

INVENTORY MODEL WITH CASH FLOW ORIENTED AND TIME-DEPENDENT HOLDING COST UNDER PERMISSIBLE DELAY IN PAYMENTS

R.P. TRIPATHI

*Graphic Era Univeristy, Dehradun (UK) INDIA
tripathi_rp0231@rediffmail.com*

Received: Oktober 2012 / Accepted: January 2013

Abstract: This study develops an inventory model for determining an optimal ordering policy for non-deteriorating items and time-dependent holding cost with delayed payments permitted by the supplier under inflation and time-discounting. The discounted cash flows approach is applied to study the problem analysis. Mathematical models have been derived under two different situations i.e. case I: The permissible delay period is less than cycle time for settling the account, and case II: The permissible delay period is greater than or equal to cycle time for settling the account. An algorithm is used to obtain minimum total present value of the costs over the time horizon H . Finally, numerical example and sensitivity analysis demonstrate the applicability of the proposed model. The main purpose of this paper is to investigate the optimal cycle time and optimal payment time for an item so that annual total relevant cost is minimized.

Keywords: Inventory, time-dependent, cash flow, delay in payments.

MSC: 90B05.

1. INTRODUCTION

In traditional economical ordering quantity (EOQ) model, it is assumed that retailer must pay for the items as soon as the items are received. However, in practice, the supplier may offer the retailer a delay period in paying for the amount of purchasing cost. To motivate faster payment, stimulate more sales or reduce credit expanses, the supplier also often provides its customers a cash discount. The permissible delay is an important source of financing for intermediate purchasers of goods and services. The permissible delay in payments reduces the buyer's cost of holding stock, because it reduces the

amount of capital invested in stock for the duration of the permissible period. Thus, it is a marketing strategy for the supplier to attract new customers who consider it to be a type of price reduction. Most of the classical inventory models did not take into account the effects of inflation and time value of money. But during the last three decades, the economic situation of most of the countries has changed to such an extent due to large scale inflation and consequent sharp decline in the purchasing power of money, that it has not been possible to ignore the effects of inflation and time value of money any further. In supermarkets, it has been observed that the demand rate may go up and down if the on-hand inventory level increases or decreases. This type of situation generally arises for consumer goods type of inventory.

The economic order quantity (EOQ) model is widely used by practitioners as a decision making tool for the control of inventory. In general, the objective of inventory management deals with minimization of the inventory carrying cost. Therefore it is important to determine the optimal stock and optimal time of replenishment of inventory to meet the future demand. An inventory model with stock at the beginning and shortages allowed, but then partially backlogged was developed by Lin et al. [15]. Urban [23] developed an inventory model that incorporated financing agreements with both suppliers and customers using boundary condition. Yadav et al. [26] established an inventory model of deteriorating items with two warehouse and stock dependent demand. Wu et al. [25] applied the Newton method to locate the optimal replenishment policy for EPQ model with present value. Roy and Chaudhuri [18] established an EPLS model with a variable production rate and demand depending on price. Huang [11] developed an EOQ model to compare the interior local minimum and the boundary local minimum.

Various models have been proposed for inflation dependent inventory models. Buzacott [5] was first who developed EOQ model taking inflation into account. In the same year Misra [17] also developed EOQ model incorporating inflationary effects. Both models assume a uniform inflation rate for all the associated costs, and minimize the average annual cost to obtain expression for the EOQ. Hou and Lin [9] developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments. In this paper Hou and Lin [9] obtained optimal (minimum) total present value of costs. The model of Hou and Lin [9] was extended by Tripathi etc [22] by taking time-dependent demand rate for non-deteriorating items. Tripathi and Kumar [20] discussed EOQ model credit financing in economic ordering policies of time-dependent deteriorating items. Aggarwal *et al.* [2] developed a model on integrated inventory system with the effect of inflation and credit period. In this model, the demand rate is assumed to be a function of inflation. Tripathi and Misra [17] developed EOQ model credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach. Jaggi *et al.* [14] developed a model retailer's optimal replenishment decision with credit-linked demand under permissible delay in payments. This paper incorporates the concepts of credit linked demand and developed a new inventory model under two levels of trade credit policy to reflect the real-life situation. An EOQ model under conditionally permissible delay in payments was developed by Huang [12] and obtained the retailer's optimal replenishment policy under permissible delay in payments. Optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chain were developed by Mahata and Mahata [16]. In this paper, the authors obtained a unique optimal cycle time to minimize the total variable

cost per unit time. Hou and Lin [10] considered an ordering policy with a cost minimizing procedure for deteriorating items under trade credit and time-discounting. Several other researchers have extended their approach to various interesting situations by considering the time-value of money, different inflation rates for the internal and external costs, finite replenishment rate, shortage etc. The models of Van Hees and Monhemius [24], Aggarwal [1], Bierman and Thomas [3], Sarker and Pan [19] etc. are worth mentioning in this direction. Brahmabhatt [4] developed an EOQ model under a variable inflation rate and marked-up prices. Gupta and Vart [8] developed a multi-item inventory model for a resource constant system under variable inflation rate. Chung [7] developed a model inventory control and trade credit revisited. Jaggi and Aggarwal [13] developed a model credit financing in economic ordering policies of deteriorating items by using discounted cash-flows (DCF) approach. Chen and Kang [6] discussed integrated vendor-buyer cooperative inventory models with variant permissible delay in payments.

For generality, this study develops an inventory model for non-deteriorating items under permissible delay in payments in which holding cost is a function of time. The discounted cash flows approach is also consider to build-up the model. We then establish algorithm to find the optimal order cycle, optimal order quantity, optimal total present value of the cost over the time-horizon H . Also, we provide numerical example and sensitivity analysis as illustrations of the theoretical results.

The rest of this paper is organized as follows. In section 2, we describe the notation and assumptions used throughout this study. In section 3, the model is mathematically formulated. In section 4, an algorithm is given for finding optimal solution. Numerical example is provided in section 5, followed by sensitivity analysis in section 6 to illustrate the features of the theoretical results. Finally, we draw the conclusions and the idea of future research in the last section 7.

2. NOTATIONS AND ASSUMPTIONS

The following notations are used throughout the manuscript:

- H : Length of planning horizon
- n : Number of replenishment during the planning horizon, $n = H/T$
- T : Replenishment cycle time
- D : Demand rate per unit time, units/unit time
- Q : Order quantity, units/cycle
- s : Ordering cost at time zero, \$/order
- c : Per unit cost of the item, \$/unit
- h : Holding cost per unit per unit time excluding interest charges, \$/unit/unit time
- r : Discount rate
- f : Inflation rate
- k : The net discount rate of inflation ($k = r - f$)
- I_e : The interest earned per dollar per unit time
- I_c : The interest charged per dollar in stocks per unit time by the supplier $I_c > I_e$
- m : The permissible delay in settling account
- $Z_1(n)$: The total present value of the costs over the time horizon H , for $m < T = H/n$

- $Z_2(n)$: The total present value of the costs for $m \geq T = H/n$
 E : The interest earned during the first replenishment cycle
 E_1 : The present value of the total interest earned over the time horizon H
 $I(t)$: The inventory level at time t
 I_p : The total interest payable over the time horizon H
 E_2 : The present value of the interest earned over the time horizon H
 E_3 : The present value of the total interest earned over the time horizon H

In addition, the following assumptions are being made:

- (1) The demand rate D is constant and downward sloping function.
- (2) Shortages are not allowed.
- (3) Lead time is zero.
- (4) The net discount rate of inflation is constant.
- (5) The holding cost h is time-dependent i.e. $h = h(t) = a + bt$, $a > 0$, and $0 < b > 1$.

3. MATHEMATICAL FORMULATION

The inventory level $I(t)$ at any time t is depleted by the effect of demand only. Thus the variation of $I(t)$ with respect to 't' is governed by the following differential equation:

$$\frac{dI(t)}{dt} = -D, 0 \leq t \leq T = H/n \quad (1)$$

The present value of the total replenishment costs is given by:

$$C_1 = s \sum_{i=0}^{n-1} e^{-ikT} = s \left(\frac{1 - e^{-kH}}{1 - e^{-kT}} \right), 0 \leq t \leq T = H/n \quad (2)$$

The present value of the total purchasing costs is given by

$$C_2 = c \sum_{i=0}^{n-1} Qe^{-ikT} = cDT \left(\frac{1 - e^{-kH}}{1 - e^{-kT}} \right), 0 \leq t \leq T = H/n \quad (3)$$

The present value of the total holding costs over the time horizon H is given by

$$\begin{aligned}
 A &= \sum_{i=0}^{n-1} e^{-ikT} \int_0^T h(t)I(t)e^{-kt} dt \\
 &= \frac{D}{k} \left\{ aT + \frac{(bT - a) + (a + bT)e^{-kT}}{k} + \frac{2b}{k^2} (e^{-kT} - 1) \right\} \left(\frac{1 - e^{-kH}}{1 - e^{-kT}} \right)
 \end{aligned} \quad (4)$$

Case I. $m < T = H/n$

The present value of the interest payable during the first replenishment cycle is

$$i_p = cI_c \int_m^T I(t)e^{-kt} dt = cI_c D \left\{ \frac{(T-m)e^{-km}}{k} + \frac{e^{-kT} - e^{-km}}{k^2} \right\} \quad (5)$$

Thus, the present value of the total interest payable over the time horizon H is

$$I_p = \sum_{i=0}^{n-1} i_p e^{-ikT} = \frac{cI_c D}{k^2} \left\{ k(T-m)e^{-km} + e^{-kT} - e^{-km} \right\} \left(\frac{1-e^{-kH}}{1-e^{-kT}} \right), T = H/n \quad (6)$$

The present value of the interest earned during the first replenishment cycle is

$$E = cI_e \int_0^T Dte^{-kt} dt = \frac{cDI_e}{k^2} (1 - e^{-kT} - kTe^{-kT}), T = H/n \quad (7)$$

Therefore, the present value of the interest earned over the time horizon H is

$$E_1 = \sum_{i=0}^{n-1} Ee^{-ikT} = \frac{cDI_e}{k^2} (1 - e^{-kT} - kTe^{-kT}) \left(\frac{1-e^{-kH}}{1-e^{-kT}} \right), T = H/n \quad (8)$$

Thus, the total present value of the costs over the time horizon H is

$$Z_1(n) = C_1 + C_2 + A + I_p - E_1 \quad (9)$$

Case II. $m \geq T = H/n$

In this case, the interest earned in the first cycle is the interest during the time period $(0, H/n)$ plus the interest earned from the cash invested during the time period (T, m) after the inventory is exhausted at time T and it is given by

$$E_2 = cI_e \left[\int_0^T Dte^{-kt} dt + (m-T)e^{-kT} \int_0^T Ddt \right] = cDI_e \left\{ \frac{1-e^{-kT}}{k^2} - \frac{Te^{-kT}}{k} + (m-T)Te^{-kT} \right\} \quad (10)$$

and the present value of the total interest earned over the time horizon H is

$$E_3 = \sum_{i=0}^{n-1} E_2 e^{-ikT} = cDI_e \left\{ \frac{1-e^{-kT}}{k^2} - \frac{Te^{-kT}}{k} + (m-T)Te^{-kT} \right\} \left(\frac{1-e^{-kH}}{1-e^{-kT}} \right), T = H/n \quad (11)$$

Therefore, the total present value of the costs is given by

$$Z_2(n) = C_1 + C_2 + A - E_3 \quad (12)$$

From equations (9) and (12), it is difficult to obtain the optimal solution in explicit form. Therefore, the model will be solved approximately by using a truncated Taylor's series for the exponential terms i.e.

$$e^{-kT} \approx 1 - kT + \frac{k^2 T^2}{2}, e^{-km} \approx 1 - km + \frac{k^2 m^2}{2} \text{ etc.} \quad (13)$$

This is a valid approximation for smaller values of kT and km etc.

With the above approximation, the present value of the cost over the time horizon H is

$$Z_1(n) \approx \frac{1}{k} \left\{ \frac{s}{T} + cD + \frac{DT(a+bT)}{2} + \frac{cI_e D(T-m)(T-m+km^2)}{2T} - \frac{cDI_e T(1-kT)}{2} \right\} \left(1 + \frac{kT}{2} + \frac{k^2 T^2}{4} \right) (1 - e^{-kH}) \quad (14)$$

and

$$Z_2(n) \approx \frac{1}{k} \left[\frac{s}{T} + cD + \frac{DT(a+bT)}{2} - cDI_e \left\{ m - \left(\frac{1}{2} + mk \right) T + \frac{k}{2} (1+mk) T^2 - \frac{k^2 T^3}{2} \right\} \right] \left(1 + \frac{kT}{2} + \frac{k^2 T^2}{4} \right) (1 - e^{-kH}) \quad (15)$$

Note that the purpose of this approximation is to obtain the unique closed form value for the optimal solution. By taking first and second order derivatives of $Z_1(n)$ and $Z_2(n)$ with respect to 'n', we obtain

$$\begin{aligned} \frac{\partial Z_1(n)}{\partial n} = & \left[\frac{s}{k} \left(\frac{1}{H} - \frac{k^2 H}{4n^2} \right) - \frac{cDH}{2n^2} \left(1 + \frac{kH}{n} \right) - \frac{DH}{2kn^2} \left\{ a + \left(b + \frac{ak}{2} \right) \left(2 + \frac{3kH}{n} \right) \frac{H}{n} + \frac{bk^2 H^3}{n^3} \right\} \right. \\ & - \frac{cI_e D}{2k} \left\{ \left(1 - mk + \frac{3m^2 k^2}{4} - \frac{m^3 k^3}{4} \right) + k \left(1 - mk + \frac{m^2 k^2}{2} \right) \frac{H}{n} + \frac{k^2 H^2}{4n^2} - \frac{m^2 (1 - km)}{H^2} n^2 \right\} \frac{H}{n^2} \\ & \left. - \frac{cDI_e H}{2kn^2} \left(-1 + \frac{kH}{n} + \frac{3k^2 H^2}{4n^2} + \frac{k^3 H^3}{n^3} \right) \right] (1 - e^{-kH}) \quad (16) \end{aligned}$$

$$\begin{aligned} \frac{\partial Z_2(n)}{\partial n} = & \left[\frac{s}{k} \left(\frac{1}{H} - \frac{k^2 H}{4n^2} \right) - \frac{cDH}{2n^2} \left(1 + \frac{kH}{n} \right) - \frac{DH}{2kn^2} \left\{ a + \left(b + \frac{ak}{2} \right) \left(2 + \frac{3kH}{2n} \right) \frac{H}{n} + \frac{bk^2 H^3}{n^3} \right\} \right. \\ & \left. - \frac{cDI_e H}{2kn^2} \left\{ \frac{(1+mk)}{2} \left(1 - \frac{kH}{n} \right) + \frac{9k^2 H^2}{n^2} + \frac{k^3 H^3}{2n^3} (1 - mk) + \frac{5k^4 H^4}{8n^4} \right\} \right] (1 - e^{-kH}) \quad (17) \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 Z_1(n)}{\partial n^2} &= \frac{H}{n^3} \left[\frac{ks}{2} + CD \left(1 + \frac{3kH}{n} \right) + \frac{D}{2k} \left\{ 2a + 6 \left(b + \frac{ak}{2} \right) \left(1 + \frac{kH}{n} \right) H + \frac{5bk^2 H^3}{n^3} \right\} \right. \\
&+ \left. \frac{cDI_e}{2kn^3} \left(-2 + \frac{3kH}{n} + \frac{3k^2 H^2}{n^2} + \frac{5k^3 H^3}{n^3} \right) \right. \\
&+ \left. \frac{cI_e DH}{2kn^3} \left\{ 2 \left(1 - mk + \frac{3m^2 k^2}{4} - \frac{m^3 k^3}{4} \right) + \frac{3kH}{n} \left(1 - mk + \frac{m^2 k^2}{4} \right) + \frac{k^2 H^2}{n^2} \right\} \right] (1 - e^{-kH}) \\
&> 0
\end{aligned} \tag{18}$$

$$\begin{aligned}
\frac{\partial^2 Z_2(n)}{\partial n^2} &= \frac{H}{n^3} \left[\frac{ks}{2} + cD \left(1 + \frac{3kH}{2n} \right) + \frac{D}{2k} \left\{ 2a + \frac{6 \left(b + \frac{ak}{2} \right) \left(1 + \frac{kH}{n} \right) H}{n} + \frac{5bk^2 H^3}{n^3} \right\} \right. \\
&+ \left. \frac{cDI_e}{2k} \left\{ (1 + mk) \left(2 - \frac{3kH}{n} \right) + \frac{9k^2 H^2}{n^2} + \frac{5k^3 H^3 (1 - mk)}{n^3} + \frac{15k^4 H^4}{2n^4} \right\} \right] (1 - e^{-kH}) > 0
\end{aligned} \tag{19}$$

Since $\frac{\partial^2 Z_1(n)}{\partial n^2} > 0$ and $\frac{\partial^2 Z_2(n)}{\partial n^2} > 0$, for fixed H , $Z_1(n)$ and $Z_2(n)$ are strictly convex functions of n . Thus, there exists a unique value of ' n ' which minimize $Z_1(n)$ and $Z_2(n)$. If we draw a curve between $Z(n)$ and ' n ', the curve is convex.

At $m = T = H/n$, we find $Z_1(n) = Z_2(n)$, we have

$$Z(n) = \begin{cases} Z_1(n), & \text{if } T = H/n \geq m \\ Z_2(n), & \text{if } T = H/n \leq m \end{cases}$$

where $Z_1(n)$ and $Z_2(n)$ are as expressed in equations (14) and (15), respectively.

Based on the above discussion, the following algorithm is developed to derive the optimal n , T , Q and $Z(n)$ values.

4. ALGORITHM

Step 1: Start by choosing positive integer ' n ', where n is equal or greater than one.

Step 2: If $T = H/n \geq m$, for different ' n ', then we determine $Z_1(n)$ from (14), if $T = H/n \leq m$, for different ' n ', then determine $Z_2(n)$ from (15).

Step 3: Repeat step 1 and 2 for all possible values of n with $T = H/n \geq m$ until the minimum $Z_1(n)$ is found from (14) and let $n_1^* = n$. For all possible values of n with $T = H/n \leq m$ until the minimum $Z_2(n)$ is found from (15) and let $n_2^* = n$. The n_1^* and n_2^* , $Z_1(n^*)$ and $Z_2(n^*)$ values form the optimal solution.

Step 4: Select the optimal number of replenishment n^* such that

$$Z(n^*) = \min \begin{cases} Z_1(n^*), & \text{if } T = H/n^* \geq m \\ Z_2(n^*), & \text{if } T = H/n^* \leq m \end{cases}$$

Hence the optimal order quantity Q^* is obtained by putting $T^* = H / n^*$

5. NUMERICAL RESULTS

An example is given to illustrate the results of the model developed in this study with the following data: $a = 2.0$ unit, $b = 0.5$ unit/time, $D = 600$ unit/year, $s = \$ 80$ /order, the net discount rate of inflation, $k = \$0.12$ /\$/year, the interest charged per dollar in stocks per year by the supplier, $I_c = \$0.18$ /\$/year, the interest earned per \$ per year, $I_e = \$0.16$ /\$/year, $c = \$15$ /unit and the planning horizon, $H = 5$ year. The permissible delay in settling the account, $m = 60$ days = $60/360$ years (assume 360 days in a year). Using the solution algorithm procedure, the computational results are shown in Table 1. We find the case is the I optimal option in credit policy. The minimum total present value of costs is obtained when the number of replenishment 'n' is 18. With 18 replenishments, the optimal cycle time T is 0.277778 years, the optimal order quantity, $Q = 166.666667$ units, and the optimal total present value of costs, $Z(n) = \$ 35597.78$ (approximately).

Table 1. The computational results: Variation of the optimal solution for different values of 'n'

Case	Order No. (n)	Cycle Time 'T' year	Order Quantity (Q) units	Total costs Z(n) (approx.)	
I	10	0.500000	300.000000	36206.11	
	11	0.454545	272.727273	36022.86	
	12	0.416667	250.000000	35886.89	
	13	0.384615	230.769231	35786.44	
	14	0.357143	214.285714	35713.32	
	15	0.333333	200.000000	35661.69	
	16	0.312500	187.500000	35627.25	
	17	0.294118	176.470588	35606.77	
	18*	0.277778*	166.666667*	35597.78*	
	19	0.263158	157.894737	35598.37	
	20	0.250000	150.000000	35607.02	
	21	0.238095	142.857143	35622.51	
	22	0.227273	136.363636	35643.87	
	23	0.217391	130.434783	35670.30	
	24	0.208333	125.000000	35701.16	
	25	0.200000	120.000000	35735.78	
	26	0.192308	115.384615	35773.84	
	27	0.185185	111.111111	35814.88	
	28	0.178571	107.142857	35858.59	
	29	0.172414	103.448276	35904.66	
	II	30	0.166667	100.000000	35952.87
		31	0.161290	96.774194	35973.94
		32	0.156250	93.750000	35997.50
		33	0.151515	90.909091	36023.30
		34	0.147059	88.235294	36051.15
		35	0.142857	85.714286	36080.88
		36	0.138889	83.333333	36112.32
		37	0.135135	81.081081	36145.33
		38	0.131579	78.947368	36179.78
39		0.128205	76.923077	36215.50	
40		0.125000	75.000000	36252.60	
45	0.111111	66.666667	36453.30		
50	0.100000	60.000000	36674.21		

* Optimal solution

6. SENSITIVITY ANALYSIS

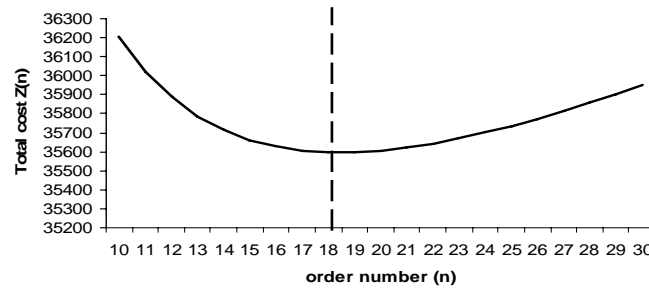
Taking all the parameters as in the above numerical example, the variation of the optimal solution for different values of net discount rate of inflation k is given in Table 2.

Table 2: Variation of the optimal solution for different values of net discount rate of inflation ‘ k ’

n ↓	$k \rightarrow$	0.12	0.15	0.18	0.21	0.24	0.27	0.30
16	C_1	980.6955	921.7646	867.9496	818.7437	773.6944	732.3970	694.4892
	C_2	34477.5766	32405.7879	30513.8517	28783.9700	27200.1955	25748.3321	24415.6381
	A	727.7469	681.8692	604.0485	601.8746	566.9816	535.0443	505.7717
	I_p	205.7914	192.1809	179.7970	168.5133	158.2174	148.8094	140.2003
	E_1	840.6909	785.2672	734.3340	688.8782	646.9412	613.5497	573.5498
	$Z(n)$	35551.1195	33416.3354	31466.8128	29684.2234	28052.1468	26551.0331	25182.5495
17	C_1	1040.8461	978.0346	920.6848	868.2545	820.2600	776.2685	735.8929
	C_2	34439.8021	32393.6070	30463.8704	28729.0560	27140.9886	25685.3846	24349.4253
	A	683.6784	640.5240	601.1874	565.2829	532.4655	502.4300	474.9014
	I_p	166.9449	155.8897	145.8314	136.5573	128.3059	120.6660	113.6753
	E_1	791.5322	739.4182	691.9943	648.7784	609.3406	573.3000	540.3160
	$Z(n)$	35539.7393	33428.6371	31439.5797	29650.4823	28012.6794	26511.4491	25133.5859
18	C_1	1100.9998	1034.3073	974.0789	918.0711	866.8291	820.1438	777.3008
	C_2	34406.2467	32322.1299	30419.3624	28680.2999	27088.4303	25629.5151	24290.6692
	A	644.6379	603.9015	566.7706	532.9101	501.9275	473.5584	447.5778
	I_p	134.2662	125.3719	117.2984	109.9188	103.1721	97.0228	91.3959
	E_1	747.8036	698.6265	653.8731	613.0889	575.8687	541.8531	510.7212
	$Z(n)$	35538.347	33387.0841	31423.6372	29628.111	27984.4903	26478.387	25096.2225
19	C_1	1161.1526	1090.5809	1026.1621	967.2841	913.4001	864.0215	818.7114
	C_2	34376.2416	32286.947	30379.8119	28636.7227	27041.4611	25579.5945	24238.1770
	A	601.8125	571.2373	536.0778	503.9887	474.6613	447.8215	423.2242
	I_p	106.8810	99.7882	93.3357	87.4575	82.0946	77.1951	72.7112
	E_1	708.6535	662.09997	619.7324	581.1209	545.8821	513.6762	484.1995
	$Z(n)$	35545.4342	33386.4534	31415.6551	29614.3321	27965.735	25591.4532	25068.6253
20	C_1	1221.3068	1446.8553	1078.9025	1016.8015	959.9728	907.9012	860.1244
	C_2	34349.2526	32255.3045	30344.1332	28597.5410	26999.23249	25534.7207	24190.9977
	A	578.5549	541.9232	508.5364	478.0662	450.2194	424.7355	401.3815
	I_p	83.8929	78.3206	73.2516	68.6339	64.4212	60.5726	57.0515
	E_1	673.3987	629.2030	588.9798	552.3212	518.8636	488.2846	460.2959
	$Z(n)$	35559.6085	33393.2006	31415.8439	29608.7214	27954.9847	26439.6454	25049.2592

From Table 2, all the observations can be summed up as follows:

- (i) An increase in the net discount rate of inflation ‘ k ’ leads to a decrease of total replenishment cost, in total purchasing cost, in total holding cost, in total interest payable, in total interest earned, and also a decrease in total present value of the costs C_1, C_2, A, I_p, E_1 and $Z(n)$ respectively.
- (ii) If the number of replenishment ‘ n ’ increases, then there is increase in total replenishment cost C_1 , but total purchasing cost C_2 , total holding cost ‘ A ’, total interest payable ‘ I_p ’ and total interest earned ‘ E_1 ’ decreases, keeping net discount rate of inflation ‘ k ’ constant.

Figure 1: Graph between $Z(n)$ Vs n

7. CONCLUSION AND FUTURE RESEARCH

This study develops an inventory model for non-deteriorating and time-dependent holding cost items over a finite planning horizon, when the supplier provides a permissible delay in payments. The model considers the effects of inflation and permissible delay in payments. The optimal solution procedure is given to obtain the optimal number of replenishment, cycle time and order quantity to minimize the total present value of costs. Numerical example is given to illustrate the model for case I and case II. The obtained results show that the case I is the optimal (minimum) option in credit policy. The minimum total present value of the costs is obtained when the number of replenishments n is 18. With 18 replenishments, the optimal (minimum) order quantity $Q = 166.666667$ units and the optimal (minimum) total present value of the costs $Z = \$ 35597.78$ (approximately).

The model proposed in this paper can be extended in several ways. For instance, we may extend the time dependent deterioration rate. We could also consider the demand as a function of quantity as well as a function of inflation. Finally, we could generalize the model with stochastic demand when the supplier provides a permissible delay in payments and cash discount.

REFERENCES

- [1] Aggarwal, S.C., "Purchase-inventory decision models for inflationary conditions", *Interfaces*, 11 (1981) 18-23.
- [2] Agrawal, R. and Rajput, D. and Varshney, N.K. "Integrated inventory system with the effect of inflation and credit period", *International Journal of Applied Engineering Research*, 4(11) (2009) 2334-2348.
- [3] Bierman, H. and Thomas, J. "Inventory decisions under inflationary conditions", *Rec. Sci.*, 8(1) (1977), 151-155.
- [4] Brahmabhatt, A.C., "Economic order quantity under variable rate of inflation and mark-up prices", *Productivity*, 23 (1982) 127-130.
- [5] Buzacott, J.A., "Economic order quantities with inflation", *Oper. Res. Quart.*, 26(3) (1975) 553-558.

- [6] Chen, L.H., and Kang, F.S., "Integrated Vendor-buyer cooperative inventory models variant permissible delay in payments", *European Journal of Operational Research*, 183(2) (2007) 658-673.
- [7] Chung, K.H., "Inventory control and trade credit revisited", *J. Oper. Res. Soc.*, 40 (1989) 495-498.
- [8] Gupta, R., Vrat, R., "Inventory model with multi items under constraint systems for stock dependent consumption rate", *Proceedings of XIX Annual Convention of Operation Research Society of India*, 2 (1986) 579-609.
- [9] Hou, L., and Lin, L.C., "A cash flow oriented EOQ model with deteriorating items under permissible delay in payments", *Journal of Applied Sciences*, 9(9) (2009) 1791-1794.
- [10] Hou, K.L., and Lin, L.C. "An ordering policy with a cost minimization procedure for deterioration items under trade-credit and time-discounting", *Accepted J. Stat. Manage. Syst.*, 11(6)(2008) 1181-1194.
- [11] Huang, K.C., "Continuous review inventory model under time value of money and crashable lead time consideration", *Yugoslav Journal of Operations Research*, 21(2) (2011) 293-306.
- [12] Huang, Y.F., "Economic order quantity under conditionally permissible delay in payments", *European Journal of Operational Research*, 176 (2007) 911-924.
- [13] Jaggi, C.K., and Aggarwal, S.P., "Credit financing in economic ordering policies of deteriorating items", *International Journal of Production Economics*, 34 (1994) 151-155.
- [14] Jaggi, C.K., Goyal, S.K., and Goel, S.K., "Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments", *European Journal of Operational Research*, 190 (2008) 130-135.
- [15] Lin, J., Chao, H., and Julin, P., "A demand independent inventory model. Yugoslav Journal of Operations Research", 22(2012) 1-7.
- [16] Mahata, C., and Mahata, P., "Optimal retailer's ordering policies in the EOQ model for deteriorating items under trade credit financing in supply chains", *International Journal of Mathematical, Physical and Engineering Sciences*, 3(1) (2009) 1-7.
- [17] Misra, R.B., "A study of inflationary effects on inventory systems", *Logist Spectrum*, 9(3) (1975) 260-268.
- [18] Roy, T., and Chaudhuri, K.S., "An EPLS model for a variable production rate with stockprice sensitive demand and deterioration", *Yugoslav Journal of Operations Research*, 22(1)(2012) 19-30.
- [19] Sarkar, B.R., and Pan, H., "Effect of inflation and the time value of money on order quantity and allowable shortages", *International Journal Production Economics*, 34 (1997) 65-72.
- [20] Tripathi, R.P., and Kumar, M., "Credit financing in economic ordering policies of time-dependent deteriorating items", *International Journal of Business, Management and Social Sciences*, 2(3) (2011) 75-84.
- [21] Tripathi, R.P., and Misra, S.S., "Credit financing in economic ordering policies of non-deteriorating items with time-dependent demand rate", *International Review of Business and Finance*, 2(1) (2010) 47-55.
- [22] Tripathi, R.P., Misra, S.S., and Shukla, H.S., "A cash flow oriented EOQ model under permissible delay in payments", *International Journal of Engineering, Science and Technology*, 2(11) (2010) 123-131.
- [23] Urban, T.L., "An extension of inventory models incorporating financing agreements with both suppliers and customers", *Applied Mathematical Modelling*, 36(2012) 6323-6330.
- [24] Van Hees, R.N., and Monhemius, W., *Production and Inventory Control: Theory and Practice*, In Barnes and Noble Macmillan. New York, (1972) 81-101.
- [25] Wu, J.K.J., Lei, H.L., Jung, S.T., and Chu, P., "Newton method for determining the optimal replenishment policy for EPQ model with present value", *Yugoslav Journal of Operations Research*, 18(1)(2008) 53-61.

- [26] Yadav, D., Singh, S.R., and Kumari, R., "Inventory model of deteriorating items with two Warehouse and stock dependent demand using genetic algorithm in fuzzy environment", *Yugoslav Journal of Operations Research*, 22(1) (2012)51-78.