Yugoslav Journal of Operations Research
23 (2013) Number 3, 343-354
DOI:10.2298/YJOR120119016A

STABILITY OF MULTI-OBJECTIVE BI- LEVEL LINEAR PROGRAMMING PROBLEMS UNDER FUZZINESS

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Received: January 2012 / Accepted: August 2012

Abstract: This paper deals with multiobjective bi-level linear programming problems under fuzzy environment. In the proposed method, tentative solutions are obtained and evaluated by using the partial information on preference of the decision-makers at each level. The existing results concerning the qualitative analysis of some basic notions in parametric linear programming problems are reformulated to study the stability of multiobjective bi-level linear programming problems. An algorithm for obtaining any subset of the parametric space, which has the same corresponding Pareto optimal solution, is presented. Also, this paper established the model for the supply-demand interaction in the age of electronic commerce (EC). First of all, the study uses the individual objectives of both parties as the foundation of the supply-demand interaction. Subsequently, it divides the interaction, in the age of electronic commerce, into the following two classifications: (i) Market transactions, with the primary focus on the supply demand relationship in the marketplace; and (ii) Information service, with the primary focus on the provider and the user of information service. By applying the bi-level programming technique of interaction process, the study will develop an analytical process to explain how supply-demand interaction achieves a compromise or why the process fails. Finally, a numerical example of information service is provided for the sake of illustration.

Keywords: Multi-objective bi-level linear programming problems; partial information of preference; stability.

MSC: 90C29, 90C08.

1. INTRODUCTION

Real decision making problems in which there are multiple decision makers (DMs), who make decisions successively, have been often formulated as multi-level programming problems [2,5,6,9]. Assuming that each DM makes a decision without any communication with some other DMs, as a solution concept to the problems, Stackeberg solutions have been employed [1,3,4,16,17,18]. However, for decision-making problems in decentralized firms, it is quite natural to assume that there exists communication and some cooperative relationship among the DMs.

Recently, for multi-level linear programming problems, Lai [7] and Shih, Lai and Lee [15] proposed solution concepts assuming cooperative communication among the DMs. Their methods are based on the idea that the DM at the lower level optimizes his or her objective function, taking a goal or preference of the DM at the upper level into consideration. The DMs identify membership functions of fuzzy goals for their objective function, and especially, the DM at the upper level also specifies those of fuzzy goals for decision variables. The DM at the lower level solves a fuzzy programming problem with a constraint on a satisfactory degree of the DM at the upper level. Unfortunately, however, there is a possibility that their methods produce an undesirable final solution because of inconsistency between the fuzzy goals of the objective function and the decision variables.

In this paper, we deal with the bi-level linear programming problem in multiobjective environments, and assume that the DMs at the upper level and at the lower level have own fuzzy goals with respect to their multiple objective functions, and also that they have partial information on their preferences [13].

Osman [10,11] introduced the notions of the solvability set, stability set of the first kind and the stability set of the second kind, and analyzed these concepts for parametric convex linear programming problems. The relation to and importance of these results are presented in [12]. In this paper, qualitative analysis of the stability for multiobjective bi-level linear programming problems is presented. Also, an algorithm for obtaining the subset of the parametric space which has the same corresponding Pareto optimal solution is introduced, and aspects of supply-demand interaction in the EC environment are analyzed. Firstly, based on the conceptual model proposed in[14], our analysis will be conducted on the following two kinds of supply-demand interaction: (i) the interaction between the user and the provider of the information services and (ii) the buyer-supplier interaction occurring in the market transaction. Secondly, the bi-level programming technique will be utilized to model the aforementioned interactions. Finally, numerical example is given to clarify the developed algorithm.

2. PROBLEM FORMULATION

In this paper, we deal with bi-level linear programming problems in which each DM has multiple objective functions to be minimized and the two DMs can determine their decisions cooperatively. Such a multi-objective bi-level linear programming problem is formally formulated as [13]:

$$\begin{aligned}
&\underset{\text{upper level}}{\text{minimize}}\ z_i^1(x_1,x_2)=c_{1i}^1x_1+c_{2i}^1x_2,\quad i=1,\dots,k_1\\
&\underset{\text{lower level}}{\text{minimize}}\ z_i^2(x_1,x_2)=c_{1i}^2x+c_{2i}^2x_2,\quad i=1,\dots,k_2
\end{aligned} \tag{1}$$

Subject to:

$$A_1x_1+A_2x_2\le b,$$
$$x_1,x_2\ge 0.$$

where x_1 and x_2 are n_1- and n_2-dimensional decision variable vectors of the DM*s* at the upper level and the lower level, respectively; $z_i^1(x_1,x_2), i=1,\dots,k_1$ and $z_i^2(x_1,x_2), i=1,\dots,k_2$ are the *i*th objective function of the DMs at the upper level and lower level, respectively; c_{ij}^1 and c_{ij}^2 are n_i- dimensional constant row vectors; $A_i, i=1,2$ are $m\times n_i$ matrices; and b is an $m-$ dimensional constant column vector.

In the following, for compact notation, we use $x=(x_1^T,x_2^T)\in R^{n_1+n_2}$ and denote the feasible region satisfying all of the constraints of problem (1) by *X*. Also, for notational convenience, let DM1 denote the DM at the upper level and DM2 denote the DM at the lower level.

In a minimization problem, the i -th fuzzy goal stated by DM*j* may be to achieve " substantially less than or equal to some value p_i^j ". This type of statement can be quantified by eliciting a corresponding membership function $\mu_i^j(z_i^j(x))$, which is a strictly monotone decreasing function with respect to $z_i^j(x)$.

To elicit a membership function $\mu_i^j(z_i^j(x))$ from DM*j* for each of the objective functions $z_i^j(x)$ in problem (1), we first calculate the following individual minimum and maximum of each objective function under the given constraints:

$$z_i^{j\min}=\min_{x\in X} z_i^j(x), i=1,\dots,k_j, j=1,2 \tag{2}$$

$$z_i^{j\max}=\max_{x\in X} z_i^j(x), i=1,\dots,k_j, j=1,2 \tag{3}$$

The following linear function is an example of the membership function $\mu_i^j(z_i^j(x))$, which characterizes the fuzzy goal for the *i*th objective function of DM*j*.

$$\mu_i^j(z_i^j(x)) = \begin{cases} 0 & if\ z_i^j(x) > z_i^{j0} \\ \dfrac{z_i^j(x) - z_i^{j0}}{z_i^{j1} - z_i^{j0}} & if\ z_i^{j1} < z_i^j(x) \le z_i^{j0} \\ 1 & if\ z_i^{j1} \ge z_i^j(x) \end{cases} \tag{4}$$

where z_i^{j0} and z_i^{j1} denote the value of the objective function $z_i^j(x)$ such that the degrees of membership function are 0 and 1, respectively.

An optimal solution to problem (2) can be denoted *by* x^{ij} and by using the individual minimum together with

$$z_i^{jm} = \max\ (z_i^j(x^{ij})), i = 1,...,k_j\ , j = 1, 2, \tag{5}$$

DM*j* determines the linear membership function as in (4) by choosing $z_i^{jm} = z_i^{j0}$ and $z_i^{j\min} = z_i^{j1}$.

We assume that each DM evaluates a solution x by aggregating the weighted

membership functions additively, and the aggregated membership function of DMj is represented as:

Where $\lambda = \left(\lambda_1^j, ..., \lambda_{k_j}^j\right)$ denotes a weighting coefficient vector

such that $\sum_{i=1}^{k_j} \lambda_i^j = 1$ and $\lambda_i^j \ge 0,\ i = 1,..., k_j$.

$$\sum_{i=1}^{k_j} \lambda_i^j\ \mu_i^j(z_i^j(x)), \tag{6}$$

Moreover, if each DM cannot identify the weighting coefficients and has some partial information on his preference, such that it can be represented by the following two inequalities with respect to DMj:

$$LB_i^j \le \lambda_i^j \le UB_i^j, \tag{7}$$

$$\lambda_p^j + \varepsilon \ge \lambda_q^j\ , p \ne q \tag{8}$$

Where ε is a small positive constant or a zero.

The upper bound UB_i^j and the lower bound LB_i^j are specified for the weight λ_i^j to the membership function $\mu_i^j(z_i^j(x))$ of the fuzzy goal for the *i*th objective function as in condition (7) but condition (8) represents as order relation between *p*th and *q*th fuzzy goals. Let Λ^j denote a set of weighting coefficient vectors $\lambda^j = \left(\lambda_1^j, ..., \lambda_{k_j}^j\right)$ of DMj satisfying condition (7) and (8).

Problem (1) will be treated using one of the existing parametric approaches, i.e. by considering the following linear program scalar objective.

(p_λ)

$$\text{minimize } \sum_{i=1}^{k_j} \lambda_i^j \, \mu_i^j (z_i^j(x))$$

Subject to :

$$G(x) = A_1 x_1 + A_2 x_2 - b \le 0$$

$$\text{for some } \lambda \in \Lambda = \{\lambda \in R^{k_j} \mid \sum_{i=1}^{k_j} \lambda_i^j = 1, \lambda_i^j \ge 0\}.$$

It is easy to see that the stability of problem (1) implies the stability of (p_λ). It is well known that (x^o) is a Pareto optimal solution of problem (1) if there exists $\lambda^o \ge 0, \lambda^o \ne 0$ such that x^o is the unique optimal solution iff there exists $\lambda^o = (\lambda_1^o, ..., \lambda_{k_j}^o) > 0,$ provided every $x \in X$ is closed and convex.

3.THE STABILITY SET OF THE FIRST KIND

Definition 1. The solvability set of problem (pλ) denoted by B, is defined by

$$B = \left\{ \lambda \in R_+^{k_j} \,\middle|\, \min_{x \in X} \sum_{i=1}^{k_j} \lambda_i^j \, \mu_i^j (z_i^j(x)) \text{ exists} \right\},$$

where $R_+^{k_j}$ denotes the nonnegative orthant of the R^m vector space of parameters.

Definition 2. Suppose that (p_λ) is solvable at $\lambda^o \in \Lambda$ with a corresponding Pareto optimal solution x^o. Then the stability set of the first kind of problem (1) corresponding to x^o, denoted by $S(x^o)$, is defined by Osman [10,11]

Determination of the stability set of the first kind $S(x^o) = \{\lambda \in \Lambda \mid x^o \text{ is a pareto optimal solution for problem (1)}\}$.

If a point $x^o \in X$ is a Pareto optimal solution of problem (1), then there exists $\lambda^o \in \Lambda$ such that x^o is a Pareto optimal solution of (p_λ^o). Therefore from the stability of problem (p_λ), it follows that there exists $\mu \in R^m, \mu \ge 0,$ such that Mangasarian [8].

$$\lambda^T \frac{\partial \mu_i^j (z_i^j (x^o))}{\partial x} + \mu^T \frac{\partial G(x^o)}{\partial x} = 0, \; G(x^o) \le 0, \; \mu^T G(x^o) = 0, \tag{9}$$

Let the set of active constraints at x^o be denoted by

$$A(x^o) = \left\{ n \,\middle|\, G_n(x_1^o, x_2^o) = 0 \right\}.$$

Then the linear independent system of equations

$$\lambda^T \frac{\partial \mu_i^j(z_i^j(x^o))}{\partial x} + \sum_{n \in A(x^o)} \mu_n \frac{\partial G_n}{\partial x}(x^o) = 0$$

can be written in the following matrix form:

$$\begin{bmatrix} C' & D' \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = 0 \tag{10}$$

where $C' = [c'_{ai}]$ is an $s \times k_j$ matrix, $D' = [d'_{ai}]$ is an $s \times k$ matrix, $\lambda \in R^{k_j}, \mu \in R^k, \lambda \geq 0; \lambda \neq 0$ and $\mu \geq 0$, where s is the cardinalities of $A(x^o)$ and k is the cardinal number of the set $A(x^o)$.

Suppose $d'_{ai} = 0, i = 1,...,k, a \in I \subset \{1,...,s\}$, where the cardinal number of I is assumed to be equal to $s - l$.

Then, we ignore for the moment these rows and consider the remaining system, which will have the form

$$\begin{bmatrix} C & D \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = 0 \tag{11}$$

Here C and D are matrices of order $l \times k_j$ and $l \times k$, respectively. Therefore, system (11) together with the condition

$$\sum_{i=1}^{k_j} C'_{ai} \lambda_i^j = 0, \quad a \in I \tag{12}$$

gives system (10), which is equivalent to system (9). Hence, we give the following two propositions Osman [10,11].

Proposition 1. If $k \geq l$, then

$$S(x^o) = \left\{ \lambda \in R^{k_j} \,\middle|\, \left(\lambda^T C^T \left(D_1^T \right)^{-1} \right)_i \leq 0, i = 1,....,l, \sum_{i=1}^{k_j} C'_{ai} \lambda_i^j = 0, a \in I \right\}, \tag{13}$$

where $[D_1 \;\; D_2]$, D_1 and D_2 are respectively $l \times l$ and $l \times k - l$ matrices.

Proposition 2. If $k < l$, then

$$S(x^o) = \left\{ \lambda \in R^{k_j} \left| \begin{array}{l} \left(\lambda^T C_2^T - C_1^T \left(D_1^T\right)^{-1} D_2^T \right)_i = 0, i = 1,...,k-l, \\ \left(\lambda^T C_1^T \left(D_1^T\right)^{-1} \right)_i \leq 0, i = 1,...,k, \sum_{i=1}^{k_j} C'_{ai} \lambda_i^j = 0,\ a \in I \end{array} \right. \right\} \tag{14}$$

4. THE ALGORITHM

Now, we can construct an algorithm for decomposing the parametric space in multi-objective bi-level linear programming (MOBLP) problems, which is considered in this paper as:

Step 1. Elicit the partial information of DM1's and DM2's preference

Step 2. Elicit the membership functions $\mu_i^1(z_i^1), i = 1,...,k_1$ *of the fuzzy goal from DM1. Similarly, elicit the membership functions* $\mu_i^2(z_i^2), i = 1,...,k_2$ *of the fuzzy goal from DM2.*

Step 3. Start with $\lambda^o \in \Lambda$ *and using the fuzzy programming for multiobjective bi-level programming problem for solving* (p_λ^o)*, we obtain a Pareto optimal solution* x^o *of problem (1).*

Step 4. Substituting in the Kuhn-Tuker condition, we obtain system (9).

Step 5. At the end of step 4, system (11) can be easily found.

Determine the set *I.*

Step 6. If $k \geq l$*, then* $S(x^o)$ *is given by (13), and if* $k < l$*, then* $S(x^o)$ *is given by (14).*

To illustrate the above algorithm, consider the following example.

5. AN EXAMPLE

In this example, we have established DM1 as a Company A (the user of information service) whose manpower participation is of two types (the variables it controls) divided into x_{11} and x_{12}. And a Company B is the DM2 (the information service provider), it also provides two types of manpower participation: represented as x_{21} and x_{22}. The man-hour is a unit of manpower. Both DM1 and DM2 have two objective functions. Speaking from the minimum objective, both DM1 and DM2 wish that manpower costs be minimized. Each variable's prior coefficient is its manpower cost per unit where a single unit is represented by 1000 dollars. Especially worth mentioning is that, every unit of x_{22} personnel that DM2 employs is eligible to receive governmental financial assistance, and in this way it's costs will be negative. From the perspective of the maximum objective, both DM1 and DM2 wish that the system had the highest level of quality. DM1's considerations involve the greater use of the personnel (represented by x_{11} and x_{12}) in order to sufficiently respond to the requirements of his information system. DM2, from the perspective of achieving the highest level of quality, considers the greater use of the pecialized technicians (x_{21}) as necessary for achieving the greatest system effectiveness. In regards to constraints, there are two types: resource constraints and quality guarantee constraints. The resource constraint type is of five kinds: man-hour, working space, computing resource, administrative resource, and other supporting resources. Quality guarantee constraints are to be approved by both parties. In order to maintain a defined system of development quality, every type of personnel combination must satisfy the time constraints. Combining all the above elements, this problem can be presented using the following bi-level multiple objectives programming problem [14]:

$$\underset{\text{upper level}}{\text{maximum}} \; z_1^1(x_1, x_2) = x_{11} + x_{12} \tag{15}$$

$$x_{11} \le 530$$

$$\underset{\text{upper level}}{\text{minimize}} \; z_2^1(x_1, x_2) = 1.4\,x_{11} + 0.9\,x_{12} + 0.8\,x_{21} + 0.3\,x_{22}$$

$$\underset{\text{lower level}}{\text{maximum}} \; z_1^2(x_1, x_2) = x_{21}$$

$$\underset{\text{lower level}}{\text{minimize}} \; z_2^2(x_1, x_2) = 0.4\,x_{11} + 0.2\,x_{12} + 1.8\,x_{21} - 0.2\,x_{22}$$

$$x_{12} \le 690$$

Subject to:

$x_{21} \le 430$ (man-hour available per month)

$x_{22} \le 550$ $2.2x_{11} + 1.9x_{12} + 1.8x_{21} + 1.3x_{22} \le 3400$ (working space)

$0.8x_{11} + 0.5x_{12} + 2.5x_{21} + 1.2x_{22} \le 2350$ (computing resource)

$2x_{11} + 3.5x_{12} + 1.4x_{21} + 2.2x_{22} \le 4000$ (administrative resource)

$1.5x_{11} + 0.8x_{12} + 3.3x_{21} + 1x_{22} \le 2750$ (other supporting resource)

$x_{11} + x_{12} \ge 800$

$x_{21} + x_{22} \ge 650$ (quality guarantee constraints)

$x_{11} + x_{21} \ge 700$

$x_{12} + x_{22} \ge 950$

$x_{11}, x_{12}, x_{21}, x_{22} \ge 0$

Where $x_1 = (x_{11}, x_{12})^T, x_2 = (x_{21}, x_{22})^T$.

Solution:

i) Suppose that the partial information of DM1's preference can be represented by the set

$\Lambda^1 = \left\{ \lambda^1 \in R^2 \;\; \middle| \lambda_1^1 \geq \lambda_2^1,\, 0.2 < \lambda_2^1 \right\}$. Consider $\lambda_2^1 = 0.3$ and $\lambda_1^1 = 0.7$.

And the partial information on DM2's preference can be represented by the set By using (2) and (5), we can get:

$$z_1^{1\,\min} = -1216.316, \qquad z_1^{1m} = -800$$
$$z_2^{1\,\min} = 1325, \qquad z_2^{1m} = 1684.75$$
$$z_1^{2\,\min} = -430, \qquad z_1^{2m} = -170$$
$$z_2^{2\,\min} = 488, \qquad z_2^{2m} = 998.4.$$

Then the membership functions of these objective functions can be written as:

$\Lambda^2 = \left\{ \lambda^2 \in R^2 \;\; \middle| \lambda_1^2 \geq \lambda_2^2,\; 0.2 < \lambda_2^2 \right\}$. Consider $\lambda_2^2 = 0.3, \lambda_1^2 = 0.7$.

$$z_1^{1\,\min} = -1216.316, \qquad z_1^{1m} = -800$$
$$z_2^{1\,\min} = 1325, \qquad z_2^{1m} = 1684.75$$
$$z_1^{2\,\min} = -430, \qquad z_1^{2m} = -170$$
$$z_2^{2\,\min} = 488, \qquad z_2^{2m} = 998.4.$$

$$\mu_1^1\,(z_1^1(x)) = \frac{-x_{11} - x_{12} + 800}{-416.316},$$
$$\mu_2^1\,(z_2^1(x)) = \frac{1.4x_{11} + 0.9x_{12} + 0.8x_{21} + 0.3x_{22} - 1684.75}{-359.75},$$
$$\mu_1^2\,(z_1^2(x)) = \frac{-x_{21} + 170}{-260},$$
$$\mu_2^2\,(z_2^2(x)) = \frac{0.4x_{11} + 0.2x_{12} + 1.8x_{21} - 0.2x_{22} - 998.4}{-510.4}$$

And so problem p_λ can be identified as:

minimize $0.0008x_{11} + 0.001x_{12} + 0.0002x_{21} - 0.0001x_{22} - 0.134$

Subject to:

$x \in X$.

where X is a feasible region satisfying the constraints of problem (15).
Then, we get the Pareto optimal solution to p_λ

$$(x_{11}^0, x_{12}^0, x_{21}^0, x_{22}^0) = (400, 400, 300, 550).$$

ii) Determination $S(x^o)$

From Kuhn -Tuker optimality conditions we get:

$0.002\lambda_1 - 0.004\lambda_2 - 0.001\lambda_4 + \mu_1 + 2.2\mu_5 + 0.8\mu_6 + 2\mu_7 + 1.5\mu_8 - \mu_9 -$
$\mu_{11} - \mu_{13} = 0$

$0.002\lambda_1 - 0.003\lambda_2 - 0.0003\lambda_4 + \mu_2 + 1.9\mu_5 + 0.5\mu_6 + 3.5\mu_7 + 0.8\mu_8 - \mu_9$
$-\mu_{12} - \mu_{14} = 0$

$-0.002\lambda_2 + 0.004\lambda_3 - 0.021\lambda_4 + \mu_3 + 1.8\mu_5 + 2.5\mu_6 + 1.4\mu_7 + 3.3\mu_8 - \mu_{10}$
$-\mu_{11} - \mu_{15} = 0$

$-0.001\lambda_2 + 0.0004\lambda_4 + \mu_4 + 1.3\mu_5 + 1.2\mu_6 + 2.2\mu_7 + \mu_8 - \mu_{10}$
$-\mu_{12} - \mu_{16} = 0$

$\mu_1(x_{11} - 530) = 0$ *i.e.* $\mu_1 = 0$
$\mu_2(x_{12} - 690) = 0$ *i.e.* $\mu_2 = 0$
$\mu_3(x_{21} - 430) = 0$ *i.e..* $\mu_3 = 0$
$\mu_4(x_{22} - 550) = 0$ *i.e.* $\mu_4 \neq 0$
$\mu_5(880 + 760 + 540 + 715 - 3400) = 0$ *i.e.* $\mu_5 = 0$
$\mu_6(320 + 200 + 750 + 660 - 2350) = 0$ *i.e.* $\mu_6 = 0$
$\mu_7(800 + 1400 + 420 + 660 - 4000) = 0$ *i.e.* $\mu_7 = 0$
$\mu_8(600 + 320 + 990 + 550 - 2750) = 0$ *i.e.* $\mu_8 = 0$
$\mu_9(-400 - 400 + 800) = 0$ *i.e.* $\mu_9 \neq 0$

$\mu_{10}(-300 - 550 + 650) = 0$ *i.e.* $\mu_{10} = 0$
$\mu_{11}(-400 - 300 + 700) = 0$ *i.e.* $\mu_{11} \neq 0$

$\mu_{12}(-400 - 550 + 950) = 0$ *i.e.* $\mu_{12} \neq 0$

Similarly, $\mu_{13} = \mu_{14} = \mu_{15} = \mu_{16} = 0$

System (9) takes the form

$$
\begin{aligned}
0.002\lambda_1 - 0.004\lambda_2 - 0.001\lambda_4 - \mu_9 - \mu_{11} &= 0\\
0.002\lambda_1 - 0.003\lambda_2 - 0.0003\lambda_4 - \mu_9 - \mu_{12} &= 0\\
-0.002\lambda_2 + 0.004\lambda_3 - 0.021\lambda_4 - 3\mu_{11} &= 0\\
-0.001\lambda_2 + 0.0004\lambda_4 + \mu_4 - \mu_{12} &= 0,
\end{aligned}
$$

$$C=\begin{bmatrix}0.002 & -0.004 & 0 & -0.001\\ 0.002 & -0.003 & 0 & -0.0003\\ 0 & -0.002 & 0.004 & -0.021\\ 0 & -0.001 & 0 & 0.0004\end{bmatrix},\ D=D_1=\begin{bmatrix}0 & -1 & -1 & 0\\ 0 & -1 & 0 & -1\\ 0 & 0 & -1 & 0\\ 1 & 0 & 0 & -1\end{bmatrix},$$

$$C^T(D_1^T)^{-1}=\begin{bmatrix}0 & -0.002 & 0 & 0\\ 0 & 0.002 & 0.002 & 0.0010\\ -0.004 & 0.004 & -0.004 & -0.0040\\ 0.0207 & -0.020 & 0.021 & 0.0203\end{bmatrix},$$

$$\lambda^T C^T\left(D_1^T\right)^{-1}=(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\begin{bmatrix}0 & -0.002 & 0 & 0\\ 0 & 0.002 & 0.002 & 0.0010\\ -0.004 & 0.004 & -0.004 & -0.0040\\ 0.0207 & -0.020 & 0.021 & 0.0203\end{bmatrix}=$$

$$=\begin{pmatrix}-0.004\lambda_3+0.0207\lambda_4,\ -0.002\lambda_1+0.002\lambda_2+0.004\lambda_3-0.020\lambda_4,\\ 0.002\lambda_2-0.004\lambda_3+0.021\lambda_4\\ 0.001\lambda_2-0.004\lambda_3+0.0203\lambda_4\end{pmatrix}\le 0$$

i.e. $-0.004\lambda_3+0.0207\lambda_4\le 0$
$-0.002\lambda_1+0.002\lambda_2+0.004\lambda_3-0.020\lambda_4\le 0$
$0.002\lambda_2-0.004\lambda_3+0.021\lambda_4\le 0$
$0.001\lambda_2-0.004\lambda_3+0.0203\lambda_4\le 0$

Therefore,

$$S(400,400,300,550)=\left\{\begin{array}{l}\lambda\in R^4\ \ \middle|\ \lambda_3\ge 5.17\lambda_4,\lambda_1-\lambda_2\ge 2\lambda_3-10\lambda_4,\\ 2\lambda_2\le 4\lambda_3-21\lambda_4\end{array}\right\}.$$

By repeating this procedure many times, we can cover the entire parametric space (Λ).

6. CONCLUSIONS

This paper has shown that multiobjective bi-level linear programming (MOBLP) problems can be reformulated as finding points parametrically by using the nonnegative weighted approach. In addition, an algorithm for determining the stability set of the first kind is presented. Also, we discuss the stability set of the first kind for finding the set of Pareto optimal solutions and decomposing the parametric space in MOBLP problems.

REFERENCES

[1] Bard, J.F., "An Efficient Point Algorithm for a Linear Two-Stage Optimization Problem", *Operations Research,* 38 (1983) 556-560.

[2] Bard, J.F. "Coordination of a Multidivisional Firm through Two Levels of Management", *Omega,* 11 (1983) 457-465.

[3] Bard, J.F. and Falk, J.E., "An Explicit Solution to the Multi-Level Programming Problems", *Computers and Operations Research*, 9 (1982) 77-100.

[4] Bialas, W.F. and Karwan, M.H., "Two-Level Linear Programming", *Management Science*, 30 (1984) 1004-1020.

[5] Bard, J. F., Plummer, J., and Sourie, J. C. "Determining tax credits for converting nonfood crops to biofuels: application of bilevel programming", *Multilevel Optimization: Algorithms and Applications (A. Migdalas, P.Pardalos and P. V.arbrand, Eds.), Dordrecht: Kluwer Academic Publishers,* 1997, 23-50.

[6] Fortuny-Amat, J. and McCarl, B., "A Representation and Economic Interpretation of a Two-level Programming Problem", *Journal of the operational Research Society,* 32 (1981) 783-792.

[7] Lai, Y. J. "Hierarchical Optimization: a Satisfactory Solution", *Fuzzy Sets and Systems,* 77 (1996) 321-335.

[8] Mangasarian, O.L., *Nonlinear Programming*, McGraw- Hill New York (1969).

[9] Marccote, P. "Network Design Problem with Congestion Effects: a Case of Bilevel Programming", *Mathematical Programming,* 34 (1986) 142-162.

[10] Osman, M., "Qualitative Analysis of Basic Notions in Parametric Convex Programming, (Parameters in The Objective Function)", *Aplikace Matematiky,* 22 (1977) 333- 348.

[11] Osman, M., "Qualitative Analysis of Basic Notions in Parametric Convex Programming, (Parameters in The Constraints)", *Aplikace Matematiky,* 22 (1977) 318- 332.

[12] Osman, M., and Dauer, J., "Characterization of Basic Notions in Multiobjective Convex Programming Problems", Technical Report. Lincoln, NF.U.S.A, 1983.

[13] Sakawa, M., Nishizaki, I., and Oka, Y., "Interactive Fuzzy Programming for Multiobjective Two-Level Linear Programming Problems with Partial Information of Preference", *International Journal of Fuzzy System*, 2 (2000) 79-86.

[14] Shee, D.Y., Tang, T.I. and Tzeng, G.H., "Modeling the Supply-Demand Interaction in Electronic Commerce: A Bi-Level Programming Programming Approach", *Journal of Electronic Commerce Research,* 1 (2000) 79-93

[15] Shih, H. S., Lai, Y. J. and Lee, E. S., "Fuzzy Approach for Multi-Level Programming Problems", *Computers and Operation Research*, 23 (1996) 73-91.

[16] Shimizu, K., Ishizuka, Y. and Bard, J.F., *Nondifferentiable and Two-Level Mathematical Programming*, Boston: Kluwer Academic Publishers (1997).

[17] Wen, U.P., and Hsu, S.T., "Linear Bi-Level Programming Problems- a Review", *Journal of Operational Research Society*, 42 (1991) 125-133.

[18] White, D.J. and Anandalingam, G., "A Penalty Function Approach for Solving Bi-Level Linear Programs", *Journal of Global Optimization*, 3 (1993) 397-419.