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EPQ MODEL FOR IMPERFECT PRODUCTION PROCESSES WITH REWORK AND RANDOM PREVENTIVE MACHINE TIME FOR DETERIORATING ITEMS AND TRENDED DEMAND

Nita H. SHAH
Department of Mathematics, Gujarat University, India
[1]*nitahshah@gmail.com*
Dushyantkumar G. PATEL
Department of Mathematics, Govt. Poly.for Girls, India
dushyantpatel_1981@yahoo.co.in
Digeshkumar B. SHAH
Department of Mathematics, L. D. College of Engg., India
digeshshah2003@yahoo.co.in

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Abstract: Economic production quantity (EPQ) model is analyzed for trended demand and the units which are subject to constant rate of deterioration. The system allows rework of imperfect units and preventive maintenance time is random. The proposed methodology, a search method used to study the model, is validated by a numerical example. Sensitivity analysis is carried out to determine the critical model parameters. It is observed that the rate of change of demand and the deterioration rate have a significant impact on the decision variables and the total cost of an inventory system. The model is highly sensitive to the production and demand rate.

Keywords: EPQ, Deterioration, Time-dependent Demand, Rework, Preventive Maintenance, Lost Sales.

MSC: 90B05.

1. INTRODUCTION

Due to out-of-control of a machine, the produced items do not satisfy the codes set by the manufacturer, but can be recovered for the sale in the market after reprocessing. This phenomenon is known as rework, Schrady(1967). At manufacturer's end, the rework is advantageous because this will reduce the production cost. Khouja (2000) modeled an optimum procurement and shipment schedule when direct rework is carried out for defective items. Koh*et al.* (2002) and Dobos and Richter (2004) discussed two optional production models in which either opt to order new items externally or recover existing product. Chiu *et al*. (2004) studied an imperfect production processes with repairable and scrapped items. Jamal et al. (2004), and later Cardenas – Barron (2009) analyzed the policies of rework for defective items in the same cycle and the rework after N cycles. Teunter (2004), and Widyadana and Wee (2010) modeled an optimal production and a rework lot-size inventory models for two lot-sizing policies. Chiu (2007), and Chiu *et al*. (2007) incorporated backlogging and service level constraint in EPQ model with imperfect production processes. Yoo et al. (2009) studied an EPQ model with imperfect production quality, imperfect inspection, and rework.

The rework and deterioration phenomena are dual of each other. In other words, the rework processes is useful for the products subject to deterioration such as pharmaceuticals, fertilizers, chemicals, foods etc., that lose their effectivity with time due to decay. Flapper and Teunter (2004), and Inderfuth*et al*. (2005) discussed a logistic planning model with a deteriorating recoverable product. When the waiting time of rework process of deteriorating items exceeds, the items are to be scrapped because of irreversible process. Wee and Chung (2009) analyzed an integrated supplier-buyer deteriorating production inventory by allowing rework and just-in-time deliveries. Yang *et al*. (2010) modeled a closed-loop supply chain comprising of multi-manufacturing and multi-rework cycles for deteriorating items. Some more studies on production inventory model with preventive maintenance are by Meller and Kim (1996), Sheu and Chen (2004), and Tsou and Chen (2008).

Abboud*et al* (2000) formulated an economic lot-size model when machine is under repair resulting shortages. Chung *et al.* (2011), Wee and Widyadana (2012) developed an economic production quantity model for deteriorating items with stochastic machine unavailability time and shortages.

In the above cited survey, the researchers assumed demand rate to be constant. However, the market survey suggests that the demand hardly remains constant. In this paper, we considered demand rate to be increasing function of time. The items are inspected immediately on production. The defective items are stored and reworked immediately at the end of the production up time. These items will be labeled as recoverable items. After rework, some recoverable items are declared as 'good' and some of them are scrapped. Preventive maintenance is performed at the end of the rework process and the maintenance time is considered to be random. Here, shortages are considered as lost sales. Two different preventive maintenance time distributions are explored viz. the uniform distribution and the exponential distribution. The paper is organized as follows. In section 2, notations and assumptions are given. The mathematical model is developed in section 3. An example and the sensitivity analysis are given in section 4. Section 5 concludes the study.

2. ASSUMTIONS AND NOTATIONS

2.1.Assumptions

1. Single item inventory system is considered.
2. Good quality items must be greater than the demand.
3. The production and rework rates are constant.
4. The demand rate, (say) $R(t)=a(1+bt)$ is function of time where $a>0$ is scale demand and $0<b<1$ denotes the rate of change of demand.
5. The units in inventory deteriorate at a constant rate θ; $0<\theta<1$.
6. Set-up cost for rework process is negligible or zero.
7. Recoverable items are obtained during the production up time and scrapped items are generated during the rework up time.

2.2 Notations

I_{1a}	:	serviceable inventory level in a production up time
I_{2a}	:	serviceable inventory level in a production down time
I_{3a}	:	serviceable inventory level in a rework up time
I_{3r}	:	serviceable inventory level from rework up time
I_{4r}	:	serviceable inventory level from rework process in rework down time
I_{r1}	:	recoverable inventory level in a production up time
I_{r3}	:	recoverable inventory level in a rework up time
TI_{1a}	:	total serviceable inventory in a production up time
TI_{2a}	:	total serviceable inventory in a production down time
TI_{3a}	:	total serviceable inventory in a rework up time
TI_{3r}	:	total serviceable inventory from a rework up time
TI_{4r}	:	total serviceable inventory from rework process in a rework down time
TTI_{r1}	:	total recoverable inventory level in a production up time
TTI_{r3}	:	total recoverable inventory level in a rework up time
T_{1a}	:	production up time
T_{2a}	:	production down time
T_{3r}	:	rework up time
T_{4r}	:	rework down time
T_{sb}	:	total production down time
T_{1aub}	:	production up time when the total production down time is equal to the upper bound of uniform distribution parameter

I_m	:	inventory level of serviceable items at the end of production up time
I_{mr}	:	maximum inventory level of recoverable items in a production up time
I_w	:	total recoverable inventory
P	:	production rate
P_1	:	rework process rate
$R = R(t)$	:	demand rate; $a(1+bt), a > 0, 0 \le b < 1$
x	:	product defect rate
x_1	:	product scrap rate
θ	:	deteriorate at a constant rate θ; $0 < \theta < 1.$
A	:	production setup cost
h	:	serviceable items holding cost
h_1	:	recoverable items holding cost
S_C	:	scrap cost
S_L	:	lost sales cost
C_d	:	Cost of deteriorated units
TC	:	total inventory cost
T	:	cycle time
TCT	:	total inventory cost per unit time for lost sales model
TCT_{NL}	:	total inventory cost per unit time for without lost sales model
TCT_U	:	total inventory cost per unit time for lost sales model with uniform distribution preventive maintenance time
TCT_E	:	total inventory cost per unit time for lost sales model with exponential distribution preventive maintenance time

3. MATHEMATICAL MODEL

The rate of change of inventory is depicted in Figure1.During production period$\left[0, T_{1a}\right]$, x defective items per unit time are to be reworked. The rework process starts at the end of time T_{1a}. The rework time ends at T_{3r} time period. The production rates of good items and defective items are different. During the rework process, some recoverable and some scrapped items are obtained. LIFO policy is considered for the production system. So, serviceable items during the rework up time are utilized before the fresh (new) items from the production up time. The new production cycle starts when the inventory level reaches zero at the end of T_{2a} time period. Because machine is under maintenance, which is randomly distributed with probability density function $f\left(t\right)$, the new production cycle may not start at time T_{2a}. The production down time may result in shortage T_3 - time period. The production will start after the T_3 time period.

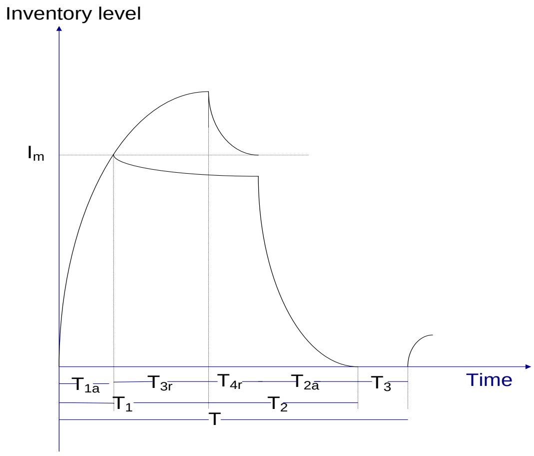

Figure 1: Inventory status of serviceable items with lost sales

Under above mentioned assumptions, the inventory level during production up time can be described by the differential equation

$$\frac{d\,I_{1a}\left(t_{1a}\right)}{dt_{1a}} = P - R\left(t_{1a}\right) - x - \theta I_{1a}\left(t_{1a}\right),\ 0 \le t_{1a} \le T_{1a} \tag{1}$$

The inventory level during rework up time is governed by the differential equation

$$\frac{d\,I_{3r}\left(t_{3r}\right)}{dt_{3r}} = P_1 - R\left(t_{3r}\right) - x_1 - \theta I_{3r}\left(t_{3r}\right),\ 0 \le t_{3r} \le T_{3r} \tag{2}$$

The rate of change of inventory level during production down time is

$$\frac{d\,I_{2a}\left(t_{2a}\right)}{dt_{2a}} = -R\left(t_{2a}\right) - \theta I_{2a}\left(t_{2a}\right),\ 0 \le t_{2a} \le T_{2a} \tag{3}$$

and during rework down time is

$$\frac{d\,I_{4r}\left(t_{4r}\right)}{dt_{4r}} = -R\left(t_{4r}\right) - \theta I_{4r}\left(t_{4r}\right),\ 0 \le t_{4r} \le T_{4r} \tag{4}$$

Under the assumption of LIFO production system, the rate of change of inventory of good items during rework up time and down time is governed by

$$\frac{d\,I_{3a}\left(t_{3a}\right)}{dt_{3a}} = -\theta I_{3a}\left(t_{3a}\right),\ \ 0 \le t_{3a} \le T_{3r} + T_{4r}$$

(5)

Using $I_{1a}\left(0\right) = 0$, the inventory level in a production up time is

$$I_{1a}\left(t_{1a}\right) = \frac{1}{\theta}\left(P - a - x\right)\left(1 - e^{-\theta t_{1a}}\right) - \frac{ab}{\theta^2}\left(e^{-\theta t_{1a}} + \theta t_{1a} - 1\right) \tag{6}$$

The total inventory in a production up time is

$$TI_{1a} = \int_0^{T_{1a}} I_{1a}\left(t_{1a}\right) dt_{1a} \tag{7}$$

Using, $I_{3r}\left(0\right) = 0$, solution of (2) is

$$I_{3r}\left(t_{3r}\right) = \frac{1}{\theta}\left(P_1 - a - x_1\right)\left(1 - e^{-\theta t_{3r}}\right) - \frac{ab}{\theta^2}\left(e^{-\theta t_{3r}} + \theta t_{3r} - 1\right) \tag{8}$$

and total inventory in a rework up time is

$$TI_{3r} = \int_0^{T_{3r}} I_{3r}\left(t_{3r}\right) dt_{3r} \tag{9}$$

Using $I_{4r}\left(t_{4r}\right) = 0,$ solution of (4) is

$$I_{4r}\left(t_{4r}\right) = \frac{a}{\theta}\left(e^{\theta(T_{4r} - t_{4r})} - 1\right) - \frac{ab}{\theta^2}\left(e^{\theta(T_{4r} - t_{4r})} - 1\right) + \frac{ab}{\theta}\left(T_{4r} e^{\theta(T_{4r} - t_{4r})} - 1\right) \tag{10}$$

And hence the total inventory of serviceable items during rework down time is

$$TI_{4r} = \int_0^{T_{4r}} I_{4r}\left(t_{4r}\right) dt_{4r} \tag{11}$$

Similarly, using $I_{2a}\left(T_{2a}\right) = 0,$ the total inventory during production down time is

$$TI_{2a} = \int_0^{T_{2a}} I_{2a}\left(t_{2a}\right)dt_{2a} \tag{12}$$

Now, the maximum inventory level is

$$I_m = I_{1a}\left(T_{1a}\right) = \frac{1}{\theta}\left(P-a-x\right)\left(1-e^{-\theta T_{1a}}\right) - \frac{ab}{\theta^2}\left(e^{-\theta T_{1a}} + \theta T_{1a} - 1\right) \tag{13}$$

Hence, the total inventory in a rework up time is

$$TI_{3a} = I_m\left(T_{3r} + T_{4r} - \frac{\theta}{2}\left(T_{3r} + T_{4r}\right)^2\right) \tag{14}$$

Next, we analyze the inventory level of recoverable items. (Figure

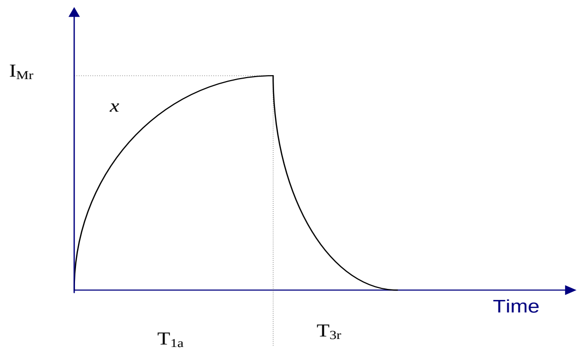

Figure 2: Inventory status of recoverable items

The rate of change of recoverable items in a production up time is

$$\frac{d\,I_{r1}\left(t_{r1}\right)}{dt_{r1}} = x - \theta I_{r1}\left(t_{r1}\right),\ 0 \le t_{r1} \le T_{1a} \tag{15}$$

Using $I_{r1}\left(0\right)=0$, the inventory level of the recoverable items during the production up time is

$$I_{r1}\left(t_{r1}\right) = \frac{x}{\theta}\left(1-e^{-t_{r1}}\right),\ 0 \le t_{r1} \le T_{1a} \tag{16}$$

hence, total recoverable items in a production up time is

$$TTI_{r1} = \int_0^{T_{1a}} I_{r1}(t_{r1})dt_{r1} \tag{17}$$

Initially, the recoverable inventory is

$$I_{Mr} = I_{r1}\left(T_{1a}\right) = \frac{x}{\theta}\left(1 - e^{-\theta T_{1a}}\right)T_{1a}$$
$$= x\left(T_{1a} - \frac{\theta T_{1a}{}^{2}}{2}\right) \quad (18)$$

The rate of change of inventory level of recoverable item during the rework up time is governed by differential equation

$$\frac{d\,I_{r3}\left(t_{r3}\right)}{dt_{r3}} = -P_1 - \theta I_{r3}\left(t_{r3}\right),\ \ 0 \le t_{r3} \le T_{3r} \quad (19)$$

Using, $I_{r3}\left(t_{3r}\right) = 0$, the solution of equation (19) is

$$I_{r3}\left(t_{r3}\right) = \frac{P_1}{\theta}\left(e^{\theta\left(T_{r3} - t_{r3}\right)} - 1\right) \quad (20)$$

The total inventory of recoverable item during rework up time is

$$TTI_{r3} = \int_{0}^{T_{3r}} I_{r3}(t_{r3})dt_{r3} \quad (21)$$

The number of recoverable items is

$$I_{Mr} = I_{r3}\left(0\right) = \frac{P_1}{\theta}\left(e^{\theta T_{3r}} - 1\right) \quad (22)$$

Since $\theta T_{3r} << 1$ and using Taylor's series approximation, equation (22) gives

$$T_{3r} = \frac{I_{Mr}}{P_1} \quad (23)$$

Substituting I_{Mr} from equation (18) in equation (23), we get

$$T_{3r} = \frac{x}{P_1}\left(T_{1a} - \frac{\theta T_{1a}{}^{2}}{2}\right) \quad (24)$$

Total recoverable items

$$I_w = TTI_{r1} + TTI_{r3} \quad (25)$$

Total number of units deteriorated is

$$DU = \left(P - \int_0^{T_{1a}} R(t)dt \right) + \left(P_1 - \int_0^{T_{3r}} R(t)dt \right) - \int_0^{T_{2a}+T_{4r}} R(t)dt - x_1 T_{3r} \tag{26}$$

Since the inventory level at the beginning of the production down time is equal to the inventory level at the end of the production up time minus the deteriorated units at $T_{3r} + T_{4r}$, using Misra(1975), the approximation concept, we have

$$T_{2a} \approx \frac{1}{a}\left[(P - a - x)T_{1a} - \frac{ab}{2} T_{1a}{}^2 \right]\left[1 - \theta(T_{3r} + T_{4r}) + \frac{1}{2}\theta(T_{3r} + T_{4r})^2 \right] \tag{27}$$

The inventoryfor serviceable item in rework process is

$$\boldsymbol{I_{3r}(T_{3r}) = I_{4r}(0)}$$

by simple calculations

$$T_{4r} \approx \frac{1}{a}(P_1 - a - x_1)T_{3r}\left(1 - \frac{1}{2}\theta T_{3r}\right) \tag{28}$$

Using equations (24) and (28), T_{2a} given in equation (27) is only a function of T_{1a} . The total production cost of inventory system is sum of production set up cost, holding cost of serviceable inventory, deteriorating cost of recoverable inventory cost, and scrap cost.
Therefore,

$$TC = A + h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] + h_1 I_w + C_d \ DU + S_C \ x_1 T_{3r} \tag{29}$$

total replenishment time is

$$T = T_{1a} + T_{3r} + T_{2a} + T_{4r} \tag{30}$$

The total cost per unit time without lost sales is given by

$$TCT_{NL} = \frac{TC}{T} \tag{31}$$

The optimal production up time for the EPQ model without lost sales is the solution of

$$\frac{dTCT_{NL}(T_{1a})}{dT_{1a}} = 0 \tag{32}$$

Lost sales will occur when maintenance time of machine is greater than the production down-time period. So the total inventory cost in this case is

$$E(TC) = TC + S_L \int_{t=T_{2a}+T_{4r}}^{\infty} R(t)\left(t-\left(T_{2a}+T_{4r}\right)\right) f(t)\,dt \tag{33}$$

the total cycle time for lost sales scenario is

$$E(T) = T + \int_{t=T_{2a}+T_{4r}}^{\infty} \left(t-\left(T_{2a}+T_{4r}\right)\right) f(t)\,dt \tag{34}$$

Using equations (33) and (34), the total cost per unit time for lost sales scenario is

$$E(TCT) = \frac{E(TC)}{E(T)} \tag{35}$$

3.1. Uniform distribution Case

Define the probability distribution function $f(t)$, when the preventive maintenance time t follows uniform distribution as follows.

$$f(t) = \begin{cases} \dfrac{1}{\tau}, & 0 \le t \le \tau \\ 0, & \text{otherwise} \end{cases}$$

substituting $f(t)$ in equation (35) gives total cost per unit time for uniform distribution as

$$TCT_U = \frac{A + h\left[TI_{1a} + TI_{3r} + TI_{2a} + TI_{4r} + TI_{3a}\right] + h_1 I_w + C_d\,DU + S_C\,x_1 T_{3r} + \dfrac{S_L}{\tau} a \int_0^{\tau} (1+bt)\left(t-\left(T_{2a}+T_{4r}\right)\right)dt}{T_{1a} + T_{3r} + T_{2a} + T_{4r} + \dfrac{1}{\tau}\int_0^{\tau}\left(t-\left(T_{2a}+T_{4r}\right)\right)dt} \tag{36}$$

The optimal production up time for lost sales case is solution of

$$\frac{dTCT_U\left(T_{1a}\right)}{dT_{1a}} = 0 \tag{37}$$

To decide whether manufacturer should allow lost sales or not, we propose following steps (Wee and Widyadana (2011)):

Step 1: Calculate T_{1a} from equation (32). Hence calculate T_{2a} from equation (27) and T_{4r} from equation (28). Set $T_{sb} = T_{2a} + T_{4r}$.

Step 2: If $T_{sb} < \tau$, then non lost sales case is not feasible, and go to step 3; otherwise the optimal solution is obtained.

Step 3: Set $T_{sb} = \tau$. Find T_{1aub} using equations (27) and (28). Calculate $TCT_{NL}\left(T_{1aub}\right)$ using equation (31).

Step 4: Calculate T_{1a} from equation (37), hence T_{2a} from equation (27) and T_{4r} from equation (28), and set $T_{sb} = T_{2a} + T_{4r}$.

Step 5: If $T_{sb} \ge \tau$ then optimal production up time T_{1a} is T_{1aub} and $TCT_{NL}\left(T_{1aub}\right)$ If $T_{sb} \le \tau$, then, calculate $TCT_U\left(T_{1a}\right)$ using equation (36).

Step 6: If $TCT_{NL}\left(T_{1aub}\right) \le TCT_U\left(T_{1a}\right)$, then optimal production up time T_{1aub} ; otherwise it is T_{1a}

3.2. Exponential distribution case

Define the probability distribution function $f\left(t\right)$, when the preventive maintenance time t follows exponential distribution with mean $\frac{1}{\lambda}$ as

$f\left(t\right) = \lambda e^{-\lambda t}, \lambda > 0.$

Here, the total cost per unit time for lost sales scenario is

$$TCT_E = \frac{TC + S_L \int\limits_{t=T_{2a}+T_{4r}}^{\infty} R\left(t\right)\left(t - \left(T_{2a} + T_{4r}\right)\right)\lambda e^{-\lambda t}\, dt}{T + \frac{1}{\lambda} e^{-\lambda\left(T_{2a}+T_{4r}\right)}} \tag{38}$$

The optimal T_{1a} can be obtained by setting

$$\frac{dTCT_E\left(T_{1a}\right)}{dT_{1a}} = 0 \tag{39}$$

The convexity of TCT_{NL}, TCT_U and/or TCT_E has been established graphically with suitable values of inventory parameters.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, we validate the proposed model by numerical example. First, we consider uniform distribution case. Take $A = \$200$ per production cycle, $P = 10,000$ units per unit time, $P_1 = 4000$ units per unit time, $a = 5000$ units per unit time, $b = 10\%$, $x = 500$ units per unit time, $x_1 = 400$ units per unit time, $h = \$15$ per unit per unit time. $h_1 = \$3$ per unit per unit time, $S_L = \$10$ per unit, $S_C = \$12$ per unit, $C_d = \$0.01$ per unit , $\theta = 10\%$ and the preventive maintenance time is uniformly distributed over the interval $[0, 0.1]$. Following algorithm with Maple 14, the optimal production up time $T_{1a} = 0.109$ years and the corresponding optimal total cost per unit time is $TCT_U = \$4448$. The convexity of TCT_U is exhibited in Figure 3.

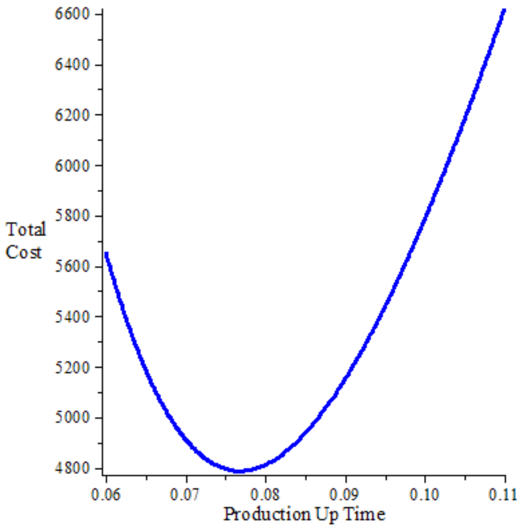

Figure 3: Convexity of Total Optimal Cost with Uniform Distribution

The sensitivity analysis is carried out by changing one parameter at a time by $-40\%, -20\%, +20\%$ and $+40\%$. The optimal production up time and the total cost per unit time for different inventory parameters are exhibited in Table 1.

Table 1 Sensitivity analysis of T_{1a} and total cost when preventive maintenance time for uniform and exponential distribution.

Parameter	Percentage change	Uniform Distribution		Exponential Distribution λ = 20	
		T1a	***TCT***	***T1a***	***TCT***
A	-40%	0.107791969	4078.521832	0.139574511	5113.723713
	-20%	0.108600011	4263.884212	0.139810498	5115.079416
	0	0.10940717	4447.906835	0.140048682	5116.43044
	20%	0.110213455	4630.606199	0.140289094	5117.776849
	40%	0.111018878	4811.998518	0.140531765	5119.118693
P	-40%	Not Feasible	Not Feasible	0.80333702	4497.769808
	-20%	0.191001903	4040.012884	0.230211542	4821.961079
	0	0.10940717	4447.906835	0.140048682	5116.43044
	20%	0.076808684	4787.228599	0.101644181	5282.346729
	40%	0.059274011	5086.939301	0.08003233	5389.345959
P_1	-40%	0.118683393	4700.523848	0.146305758	5506.08129
	-20%	0.11269291	4526.771856	0.14232037	5258.800083
	0	0.10940717	4447.906835	0.140048682	5116.43044
	20%	0.107362559	4412.358408	0.138580965	5023.818806
	40%	0.105989146	4400.192947	0.137554578	4958.738514
A	-40%	Not Feasible	Not Feasible	0.065854221	3719.215509
	-20%	0.07246471	4196.531895	0.097126915	4449.530627
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.16785454	4733.262382	0.204019241	5718.553561
	40%	0.273755232	5081.486552	0.314035664	6269.664841
B	-40%	0.109157125	4445.377023	0.13955077	5104.426062
	-20%	0.10928183	4446.642683	0.139799146	5110.431143
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.109533149	4449.169487	0.140299393	5122.423958
	40%	0.109659774	4450.430641	0.140551295	5128.411696
X	-40%	0.104065638	4318.549344	0.135769095	4949.255156
	-20%	0.106673421	4382.428699	0.137864251	5032.509567
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.112275606	4515.097081	0.142328128	5201.082414
	40%	0.115288203	4584.123734	0.144708828	5286.537699

Parameter	Percentage change	Uniform Distribution		Exponential Distribution λ = 20	
		T1a	***TCT***	***T1a***	***TCT***
	-40%	0.109193541	4386.01374	0.139884967	5054.344838
	-20%	0.109300267	4416.943204	0.139966795	5085.371346
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.109514249	4478.904682	0.140130627	5147.5222
	40%	0.109621504	4509.936823	0.14021263	5178.646666
h	-40%	0.111814445	3929.608716	0.150159236	4453.225302
	-20%	0.110597091	4190.190873	0.144782689	4790.825835
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.108243755	4702.808287	0.135826709	5431.367706
	40%	0.107105962	4954.945599	0.132021882	5736.710556
h_1	-40%	0.109492258	4429.380983	0.140379577	5093.016012
	-20%	0.109449696	4438.645749	0.140213774	5104.730522
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.109364678	4457.164251	0.139884294	5128.115837
	40%	0.10932222	4466.417991	0.139720605	5139.78676
S_L	-40%	0.106222274	4429.303489	0.122003914	4700.202185
	-20%	0.108157269	4440.669141	0.131984403	4930.302135
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.110281085	4452.91983	0.146835261	5273.171982
	40%	0.110926483	4456.597219	0.152705659	5408.808796
S_c	-40%	0.10941866	4327.54245	0.140134136	4997.933007
	-20%	0.109412921	4387.724673	0.140091431	5057.182213
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.109401406	4508.088927	0.140005888	5175.677688
	40%	0.109395631	4568.270952	0.13996305	5234.923957
Θ	-40%	0.109485646	4454.425581	0.139574511	5113.723718
	-20%	0.109468915	4451.150998	0.139810498	5115.079422
	0	0.10940717	4447.906833	0.140048682	5116.43044
	20%	0.109246098	4444.692714	0.140289094	5117.776842
	40%	0.109085698	4441.508265	0.140531765	5119.118694

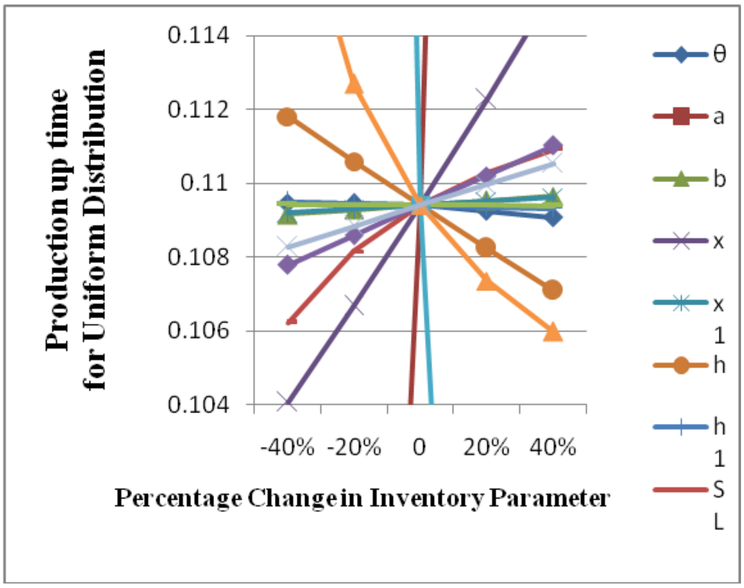

Figure 4: Sensitivity analysis of production up time for uniform distribution

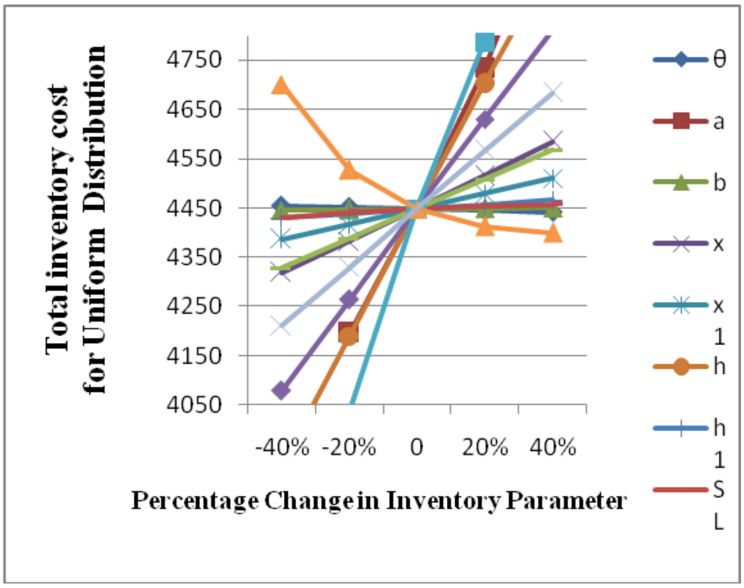

Figure 5: Sensitivity analysis of total cost for uniform distribution

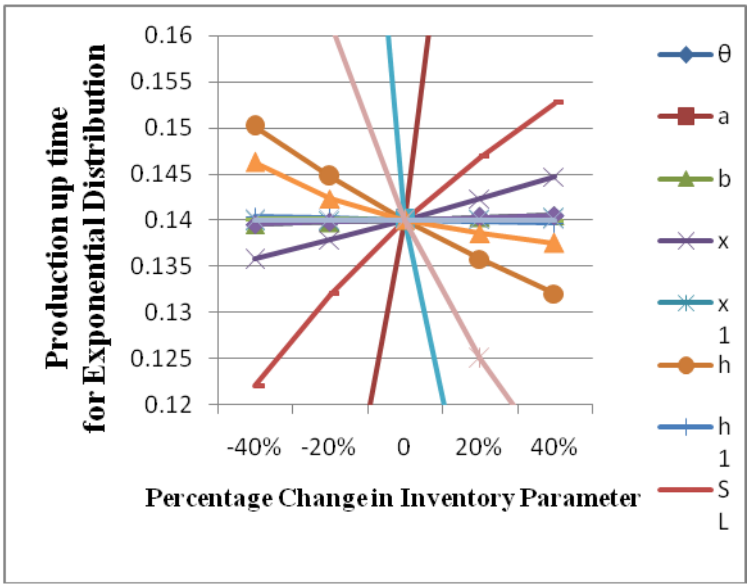

Figure 6: Sensitivity analysis of production up time for exponential distribution

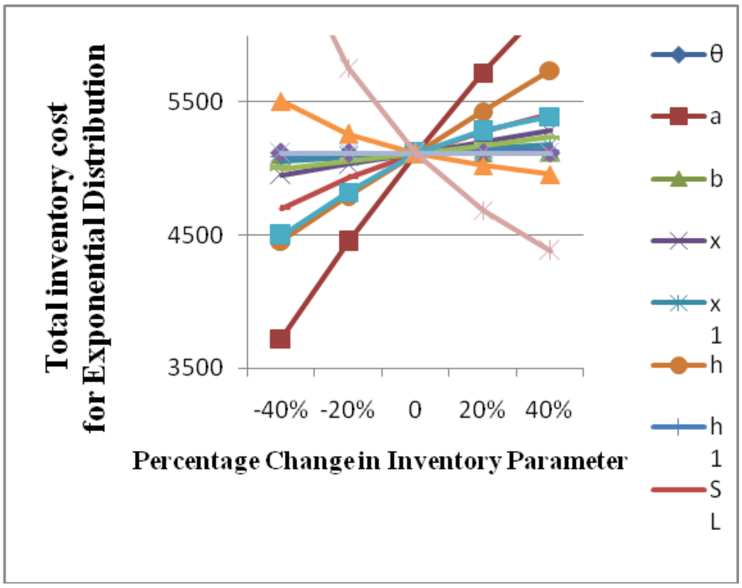

Figure: 7 Sensitivity analysis of total cost for exponential distribution

It is observed from figure 5 that the optimal production up time is slightly sensitive to changes in P, θ and a, moderately sensitive to changes in τ and b, and insensitive to changes in other parameters. T_{1a} is negative related to P and θ and positively related to a and T_{1a}. Figure 6 exhibits variations in the optimal total cost per unit time with uniform distribution. The optimal total cost is slightly sensitive to changes in a, P, x, C_d and h; moderately sensitive to changes in A, τ, Sc, θ, x_1 and S_L and insensitive to changes in other parameters.

Take mean of exponential distribution as 20. The optimum total cost is $ 5116 when the optimal production up time is 0.14 years. The convexity of the total cost is shown in Figure 8.

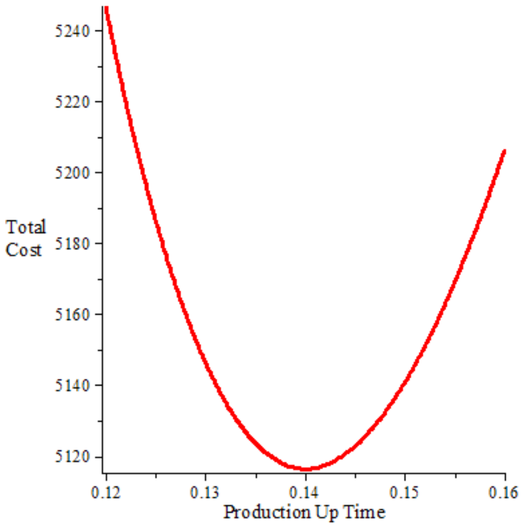

Figure 8: Convexity of Total Optimal Cost with Exponential Distribution

From Figure6, for lost sales with exponential distribution, it is observed that the optimal production up time is slightly sensitive to changes in P and a, moderately sensitive to changes in the parameter λ and θ, and insensitive to changes in other parameters. The optimal production up time is negatively related to P, b and λ, and positively related to the value of a. In Figure7, variations in the optimal total cost is studied. Observations are similar tothe uniform distribution case.

5. CONCLUSIONS

In this research, the economic production quantity model for deteriorating items is studied when demand is time dependent. The rework of the items is allowed and random preventive maintenance time is incorporated. The model considers lost sales. The probability of machine preventive maintenance time is assumed to be uniformly and exponentially distributed. It is observed that the production up time is sensitive to demand rate and deterioration. It suggests that the manufacturer should control deterioration of units in inventory by using proper storage facilities. The optimal total cost per unit time is sensitive to changes in the holding cost, the product defect rate, and the production rate in both the distributions. This suggests that the manufacture should depute an efficient technician to reduce preventive maintenance time. This model has wide applications in manufacturing sector. Because of using machines for a long period of time,manufacturer facesimproper production, some customers are not satisfied with the quality of the production so, manufacturer has to adopt rework policy.Future research by considering constraints on the machine's output, machine's life time will be worthy.

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