

A STUDY OF INFLATION EFFECTS ON AN EOQ MODEL FOR WEIBULL DETERIORATING/AMELIORATING ITEMS WITH RAMP TYPE OF DEMAND AND SHORTAGES

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Abstract: This paper deals with the effects of inflation and time discounting on an inventory model with general ramp type demand rate, time dependent (Weibull) deterioration rate and partial backlogging of unsatisfied demand. The model is studied under the replenishment policy, starting with shortages under two different types of backlogging rates, and their comparative study is also provided. We then use the computer software, MATLAB to find the optimal replenishment policies. Duration of positive inventory level is taken as the decision variable to minimize the total cost of the proposed system. Numerical examples are then taken to illustrate the solution procedure. Finally, sensitivity of the optimal solution to changes of the values of different system parameters is also studied.

Keywords: Inventory, ramp type of demand, inflation, partial backlogging.

MSC: 90B05.

1. INTRODUCTION

For long, inventory models have dealt with the case where demand is either a constant or a monotonic function. Constant demands take place in the fully developed

stage of the item, and monotonic in the beginning or at the last stage of the cycle of life. Existing inventory models deal with these types of demand functions, but in practice, it is not possible. The demands of fashionable goods increase up to a certain level and after that become steady. Such type of demand functions is known as ramp type of demand. (See Skouri et al. [20]). Researches related to this field are: Goyal and Giri [9], Mahata and Goswami [12], Manna and Chaudhuri [13], Panda et al. [18], etc.

Now- a- days many countries have been confronted with fluctuating inflation rates that are often far from being negligible. Buzzacott [4] developed the first EOQ model taking inflation into account. Researches related to this field are: Mishra [14], [15], Biermann and Thomas [2], Yang et al. [21], Moon and Yun [16], Brahmhat [3] and so on.

Some customers would like to wait during the shortage period, but others would not. In the lost sales case, the customer's demand for the item is lost and presumably filled by a competitor, which can be contemplated as the loss of profit on the sales. Consequently, the opportunity cost resulting from lost sales should be deliberated in the model. In the previous papers on the topic, shortages were endorsed and assumed to be completely backlogged or completely lost. Many recent studies have mutated inventory policies by considering the 'partial backlogging rate'. In some inventory systems, for many stocks like fashionable commodities, the amount of demand backlogged becomes the main factor of determining if it is to be accepted or not. Mainly, researchers use two types of backlogging such as constant backlogging and time dependent partial backlogging, i.e. decreasing function of a waiting time up to the next replenishment. Abad [1] and Dye and Ouyang [7] have considered the partial backlogging rate dependent on the waiting time up to the next replenishment. Recently, many researchers have studied the effects of both types of backlogging on the optimal replenishment policies of the inventory system. Some of the recent works related to this concept are: Chang et al [5], Ouyang et al. [17], San Jose et al. [19], Wu et al. [22] and Dye et al. [8], etc.

Here, we extend and modify Chern et al. [6] in two directions: Firstly, by considering the effect of inflation and time discounting on the replenishment policies. Secondly, we also discuss ameliorating items in the described model. (See Hwang [10], [11]). Comparative analysis between the models of non-exponential backlogging rate with linear demand function and exponential backlogging rate with non-linear demand function through numerical results is also given. The rest of this paper is organized as follows. In section 2, the notations and assumptions are given. In section 3, we present the mathematical model. In section 4, numerical examples are given to illustrate the model. Finally, we give the conclusion of this paper.

2. NOTATIONS AND ASSUMPTIONS

To develop the Mathematical model, the following notations and assumptions are being made.

2.1 Notations

K	the ordering cost of inventory per order
d	the deterioration cost per unit per unit time
h	the holding cost per unit per unit time

- s the shortage cost per unit per unit time
- μ the parameter of the ramp type demand function (time point)
- π the opportunity cost due to lost sales per unit
- r the net inflation rate
- $I(t)$ the inventory level at time t, $t \in [0, T]$
- $f(t)$ the demand rate at time t, $t \in [0, T]$
- $\theta(t)$ the deterioration rate at time t, $t \in [0, T]$
- T the length of the replenishment cycle
- T_1 the time at which the shortage starts, $0 \leq T_1 \leq T$
- TC the total cost per unit time.

2.2. Assumptions

- The replenishment rate is infinite and lead time is zero.
- The unit cost and the inventory carrying cost are known and constant.
- The selling price per unit and the ordering cost per order are known and constant
- Shortages are allowed. The fraction of shortages backlogged is a non-increasing function $B(x)$, where x is the waiting time up to the next replenishment, and $0 \leq B(x) \leq 1$ with $B(0) = 1$. Together with this, we assume that $B(x) + T B'(x) > 0$, where $B'(x) \leq 0$ is the derivative of $B(x)$. When $B(x) = 1$ (or 0), it corresponds to a complete backlogging (or complete lost sales) model.
- There is no repair or replacement of the deteriorated items during the production cycle.

3. MODEL FORMULATION

Due to the position of the time point μ , the model can be classified into two cases as follows:

Case 1: $0 \leq T_1 \leq \mu \leq T$ and

Case 2: $0 \leq \mu \leq T_1 \leq T$

We shall discuss first Case 1, and then Case 2.

3.1 Case 1: $0 \leq T_1 \leq \mu \leq T$

A typical behavior of the inventory in a cycle is depicted in Figure 1. The instantaneous states of the inventory level $I(t)$ at time t ($0 \leq t \leq T_1$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(t)B(T_1 - t), \quad 0 \leq t \leq T_1 \tag{1}$$

with the boundary condition $I(0) = 0$. Solving the differential equation (1), the inventory level is

$$I(t) = - \int_0^t f(u)B(T_1 - u)du, \quad 0 \leq t \leq T_1 \tag{2}$$

The instantaneous states of the inventory level $I(t)$ at time t ($T_1 \leq t \leq \mu$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(t) - \theta(t)I(t), \quad T_1 \leq t \leq \mu, \quad I(\mu_+) = I(\mu_-) \tag{3}$$

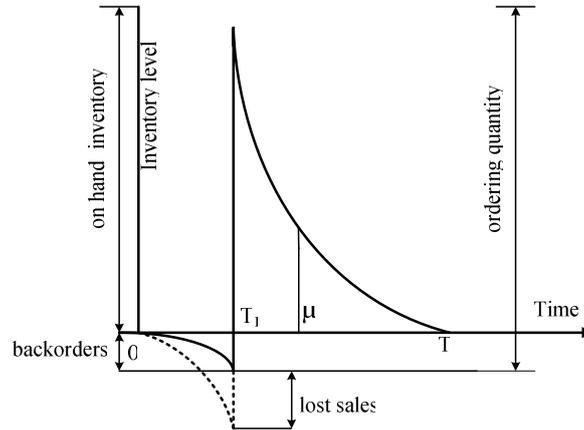


Figure 1. Graphical representation of inventory system

with the boundary condition $I(\mu_+) = I(\mu_-)$. Solving the differential equation (3), the inventory level is

$$I(t) = e^{-at^b} \left(\int_t^\mu e^{ax^b} f(x) dx + f(\mu) \int_\mu^T e^{ax^b} dx \right), \quad T_1 \leq t \leq \mu, \quad I(\mu_+) = I(\mu_-) \tag{4}$$

The instantaneous states of the inventory level $I(t)$ at time t ($\mu \leq t \leq T$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(\mu) - \theta(t)I(t), \quad \mu \leq t \leq T, \quad I(T) = 0 \tag{5}$$

with the boundary condition $I(T) = 0$. Solving the differential equation (5), the inventory level is

$$I(t) = e^{-at^b} f(\mu) \int_t^T e^{ax^b} dx, \quad \mu \leq t \leq T \tag{6}$$

The present worth of the holding cost for carrying inventory over the period $[T_1, T]$ is given by

$$\begin{aligned}
 HC &= \int_{T_1}^T hI(t)e^{-rt} dt \\
 &= \int_{T_1}^{\mu} hI(t)e^{-rt} dt + \int_{\mu}^T hI(t)e^{-rt} dt \tag{7} \\
 &= \int_{T_1}^{\mu} h e^{-at^b} \left(\int_t^{\mu} e^{ax^b} f(x) dx + f(\mu) \int_{\mu}^T e^{ax^b} dx \right) e^{-rt} dt + \int_{\mu}^T h \left(e^{-at^b} f(\mu) \int_t^T e^{ax^b} dx \right) e^{-rt} dt
 \end{aligned}$$

The present worth of the shortage cost over the period [0, T₁] is given by

$$\begin{aligned}
 SC &= s \int_0^{T_1} [-I(t)]e^{-rt} dt \tag{8} \\
 &= s \int_0^{T_1} \left(\int_0^t f(u)B(T_1 - u)du \right) e^{-rt} dt
 \end{aligned}$$

The present worth of the opportunity cost due to lost sales during the period [0, T₁] is given by

$$\begin{aligned}
 OC &= \pi \int_{T_1}^T f(t)[1 - B(T - t)]e^{-rt} dt \tag{9} \\
 &= \pi \int_{T_1}^T f(t)[1 - B(T - t)]e^{-rt} dt
 \end{aligned}$$

The present worth of the deterioration cost over the period [T₁, T] is given by

$$DC = d \left(e^{-aT_1^b} \left(f(\mu) \int_{\mu}^T e^{ax^b} dx + \int_{T_1}^{\mu} e^{ax^b} f(x) dx \right) e^{-rT_1} - \int_{T_1}^{\mu} f(\mu) e^{-rt} dt - \int_{\mu}^T f(t) e^{-rt} dt \right) \tag{10}$$

Therefore, the present worth of the total cost during the period [0, T] is given by

$$TC_1(T_1) = \text{the present worth of the } \begin{cases} \text{holding cost} \\ + \text{shortage cost} \\ + \text{opportunity cost due to lost sales} \\ + \text{deterioration cost} \end{cases}$$

$$\begin{aligned}
&= \left\{ \int_{T_1}^{\mu} h e^{-at^b} \left(\int_t^{\mu} e^{ax^b} f(x) dx + f(\mu) \int_{\mu}^T e^{ax^b} dx \right) e^{-rt} dt + \int_{\mu}^T h \left(e^{-at^b} f(\mu) \int_t^T e^{ax^b} dx \right) e^{-rt} dt \right. \\
&+ s \int_0^{T_1} \left(\int_0^t f(u) B(T_1 - u) du \right) e^{-rt} dt + \pi \int_{T_1}^T f(t) [1 - B(T - t)] e^{-rt} dt \\
&\left. + d \left(e^{-aT_1^b} \left(f(\mu) \int_{\mu}^T e^{ax^b} dx + \int_{T_1}^{\mu} e^{ax^b} f(x) dx \right) e^{-rT_1} - \int_{T_1}^{\mu} f(\mu) e^{-rt} dt - \int_{\mu}^T f(t) e^{-rt} dt \right) \right\} \quad (11)
\end{aligned}$$

The solutions for the optimal values of T_1 (say T_1^*) can be found by solving the following equations simultaneously:

$$\frac{\partial TC(T_1)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial TC(T_1)}{\partial T_1} = 0 \quad (12)$$

Provided they satisfy the conditions:

$$\left[\frac{\partial^2 TC_1(T_1)}{\partial T_1^2} \right]_{(T_1^*)} > 0 \quad (13)$$

$$\frac{\partial TC_1(T_1)}{\partial T_1} = 0 \quad \Rightarrow$$

$$\begin{aligned}
&\left\{ h \left[\left(-e^{-aT_1^b} \right) \left(\int_{T_1}^{\mu} f(x) e^{ax^b} dx + f(\mu) \int_{\mu}^T e^{ax^b} dx \right) e^{-rT_1} \right] + \right. \\
&s \left[\int_0^{T_1} f(u) B(T_1 - u) du \right] e^{-rT_1} + \pi \left((1 - B(T_1 - T_1)) f(T_1) \right) e^{-rT_1} + df(T_1) e^{-rT_1} \\
&+ d \left[e^{-aT_1^b} (-r) e^{-rT_1} + e^{-rT_1} (-a) b T_1^{(b-1)} e^{-aT_1^b} \right] \left[\int_{\mu}^T f(\mu) e^{ax^b} dx + \int_{T_1}^{\mu} f(x) e^{ax^b} dx \right] \\
&\left. + \pi \left(\int_0^{T_1} - \left(\frac{dB(T_1 - t)}{dT_1} \right) f(t) e^{-rt} dt \right) + s \left[\int_0^{T_1} \left[\int_0^t f(u) \frac{dB(T_1 - u)}{dT_1} du \right] e^{-rt} dt \right] \right\} = 0 \quad (14)
\end{aligned}$$

$$\begin{aligned}
 \frac{\partial^2 TC_1(T_1)}{\partial T_1^2} = & \left\{ h \left[\left(-e^{-aT_1^b} \right) \left(\int_{T_1}^{\mu} f(x)e^{ax^b} dx + f(\mu) \int_{\mu}^T e^{ax^b} dx \right) e^{-rT_1} \right] + \right. \\
 & s \left[\left(-r \right) \int_0^{T_1} f(u)B(T_1-u)du + \int_0^{T_1} f(u) \frac{dB(T_1-u)}{dT_1} du + f(T_1) \right] e^{-rT_1} + \\
 & d \left(e^{-aT_1^b} e^{-rT_1} \left[\begin{aligned} & (-r)^2 + (-a)(b-1)bT_1^{(b-2)} \left[\int_{\mu}^T f(\mu)e^{ax^b} dx + \int_{T_1}^{\mu} f(x)e^{ax^b} dx \right] \\ & + 2(-r)(-a)bT_1^{(b-1)} \end{aligned} \right] \right. \\
 & \left. + \left((-r) \right) f(T_1)e^{-rT_1} + f'(T_1)e^{-rT_1} + e^{-rT_1} \left[(-r) + (-a)bT_1^{(b-1)} f(T_1) \right] \right) \\
 & \left. + \pi \left(\int_0^{T_1} - \left(\frac{d^2 B(T_1-t)}{dT_1^2} \right) f(t)e^{-\pi t} dt \right) + s \left[\int_0^{T_1} \int_0^t f(u) \frac{d^2 B(T_1-u)}{dT_1^2} du \right] e^{-\pi t} dt \right\} > 0 \tag{15}
 \end{aligned}$$

Summarizing the above results, we now establish the following solution procedure to obtain the optimal solution of our problem.

3.1.1 Solution procedure for Case 1

Step 1: Take $T = 1$.

Step 2: Solving (14) find T_1 .

Step 3: Check equation (13); if yes, go to step 2; otherwise, go back to step 1.

Step 4: Using equation (11) to find TC_1 Value.

3.2 Case 2: $0 \leq \mu \leq T_1 \leq T$

A typical behavior of the inventory in a cycle is depicted in Figure.2

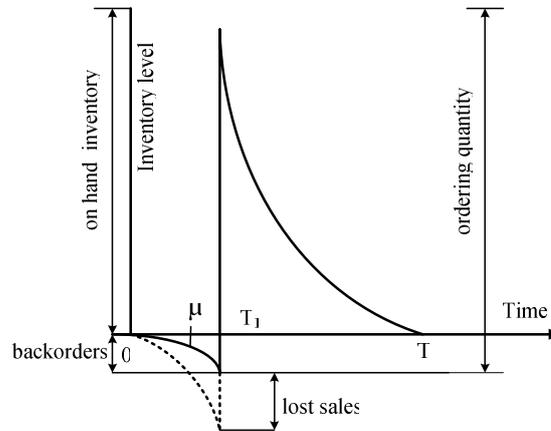


Figure 2 Graphical representation of inventory system

The instantaneous states of the inventory level $I(t)$ at time t ($T_1 \leq t \leq T$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(\mu) - \theta(t)I(t), \quad T_1 \leq t \leq T \quad (16)$$

with the boundary condition $I(T) = 0$. Solving the differential equation (16), the inventory level is as

$$I(t) = e^{-at} f(\mu) \int_t^T e^{ax} dx, \quad T_1 \leq t \leq T \quad (17)$$

The instantaneous states of the inventory level $I(t)$ at time t ($0 \leq t \leq \mu$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(t)B(T_1 - t)], \quad 0 \leq t \leq \mu \quad (18)$$

with the boundary condition $I(0) = 0$. Solving the differential equation (18), the inventory level is as

$$I(t) = -\int_0^t f(u)B(T_1 - u)du, \quad 0 \leq t \leq \mu \quad (19)$$

The instantaneous states of the inventory level $I(t)$ at time t ($\mu \leq t \leq T$) can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -f(\mu)B(T_1 - t)], \quad \mu \leq t \leq T_1 \quad (20)$$

with the boundary condition $I(\mu_-) = I(\mu_+)$. Solving the differential equation (20), the inventory level is as

$$I(t) = -\int_0^{\mu} f(u)B(T_1 - u)du + \int_t^{\mu} f(\mu)B(T_1 - u)du, \quad \mu \leq t \leq T_1 \quad (21)$$

The present worth of the holding cost for carrying inventory over the period $[T_1, T]$ is given by

$$\begin{aligned} HC &= \int_{T_1}^T hI(t)e^{-rt} dt \\ &= \int_{T_1}^T h \left(e^{-at} f(\mu) \int_t^T e^{ax} dx \right) e^{-rt} dt \end{aligned} \quad (22)$$

The present worth of the shortage cost over the period $[0, T_1]$ is given by

$$\begin{aligned}
 SC &= s \int_0^{T_1} [-I(t)] e^{-rt} dt \\
 &= s \int_0^{\mu} \left[\int_0^t f(u)B(T_1-u)du \right] e^{-rt} dt + s \int_{\mu}^{T_1} \left(\int_{\mu}^{\mu} f(u)B(T_1-u)du - \int_t^{\mu} f(\mu)B(T_1-u)du \right) e^{-rt} dt
 \end{aligned} \tag{23}$$

The present worth of the opportunity cost due to lost sales during the period [0, T₁] is given by

$$OC = \pi \left(\int_0^{\mu} f(t)[1-B(T-t)] e^{-rt} dt + \int_{\mu}^{T_1} f(\mu)[1-B(T-t)] e^{-rt} dt \right) \tag{24}$$

The present worth of the deterioration cost over the period [T₁, T] is given by

$$DC = d \left(e^{-aT_1^b} e^{-rT_1} f(\mu) \int_{T_1}^T e^{ax^b} dx - \int_{T_1}^T e^{-rt} f(\mu) dt \right) \tag{25}$$

Therefore, the present worth of the total cost during the period [0,T] is given by

$$\begin{aligned}
 TC_2(T_1) &= \text{the present worth of the} \begin{cases} \text{holding cost} \\ + \text{shortage cost} \\ + \text{opportunity cost due to lost sales} \\ + \text{deterioration cost} \end{cases} \\
 &= \left\{ \int_{T_1}^T h \left(e^{-at^b} f(\mu) \int_t^T e^{ax^b} dx \right) e^{-rt} dt + s \int_0^{\mu} \left[\int_0^t f(u)B(T_1-u)du \right] e^{-rt} dt \right. \\
 &\quad + s \int_{\mu}^{T_1} \left(\int_0^{\mu} f(u)B(T_1-u)du - \int_t^{\mu} f(\mu)B(T_1-u)du \right) e^{-rt} dt \\
 &\quad + \pi \left(\int_0^{\mu} f(t)[1-B(T-t)] e^{-rt} dt + \int_{\mu}^{T_1} f(\mu)[1-B(T-t)] e^{-rt} dt \right) \\
 &\quad \left. + d \left(e^{-aT_1^b} e^{-rT_1} f(\mu) \int_{T_1}^T e^{ax^b} dx - \int_{T_1}^T e^{-rt} f(\mu) dt \right) \right\} \tag{26}
 \end{aligned}$$

The solutions for the optimal values of T₁ (say T₁^{*}) can be found by solving the following equations simultaneously:

$$\frac{\partial TC_2(T_1)}{\partial T_1} = 0 \tag{27}$$

$$\text{Provided they satisfy the conditions: } \left[\frac{\partial^2 TC_2(T_1)}{\partial T_1^2} \right]_{(T_1^*)} > 0 \tag{28}$$

$$\begin{aligned}
\frac{\partial TC_2(T_1, T)}{\partial T_1} = 0 \Rightarrow & \\
& \left\{ h \left[(-e^{-rT_1}) \left(f(\mu) e^{-aT_1^b} \int_{T_1}^T e^{ax^b} dx \right) \right] + s \left(\int_0^\mu \left(\int_0^t f(u) \frac{dB(T_1-u)}{dT_1} du \right) e^{-rt} dt \right. \right. \\
& \quad \left. \left. + \int_\mu^{T_1} e^{-rt} \left(\int_0^\mu f(u) \frac{dB(T_1-u)}{dT_1} du - \int_t^\mu f(\mu) \frac{dB(T_1-u)}{dT_1} du \right) dt \right. \right. \\
& \quad \left. \left. + e^{-rT_1} \left(\int_0^\mu f(u) B(T_1-u) du - \int_{T_1}^\mu f(\mu) B(T_1-u) du \right) \right) \right. \quad (29) \\
& \quad \left. + \pi \left(\int_0^\mu \left(\frac{-dB(T_1-t)}{dT_1} \right) f(t) e^{-rt} dt + \int_\mu^{T_1} \left(\frac{-dB(T_1-t)}{dT_1} \right) f(\mu) e^{-rt} dt \right) + 2df(\mu) e^{-rT_1} \right. \\
& \quad \left. + d \left[e^{-aT_1^b} (-r) e^{-rT_1} + e^{-rT_1} (-a) b T_1^{(b-1)} e^{-aT_1^b} \right] \left[\int_\mu^T f(\mu) e^{ax^b} dx \right] \right\} = 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 TC_2(T_1, T)}{\partial T_1^2} = & \\
& \left\{ h \left[(-e^{-rT_1}) (-f(\mu)) + (re^{-rT_1} e^{-aT_1^b} + abT_1^{b-1}) \left(f(\mu) \int_{T_1}^T e^{ax^b} dx \right) \right] \right. \\
& \quad \left. + s \left[\int_0^\mu \left(\int_0^t f(u) \frac{d^2 B(T_1-u)}{dT_1^2} du \right) e^{-rt} dt \right. \right. \\
& \quad \left. \left. + \int_\mu^{T_1} e^{-rt} \left(\int_0^\mu f(u) \frac{d^2 B(T_1-u)}{dT_1^2} du - \int_t^\mu f(\mu) \frac{d^2 B(T_1-u)}{dT_1^2} du \right) dt \right. \right. \\
& \quad \left. \left. + e^{-rT_1} \left(\int_0^\mu f(u) \frac{dB(T_1-u)}{dT_1} du - \int_{T_1}^\mu f(\mu) \frac{dB(T_1-u)}{dT_1} du \right) \right. \right. \\
& \quad \left. \left. - re^{-rT_1} \left(\int_0^\mu f(u) B(T_1-u) du - \int_{T_1}^\mu f(\mu) B(T_1-u) du \right) \right] \right.
\end{aligned}$$

$$\begin{aligned}
 & + e^{-rT_1} \left(\int_0^\mu f(u) \frac{dB(T_1-u)}{dT_1} du - \int_{T_1}^\mu f(\mu) \frac{dB(T_1-u)}{dT_1} du + f(\mu) \right) \\
 & + \pi \left(\int_0^\mu \left(\frac{-d^2B(T_1-t)}{dT_1^2} \right) f(t) e^{-rt} dt + \int_\mu^{T_1} \left(\frac{-d^2B(T_1-t)}{dT_1^2} \right) f(\mu) e^{-rt} dt \right) - 2rdf(\mu)e^{-rT_1} \quad (30) \\
 & + d \left[e^{-aT_1^b} (-r) e^{-rT_1} + e^{-rT_1} (-a) b T_1^{(b-1)} e^{-aT_1^b} \right] \left[\int_\mu^T f(\mu) e^{ax^b} dx \right] > 0
 \end{aligned}$$

Summarizing the above results, we now establish the following solution procedure to obtain the optimal solution of our problem.

3.3 Solution procedure

- Step1: Take T = 1.
 - Step 2: Solving (29) find T_1 .
 - Step 3: Check equation (28); if yes, go to step 2; otherwise go back to step 1.
 - Step 4: Using equation (26) to find TC_2 Value.
- Find the optimum order quantity using the relation,

$$Q = \begin{cases} \int_0^{T_1} f(x)B(T_1-x)dx + e^{-aT_1^b} \left[\int_{T_1}^\mu e^{ax^b} f(x) dx + f(\mu) \int_\mu^T e^{ax^b} dx \right] & T_1 \leq \mu \\ \int_0^\mu B(T_1-x) f(x) dx + f(\mu) \int_\mu^{T_1} B(T_1-x) dx + e^{-aT_1^b} f(\mu) \int_{T_1}^T e^{ax^b} dx & T_1 > \mu \end{cases} \quad (31)$$

4. NUMERICAL EXAMPLES

The purpose of this section is to illustrate the results of our models and demonstrate the performance of the solution procedures presented in sections 3.2 and 3.4. Example 1 presented below is for a non-linear demand function with exponential partial backlogging. Example 2 presented below is for a linear demand function with non-exponential partial backlogging.

4.1 Example 1

To illustrate the proposed model, let us consider the following example: Let $s =$

$$15, d = 5, f(t) = 3e^{4.5t}, h = 3, \pi = 20, a = 0.001, b = 2, T = 1, \mu = 0.12, r = 0.14, B(T_1 -$$

$t) = e^{-0.2(T_1-t)}$ in appropriate units. Numerical values for deteriorating items are shown in Table 1, and that of ameliorating items are shown in Table 2.

Table 1 Optimal replenishment schedule for deteriorating items

T_1^*	Q^*	TC^*
0.02468	5.0091	5.818

Table 2 Optimal replenishment schedule for ameliorating items

T_1^*	Q^*	TC^*
0.02458	5.0057	5.8016

4.2 Example 2

To illustrate the proposed model, let us consider the following example: Let $s = 15$, $d = 5$, $f(t) = 3 + 5t$, $h = 3$, $\pi = 20$, $a = 0.001$, $b = 2$, $T = 1$, $\mu = 0.12$, $r = 0.14$,

$$B(T_1 - t) = \frac{1}{(1+0.2(T_1-t))}$$

in appropriate units. Numerical values for deteriorating items are shown in Table 3, and that of ameliorating items are shown in Table 4.

Table 3. Optimal replenishment schedule for deteriorating items

T_1^*	Q^*	TC^*
0.0001	3.5652	4.3723

Table 4. Optimal replenishment schedule for ameliorating items

T_1^*	Q^*	TC^*
0.0001	3.5628	4.3589

4.3 Comparative Study

From the numerical values displayed in Table 1 and Table 3, we conclude that the results of non-exponential backlogging rate with linear demand function are better than that of exponential backlogging rate with non-linear demand function. Optimum order quantity and the total cost of the first one are lesser than of the second one. Duration of shortage period of the previous model is longer than the latter one. So, exponential backlogging model shortens the shortage time and minimizes the total profit. The above results are the same as for that of ameliorating items.

4.4 Parametric Study

In order to study how various parameters affect the optimal solution of the proposed inventory model, sensitivity analysis is performed, keeping all the other parameters fixed and varying a single parameter at a time, for the same set of values. Sensitivity analyses for various parameters involved in the model mentioned above are shown in Table 5.

Table 5. Effect of various parameters on the total cost for deteriorating items

Parameter	value	TC
r	0.04	5.0486
	0.24	6.1204
h	2	4.7357
	4	6.5926
π	15	5.8089
	25	5.8280
s	10	5.7679
	20	5.8595
B	0.1	5.8155
	0.4	5.8188

The purpose of this section is to give some practical applications of our model.

4.5 Managerial Implications

Based on our numerical results, we obtain the following managerial phenomena:

- Increasing inflation rate is not suitable for the retailer.
- Try to reduce the holding cost.
- Maximize the backlogging rate and minimize the lost sales case.
- Reduce the shortage cost.

The above applications are also applicable to suppliers. Some special cases of our model are listed below:

4.6 Some special cases

- If we take $r = 0$, the described model is reduced to that of an EOQ model for Weibull deteriorating/ameliorating items with partial backlogging and ramp type of demand.
- If we take $\theta(t) = -\theta(t)$, the described model is reduced to that of an EOQ

model for Weibull ameliorating items with partial backlogging and ramp type of demand.

5. CONCLUSIONS

This paper discussed an effect of inflation and time discounting on an EOQ model for Weibull deteriorating/ameliorating items with partial backlogging and ramp type of demand. Non-exponential backlogging rate with linear demand function gives better results than exponential backlogging rate with non-linear demand function.

Optimum order quantity and the total cost of the first one are lesser than that of the second one. From our numerical values, we concluded that inflation definitely plays a major role on the replenishment policies and the optimum inventory cost. Here, total cost is more sensitive to inflation than optimum order quantity.

The proposed model can be extended in several ways. We could extend the deterministic demand function to stochastic demand patterns, as a function of selling price, etc.

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