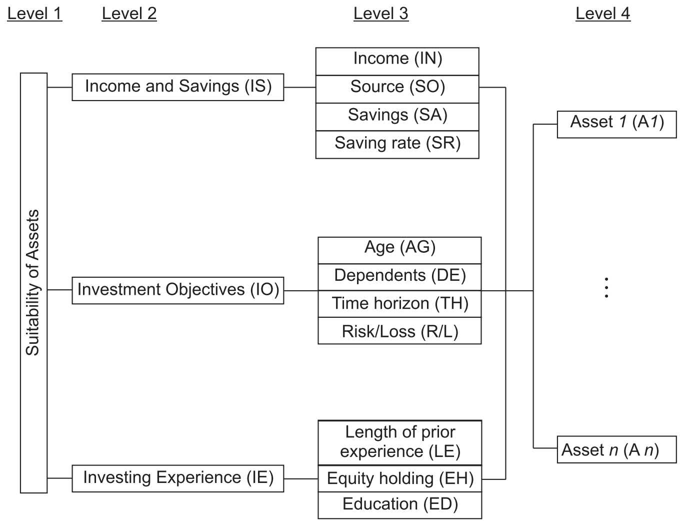

Figure 1. Structural hierarchy for suitability of assets

2.1.2 Computational procedure of AHP

In AHP, the elements of each level of the decision hierarchy are rated using pair-wise comparison based on a nine-point scale, see Table 1 [21]. After all the elements have been compared pair by pair, a paired comparison matrix is formed. The order of the matrix depends on the number of elements at each level. The number of such matrices at each level depends on the number of elements at the immediate upper level that it links to. After developing all the paired comparison matrices, the eigenvector or the relative weights representing the degree of the relative importance amongst the elements and the maximum eigenvalue (λ_{max}) are calculated for each matrix.

Table 1. Fundamental scale for pair-wise comparisons

Verbal Scale	Numerical Values
Equally important, likely or preferred	1
Moderately more important, likely or preferred	3
Strongly more important, likely or preferred	5
Very strongly more important, likely or preferred	7
Extremely more important, likely or preferred	9
Intermediate values to reflect compromise	2,4,6,8
Reciprocals for inverse comparison	Reciprocals

The λ_{max} value is an important validating parameter in AHP. It is used as a reference index to screen information by calculating the consistency ratio of the estimated vector (eigenvector) in order to validate whether the paired comparison matrix provides a completely consistent evaluation. The consistency ratio is calculated as per the following steps:

1. Calculate the eigenvector or the relative weights and λ_{max} for each matrix of order n.
2. Compute the consistency index (CI) for each matrix of order n as follows:
$CI = (\lambda_{max} - n)/(n-1)$.
3. The consistency ratio (CR) is calculated as follows:
$CR = CI/RI$

where RI is a known random consistency index that has been obtained from a large number of simulation runs and varies according to the order of matrix.

If CI is sufficiently small, then pair-wise comparisons are probably consistent enough to give useful estimates of the weights. If $CI/RI \leq 0.10$, then the degree of consistency is satisfactory. However, if $CI/RI > 0.10$, then serious inconsistencies may exist and hence, AHP may not yield meaningful results. The evaluation process should therefore, be reviewed and improved. The eigenvectors are used to calculate the global weights if there is an acceptable degree of consistency for the selection criteria.

3. FINANCIAL PERFORMANCE SCORE USING FUZZY-MCDM

The financial quality of the assets is usually measured in terms of their potential short and long term returns, liquidity and risk related characteristics, see for details Gupta, Mehlawat and Saxena [15]. An estimation of these characteristics by extrapolation of historical data is fraught with the possibility of measurement and judgmental errors. Moreover, the investors are more comfortable in articulating their preferences linguistically, for example, high return, low risk, medium liquidity. Such type of vagueness in expression necessitates recourse to Fuzzy-MCDM for determining the financial quality of the assets under consideration.

In traditional multiple criteria decision making (MCDM) methods [10, 17, 23], performance ratings and weights are measured in crisp numbers. In Fuzzy-MCDM methods [3, 4, 5, 11, 16, 24, 25], performance ratings and criteria weights are usually represented by fuzzy numbers. In dealing with fuzzy numbers, ranking [6, 7, 19, 26] is an important issue. In the following discussion, we present details of the fuzzy-MCDM method developed by Lee [18] and recently used by Gupta, Mehlawat and Saxena [15]. We include all the major details here for the sake of completeness. It may be noted that the method is appropriately modified to suit the purpose of this paper. We first present some basic definitions and results.

Definition 1. Fuzzy set $\tilde{A}$ in $X \subset R$, the set of real numbers, is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$, where x is the generic element of X and $\mu_{\tilde{A}}(x)$ is the membership function or grade of membership, or degree of compatibility or degree of truth of $x \in X$ which maps $x \in X$ on the real interval $[0, 1]$.

Definition 2. The crisp set A_α of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree $\alpha \in [0, 1]$ is called the α-cut (α-level set) of fuzzy set $\tilde{A}$ and is given by $A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$. The support of a fuzzy set $\tilde{A}$ is the crisp set,

$S(A)$, that contains all the elements of X that have nonzero grades of membership in $\tilde{A}$ and is given by $S(A) = \{x \in X | \mu_{\tilde{A}}(x) \geq 0\}$. The fuzzy set $\tilde{A}$ is normal iff $\sup_{x \in X} \mu_{\tilde{A}}(x) = 1$, where sup is the operator used to find the maximum membership value obtained by any element in that set.

Definition 3. A fuzzy set $\tilde{A}$ is said to be a convex set if $\mu(\lambda x_1 + (1-\lambda)x_2)) \geq \min(\mu(x_1), \mu(x_2)), x_1, x_2 \in X, \lambda \in [0,1]$.

Definition 4. A fuzzy set $\tilde{A}$, which is both convex and normal is defined to be a fuzzy number on R.

Definition 5. If $\tilde{A}$ be a fuzzy number, the α-level sets A_α can be written as $A_\alpha = [A^L_\alpha, A^R_\alpha]$. A^L_α and A^U_α are called lower and upper α-level cuts and are defined as $A^L_\alpha = \inf_{\mu_{\tilde{A}}(x) \geq \alpha}(x)$ and $A^U_\alpha = \sup_{\mu_{\tilde{A}}(x) \geq \alpha}(x)$, respectively. Here, inf and sup are used to find the minimum and maximum α-level cuts, respectively.

Definition 6. A triangular fuzzy number is denoted as $\tilde{A} = (l, m, u)$ and its membership function is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-l)/(m-l), & \text{if } \ l \leq x \leq m\,, \\ (u-x)/(u-m), & \text{if } \ m \leq x \leq u\,, \\ 0, & \text{otherwise}\,, \end{cases}$$

where l and u represent the lower and upper bounds of the fuzzy number $\tilde{A}$, respectively, and m is the median value.

Definition 7 [18]. For fuzzy numbers $\tilde{A}$ and $\tilde{B}$, the extended fuzzy preference relation $F(\tilde{A}, \tilde{B})$ is defined by the membership function

$$\mu_F(\tilde{A}, \tilde{B}) = \int_0^1 ((\tilde{A} - \tilde{B})^L_\alpha + (\tilde{A} - \tilde{B})^U_\alpha) d\alpha \tag{3.1}$$

Remark 1. If $\tilde{A} = (l_1, m_1, n_1)$ and $\tilde{B} = (l_2, m_2, n_2)$ are two triangular fuzzy numbers then

$$\mu_F(\tilde{A}, \tilde{B}) = (l_1 + 2m_1 + n_1 - l_2 - 2m_2 - n_2)/2.$$

Proposition 1 [18]. For the extended fuzzy preference relation F, the following statements hold true :

(i) F is reciprocal, i.e. $\mu_F(\tilde{B}, \tilde{A}) = -\mu_F(\tilde{A}, \tilde{B})$.

(ii) F is additive, i.e. $\mu_F(\tilde{A}, \tilde{B}) + \mu_F(\tilde{B}, \tilde{C}) = \mu_F(\tilde{A}, \tilde{C})$.

(iii) F is transitive, i.e. $\mu_F(\tilde{A}, \tilde{B}) \geq 0$ and $\mu_F(\tilde{B}, \tilde{C}) \geq 0 \Rightarrow \mu_F(\tilde{A}, \tilde{C}) \geq 0$.

Definition 8 [18]. The preference intensity function of one fuzzy number $\tilde{A}$ over another fuzzy number $\tilde{B}$ is defined as:

$$Q(\tilde{A}, \tilde{B}) = \begin{cases} \mu_F(\tilde{A}, \tilde{B}) & \text{if } \ \mu_F(\tilde{A}, \tilde{B}) \geq 0\,, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

We assume that there are n assets under evaluation against m criteria. Let the indices i and k denote the assets under consideration and the index j denote the evaluation criteria. Let fuzzy number $\tilde{A}_{ij}$ be rating of the i-th asset on the j-th criterion and fuzzy number $\tilde{w}_j$ be the weight of the j-th criterion. Let J be the set of benefit criteria (i.e. larger the value is, the better the asset is) and J' be the set of negative criteria (i.e. smaller the value is, the better the asset is) with $J \cup J' = \{1, 2, \ldots, m\}$ and $J \cap J' = \emptyset$.

The crisp advantage of the i-th asset on the j-th criterion is given as:

$$a_{ij} = \begin{cases} \sum\limits_{k \neq i} Q(\tilde{A}_{ij}, \tilde{A}_{kj}) & \text{if } \ j \in J\,, \\ \sum\limits_{k \neq i} Q(\tilde{A}_{kj}, \tilde{A}_{ij}) & \text{if } \ j \in J'\,. \end{cases} \tag{3.3}$$

Similarly, the crisp disadvantage of the i-th asset on the j-th criterion is:

$$d_{ij} = \begin{cases} \sum\limits_{k \neq i} Q(\tilde{A}_{kj}, \tilde{A}_{ij}) & \text{if } \ j \in J\,, \\ \sum\limits_{k \neq i} Q(\tilde{A}_{ij}, \tilde{A}_{kj}) & \text{if } \ j \in J'\,. \end{cases} \tag{3.4}$$

The fuzzy strength of the i-th asset is now obtained as:

$$FS_i = \sum_{j=1}^{m} a_{ij} \tilde{w}_j, \tag{3.5}$$

and the fuzzy weakness of the i-th asset is obtained as:

$$FW_i = \sum_{j=1}^{m} d_{ij} \tilde{w}_j\,. \tag{3.6}$$

The FP score of the i-th asset in crisp value can now be obtained as:

$$f_i = \sum_{k \neq i} Q(FS_i, FS_k) + \sum_{k \neq i} Q(FW_k, FW_i), \tag{3.7}$$

and its normalized value is

$$f'_i = \frac{f_i}{\sum\limits_{i=1}^{n} f_i}\,. \tag{3.8}$$

The FP scores f'_i are used to rank the assets on the basis of the financial criteria.

4. HYBRID OPTIMIZATION MODELS

We assume that investors allocate their wealth among n assets. We introduce some notations as follows:

f_i': Overall financial quality score of the i-th asset calculated using the fuzzy-MCDM method,

s_i: Overall suitability score of the i-th asset calculated using the AHP,

x_i: the proportion of total fund invested in the i-th asset,

y_i: the binary variable indicating whether the i-th asset is contained in the portfolio or not, i.e.

$$y_i = \begin{cases} 1, & \text{if } i\text{-th asset is contained in the portfolio} \\ 0, & \text{otherwise} \end{cases}$$

u_i: the maximal fraction of the capital budget allocated to the i-th asset,

l_i: the minimal fraction of the capital budget allocated to the i-th asset.

We first introduce the objective function and constraints.

• Objective

Financial goal

The objective function using FP scores based on the four key financial criteria is expressed as:

$$z(x) = \sum_{i=1}^{n} f_i' x_i \,.$$

•Constraints

Suitability constraint

When investors choose the suitability level they desire a priori, an suitability constraint is actually imposed on the portfolio selection. The suitability constraint using the SP scores is expressed as:

$$\sum_{i=1}^{n} s_i x_i \geq \beta \,,$$

where beta (β) is regarded as investor's choice for a minimum desired level of suitability in the portfolio construction.

Capital budget constraint

$$\sum_{i=1}^{n} x_i = 1 \,.$$

Maximal fraction of the capital that can be invested in a single asset

$$x_i \leq u_i y_i\,, \quad i = 1, 2, \ldots, n\,.$$

Minimal fraction of the capital that can be invested in a single asset

$$x_i \geq l_i y_i\,, \quad i = 1, 2, \ldots, n\,.$$

The constraints corresponding to lower bounds l_i and upper bounds u_i on the investment in individual assets ($0 \leq l_i, u_i \leq 1$, $l_i \leq u_i\,, \forall i$) are included to avoid a large number of very small investments (lower bounds), and at the same time to ensure a sufficient diversification of the investment (upper bounds).

Number of assets held in the portfolio

$$\sum_{i=1}^{n} y_i = h$$

where h is the number of assets that the investor chooses to include in the portfolio. Of all the assets from a given set, the investor would pick up the ones that are likely to yield the desired satisfaction of his preferences. It is not necessary that all the assets from a given set may configure in the portfolio as well. Investors would differ with respect to the number of assets they can effectively manage in a portfolio.

No short selling of assets

$$x_i \geq 0\,, \quad i = 1, 2, \ldots, n\,.$$

We now propose two optimization models for portfolio selection. The first model, namely, P-I is appropriate when investors fix a priori, the suitability level desired and maximize the financial goal while satisfying the desired suitability level. The second model, namely, P-II is appropriate when investors selects the portfolio to invest their money by trying to maximize both the financial goal and the suitability level of the investment simultaneously.

The constrained portfolio selection model P-I is formulated as follows:

$$\text{(P-I)} \qquad \max\ z(x) = \sum_{i=1}^{n} f_i' x_i$$

$$\text{subject to} \quad \sum_{i=1}^{n} s_i x_i \geq \beta\,, \tag{4.1}$$

$$\sum_{i=1}^{n} x_i = 1\,, \tag{4.2}$$

$$\sum_{i=1}^{n} y_i = h\,, \tag{4.3}$$

$$x_i \leq u_i y_i\,, \qquad i = 1, 2, \ldots, n\,, \tag{4.4}$$

$$x_i \geq l_i y_i\,, \qquad i = 1, 2, \ldots, n\,, \tag{4.5}$$

$$x_i \geq 0\,, \qquad i = 1, 2, \ldots, n\,, \tag{4.6}$$

$$y_i \in \{0, 1\}\,, \quad i = 1, 2, \ldots, n\,. \tag{4.7}$$

The problem P-I is a linear programming problem which can be solved using the LINDO software [22].

Unlike problem P-I, here suitability is considered as an objective function. Further, we formulate the constrained portfolio selection model P-II in order to consider the trade-off between the financial goal and the suitability goal as follows:

$$\text{(P-II)} \qquad \max\ z'(x) = w_1 \sum_{i=1}^{n} f_i' x_i + w_2 \sum_{i=1}^{n} s_i x_i$$
$$\text{subject to} \ \ \text{Constraints (4.2)-(4.7)}\,.$$

where w_1 is the relative weight of the financial criteria and w_2 is the relative weight of the suitability criteria given by investors such that $w_1 + w_2 = 1$.

5. NUMERICAL ILLUSTRATIONS

We present an empirical study of 10 randomly selected assets listed on National Stock Exchange (NSE), Mumbai, India, the premier market for financial assets.

5.1 SP scores

We calculate the SP scores using AHP. For the data in respect of pair-wise comparison matrices, we have relied on inputs from investors via questionnaire that are based on the verbal scale provided in Table 1. At level 2, we determine local weights (see Table 2) of the three main criteria with respect to the overall goal of

SP score. At level 3, we determine local weights (see Table 3) of the sub-criteria with respect to their respective parent criterion in the level 2. For example, the sub-criteria, IN, SO, SA and SR are pair-wise compared with respect to the parent criterion IS. At level 4, we determine the local weights (see Tables 4-6) of all the 10 assets with respect to each of the eleven sub-criteria of suitability in the level 3. These local weights are aggregated in respect of each asset by following, what in terms of the AHP hierarchy may be regarded as a bottom-up process of successive multiplication. Illustratively speaking, the local weight of an asset in relation to a sub-criterion is multiplied with the local weight of that sub-criterion in relation to its parent criterion, which in turn, is multiplied with the local weight of the parent criterion in relation to the overall goal of SP score. Thus, we obtain 11 aggregated local weights for each asset. The global weight of an asset in relation to each main criterion, involving all its sub-criteria, is obtained by adding the aggregated local weights of the asset in relation to the said criterion through its sub-criteria (rows 3, 4 and 5 of the Table 7 presents the global weights of the assets in respect of the three main criteria). In order to calculate the SP score, the global weights of each asset are summed over the three main criteria. The SP scores of the 10 assets are listed in row 6 of Table 7.

Table 2. Pair-wise comparisons of the main criteria in relation to the overall goal

Criteria	IS	IO	IE	Local weight
IS	1	4	4	0.57143
IO	1/2	1	2	0.28571
IE	1/4	1/2	1	0.14286

Table 3. Pair-wise comparisons of the sub-criteria in relation to the main criteria

IS	IN	SO	SA	SR	Local weight
IN	1	4	1/2	3	0.27763
SO	1/4	1	1/8	1/2	0.06346
SA	2	8	1	6	0.55526
SR	1/3	2	1/6	1	0.10365
IO	**AG**	**DE**	**TH**	**R/L**	**Local weight**
AG	1	1/2	2	3	0.24956
DE	2	1	5	7	0.54777
TH	1/2	1/5	1	2	0.12761
R/L	1/3	1/7	1/2	1	0.07506
IE	**LE**	**EH**	**ED**		**Local weight**
LE	1	4	2		0.55714
EH	1/4	1	1/3		0.12262
ED	1/2	3	1		0.32024

Table 4. Pair-wise comparisons of the alternatives in relation to the sub-criteria IN, SO, SA and SR

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Local weight
IN											
A1	1	1	1	3	6	3	3	3	4	1	0.17738
A2	1	1	1/2	2	5	3	3	2	3	1	0.14760
A3	1	2	1	2	5	2	3	2	3	1	0.16376
A4	1/3	1/2	1/2	1	2	1	1	1	2	1/3	0.06690
A5	1/6	1/5	1/5	1/2	1	1/2	1/2	1	1/2	1/6	0.03372
A6	1/3	1/3	1/2	1	2	1	1	1	2	1/3	0.06458
A7	1/3	1/3	1/3	1	2	1	1	1	2	1/3	0.06174
A8	1/3	1/2	1/2	1	1	1	1	1	4	1	0.08201
A9	1/4	1/3	1/3	1/2	2	1/2	1/2	1/4	1	1/4	0.04001
A10	1	1	1	3	6	3	3	1	4	1	0.16229
SO											
A1	1	1	2	2	5	3	3	3	4	1	0.18376
A2	1	1	2	2	4	3	3	2	3	1	0.16874
A3	1/2	1/2	1	1	3	2	1	1	2	1/2	0.08982
A4	1/2	1/2	1	1	3	2	1	1	2	1/2	0.08982
A5	1/5	1/4	1/3	1/3	1	1/2	1/2	1/2	1	1/5	0.03541
A6	1/3	1/3	1/2	1/2	2	1	1	1	2	1/3	0.06253
A7	1/3	1/3	1	1	2	1	1	1	2	1/3	0.07136
A8	1/3	1/2	1	1	2	1	1	1	1	1/3	0.06971
A9	1/4	1/3	1/2	1/2	1	1/2	1/2	1	1	1/4	0.04508
A10	1	1	2	2	5	3	3	3	4	1	0.18376
SA											
A1	1	1/2	1	1	4	1	2	1/2	3	2	0.10606
A2	2	1	2	3	6	2	3	2	5	3	0.21292
A3	1	1/2	1	2	5	2	2	2	4	2	0.15035
A4	1	1/3	1/2	1	4	1	3	1/2	2	3	0.10259
A5	1/4	1/6	1/5	1/4	1	1/4	1/2	1/6	1	1/2	0.02816
A6	1	1/2	1/2	1	4	1	2	1	3	2	0.10509
A7	1/2	1/3	1/2	1/3	2	1/2	1	1/3	2	1	0.05630
A8	2	1/2	1/2	2	6	1	3	1	5	3	0.14825
A9	1/3	1/5	1/4	1/2	1	1/3	1/2	1/5	1	1/2	0.03399
A10	1/2	1/3	1/2	1/3	2	1/2	1	1/3	2	1	0.05630
SR											
A1	1	2	1/2	1/2	1/4	2	1	2	1/3	1/2	0.06244
A2	1/2	1	1/2	1/3	1/6	1	1/2	1	1/5	1/3	0.03741
A3	2	2	1	1	1/2	3	2	4	1/2	1	0.11081
A4	2	3	1	1	1/3	3	2	4	1/2	1	0.11035
A5	4	6	2	3	1	6	4	5	2	2	0.24555
A6	1/2	1	1/3	1/3	1/6	1	1/2	1	1/5	1/4	0.03462
A7	1	2	1/2	1/2	1/4	2	1	2	1/2	1/3	0.06335
A8	1/2	1	1/4	1/4	1/5	1	1/2	1	1/5	1/3	0.03468
A9	3	5	2	2	1/2	5	2	5	1	2	0.18008
A10	2	3	1	1	1/2	4	3	3	1/2	1	0.12072

Table 5. Pair-wise comparisons of the alternatives in relation to the sub-criteria AG, DE, TH and R/L

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Local weight
AG											
A1	1	1/2	1	1	3	1	1	2	3	1	0.10750
A2	2	1	2	3	5	2	3	4	5	3	0.23322
A3	1	1/2	1	1	3	1	2	1	3	2	0.11784
A4	1	1/3	1	1	3	1	2	1	3	1	0.10561
A5	1/3	1/5	1/3	1/3	1	1/3	1/2	1	1	1/3	0.03993
A6	1	1/2	1	1	3	1	1	2	4	2	0.11957
A7	1	1/3	1/2	1/2	2	1	1	2	2	1	0.08496
A8	1/2	1/4	1	1	1	1/2	1/2	1	1	1/2	0.06052
A9	1/3	1/5	1/3	1/3	1	1/4	1/2	1	1	1/3	0.03896
A10	1	1/3	1/2	1	3	1/2	1	2	3	1	0.09189
DE											
A1	1	1/2	1/2	3	6	3	3	3	4	2	0.14973
A2	2	1	1	5	6	4	5	4	5	4	0.23687
A3	2	1	1	5	6	5	4	5	6	4	0.24592
A4	1/3	1/5	1/5	1	1/2	1	1	1/2	2	1/2	0.04528
A5	1/6	1/6	1/6	2	1	1	1/2	1/2	1/2	1/3	0.03773
A6	1/3	1/4	1/5	1	1	1	1	1	2	1/2	0.05085
A7	1/3	1/5	1/4	1	2	1	1	1	2	1/2	0.05427
A8	1/3	1/4	1/5	2	2	1	1	1	1	1/2	0.05477
A9	1/4	1/5	1/6	1/2	2	1/2	1/2	1	1	1/2	0.03966
A10	1/2	1/4	1/4	2	3	2	2	2	2	1	0.08492
TH											
A1	1	3	3	2	1/3	1	1/2	2	1/3	1	0.08366
A2	1/3	1	1	1	1/6	1/4	1/4	1	1/6	1/3	0.03388
A3	1/3	1	1	1	1/5	1/3	1/4	1	1/5	1/4	0.03539
A4	1/2	1	1	1	1/4	1/5	1/3	1	1/5	1/2	0.04221
A5	3	6	5	4	1	4	2	5	1	3	0.22731
A6	1	4	3	2	1/4	1	1/2	2	1/3	1	0.08502
A7	2	4	4	3	1/2	2	1	3	1/2	2	0.13695
A8	1/2	1	1	1	1/5	1/2	1/3	1	1/5	1/3	0.03969
A9	3	6	5	5	1	3	2	5	1	3	0.22450
A10	1	3	4	2	1/3	1	1/2	3	1/3	1	0.09140
R/L											
A1	1	1/2	1	1	5	1	2	1/2	3	3	0.11205
A2	2	1	2	3	6	3	4	1	5	4	0.21965
A3	1	1/2	1	1	4	1	1	1/2	2	3	0.09947
A4	1	1/3	1	1	4	1	2	1/2	3	2	0.10095
A5	1/5	1/6	1/4	1/4	1	1/4	1/2	1/6	1/2	1/2	0.02633
A6	1	1/3	1	1	4	1	2	1	3	2	0.11012
A7	1/2	1/4	1	1/2	2	1/2	1	1/3	2	1	0.06076
A8	2	1	2	2	6	1	3	1	5	4	0.18345
A9	1/3	1/5	1/2	1/3	2	1/3	1/2	1/5	1	1	0.04030
A10	1/3	1/4	1/3	1/2	2	1/2	1	1/4	1	1	0.04692

Table 6. Pair-wise comparisons of the alternatives in relation to the sub-criteria LE, EH and ED

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	Local weight
LE											
A1	1	2	2	1	4	1	1	1	2	1	0.12829
A2	1/2	1	1	1/2	2	1/2	1/2	1/3	1/2	1/2	0.05840
A3	1/2	1	1	1/2	2	1/2	1/2	1/3	1	1/2	0.06173
A4	1	2	2	1	3	1	1	1/2	2	1	0.11749
A5	1/4	1/2	1/2	1/3	1	1/4	1/3	1/4	1/2	1/3	0.03514
A6	1	2	2	1	4	1	1	1	2	1	0.12829
A7	1	2	2	1	3	1	1	1	2	1	0.12471
A8	1	3	3	2	4	1	1	1	2	1	0.15138
A9	1/2	2	1	1/2	2	1/2	1/2	1/2	1	1/2	0.06986
A10	1	2	2	1	3	1	1	1	2	1	0.12471
EH											
A1	1	2	1/2	1/2	1/4	2	1	2	1/3	1/2	0.06285
A2	1/2	1	1/3	1/4	1/6	1	1/2	1	1/5	1/3	0.03489
A3	2	3	1	1	1/2	3	2	3	1/2	1	0.11185
A4	2	4	1	1	1/2	3	2	4	1/2	1	0.11927
A5	4	6	2	2	1	6	4	5	2	2	0.23704
A6	1/2	1	1/3	1/3	1/6	1	1/2	1	1/5	1/4	0.03490
A7	1	2	1/2	1/2	1/4	2	1	2	1/3	1/2	0.06285
A8	1/2	1	1/3	1/4	1/5	1	1/2	1	1/4	1/3	0.03658
A9	3	5	2	2	1/2	5	3	4	1	2	0.18434
A10	2	3	1	1	1/2	4	2	3	1/2	1	0.11542
ED											
A1	1	2	2	1/2	3	1	1/2	1	1/2	1	0.08971
A2	1/2	1	1	1/3	2	1/2	1/4	1/2	1/4	1/2	0.04813
A3	1/2	1	1	1/3	1	1/2	1/4	1/2	1/3	1/2	0.04612
A4	2	3	3	1	5	2	2	2	1	2	0.18137
A5	1/3	1/2	1	1/5	1	1/2	1/4	1/3	1/5	1/3	0.03460
A6	1	2	2	1/2	2	1	1/2	1	1/2	1	0.08626
A7	2	4	4	1/2	4	2	1	2	1	2	0.16320
A8	1	2	2	1/2	3	1	1/2	1	1/2	1	0.08971
A9	2	4	3	1	5	2	1	2	1	2	0.17120
A10	1	2	2	1/2	3	1	1/2	1	1/2	1	0.08971

Table 7. SP scores of the assets

	Global weight									
Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
IS	0.07215	0.09931	0.08351	0.05296	0.03011	0.04791	0.03400	0.06463	0.02943	0.05742
IO	0.03655	0.05965	0.05031	0.01832	0.01760	0.02195	0.02085	0.01827	0.01803	0.02418
IE	0.01542	0.00746	0.00898	0.01974	0.00853	0.01477	0.01849	0.01679	0.01662	0.01605
SP Score	0.1241	0.1664	0.1428	0.0910	0.0562	0.0846	0.0733	0.0997	0.0641	0.0977

5.2 FP scores

The linguistic variables employed to represent relative importance and ratings are shown in Table 8. We use the following four evaluation criteria as considered in Gupta, Mehlawat and Saxena [15]:

Short term return (C_1)*; Long term return* (C_2)*; Risk* (C_3)*; Liquidity* (C_4).

Here, C_1, C_2 and C_4 are benefit criteria, whereas C_3 is a negative criterion. The weights of these criteria and rating of the assets from investors collected via questionnaire are shown in Tables 9. The evaluation procedure followed to arrive at the FP score of each asset is as per the description given in Section 3. Table 10 presents the FP score and its normalized value for 10 asset. For the details of the evaluation procedure, one may refer to Gupta, Mehlawat and Saxena [15].

Table 8. Linguistic variables for relative importance of the criteria and the performance ratings

Linguistic variables	Fuzzy number
Very low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)
Very high (VH)	(0.9, 1.0, 1.0)

Table 9. The weights of the evaluation criteria

	C_1	C_2	C_3	C_4
Weight	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)

Table 10. FP scores of the assets

	Assets									
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
FP score	19.375	126.0475	116.445	28.75	13.025	0	62.1575	58.6575	101.65	184.125
Normalized score	0.0273	0.1775	0.1640	0.0405	0.0183	0	0.0875	0.0826	0.1431	0.2592

5.3 Asset allocation

Here, we consider the maximization of preferences on both financial and suitability considerations, i.e. we try to maintain trade-off between financial optimality and the suitability level of the portfolio.

• **Portfolio selection using P-I**

We use $\beta = 0.125, h = 6, l_1 = 0.1, l_2 = 0.2, l_3 = 0.02, l_4 = 0.025, l_5 = 0.019, l_6 = 0.025, l_7 = 0.02, l_8 = 0.028, l_9 = 0.035, l_{10} = 0.026, u_1 = 0.4, u_2 = 0.3, u_3 =$

$0.35, u_4 = 0.4, u_5 = 0.35, u_6 = 0.4, u_7 = 0.3, u_8 = 0.42, u_9 = 0.4, u_{10} = 0.35$ to construct P-I. The corresponding computational results are presented in Table 11.

Table 11. The proportion of the assets in the portfolio

Assets	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Proportions	0	0.30	0.267	0	0	0	0.02	0.028	0.035	0.35

Note that while investors seek to maximize their overall financial goal, they want to be sure of an acceptable level of suitability of their portfolio as well. Further, as the desired level of suitability increases, the achievement level of the financial goal becomes smaller (see Fig. 2). This is in sync with the trade-off between financial and suitability performance of the portfolio. However, they would be able to achieve suitability only up to a particular level as the portfolio selection model becomes infeasible on increasing the desired level of suitability any further. The computational results to highlight this relationship are listed in Table 12.

Table 12. The proportion of the assets using model P-I

		Assets									
Suitability β	Financial goal	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
0.125	0.1968	0	0.30	0.267	0	0	0	0.02	0.028	0.035	0.35
0.130	0.1945	0	0.30	0.2863	0	0.019	0	0.02	0.028	0	0.3467
0.135	0.1726	0.10	0.30	0.3211	0	0	0	0.02	0.028	0	0.2309
0.140	0.1381	0.2279	0.30	0.35	0.025	0	0	0	0.028	0	0.0691
0.141	0.1293	0.2658	0.30	0.35	0.025	0	0	0	0.028	0	0.0312
> 0.142	Infeasible										

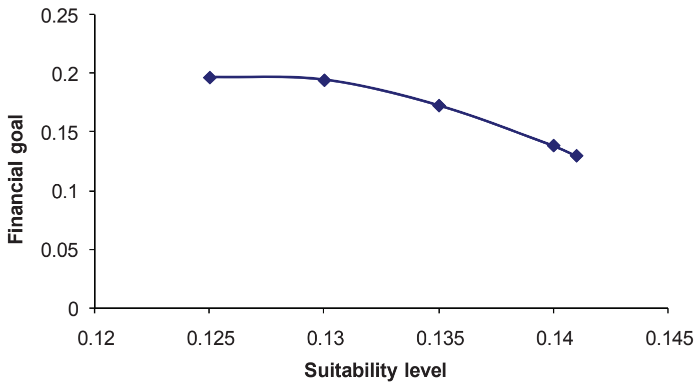

Figure 2. Efficient financial goal-suitability goal frontier using P-I

• Portfolio selection using P-II

We use different values of w_1 and w_2 to construct P-II. It is worth mentioning that these weights can be obtained either by using the investor-preferences or by using some exact method such as AHP, TOPSIS, etc. The computational results presented in Table 13 shows that as the importance of the suitability goal increases, i.e. w_2 increases, the achievement level of the financial goal becomes smaller (see Fig. 3).

Table 13. The proportion of the assets using model P-II

				Assets									
w_1	w_2	Suitability goal	Financial goal	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
0.5	0.5	0.1287	0.1968	0	0.30	0.267	0	0	0	0.02	0.028	0.035	0.35
0.3	0.7	0.1335	0.1875	0	0.30	0.35	0.025	0	0	0.02	0.028	0	0.277
0.1	0.9	0.1411	0.1281	0.271	0.30	0.35	0.025	0	0	0	0.028	0	0.026

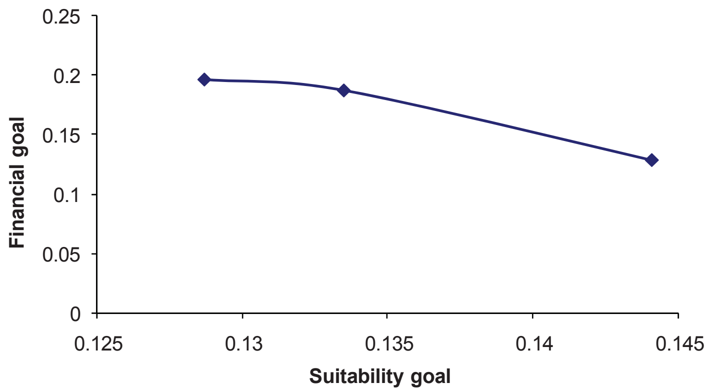

Figure 3. Efficient financial goal-suitability goal frontier using P-II

It may be noted that both models proposed herein manifest the trade-off between the financial goal and suitability goal. So, the investor may rely on either of these for portfolio optimization. However, in P-I the investor does not require to assign weights for the two goals and therefore, it would be less cumbersome to implement.

6. CONCLUSIONS

Suitability consideration for investment has recently become an important issue in portfolio selection. This implies that increasingly investment decisions are likely to be influenced both by financial and suitability considerations. The focus of the present research has been to incorporate the suitability considerations along with financial optimization in portfolio selection by using multiple methodologies. Two optimization models have been developed to incorporate the suitability considerations along with financial optimization in portfolio selection. The models differ in the way that suitability goal is assumed to be pursued by investors. The model P-I is appropriate when investors choose the suitability level a priori and try to maximize the financial goal of their investment while satisfying the desired suitability level; whereas, the model P-II is appropriate in the case in which investors try to maximize both the financial goal and the suitability level of the investment at the same time. Also, if the investor is not satisfied with the suitability level of the portfolio obtained, more portfolios can be generated by varying the preset suitability value in the proposed model. Our results demonstrate the effectiveness of the proposed models to effectively manage the trade-off between financial optimality and suitability.

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