REVIEW
AN ANNOTATED BIBLIOGRAPHY OF COMBINED ROUTING AND LOADING PROBLEMS

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Abstract: Transportation problems involving routing and loading at the same time are currently a hot topic in combinatorial optimization. The interest of researchers and practitioners is motivated by the intrinsic difficulty of this research area, which combines two computationally hard problems, and by its practical relevance in important real world applications. This annotated bibliography aims at collecting, in a systematic way, the most relevant results obtained in the area of vehicle routing with loading constraints, with the objective of stimulating further research in this promising area.

Keywords: Vehicle routing, Loading, Two-dimensional packing, Three-dimensional packing, Traveling salesman, Pickup and delivery.

MSC: 90B06, 90C10, 90C27, 90C59.

1. INTRODUCTION

Many activities in freight transportation involve two basic optimization issues that have been intensively studied in the last decades: finding the optimal routes to deliver goods, and determine the best way for loading such goods on the vehicles used

for transportation. The great majority of problems arising in these two areas belong to the class of (strongly) NP-hard problems, and are very challenging in practice. Up to recent years, most of the research was concentrated on solving these problems separately, while now, their combined solution has attracted a number of researchers and practitioners and led to interesting theoretical, as well as practical results. To our knowledge, the only complete survey on this novel research area is

M. Iori and S. Martello. Routing problems with loading constraints. *TOP* 18:4-27, 2010. This survey reviews the capacitated vehicle routing problem with two-and three-dimensional loading constraints. It also covers a number of relevant variants, such as the multi-pile vehicle routing problem and traveling salesman problems with pickup and delivery plus loading constraints.

The present article reviews, in a systematic way, the most relevant results obtained in the area of vehicle routing with loading constraints, with special emphasis on contributions appeared in recent years.

The following sections present an annotated bibliography of results obtained in the following main fields:

- Section 2: a brief description of the (separate) routing and loading problems, with emphasis on recent surveys;
- Section 3: the capacitated vehicle routing problem with two-dimensional loading constraints, and a number of its variants;
- Section 4: the capacitated vehicle routing problem with three-dimensional loading constraints, and some relevant variants;
- Section 5: other capacitated vehicle routing problems with loading constraints arising in real world applications;
- Section 6: the capacitated vehicle routing problem with pickup and delivery plus loading constraints (and a synthetic review of traveling salesman problems with pickup and delivery plus sequencing constraints).

2. ROUTING PROBLEMS AND LOADING PROBLEMS

In this section we provide a definition of the two basic problems from which the subject of the present article derives, and we comment a selection of recent related surveys.

2.1 Routing

The roots of routing problems date back to the Nineteenth century, when the Irish mathematician William Rowan Hamilton (1805-1865), defined the *Hamiltonian Circuit Problem:* decide if there exists a sequence of consecutive edges of a graph that visits each vertex exactly once.

Its extension to the weighted case gave rise to the famous *Traveling Salesman Problem* (TSP) defined in the Thirties by Karl Menger (1902-1985), on which intensive studies started in the Fifties (with Dantzig and Fulkerson, among others). Given a *graph* $G = (V, E)$, with *vertex* set $V = \{0, 1, ..., n\}$, *edge* set $E = \{(i, j) : i, j \in V, i \neq j\}$ and *cost*

c_{ij} for traveling along edge (i, j) (in either direction), the *Symmetric Traveling Salesman Problem* (STSP) is to find a sequence of consecutive edges (*circuit*) that visits all vertices of V at minimum total cost. Its *asymmetric* counterpart (ATSP) is similarly modeled on a *digraph* $G = (V, A)$ having *arc* set $A = \{(i, j) : i, j \in V, i \neq j\}$ and c_{ij} as the *cost* of traveling from i to j. The literature on the TSP includes hundreds of papers, and many surveys and books. We mention just two very recent encyclopedic entries.

C. D'Ambrosio, A. Lodi, and S. Martello. Combinatorial traveling salesman problem algorithms. In J.J. Cochran, editor, *Wiley Encyclopedia of Operations Research and Management Science*, pages 738-747. Wiley, Chichester, 2011.
A general introduction to the history of the Hamiltonian Circuit Problem and to the classical algorithmic approaches for the exact solution of the TSP.

A.N. Letchford and A. Lodi. Mathematical programming approaches to the traveling salesman problem. In J.J. Cochran, editor, Wiley Encyclopedia of Operations Research and Management Science, pages 3239-3248. Wiley, Chichester, 2011.
A review of the main ingredients of exact algorithms based on polyhedral theory for both the symmetric and the asymmetric TSP.

In real world transportation problems, the TSP is generalized to the *Capacitated Vehicle Routing Problem* (CVRP), in which we search, instead of a single circuit, a set of circuits (called *routes*) which start and end at a central depot located at vertex 0. Given K identical vehicles of capacity D, and n customers, located at vertices $1, 2, ..., n$, each having a demand $d_i (0 \leq d_i \leq D)$, the problem is to determine the routes so that each vehicle is assigned to at most one route, each customer is visited by one vehicle, the sum of the demands on each route is not greater than D, and the overall cost of all routes is a minimum. This problem has been deeply investigated since the Fifties. The interested reader is referred to the following books and surveys.

P. Toth and D. Vigo (eds.). *The Vehicle Routing Problem*. SIAM Monographs on Discrete Mathematics and Applications, Philadelphia, 2002.
A series of chapters, written by prominent researchers, covering the state-of-the-art of exact and heuristic methods for the CVRP and some of its variants. A revised reprint is currently (2013) in preparation.

B. Golden, S. Raghavan, and E. Wasil (eds.). *The Vehicle Routing Problem: Latest Advances and New Challenges*, volume 43 of *Operations Research/Computer Science Interfaces Series*. Springer, Berlin, 2008.
A collection of contributions that examine recent developments, and present novel problems and significant methodological advances.

R. Baldacci, P. Toth, and D. Vigo. Exact algorithms for routing problems under vehicle capacity constraints. In D. Bouyssou, S. Martello, and F. Plastria, editors, *Surveys in Operations Research II (Invited Surveys from 4OR, 2006-2008)*, volume 175 of *Annals of Operations Research*, pages 213-245. Springer US, 2010.

A recent survey focusing on the classical CVRP, and its extension to the case of heterogeneous fleets.

V. Schmid, K.F. Doerner, and G. Laporte. Rich routing problems arising in supply chain management. *European Journal of Operational Research*, 224:435-448, 2013.
This survey on routing issues in supply chain contexts includes a section on container and pallet loading in warehouse logistics, in which routing problems with two- and three-dimensional loading constraints are discussed.

2.2. Loading

In real world distribution problems the customers' demands are characterized not only by a weight (as in the CVRP) but also by a shape. It is then necessary to ensure that the transported items can be feasibly allocated within the vehicle loading space. Such issues are related to multi-dimensional rectangle packing problems, which arise as extensions of the classical one-dimensional bin packing problem. The latter problem can be described as that of allocating, without overlapping, a set of segments, each having a given width, to the minimum number of large identical segments. The main extensions to higher dimensions are:

- *Two-Dimensional Bin Packing Problem* (2BPP): Pack a set of rectangles into the minimum number of large identical rectangles (*bins*);
- *Two-Dimensional Strip Packing Problem* (2SPP): Pack a set of rectangles into an open-ended rectangular *strip* of given width and infinite height so as to minimize the overall height at which the strip is used;
- *Three-Dimensional Bin Packing Problem* (3BPP): Pack a set of rectangular boxes into the minimum number of large identical three-dimensional bins;
- *Three-Dimensional Strip Packing Problem* (3SPP): Pack a set of rectangular boxes into an open-ended three-dimensional strip of given width and height, and infinite length, so as to minimize the overall length at which the strip is used.

The following papers provide general introductions to the area of rectangular packing:

A. Lodi, S. Martello, and M. Monaci. Two-dimensional packing problems: A survey. *European Journal of Operational Research*, 141:3-13, 2002.
A. Lodi, S. Martello, M. Monaci, and D. Vigo. Two-dimensional bin packing problems. In V.Th. Paschos (ed.), *Paradigms of Combinatorial Optimization: Problems and New Approaches*, ISTE and John Wiley & Sons, pages 107-129, 2010.
Two reviews of various algorithmic approaches to the exact and approximate solution of two-dimensional packing problems.

G. Wäscher, H. Haußner, and H. Schumann. An improved typology of cutting and packing problems. *European Journal of Operational Research*, 183:1109-1130, 2007.
A typology and classification of the most important packing problems in one and higher dimensions.

E.G. Coffman, Jr, J. Csirik, G. Galambos, S. Martello, and D. Vigo. Bin Packing Approximation Algorithms: Survey and Classification. In D.-Z. Du, P.M. Pardalos, and

R.L. Graham (eds.), *Handbook of Combinatorial Optimization*, 2nd Edition, Springer, 2013. To appear.
An exhausting survey describing all main results on approximation algorithms for the one-dimensional bin packing problem, as well as a novel classification method of this huge area.

In addition to the need of obtaining a feasible placement of the demanded items in the loading space, a frequent relevant constraint is that the placement allows to perform the unloading operations without reshuffling the items (so that no time is wasted when the vehicle visits a customer). Some recent papers investigated such type of packing problems:

L. Junqueira, R. Morabito, and D. SatoYamashita. MIP-based approaches for the container loading problem with multi-drop constraints. *Annals of Operations Research*, 199:51-75, 2012.
The paper presents *Mixed Integer Linear Programming* (MILP) models for packing rectangular items into a single container so that, for a prefixed visiting sequence, the items can be directly unloaded, and the loading is stable.

J.L.M. da Silveira, F.K. Miyazawa, and E.C. Xavier. Heuristics for the strip packing problem with unloading constraints. *Computers & Operations Research*, 40:991-1003, 2013.
The algorithms introduced in this paper provide approximate solutions to a version of the 2SPP in which an unloading item sequence is prefixed, and the packing must allow to unload each item without moving any other item.

S. Ceschia and A. Schaerf. Local search for a multi-drop multi-container loading problem. *Journal of Heuristics,* 19: 275-294, 2013.
The paper investigates a number of real world loading constraints arising in variants of the 3BPP, and presents local search algorithms for their solution.

3. CAPACITATED VEHICLE ROUTING WITH TWO-DIMENSIONAL LOADING

Consider a generalization of the CVRP (see Section 1), in which, for the demand of a customer i $(i = 1, 2, ..., n)$, the total weight d_i is determined by m_i items. Item $I_{i\ell}(\ell = 1, 2, ..., m_i)$ has *width* $w_{i\ell}$ and *height* $h_{i\ell}$, while the loading surface of each vehicle has width W and height H. Let $S(k) \subseteq \{1, 2, ..., n\}$ denote the set of customers visited by vehicle k. In addition to the classical CVRP capacity constraint, $\sum_{i \in S(k)} d_i \leq D$, a solution requires a feasible (non-overlapping) loading of all items requested by the customers of $S(k)$ into the $W \times H$ loading area. The resulting problem, denoted as the 2L-CVRP (*Capacitated Vehicle Routing Problem with Two-Dimensional Loading Constraints*), arises in real world applications in which the loaded items cannot be stacked one on top of the other because of their fragility or weight.

A number of variants of the 2L-CVRP has been considered in the literature, basically coming from two additional criteria:

- *Orientation*: the items may have a fixed orientation, i.e., they must be packed with their *w*-edge (resp. *h*-edge) parallel to the *W*-edge (resp. *H*-edge) of the loading surface, or they can be rotated by 90°;
- *Sequential loading*: it may be imposed that the loading pattern of each vehicle is such that the demanded items of each customer can be downloaded through a sequence of a single straight movement per item, parallel to the *H*-edge of the loading area. This version is denoted as *sequential loading* (or *rear loading*, or *LIFO*), while the version in which this is not imposed is usually called *unrestricted*.

Note that the sequential loading constraint imposes that the strip going from the *w*-edge of any item demanded by a customer to the rear of the vehicle cannot contain any (portion of) item demanded by the customers that are visited later in the same route.

M. Iori. Metaheuristic algorithms for combinatorial optimization problems. *4OR*, 3:163-166, 2005.
This is the summary of Iori's PhD thesis, that contains exact and heuristic algorithms for some 2L-CVRPs as well as a set of benchmark instances, available on line at http://www.or.deis.unibo.it/research.html

M. Iori, J.J. Salazar Gonzalez, and D. Vigo. An exact approach for the vehicle routing problem with two-dimensional loading constraints. *Transportation Science*, 41:253-264, 2007.
This is the first exact algorithm for the sequential 2L-CVRP with integer edge costs. It also handles two additional restrictions that sometimes appear in the CVRP literature: (i) all *K* vehicles must be used, and (ii) no one-customer route is allowed. The algorithm is based on a branch-and-cut approach that uses valid inequalities to remove infeasible loading sequences. The feasibility of a loading pattern is evaluated through heuristics and a nested branch-and-bound procedure.

M. Gendreau, M. Iori, G. Laporte, and S. Martello. A tabu search heuristic for the vehicle routing problem with two-dimensional loading constraints. *Networks*, 51:4-18, 2007.
This is the first metaheuristic (Tabu search) algorithm for the 2L-CVRP. It handles both the sequential and the unrestricted version. The algorithm can accept moves producing infeasible routes (either because of excessive weight, or because of loading patterns exceeding the height of the loading surface).

G. Fuellerer, K.F. Doerner, R. Hartl, and M. Iori. Ant colony optimization for the two-dimensional loading vehicle routing problem. *Computers & Operations Research*, 36:655-673, 2009.
This *Ant Colony Optimization* (ACO) algorithm is initialized with a population of ants, each of which searches for a low-cost feasible solution through a generalization of the classical savings algorithm. The loading feasibility is checked through lower bounds, heuristics, and a truncated branch-and-bound.

E.E. Zachariadis, C.D. Tarantilis, and C.T. Kiranoudis. A guided tabu search for the vehicle routing problem with two-dimensional loading constraints. *European Journal of Operational Research*, 195:729-743, 2009.
This Tabu search algorithm makes use of a guiding mechanism that drives the search towards highly diversified solution areas. The information on the generated routes is stored in hash maps to avoid useless re-executions.

J. Strodl, K.F. Doerner, F. Tricoire, and R.F. Hartl. On index structures in hybrid meta-heuristics for routing problems with hard feasibility checks: An application to the 2-dimensional loading vehicle routing problem. In M.J. Blesa, C. Blum, G. Raidl, A. Roli, and M. Sampels, editors, *Hybrid Metaheuristics*, volume 6373 of *Lecture Notes in Computer Science*, pages 160-173. Springer Berlin Heidelberg, 2010.
A *Variable Neighborhood Search* (VNS) in which a parametric cross-exchange neighborhood is used to perturbate the current solution, and a standard two-opt local search determines a local optimum.

S.C.H. Leung, J. Zheng, D. Zhang, and X. Zhou. Simulated annealing for the vehicle routing problem with two-dimensional loading constraints. *Flexible Services and Manufacturing Journal*, 22:61-82, 2010.
S.C.H. Leung, X. Zhou, D. Zhang, and J. Zheng. Extended guided tabu search and a new packing algorithm for the two-dimensional loading vehicle routing problem. *Computers & Operations Research*, 38:205-215, 2011.
Two metaheuristic algorithms that use a fitness-based heuristic for producing feasible packing patterns. The former one explores the solution space through three different neighborhoods, while the latter is based on a guided local search.

C. Duhamel, P. Lacomme, A. Quilliot, and H. Toussaint. A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. *Computers & Operations Research*, 38:617-640, 2011.
This heuristic starts with a *Greedy Randomized Adaptive Search Procedure* (GRASP) that generates a single-tour solution, and splits it into separate routes. The resulting routes are then optimized through a combination of genetic tools (mutation) and local search.

Y. Shen and T. Murata. Pick-up scheduling of two-dimensional loading in vehicle routing problem by using GA. In *Proceedings of the International MultiConference of Engineers and Computer Scientists, IMECS 2012*, Vol. II, pages 1532-1537, Hong Kong, 2012.
A standard genetic algorithm is presented. No computational comparison with previous algorithms is provided.

3.1 Variants

For a number of relevant classic variants of the CVRP (time windows, heterogeneous fleet, ...), the addition of (variants of) two-dimensional loading constraints has been considered in the recent literature.

S. Khebbache-Hadji, C. Prins, A. Yalaoui, and M. Reghioui. Heuristics and memetic algorithm for the two-dimensional loading capacitated vehicle routing problem with time windows. *Central European Journal of Operations Research*, 21:307-336, 2013.
In the variant considered here each customer i $(i = 1, 2, ..., n)$ has a time window $\left[a_i, b_i\right]$: the vehicle assigned to the customer must deliver the goods not earlier than a_i and not later than b_i. The paper provides some constructive heuristics, and a genetic algorithm.

S.C.H. Leung, Z. Zhang, D. Zhang, X. Hua, and M.K. Lim. A meta-heuristic algorithm for heterogeneous fleet vehicle routing problems with two-dimensional loading constraints. *European Journal of Operational Research*, 225:199-210, 2013.
In this case, the K vehicles are not identical. There are different types of vehicles with different capacity and different length and width. According to the specific application, the cost associated with the use of a vehicle can be fixed or variable. The simulated annealing algorithm presented in this paper makes use of a heuristic local search to improve the solution found. In addition, a number of heuristic packing algorithms are used to handle the loading constraints.

K. Hamdi-Dhaoui, N. Labadie, and A. Yalaoui. Memetic algorithm with population management for the two-dimensional loading vehicle routing problem with partial conflicts. In A.C. Rosa, A.D. Correia, K. Madani, J. Filipe, and J. Kacprzyk, editors, *IJCCI 2012 - Proceedings of the 4th International Joint Conference on Computational Intelligence*, SciTePress, pages 189-195, 2012.
In this case, some pairs of items may be incompatible, i.e., they cannot be transported by the same vehicle. Typical applications arise in the transportation of hazardous materials. The paper solves the problem with a genetic algorithm.

L. Martínez and C.A. Amaya. A vehicle routing problem with multi-trips and time windows for circular items. *Journal of the Operational Research Society*, 2013. To appear.
The main variant considered in this paper comes from the fact that the transported items are circular. The authors present a constructive heuristic and a Tabu search to improve the solution quality. (The circular items considered in the real world application are paella pans ☺.)

4. CAPACITATED VEHICLE ROUTING WITH THREE-DIMENSIONAL LOADING

The *Capacitated Vehicle Routing Problem with Three-Dimensional Loading Constraints* (3L-CVRP) is the following generalization of the CVRP (see Section 1). In this case, the total weight di of the demand of customer i $(i = 1, 2, ..., n)$ is produced by m_i three-dimensional items. Item $I_{i\ell}$ $(\ell = 1, 2, ..., m_i)$ has width $w_{i\ell}$, height $h_{i\ell}$, and length $l_{i\ell}$. The loading surface of each vehicle has width W, height H, and length L. Let $S(k) \subseteq \{1, 2, ..., n\}$ be the set of customers visited by vehicle k. Besides the standard

weight constraint $\sum_{i \in S(k)} d_i \leq D$, a loading constraint imposes that there exists a feasible (non-overlapping) loading of all three-dimensional items requested by each customer of $S(k)$ into the $W \times H \times L$ loading space.

As for the two-dimensional case, a number of additional, real world, constraints appears in the literature:

- *Orientation*: the items may have a fixed orientation, or (more frequently) they can be rotated by 90° on the horizontal plane (while upside-down rotations are usually not allowed);

- *Fragility*: each item $I_{i\ell}$ may have a fragility flag $f_{i\ell}$, taking the value 1 if $I_{i\ell}$ is fragile, and the value 0 otherwise. In this case, an additional constraint imposes that non-fragile items cannot be placed over fragile items;

- *Supporting area*: when an item $I_{i\ell}$ is packed over other items, let $\bar{A}$ be the area of the bottom of $I_{i\ell}$ that touches the items below: the packing is only feasible if $\bar{A} \geq a w_{i\ell} l_{i\ell}$, where a is a given threshold $(0 \leq a \leq 1)$;

- *Sequential loading*: when customer i is visited, there must exist a sequence of straight movements (one per $I_{i\ell}$ item) in the direction of the vehicle's rear, that allows to unload the item without moving any other item. In other words, no item demanded by a customer that is visited later may be placed over $I_{i\ell}$ or between $I_{i\ell}$ and the rear of the vehicle.

M. Gendreau, M. Iori, G. Laporte, and S. Martello. A tabu search algorithm for a routing and container loading problem. *Transportation Science*, 40:342-350, 2006.
The algorithm is based on an outer Tabu search framework, in which, for each generated neighbor solution, the vehicle loads are obtained by invoking an inner Tabu search on a modified 3SPP. If the resulting load exceeds the vehicle length, the solution returned to the outer search is accepted, but penalized accordingly. The algorithm is tested both on randomly generated instances and on real world 3L-CVRP instances provided by a furniture company.

C.D. Tarantilis, E.E. Zachariadis, and C.T. Kiranoudis. A hybrid metaheuristic algorithm for the integrated vehicle routing and three-dimensional container-loading problem. *IEEE Transactions on Intelligent Transportation Systems*, 10:255-271, 2009.
The authors propose an adaptation to the three-dimensional case of their two-dimensional guided Tabu search (see Section 3).

G. Fuellerer, K.F. Doerner, R. Hartl, and M. Iori. Metaheuristics for vehicle routing problems with three-dimensional loading constraints. *European Journal of Operational Research*, 201:751-759, 2010.
This paper presents a generalization to the three-dimensional case of the authors' ACO approach for the two-dimensional problem discussed in Section 3.

Y. Tao and F. Wang. A new packing heuristic based algorithm for vehicle routing problem with three-dimensional loading constraints. In *2010 IEEE International Conference on Automation Science and Engineering, CASE 2010*, pages 972-977, 2010.
The core of the presented approach is a least waste heuristic for the solution of the loading subproblem, which is iteratively invoked by a simple outer Tabu search algorithm.

A. Bortfeldt. A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research*, 39:2248-2257, 2012.
The author presents an efficient hybrid approach based on a Tabu search algorithm for the routing subproblem. At each Tabu search iteration, the generated routes are stored in a list, that is sorted by increasing routing cost. For each solution in the resulting sequence, a heuristic tree search algorithm solves the loading subproblem. Computational experiments show the effectiveness of the proposed approach.

Q. Ruan, Z. Zhang, L. Miao, and H. Shen. A hybrid approach for the vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research*, 40:1579-1589, 2013.
L. Miao, Q. Ruan, K. Woghiren, and Q. Ruo. A hybrid genetic algorithm for the vehicle routing problem with three-dimensional loading constraints. *RAIRO - Operations Research*, 46:63-82, 2012.
These papers present hybrid approaches in which the routing subproblem is solved by a metaheuristic algorithm (bee mating in the former paper, genetic in the latter). At each iteration, the loading subproblem is solved through Tabu search or constructive heuristics, respectively. Computational experiments show that the latter approach is more efficient.

W. Zhu, H. Qin, A. Lim, and L. Wang. A two-stage tabu search algorithm with enhanced packing heuristics for the 3L-CVRP and M3L-CVRP. *Computers & Operations Research*, 39:2178-2195, 2012.
Two packing heuristics from the literature are improved and embedded in a Tabu search algorithm. Computational experiments show the efficiency of the proposed approach.

4.1 Variants

A. Attanasio, A. Fuduli, G. Ghiani, and C. Triki. Integrated shipment dispatching and packing problems: a case study. *Journal of Mathematical Modelling and Algorithms*, 6:77-85, 2007.
A multi-day integrated model for a three-dimensional freight transportation problem is considered. The authors propose a simplified *Integer Linear Programming* (ILP) model, adopting a rolling horizon technique, and a heuristic algorithm for the packing constraints. The method is validated on real world data coming from a chemical company.

A. Moura and J.F. Oliveira. An integrated approach to vehicle routing and container loading problems. *OR Spectrum*, 31:775-800, 2009.

A. Moura. A multi-objective genetic algorithm for the vehicle routing with time windows and loading. In A. Bortfeldt, J. Homberger, H. Kopfer, G. Pankratz, and R. Strangmeier, editors, *Intelligent Decision Support*, pages 187-201. Gabler, Germany, 2008.
The former paper addresses the issue of integrating the CVRP with time windows and the three-dimensional container loading problem, proposing a number of constructive heuristics. The latter paper extends the results to a multi-objective case in which the objective function considers the number of vehicles, the total traveled distance, and the volume utilization. The problem is solved through a genetic algorithm.

G. Koloch and B. Kaminski. Nested vs. joint optimization of vehicle routing problems with three-dimensional loading constraints. *Engineering Letters*, 18:193-198, 2010.
The paper concerns a vehicle routing problem with three-dimensional loading constraints, but without capacity constraints. Two approaches are proposed and compared. The authors do not mention previous results on combined routing and loading.

A. Bortfeldt and J. Homberger. Packing first, routing second - a heuristic for the vehicle routing and loading problem. *Computers & Operations Research*, 40:873-885, 2013.
A two-stage heuristic for the problem considered by Moura and Oliveira (above) is presented. The first stage optimizes the packing, while the second stage takes care of the routing aspect. Computational experiments show the high efficiency of the method.

5. OTHER CAPACITATED VEHICLE ROUTING PROBLEMS WITH REAL WORLD LOADING CONSTRAINTS

In this section we consider some real world CVRPs with additional loading constraints, which are related to the issues considered in Sections 3 and 4. In most cases, the loading aspects of the considered problems have a three-dimensional characterization, but they are solved through reduction to a two-dimensional case. Other (more traditional) vehicle routing problems with additional constraints that are not clearly identified as two- or three-dimensional packing are beyond the scope of this article. For example, vehicle routing problems with multi-compartment loading (where the transported goods are typically liquids of different kinds), that arise in land or sea transportation.

5.1 Multi-pile vehicle routing problems

In transportation problems faced by timber companies, each customer requires a set of chip-boards which may have very different sizes. In the *Multi-Pile Vehicle Routing Problem* (MP-VRP) the chipboards of similar size requested by a customer are preventively palletized, thus producing *short* and *long* pallets, all having the width of the vehicle. The items can be stacked one on top of the other, producing *piles*, but no supporting surface is required as, in the loading phase, "holes" are filled with bulk material. In addition, the constraint on sequential loading (see Section 3) is usually imposed.

K.F. Doerner, G. Fuellerer, M. Gronalt, R. Hartl, and M. Iori. Metaheuristics for vehicle routing problems with loading constraints. *Networks*, 49:294-307, 2007.

F. Tricoire, K.F. Doerner, R. Hartl, and M. Iori. Heuristic and exact algorithms for the multi-pile vehicle routing problem. *OR Spectrum*, 33:931-959, 2011.
The former paper introduces the problem, and presents two metaheuristic approaches: a Tabu search algorithm and an ACO algorithm. The latter paper proposes a combination of VNS and branch-and-cut. The neighborhood of the VNS phase is based on *cross-exchanges*, while the branch-and-cut algorithm relies on the classical two-index model of the CVRP.

F. Massen, Y. Deville, and P. Hentenryck. Pheromone-based heuristic column generation for vehicle routing problems with black box feasibility. In N. Beldiceanu, N. Jussien, and E. Pinson, editors, *Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems*, volume 7298 of *Lecture Notes in Computer Science*, pages 260-274. Springer Berlin Heidelberg, 2012.
This paper proposes an abstraction of the typology of vehicle routing problems treated in the present bibliography: the *Vehicle Routing Problem with Black Box Feasibility* (VRPBB). The black box is a function that tests the feasibility of a proposed route, hence it can be used to model any set of loading constraints. The authors propose a column generation approach for determining a locally optimal solution. The column generation is guided by an ACO construction heuristic. The approach is tested both on the 3L-CVRP and the MP-VRP.

5.2 Auto-carrier transportation problems

The *Auto-Carrier Transportation Problem* (ACTP) combines the traditional routing problem and the loading of vehicles into the auto-carrier platforms. The loading aspect is particularly difficult, as it involves the shapes of the loaded goods (cars, trucks, etc), which can be very irregular and non-convex.

R. Tadei, G. Perboli, and F. Della Croce. A heuristic algorithm for the auto-carrier transportation problem. *Transportation Science*, 36:55-62, 2002.
The paper considers the real world case of an Italian vehicle transportation company, and provides a heuristic approach based on an ILP formulation. The considered problem involves a multiple-day delivery plan, and requires the maximization of the overall profit. The loading subproblem is relaxed and heuristically solved. The routing subproblem is attacked by an ILP in which all possible destinations are grouped into clusters, followed by a local search refinement.

B.M. Miller. *Auto Carrier Transporter Loading and Unloading Improvement*. PhD thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, 2003.
This thesis considers an ACTP arising in the U.S. market, and develops a greedy heuristic followed by inter-route and intra-route local search optimization. A number of relaxations of the constraints is introduced to handle the computational difficulty of the problem.

B.O. Øystebø, L.M. Hvattum, and K. Fagerholt. Routing and scheduling of RoRo ships with stowage constraints. *Transportation Research Part C: Emerging Technologies*, 19:1225-1242, 2011.

An ACTP arising in the international ship transportation of vehicles is modeled as an MILP.

6. PICKUP AND DELIVERY PROBLEMS WITH LOADING CONSTRAINTS

In transportation problems with *Pickup and Delivery* constraints, each customer is associated with an origin location, where a demand (characterized by its weight) must be picked up, and a destination, where such a demand must be delivered. The interested reader is referred to the following recent surveys.

G. Berbeglia, J.-F. Cordeau, I. Gribkovskaia, and G. Laporte. Static pickup and delivery problems: A classiffication scheme and survey. *TOP*, 15:1-31, 2007.
This paper presents a general framework for modeling a large number of pickup and delivery problems, and classifies them according to three main categories. In *many-to-many* problems, each demand may have multiple origins and/or multiple destinations. In *one-to-many-to-one problems*, some demands must be delivered from a depot to many customers, while other demands must be collected at the customers and transported back to the depot. In *one-to-one* problems, each demand has a single origin and a single destination between which it must be transported.

S.N. Parragh, K.F. Doerner, and R.F. Hartl. A survey on pickup and delivery models. Part I: transportation between customers and depot. *Journal für Betriebswirtschaft*, 58:21-51, 2008.
S.N. Parragh, K.F. Doerner, and R.F. Hartl. A survey on pickup and delivery models. Part II: transportation between pickup and delivery locations. *Journal für Betriebswirtschaft*, 58:81-117, 2008.
A comprehensive survey that classifies pickup and delivery problems according to two classes, whose characterization is clear from the titles.

In the next section we examine results in which the two- and three-dimensional loading constraints play a consistent role, while we collectively list contributions in which the loading aspects only appear as sequencing constraints.

6.1 Vehicle routing problems with pickup and delivery, and multi-dimensional loading constraints

The following papers deal with problems in which the demands are characterized as sets of two-or three-dimensional items that have to be transported from a single origin to a single destination.

A. Malapert, C. Guerét, N. Jussien, A. Langevin, and L.-M. Rousseau. Two-dimensional pickup and delivery routing problem with loading constraints. In *Proceedings of the First CPAIOR Workshop on Bin Packing and Placement Constraints (BPPC'08)*, Paris, France, 2008.
A constraint programming model is proposed for the 2L-CVRP with pickup and delivery.

T. Bartók and C. Imreh. Pickup and delivery vehicle routing with multidimensional loading constraints. *Acta Cybernetica*, 20:17-33, 2011.
This paper introduces a CVRP with pickup and delivery and three-dimensional loading constraints, and proposes a heuristic algorithm for its solution.

E.E. Zachariadis, C.D. Tarantilis, and C.T. Kiranoudis. The pallet-packing vehicle routing problem. *Transportation Science*, 46:341-358, 2012.
In the basic problem considered in this paper, the transported three-dimensional goods are preventively palletized, and the pallets are then packed into the vehicles, subject to time windows constraints. In a more complex version, pickup and delivery is also imposed. For both problems, the authors present a Tabu search approach, enhanced by the use of hash tables.

6.2 Traveling salesman problems with pickup and delivery, and sequencing constraints on the load

In the classical TSP with pickup and delivery the sequence in which the items are loaded and unloaded from the vehicle is irrelevant. The recent literature considers cases where the reshuffle of the load is impossible, or has a relevant cost. Contributions in this field may be classified according to four types of loading/unloading policies, corresponding to the four groups of papers below. The first three groups are related to transportation from a single origin to a single destination, while the fourth group concerns one-to-many-to-one transportation.

S.P. Ladany and A. Mehrez. Optimal routing of a single vehicle with loading constraints. *Transportation Planning and Technology*, 8:301-306, 1984.
J. Pacheco. Heuristico para los problemas de ruta con carga y descarga en sistemas LIFO. *SORT, Statistics and Operations Research Transactions*, 21:153-175, 1997.
K. Levitin and R. Abezgaouz. Optimal routing of multiple-load AGV subject to LIFO loading constraints. *Computers & Operations Research*, 30:397-410, 2003.
H. Xu, Z.-L. Chen, S. Rajagopal, and S. Arunapuram. Solving a practical pickup and delivery problem. *Transportation Science*, 37:347-364, 2003.
F. Carrabs, J.-F. Cordeau, and G. Laporte. Variable neighbourhood search for the pickup and delivery traveling salesman problem with LIFO loading. *INFORMS Journal on Computing*, 19:618-623, 2007.
F. Carrabs, R. Cerulli, and J.-F. Cordeau. An additive branch-and-bound algorithm for the pickup and delivery traveling salesman problem with LIFO or FIFO loading. *INFOR*, 45:223-238, 2007.
J.-F. Cordeau, M. Iori, G. Laporte, and J.J. Salazar González. Branch-and-cut for the pickup and delivery traveling salesman problem with LIFO loading. *Networks*, 55:46-59, 2010.
Y. Li, A. Lim, W.-C. Oon, H. Qin, and D. Tu. The tree representation for the pickup and delivery traveling salesman problem with LIFO loading. *European Journal of Operational Research*, 212:482-496, 2011.
The problems treated in these eight papers arise in cases where the vehicle has a single access point (usually the rear) and the cargo (typically consisting of heavy, or fragile, or hazardous materials) cannot be reshuffled along the tour. It follows that the last loaded

demand is the only one that can be unloaded at the next stop, i.e., the operations must be performed in Last-In First-Out (LIFO) order.

G. Erdoğan, J.-F. Cordeau, and G. Laporte. The pickup and delivery traveling salesman problem with first-in-first-out loading. *Computers & Operations Research*, 36:1800-1808, 2009.
J.-F. Cordeau, M. Dell'Amico, and M. Iori. Branch-and-cut for the pickup and delivery traveling salesman problem with FIFO loading. *Computers & Operations Research*, 37:970-980, 2010.
Both papers address the variant (arising, e.g., in dial-a-ride passenger transportation systems), in which the loading/unloading operations must be performed following a First-In First-Out (FIFO) policy.

H.L. Petersen and O.B.G. Madsen. The double travelling salesman problem with multiple stacks - Formulation and heuristic solution approaches. *European Journal of Operational Research*, 198:139-147, 2009.
A. Felipe, M.T. Ortuño, and G. Tirado. The double traveling salesman problem with multiple stacks: A variable neighborhood search approach. *Computers & Operations Research*, 36:2983-2993, 2009.
A. Felipe, M.T. Ortuño, and G. Tirado. New neighborhood structures for the double traveling salesman problem with multiple stacks. *TOP*, 17:190-213, 2009.
S. Toulouse and R. Wolfer Calvo. On the complexity of the multiple stack TSP, kSTSP. *Lecture Notes in Computer Science, Theory and Applications of Models of Computation*, 5532:360-369, 2009.
R.M. Lusby, J. Larsen, M. Ehrgott, and D. Ryan. An exact method for the double TSP with multiple stacks. *International Transactions on Operations Research*, 17:637-652, 2010.
H.L. Petersen, C. Archetti, and M.G. Speranza. Exact solutions to the double travelling salesman problem with multiple stacks. *Networks*, 56:229-243, 2010.
R.M. Lusby and J. Larsen. Improved exact method for the double TSP with multiple stacks. *Networks*, 58:290-300, 2011.
A. Felipe, M.T. Ortuño, and G. Tirado. Using intermediate non-feasible solutions to approach vehicle routing problems with precedence and loading constraints. *European Journal of Operational Research*, 211:66-75, 2011.
F. Bonomo, S. Mattia, and G. Oriolo. Bounded coloring of co-comparability graphs and the pickup and delivery tour combination problem. *Theoretical Computer Science*, 412:6261-6268, 2011.
M. Casazza, A. Ceselli, and M. Nunkesser. Efficient algorithms for the double travelling salesman problem with multiple stacks. *Computers & Operations Research*, 39:1044-1053, 2012.
F. Carrabs, R. Cerulli, and M.G. Speranza. A branch-and-bound algorithm for the double travelling salesman problem with two stacks. *Networks*, 61:58-75, 2013.
M.A. Alba Martínez, J.-F. Cordeau, M. Iori, and M. Dell'Amico. A branch-and-cut algorithm for the double traveling salesman problem with multiple stacks. *INFORMS Journal on Computing*, 25: 41-55, 2013.

J.-F. Côté, M. Gendreau, and J.-Y. Potvin. Large neighborhood search for the single vehicle pickup and delivery problem with multiple loading stacks. *Networks*, 60:19-30, 2012.
J.-F. Côté, C. Archetti, M.G. Speranza, M. Gendreau, and J.-Y. Potvin. A branch-and-cut algorithm for the pickup and delivery traveling salesman problem with multiple stacks. *Networks*, 60:212-226, 2012.

These fourteen papers deal with the *double traveling salesman problem with multiple stacks*, in which the load is structured in a number of stacks, each of which must obey a LIFO loading/unloading policy. In addition, it is usually imposed that all pickups be completed before any delivery can occur, and that pickups and deliveries be performed in two separate routes. The problem is to find the two routes and the stacking plan that minimize the total transportation costs. The last two papers deal however with the variant in which pickups and deliveries may be performed in mixed order.

M. Battarra, G. Erdoğan, G. Laporte, and D. Vigo. The traveling salesman problem with pickups, deliveries, and handling costs. *Transportation Science*, 44:383-399, 2010.
G. Erdoğan, M. Battarra, G. Laporte, and D. Vigo. Metaheuristics for the traveling salesman problem with pickups, deliveries and handling costs. *Computers & Operations Research*, 39:1074-1086, 2012.

The problem treated in these two papers concerns the case where one commodity has to be delivered from the depot to the customers, and another commodity has to be picked up at the customers and returned to the depot. A LIFO policy should be followed, but reshuffling of the cargo is allowed at the expenses of an additional cost.

7. CONCLUSIONS AND OPEN PERSPECTIVES

We have classified and examined a bibliography on vehicle routing problems with two- and three-dimensional loading constraints, a recent and challenging research area in transportation science. An evident proof of the interest of researchers is the fact that about 60% of the reviewed papers appeared in the two thousand and tens. We are hoping that this annotated bibliography will stimulate further research in this lively and active field.

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