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NOTE ON INVENTORY MODELS WITH A PERMISSIBLE DELAY IN PAYMENTS

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Abstract: This note tries to provide a patch work for the paper of Chang Dye and Chuang (published in Yugoslav Journal of Operational Research 2002, number 1, 73-84). Their paper contains an important finding of smoothly connected property that can very dramatically simplify the solution procedure of many inventory models with ramp type demand and trapezoidal type demand. Our improvement will arouse attention of the researchers and help them apply their important findings in the pending research projects.

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MSC: 90B05.

1. INTRODUCTION

Hill [1] was the original researcher to develop an inventory model with ramp type demands, thereafter many papers were done on this topic. For examples, Mandal and Pal [2] considered inventory system with deterioration items. Wu et al. [3] constructed an inventory model in which the backlogging rate is relative to the waiting time. Wu and Ouyang [4] studied inventory systems with two different strategies, starting with stock or shortage. Wu [5] and Giri et al. [6] developed inventory models with the Weibull distributed deterioration. Deng [7] revised Wu et al. [3] to provide a complete solution structure. Manna and Chaudhuri [8] extended inventory models with variable deteriorating rate. Deng et al. [9] modified Mandal, and Pal [2] and Wu and Ouyang [4] to find the optimal solution. Skouri et al. [10] extended linear increasing ramp type demand to arbitrary increasing ramp type demand. Cheng and Wang [11] studied inventory models with a trapezoidal type demand. Cheng et al. [12] improved Cheng and Wang [11] from linear increasing and linear decreasing to arbitrary increasing and arbitrary decreasing trapezoidal type demand. In the above mentioned papers, the problem was solved by dividing it within different domains of the demand type; so, a detailed solution process was presented, but their solution structures contained many different cases, which resulted in very complicated solution method. We have read Chang et al. [13] and found that it contains an interesting discovery, that is, the smoothly connected property. We may predict that the smoothly connected property will help researchers to solve, previously mentioned, complex solution algorithms. However, Chang et al. [13] contained several questionable results that may set an obstacle for ordinary readers to understand their paper and then apply the smoothly connected property in their future research. The purpose of this paper is to provid a patch work for Chang et al. [13] to aid researchers absorb their important findings.

2. NOTATION AND ASSUMPTIONS

The mathematical model in this paper is developed on the basis of the following assumptions and notations.

Assumptions

1. The inventory system involves only one item.
2. Replenishment occurs instantaneously at an infinite rate.
3. Let $\theta(t)$ be the deterioration rate of the on-hand inventory at time t , where $0 << \theta(t) < 1$ and $\theta'(t) \geq 0$.
4. Shortages are not allowed.
5. Before the replenishment account has to be settled, the buyer can use sales revenue to earn interest with an annual rate I_e. However, beyond the fixed credit period, the product still in stock is assumed to be financed with an annual rate I_r, where $I_r \geq I_e$.

Notations

R = annual demand (demand rate being constant)
A = ordering cost per order
$I(t)$ = the inventory level at time t
P = unit purchase cost, \$/per unit

h = holding cost excluding interest charges, \$/unit/year
I_e = interest which can be earned, \$/year
I_r = interest charges which are invested in inventory, \$/year, $I_r \geq I_e$
M = permissible delay in settling the account
T = the length of replenishment cycle
$C(T)$ = the total reverent inventory cost
$C_1(T)$ = the total reverent inventory cost for $T > M$ in Case 1
$C_2(T)$ = the total reverent inventory cost for $T \leq M$ in Case 2
$V(T)$ = the average total inventory cost per unit time
$V_1(T)$ = the average total inventory cost per unit time for $T > M$ in Case 1
$V_2(T)$ = the average total inventory cost per unit time for $T \leq M$ in Case 2

3. OUR PATCH WORK FOR THEIR DERIVATIONS

We provide some patch work to help ordinary readers to absorb the important inventory model of Chang at all. They forgot to define $g(x)$. From the text, we can assume that

$$g(x) = \int_0^x \theta(u)du \,, \tag{1}$$

such that $g'(T) = \theta(T)$.

They divided the problem into two cases: Case 1: $T > M$ and Case 2: $T \leq M$.

For the Case 1, after they derive the total cost, $C_1(T)$, as follows

$$\begin{aligned} C_1(T) &= A + hR\int_0^T e^{-g(t)}\int_t^T e^{g(u)}dudt + PR\left(\int_0^T e^{g(t)}dt - T\right) \\ &+ PRI_r\int_M^T e^{-g(t)}\int_t^T e^{g(u)}dudt - PRI_e\int_0^M (M-t)dt \end{aligned} \tag{2}$$

they assume that $V_1(T)$ is the average total cost per unit time. However, they did not define $V_1(T)$. From the text, we can assume that

$$V_1(T) = \frac{C_1(T)}{T}. \tag{3}$$

They found that

$$\frac{dV_1(T)}{dT}=\frac{-A}{T^2}+\frac{PR}{2T^2}\left(I_eM^2+2I_re^{g(T)}\int_M^T e^{-g(t)}dt-2I_r\int_M^T e^{-g(t)}\int_t^T e^{g(t)}dudt\right)$$
$$+\frac{PR}{T^2}\left(Te^{g(T)}-\int_0^T e^{g(t)}dt\right)+\frac{hR}{T^2}\left(Te^{g(T)}\int_0^T e^{-g(t)}dt-\int_0^T e^{-g(t)}\int_t^T e^{g(u)}dudt\right) \tag{4}$$

We must point out that the second term of $\frac{dV_1(T)}{dT}$ contains questionable results. The corrected version for the second term should be revised as

$$\frac{PR}{2T^2}\left(I_eM^2+2I_rT\,e^{g(T)}\int_M^T e^{-g(t)}dt-2I_r\int_M^T e^{-g(t)}\int_t^T e^{g(t)}dudt\right). \tag{5}$$

They tried to show that $V_1(T)$ is a convex function so they obtained that

$$\frac{d^2V_1(T)}{dT^2}=\frac{2A}{T^3}+\frac{PR}{T^3}k_1(T)+\frac{hR}{T^3}k_2(T)+\frac{PRI_r}{T^3}-\frac{PRI_e}{T^3}M^2, \tag{6}$$

where

$$k_1(T)=2\int_0^T e^{g(t)}dt+\left(T\theta(T)-2\right)T\,e^{g(T)}, \tag{7}$$

$$k_2(T)=T^2+2\int_0^T e^{-g(t)}\int_t^T e^{g(u)}dudt+T\left(T\theta(T)-2\right)e^{g(T)}\int_0^T e^{-g(t)}dt \tag{8}$$

and

$$k_3(T)=T^2+2\int_M^T e^{-g(t)}\int_t^T e^{g(u)}dudt+T\left(T\theta(T)-2\right)e^{g(T)}\int_M^T e^{-g(t)}dt\ , \tag{9}$$

where we already use $g'(T)=\theta(T)$ to simplify expressions.

They tried to prove that $k_j(T)$ for $j=1,2,3$ are increasing function by showing that $\frac{dk_j(T)}{dT}>0$ for $j=1,2,3$. Their proof for $\frac{dk_1(T)}{dT}>0$ is right. However, they implied that

$$\frac{dk_2(T)}{dT}=T^2\left(\theta(T)+\left(\left(\theta(T)\right)^2+\theta'(T)\right)e^{g(T)}\int_0^T e^{-g(t)}dt\right)e^{g(T)}. \tag{10}$$

We must point that their result of $\frac{dk_2(T)}{dT}$ contains questionable result, and then revise them as follows

$$\frac{dk_2(T)}{dT} = T^2\left(\theta(T) + \left(\left(\theta(T)\right)^2 + \theta'(T)\right)e^{g(T)}\int_0^T e^{-g(t)}dt\right). \tag{11}$$

On the other hand, they obtained that

$$\frac{dk_3(T)}{dT} = T^2\left(\theta(T) + \left(\left(\theta(T)\right)^2 + \theta'(T)\right)e^{g(T)}\int_M^T e^{-g(t)}dt\right)e^{g(T)}. \tag{12}$$

We also revise their finding as

$$\frac{dk_3(T)}{dT} = T^2\left(\theta(T) + \left(\left(\theta(T)\right)^2 + \theta'(T)\right)e^{g(T)}\int_M^T e^{-g(t)}dt\right). \tag{13}$$

They tried to use the Hospital rule to show that $\lim_{T\to\infty}\frac{dV_1(T)}{dT} = \infty$, but their derivation is expressed as

$$\lim_{T\to\infty}\frac{dV_1(T)}{dT} = \lim_{T\to\infty}\frac{1}{2}\left(PR\,e^{g(T)}\theta(T) + hR\left(1 + e^{g(T)}\int_0^T e^{-g(t)}dt\right)\theta(T)\right.$$
$$\left. + PRI_r\left(1 + e^{g(T)}\int_M^T e^{-g(t)}dt\right)\theta(T)\right) = \infty \tag{14}$$

Their claim of $\lim_{T\to\infty}\frac{dV_1(T)}{dT} = \infty$ is right but their result are still questionable, so we revise them as follows

$$\lim_{T\to\infty}\frac{dV_1(T)}{dT} = \lim_{T\to\infty}\frac{1}{2}\left(PR\,e^{g(T)}\theta(T) + hR\left(1 + \theta(T)e^{g(T)}\int_0^T e^{-g(t)}dt\right)\right.$$
$$\left. + PRI_r\left(1 + \theta(T)e^{g(T)}\int_M^T e^{-g(t)}dt\right)\right) = \infty \tag{15}$$

We have checked their findings for Case 2, and found that there is no questionable results in their derivations.

4. DIRECTION FOR THE FUTURE RESEARCH

In their paper, Chang et al. [13] divided the problem into two cases: Case (1) $T > M$ and Case 2: $T \le M$. Hence, there are two objective functions $V_1(T)$ for $T \ge M$ and $V_2(T)$ for $T \le M$.

In the literature, researchers found the minimum of $V_1(T)$, say $V_1\left(T^{\#}\right)$ where $T^{\#} \geq M$ and the minimum of $V_2(T)$, say $V_2\left(T^{\Delta}\right)$ where $T^{\Delta} \leq M$ such that the optimal solution will be

$$\min\left\{V_1\left(T^{\#}\right), V_2\left(T^{\Delta}\right)\right\}. \tag{16}$$

They discovered that

$$V_1(M) = V_2(M), \tag{17}$$

and

$$\frac{dV_1(T)}{dT}\Big|_{T=M} = \frac{f(M)}{M^2} = \frac{dV_2(T)}{dT}\Big|_{T=M} \tag{18}$$

where

$$\begin{aligned} f(M) &= -A + PR\left(Me^{g(M)} - \int_0^M e^{g(t)}dt\right) + \frac{1}{2}PRI_eM^2 \\ &+hR\left(Me^{g(M)}\int_0^M e^{-g(t)}dt - \int_0^M e^{-g(t)}\int_t^M e^{g(u)}dudt\right) \end{aligned} \tag{19}$$

Hence, using the convexity property of $V_1(T)$ and $V_2(T)$, $V_1(T)$ and $V_2(T)$ can not both have interior minimum, such that depending on the sign of $f(M)$, they found the optimal solution, say $V\left(T^*\right)$, that

$$V\left(T^*\right) = \begin{cases} V_1\left(T^*\right), & f(M) < 0, \\ V_2\left(T^*\right), & f(M) > 0, \\ V_1\left(T^*\right) = V_2\left(T^*\right), & f(M) = 0. \end{cases} \tag{20}$$

We must improve their expression as follows

$$V\left(T^*\right) = \begin{cases} V_1\left(T^{\#}\right), & f(M) < 0, \\ V_2\left(T^{\Delta}\right), & f(M) > 0, \\ V_1(M) = V_2(M), & f(M) = 0. \end{cases} \tag{21}$$

We can predict that this kind of approach in showing that the interior optimal solution only happens inside one interval will be very useful when dealing with some inventory models with multiple demand patterns. For examples, inventory models with ramp type demand, Hill [1], Wu et al. [3], Wu and Ouyang [4], Wu [5], Giri et al. [6], Deng [7], Manna and Chaudhuri [8], Deng et al. [9], and Skouri et al. [10], and inventory models with trapezoidal type demand, Cheng and Wang [11], and Cheng et al. [12].

5. CONCLUSION

In Chang et al. [13], the authors abstractly treated the deterioration as a non-decreasing function, which allows their model to be applied in many possible practical applications. For examples, they provided two special cases: linear deterioration and two-parameter Weibull distribution. However, in their paper, there are several questionable results that may hamper ordinary readers to comprehend their important findings of smoothly connected properties. Hence, in this note, we provide a patch work to help practitioners to absorb their achievement and then apply their findings to future research works.

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