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INTERACTIVE FUZZY GOAL PROGRAMMING APPROACH IN MULTI-RESPONSE STRATIFIED SAMPLE SURVEYS

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Abstract: In this paper, we applied an Interactive Fuzzy Goal Programming (IFGP) approach with linear, exponential and hyperbolic membership functions, which focuses on maximizing the minimum membership values to determine the preferred compromise solution for the multi-response stratified surveys problem, formulated as a Multi-Objective Non Linear Programming Problem (MONLPP), and by linearizing the non-linear objective functions at their individual optimum solution, the problem is approximated to an Integer Linear Programming Problem (ILPP). A numerical example based on real data is given, and comparison with some existing allocations viz. Cochran's compromise allocation, Chatterjee's compromise allocation and Khowaja's compromise allocation is made to demonstrate the utility of the approach.

Keywords: Compromise Allocation, Coefficient of Variation, Interactive Fuzzy Goal Programming, Optimum Allocation.

MSC: 62D05.

1. INTRODUCTION

In statistics, one of the most commonly used technique in all fields of scientific investigation is stratified sampling. In statistical surveys, when subpopulations within an overall population vary, it is advantageous to sample each subpopulation (stratum) independently. Stratification is the process of dividing members of the population into homogeneous subgroups before sampling. The strata should be mutually exclusive: every element in the population must be assigned to only one stratum. The strata should also be collectively exhaustive: no population element can be excluded. Then, simple random sampling or systematic sampling is applied within each stratum. This often improves the representativeness of the sample by reducing sampling error. It can produce a weighted mean that has less variability than the arithmetic mean of a simple random sample of the population. After stratification, the next problem is the allocation of sample sizes in each stratum. In multivariate surveys, the problem of obtaining optimal allocation is complicated because univariate allocation methods are not optimum for all characteristics. Many authors discussed compromise criterion that provides a compromise allocation, which is optimum for all characteristics, at least in some sense. Some of these are Neyman [23], Kokan and Khan [20], Chatterjee [10], Ahsan and Khan [2,3], Chromy [11], Bethel [6], Jahan *et al.* [17], Khan *et al.* [18,19], Kozak [21], Diaz-Garcia and Ulloa [14,15], Ansari *et al*. [1], Ali *et al*. [4], Khowaja *et al*. [19], Gupta *et al.* [16] etc. Hence, in planning multivariate stratified surveys, we need a compromise criterion that gives an allocation, which is optimum for all characteristics, in some sense. Khowaja et *al.* minimize the coefficient of variation subject to budget constraint and other restrictions. In this article, based on their formulation of Integer Linear Programming Problem, it was demonstrated how the proposed approach worked in the field of sampling.

In probability theory and statistics, the coefficient of variation (CV) is a normalized measure of dispersion of a probability distribution. It is also known as unitized risk or the variation coefficient. The absolute value of the CV is sometimes known as relative standard deviation (RSD), which is expressed as a percentage. The coefficient of variation represents the ratio of the standard deviation to the mean, and it is a useful statistic for comparing the degree of variation from one data series to another, even if the means are drastically different.

An Interactive Fuzzy Goal Programming is developed by combining three approaches viz. Interactive Programming, Fuzzy Programming and Goal Programming to obtain a most preferred compromise solution of the formulated Integer Linear Programming Problem. This approach combines the advantages of three approaches to produce a powerful method. Recently De and Yadav [13] use this approach to solve a Multi-Objective Assignment Problem.

In this paper, we develop an algorithm which is characterized by linear, exponential and hyperbolic membership functions to solve a Multi-Objective Integer Linear Programming Problem and obtain a best preferred compromise solution.

This paper is organized as follows: section 1 gives the brief introduction, a survey of the work done in this area. In section 2, mathematical model is described. Interactive Fuzzy Goal Programming approach with linear, exponential & hyperbolic membership functions and the solution of Integer Linear Programming Problem using IFGP is presented in section 3. In section 4, some other existing approaches are given for the

purpose of comparison. A numerical example is presented to demonstrate the algorithm in section 5. And finally, conclusion of the work is presented in section 6.

2. PROBLEM FORMULATION

We assume that more than one characteristic ($p \geq 2$) is to be measured on each unit of a population of size N, which is divided into L non overlapping strata of size N_h, $h = 1, 2, \ldots, L$, $N = \sum_{h=1}^{L} N_h$ and the interest is in the estimation of p-population means. Let n_h, where $h=1, 2, \ldots, L$, be the number of units drawn without replacement from the h^{th} stratum. For the j^{th} character, unbiased estimates of the population mean,

$$\begin{aligned} \overline{X}_j &= \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{jhi}; \quad j = 1, 2, \ldots, p \\ &= \sum_{h=1}^{L} W_h \overline{X}_{jh} \end{aligned} \tag{1}$$

$$\text{is } \bar{x}_{j,st} = \frac{1}{n}\sum_{h=1}^{L}\sum_{i=1}^{n_h} x_{jhi} \tag{2}$$

with a sampling variance

$$V(\bar{x}_{j,st}) = \sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_{jh}^2; \quad j = 1, 2, \ldots, p \tag{3}$$

where

$$W_h = \frac{N_h}{N}; h = 1, 2, \ldots, L \text{ (stratum weights)}; \quad \overline{X}_{jh} = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{jhi} \text{ (stratum means)} \tag{4}$$

$$S_{jh}^2 = \frac{1}{(N_h - 1)}\sum_{i=1}^{N_h}(x_{jhi} - \overline{X}_{jh})^2; \; j = 1, 2, \ldots, p; \; h = 1, 2, \ldots, L \text{ (stratum variances)}$$

Assuming a linear cost function

$$C = c_0 + \sum_{h=1}^{L} c_h n_h \tag{5}$$

where

c_h = per unit cost of measurment in the h^{th} stratum

c_0 = overhead cost

The problem of finding optimum allocation may be given as the following Multi-Objective Integer Non Linear Programming Problem (MOINLPP):

$$\left.\begin{array}{ll} \textit{Minimize} & \begin{pmatrix} (CV)_1^2 \\ (CV)_2^2 \\ \vdots \\ (CV)_p^2 \end{pmatrix}, \\ \textit{Subject to} & \sum_{h=1}^{L} c_h n_h \le C_0 \\ & 2 \le n_h \le N_h \\ \text{and} & n_h \text{ are integers; } h = 1, 2, ..., L \end{array}\right\}$$

where $C_0 = C - c_0$

and

$$(CV)_j^2 = CV(\bar{x}_{j,st})^2 = \frac{\sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_{jh}^2}{\overline{X}_j^2}; \quad j = 1, 2, ..., p \tag{7}$$

is the population squared coefficient of variation for the j^{th} characteristics.

Using Eq. (7), the problem of finding individual optimum allocations that minimize the $(CV)^2$ under cost and the restrictions on n_h may be given as the following Non Linear Programming Problem (NLPP):

$$\left.\begin{array}{ll} \textit{Minimize } Z_j = & \dfrac{\sum_{h=1}^{L}\left(\dfrac{W_h^2 S_{jh}^2}{n_h}\right)}{\overline{X}_j^2} \\ \textit{Subject to} & \sum_{h=1}^{L} c_h n_h \le C_0 \\ & 2 \le n_h \le N_h \\ \text{and} & n_h \text{ are integers; } h = 1, 2, ..., L \end{array}\right\} \; j = 1, 2, ..., p \tag{8}$$

Each objective function in Eq. (8) is non-linear. The cost constraint and the upper and lower bounds on n_h are linear. The NLPP (Eq. (8)) for the p characteristics may be solved by using an appropriate Non Linear Programming method.

It can be seen that the objective function in Eq. (8) are convex. To use Interactive Fuzzy Goal Programming, Z_j are linearized at the individual optimum points.

Thus, for $j=k$ at the point $\underline{n}_k^* = (n_{k1}^*, n_{k2}^*, \ldots, n_{kh}^*), Z_k^{'}$ may be approximated by the linear function in n_h as:

$$Z_k^{'} \cong Z_k(n_{kh}^*) + \nabla^{'} Z_k(n_{kh}^*)(n_h - n_{kh}^*), \tag{9}$$

where $Z_k^{'}$ is the squared coefficient of variation and $\nabla^{'} Z_k(n_{kh}^{*})$ is the value of the vector of partial derivatives of Z_k with respect to $n_{kh}\,(h=1,2,\ldots,L)$ at the point $\underline{n}_k^{*}$ given as:

$$\nabla^{'} Z_k^{'}(n_{kh}^{*}) = \left[-\left(\frac{W_1^2 S_{k1}^2}{\overline{X}_k^2 (n_{k1}^{*})^2} \right), -\left(\frac{W_2^2 S_{k2}^2}{\overline{X}_k^2 (n_{k2}^{*})^2} \right), \ldots, -\left(\frac{W_L^2 S_{kL}^2}{\overline{X}_k^2 (n_{kL}^{*})^2} \right) \right],$$

This gives

$$\nabla^{'} Z_k^{'}(n_{kh}^{*})(n_h - n_{kh}^{*}) = \left\{ \left(\sum_{h=1}^{L} \frac{W_h^2 S_{kh}^2}{\overline{X}_k^2 n_{kh}^{*}} \right) - \left(\sum_{h=1}^{L} \left(\frac{W_h^2 S_{kh}^2}{\overline{X}_k^2 (n_{kh}^{*})^2} \right) n_h \right) \right\},$$

and

$$Z_k^{'} \cong 2\left\{ \sum_{h=1}^{L} \frac{W_h^2 S_{kh}^2}{\overline{X}_k^2 n_{kh}^{*}} \right\} - \left\{ \sum_{h=1}^{L} \left(\frac{W_h^2 S_{kh}^2}{\overline{X}_k^2 (n_{kh}^{*})^2} \right) n_h \right\} = z_k^{'} \quad \text{(say)}$$

The NLPP (Eq. (8)) can now be approximated by ILPP and after dropping the constant terms from the linear objective function, the final problem is equivalent to maximizing (- $z_k^{'}$); this gives the ILPP as:

$$\left.\begin{array}{ll} \textit{Maximize } z_j = \displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^{*})^2} \right) n_h & \\ \textit{Subject to } \displaystyle\sum_{h=1}^{L} c_h n_h \le C_0 & \\ \qquad\qquad 2 \le n_h \le N_h & \\ \text{and} \qquad n_h \text{ are integers; } h = 1,2,\ldots,L & \end{array}\right\} \; j = 1,2,\ldots,p$$

In real surveys, $\overline{X}_j$ are not known, but in this formulation, they are assumed to be known. In practice, some approximations of these parameters may be used that are known from some recent or preliminary survey (Kozak, [21]).
(For detailed formulation of the problem see Khowaja *et al*. [19]).

3. INTERACTIVE FUZZY GOAL PROGRAMMING (IFGP) APPROACH

By combining the three approaches, a powerful approach is developed, called Interactive Fuzzy Goal Programming approach. Wahed and Lee [25] presented IFGP approach for Multi-Objective Transportation Problem, and De and Yadav [13] for Multi-Objective Assignment Problem. We try to use this approach in the field of sampling. Although the three approaches are very well known and well defined in past by several

authors in various fields, but for the sake of simplicity, a brief description of the three approaches is given below:

a) Interactive approach

Interactive methods are based upon extensive employment of the decision maker, particularly throughout the solution process. Interactive methods take on a variety of forms and are discussed in the literature by Hwang and Masud [17]. Interactive approaches play an important role in deriving the best preferred compromise solution because the solution maker is involved in the solution procedure.

b) Fuzzy programming approach

Fuzzy programming offers a powerful means of handling optimization problems with fuzzy parameters and is by far, the better known concept and has, in fact, established a wide following in the multi-objective optimization and MCDM (multi criteria decision making) communities, wherein numerous real world problems have been approached and successfully solved by the methodology. In the past, Fuzzy programming has been used in different fields such as transportation, reliability, sampling, and etc. by several authors.

c) Goal programming

Goal programming is a variation of LP that permits multiple and conflicting goals with different dimensions. Multiple goals are rank-ordered and are treated as *preemptive priorities.* In the solution procedure, higher-ranked goals are not sacrificed to achieve lower-ranked goals. The solution approach is equivalent to solving a series of nested LP problems in which higher-ranked goals become constraints on lower-ranked goals. While LP optimizes a single objective, goal programming minimizes deviations from goals. In one sentence, we can say that Goal programming is the "workhorse" of the multi-objective optimization methods. Goal programming was first used by Charnes, Cooper and Ferguson in 1955, although the actual name first appeared in a 1961's text by Charnes and Cooper.

3.1. Solution using IFGP approach

First, we solve Multi-Objective Integer Linear Programming Problem as a single objective problem for each p characteristics subject to the system constraints. The optimum solution obtained for each characteristic helps us in defining the pay-off matrix as:

$$\begin{pmatrix} & z_1 & \cdots & z_j \\ n_1^{(1)} & z_1(n_1^{(1)}) & \cdots & z_j(n_1^{(1)}) \\ \vdots & \vdots & \ddots & \vdots \\ n_h^{(j)} & z_1(n_h^{(j)}) & \cdots & z_j(n_h^{(j)}) \end{pmatrix}, \quad \begin{matrix} h = 1,2,...,L \\ j = 1,2,...,p \end{matrix}$$

where $n_1^{(1)},...,n_h^{(j)};\ h = 1,2,...,L;\ j = 1,2,...,p$ are the individual optimal solution of each objective function and $z_j = \sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2(n_{jh}^*)^2} \right) n_h;\ j = 1,2,\ldots p$.

Now, we can obtain the upper and lower tolerance limits of each objective function as $U_j = \max(z_j)$ and $L_j = \min(z_j)$; $j = 1,2,...,j$. And after that, we define membership function for the j^{th} objective function as:

Case (i) Linear membership function

A linear membership function $\mu_j^L(z_j)$ for each objective function is defined as:

$$\mu_j^L(z_j) = \begin{cases} 0 & , \text{if } \sum_{h=1}^{L}\left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2(n_{jh}^*)^2}\right) n_h \le L_j \\ \dfrac{\left(\sum_{h=1}^{L}\left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2(n_{jh}^*)^2}\right) n_h\right) - L_j}{U_j - L_j} & , \quad \text{if } L_j < z_j < U_j \\ 1 & , \text{if } \sum_{h=1}^{L}\left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2(n_{jh}^*)^2}\right) n_h \ge U_j \end{cases} \tag{11}$$

where U_j and L_j are the lower and upper tolerance limits of the objective functions, such that the degrees of the membership function are 0 and 1, respectively, and it is depicted in Fig.1 as follows:

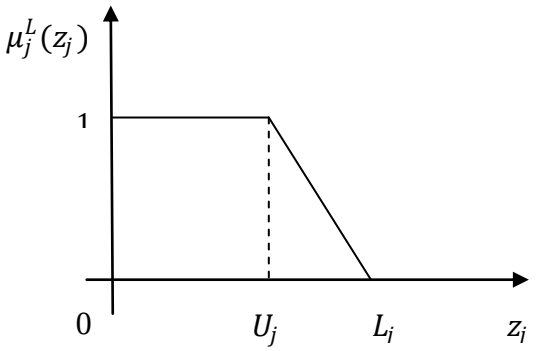

Figure 1: Linear membership function for j[th] goal

Now, following the principle of the fuzzy decision by Bellman and Zadeh [5], the ILPP is given as the following maxmin problem:

$$\left.\begin{array}{ll} \textit{Maximize} & \underset{j}{Min}\,\mu_j(z_j) \\ \textit{subject to} & \sum_{h=1}^{L} c_h n_h \le C_0 \\ & 2 \le n_h \le N_h \\ \text{and} & n_h \text{ are integers; } h = 1,2,...,L \end{array}\right\} \tag{12}$$

Problem (12) can be rewritten as a standard LPP by introducing an auxiliary variable λ as:

$$\left.\begin{array}{l} \textit{Maximize} \quad \lambda \\ \textit{subject to} \\ \lambda \le \left(\dfrac{\left(\sum_{h=1}^{L} \left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \right) - L_j}{U_j - L_j} \right), \quad j = 1,2,....,p \\ \sum_{h=1}^{L} c_h n_h \le C_0 \\ 2 \le n_h \le N_h \\ \text{and } n_h \text{ are integers; } h = 1,2,...,L \end{array}\right\} \tag{13}$$

Now, let us introduce the following deviational variables to formulate model (13) as a goal programming model:

$$z_k - \delta_k = G_k, \quad k = 1,2,...,k$$

where G_k is the aspiration level of the objective function *k*.

Therefore, model (13) can be formulated as a mixed integer goal programming as follows:

$$
\left.
\begin{array}{l}
\textit{Maximize} \quad \lambda \\
\textit{subject to} \\
\lambda \le \left(\dfrac{\left(\displaystyle\sum_{h=1}^{L} \left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \right) - L_j}{U_j - L_j} \right), \quad j = 1, 2, ..., p \\
\displaystyle\sum_{h=1}^{L} c_h n_h \le C_0 \\
z_k - \delta_k = G_k \\
2 \le n_h \le N_h \\
\delta_k \ge 0 \\
\text{and } n_h \text{ are integers; } h = 1, 2, ..., L
\end{array}
\right\} \tag{14}
$$

Case (ii) Exponential membership function

An exponential membership function can be defined as:

$$
\mu_j^E(Z_j) = \begin{cases}
0 & , \textit{if } \displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \le L_j \\
\exp\left(\dfrac{\alpha \left(\displaystyle\sum_{h=1}^{L} \left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h - U_j \right)}{U_j - L_j} \right) & , \textit{if } L_j < z_j < U_j \\
1 & , \textit{if } \displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \ge U_j \textit{ and } \alpha \to \infty
\end{cases} \tag{15}
$$

where α is a non-zero parameter, prescribed by the decision maker. Figure 2 depicts a possible shape of $\mu_j(z_j)$ with respect to the objective function.

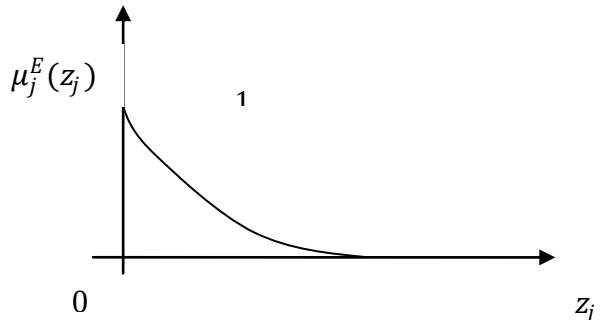

Figure 2: Exponential membership function for j[tn] goal

By using the exponential membership function as defined in (15), the equivalent non-linear model is:

$$\left.\begin{array}{l} \textit{Maximize} \quad \lambda \\ \textit{subject to} \\ \lambda \le \exp\left(\dfrac{\alpha\left(\sum_{h=1}^{L}\left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h - U_j \right)}{U_j - L_j} \right), \quad j = 1,2,\ldots,p \\ \sum_{h=1}^{L} c_h n_h \le C_0 \\ 2 \le n_h \le N_h \\ \text{and } n_h \text{ are integers; } h = 1,2,\ldots,L \end{array}\right\} \tag{16}$$

Now, let us introduce the following deviational variables to formulate model (16) as a goal programming model:

$$z_k - \delta_k = G_k, \; k = 1,2,\ldots,k$$

where G_k is the aspiration level of the objective function *k*.

Therefore, model (16) can be formulated as a mixed integer goal programming as follows:

$$
\left.
\begin{array}{l}
\textit{Maximize} \quad \lambda \\
\textit{subject to} \\
\lambda \le \left(\dfrac{\alpha \left(\displaystyle\sum_{h=1}^{L} \left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h - U_j \right)}{U_j - L_j} \right), \quad j = 1, 2, \ldots, p \\
\displaystyle\sum_{h=1}^{L} c_h n_h \le C_0 \\
z_k - \delta_k = G_k \\
2 \le n_h \le N_h \\
\delta_k \ge 0 \\
\text{and } n_h \text{ are integers; } h = 1, 2, \ldots, L
\end{array}
\right\} \qquad (17)
$$

Case (iii) Hyperbolic membership function

A hyperbolic membership function $\mu_j^H(z_j)$ for each objective function is defined as:

$$
\mu_j^H(z_j) = \begin{cases}
0 & , \quad \textit{if } \displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \le L_j \\
\dfrac{1}{2} \tanh \left(\left(\displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h - \frac{U_j + L_j}{2} \right) \alpha_j \right) + \frac{1}{2}, & \quad \textit{if } L_j < z_j < U_j \\
1 & , \quad \textit{if } \displaystyle\sum_{h=1}^{L} \left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2} \right) n_h \ge U_j
\end{cases} \qquad (18)
$$

where $\alpha_j = \dfrac{6}{(U_j - L_j)}$ is a parameter.

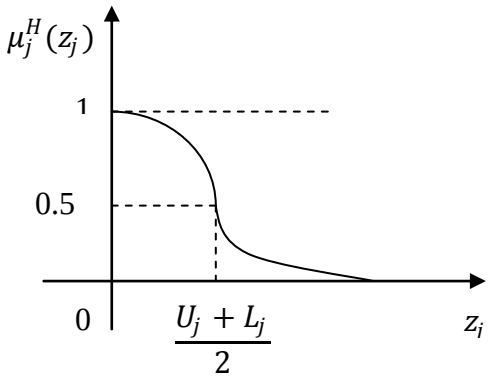

Figure 3: Hyperbolic membership function for j[th] goal

Hyperbolic membership function holds the following properties:

(i) It is strictly decreasing function.

(ii) It is strictly concave for $z_j \le (U_j + L_j)/2$

(iii) It is equal to 0.5 for $z_j = (U_j + L_j)/2$

(iv) It is strictly convex for $z_j \ge (U_j + L_j)/2$

(v) $\mu_j^H(z_j)$ satisfies $0 < \mu_j^H(z_j) < 1$ for $L_j < z_j < U_j$ and approaches asymptotically $\mu_j^H(z_j) = 0$ and $\mu_j^H(z_j) = 1$ as $z_j \to \infty$ and $-\infty$, respectively.

By using the hyperbolic membership function as defined in (18), the equivalent non-linear model is:

$$\left.\begin{array}{l} \textit{Maximize} \quad \lambda \\ \textit{subject to} \\ \lambda \le \frac{1}{2}\tanh\left(\left(\sum_{h=1}^{L}\left(\frac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2}\right) n_h - \frac{U_j + L_j}{2}\right)\alpha_j\right) + \frac{1}{2}, \quad j = 1,2,\ldots,p \\ \sum_{h=1}^{L} c_h n_h \le C_0 \\ 2 \le n_h \le N_h \\ \text{and } n_h \text{ are integers; } h = 1,2,\ldots,L \end{array}\right\} \quad (19)$$

Now, let us introduce the following deviational variables to formulate model (19) as a goal programming model:

$$z_k - \delta_k = G_k, \quad k = 1,2,\ldots,k$$

where G_k is the aspiration level of the objective function *k*.

Therefore, model (19) can be formulated as a mixed integer goal programming as follows:

$$\left.\begin{array}{l} \textit{Maximize} \quad \lambda \\ \textit{subject to} \\ \lambda \le \dfrac{1}{2}\tanh\left(\left(\sum_{h=1}^{L}\left(\dfrac{W_h^2 S_{jh}^2}{\overline{X}_j^2 (n_{jh}^*)^2}\right) n_h - \dfrac{U_j + L_j}{2}\right)\alpha_j\right) + \dfrac{1}{2}, \quad j = 1,2,\ldots,p \\ \sum_{h=1}^{L} c_h n_h \le C_0 \\ z_k - \delta_k = G_k \\ 2 \le n_h \le N_h \\ \lambda, \delta_k \ge 0 \\ \text{and } n_h \text{ are integers; } h = 1,2,\ldots,L \end{array}\right\} \quad (20)$$

3.2. Determination of aspiration level

Lastly, we determine the aspiration level. We know that $L_k \le z_k \le U_k$. For the MOILPP, we should get the optimal solution that is close to the ideal solution if we set the aspiration level equal to the lower tolerance limit (L_k). Let us now solve the model based on the above described algorithm and the corresponding solution vector is $n_h^*, h = 1,2,\ldots,L$. If this solution is accepted by the decision maker, than stop, optimal solution is found. Otherwise, modify the aspiration level as:

Let the objective functions be $z_1^*, z_2^*, \ldots, z_k^*$ corresponding to the solution vector n_h^*. Compare each objective value with existing lower bound and apply the following rules to modify the aspiration level.

(i) If $z_k^* < L_k$, then replace L_k *by* z_k^*.

(ii) If $z_k^* \ge L_k$, then keep these aspiration levels as they are and solution terminates.

4. SOME OTHER COMPROMISE ALLOCATIONS

In this section three other compromise allocations are discussed for the sake of comparison with the proposed allocation.

4.1. Cochran's compromise allocation

Cochran [12] gave the compromise criteria by averaging the individual optimum allocations n_{jh}^* that are solutions to ILPP (10) for $j = 1,2,\ldots,p,$ over the characteristics. Cochran's compromise allocation is given by

$$n_h = \frac{1}{p}\sum_{j=1}^{p} n_{jh}^{*} . \tag{21}$$

4.2. Chatterjee's compromise allocation

Chatterjee [9] obtained the compromise allocation by minimizing the sum of the relative increase E_l in the variances of the estimates $\bar{y}_{lst}$ of the population means $\bar{Y}_l ; l = 1,2,\ldots, p.$

Chatterjee formulated the problem as:

$$\left.\begin{aligned} &Minimize\ E = \sum_{l=1}^{p} E_l = \frac{1}{C_0}\sum_{l=1}^{p}\sum_{h=1}^{L}\frac{\hat{c}_h (n_{lh}^{*} - n_h)^2}{n_h} \\ &subject\ to \quad \sum_{h=1}^{L}\hat{c}_h n_h \le C_0 \\ &\text{and} \quad 2 \le n_h \le N_h ;\ h = 1,2,\ldots L \end{aligned}\right\} \tag{22}$$

where n_{lh}^{*} is the usual optimum allocation for fixed budget C_0 for the $\underline{l^{th}}$ characteristic in $\underline{h^{th}}$ stratum.

4.3. Khowaja's compromise allocation

Khowaja *et al*. [19] use Chebyshev's Goal Programming to obtain the compromise allocation. The Chebyshev's goal programming formulation of the ILPP is given as:

$$\left.\begin{aligned} &Minimize \quad \lambda \\ &subject\ to \\ &\sum_{h=1}^{L} c_h n_h \le C_0 \\ &\left\{\sum_{h=1}^{L}\left(\frac{W_h^2 S_{jh}^2}{\bar{X}_j^2 (n_{jh}^{*})^2}\right) n_h\right\} + \lambda \ge \alpha_j^{'} \\ &2 \le n_h \le N_h \\ &\text{and } n_h \text{ are integers; } h = 1,2,\ldots,L \end{aligned}\right\} \ j = 1,2,\ldots., p \tag{23}$$

where $\alpha_j^{'} = \left\{2\left(\sum_{h=1}^{L}\frac{W_h^2 S_{jh}^2}{\bar{X}_j^2 (n_{jh}^{*})}\right) - \alpha_j\right\}$, λ represents the worst deviation level and α_j are the aspiration levels that are the upper bounds.

5. NUMERICAL ILLUSTRATION

To demonstrate the practical utility and computational details of the proposed approach, the following numerical example is presented. The data are from 1997 Agricultural Censuses in Iowa State conducted by National Agricultural Statistics Service, USDA, Washington D.C. (Source: http://www.agcensus.usda.gov/) as reported in Khan *et al*. [21]. The 99 counties in the Iowa State are divided into 4 strata. The relevant data with respect to two characteristics (i) the quantity of corn harvested X_1, (ii) the quantity of oats harvested X_2 and the assumed value of the costs of measurement c_h in the four strata are given in Table 1.

And $\overline{X}_1 = 405654.19$ and $\overline{X}_2 = 2116.70.$

The total amount available for conducting the survey is assumed to be C = 350 units with an expected overhead cost $c_0 = 70$ units. This gives $C_0 = C - c_0 = 280$ units.

Table 1: Data for four Strata and two characteristics

h	N_h	W_h	c_h	S_{1h}^2	S_{2h}^2
1	8	0.0808	10	21,601,503,189.8	1,154,134.2
2	34	0.3434	5	19,734,615,816.7	7,056,074.8
3	45	0.4545	3	27,129,658,750.0	2,082,871.3
4	12	0.1212	7	17,258,237,358.5	732,004.9

Using all the above information, we get the ILPP (eq. (10)) as follows:
For *k=1*

$$\left.\begin{array}{ll} \textit{Maximize}\ z_1 = 0.00007799613n_1 + 0.0000389981n_2 + 0.0000233988n_3 + 0.0000545973n_4 & \\ \textit{subject to} \quad 10n_1 + 5n_2 + 3n_3 + 7n_4 \le 280 & \\ \qquad\qquad 2 \le n_1 \le 8,\ 2 \le n_2 \le 34,\ 2 \le n_3 \le 45,\ 2 \le n_4 \le 12 & \\ \text{and} \qquad n_h \text{ are integers; } h = 1,2,...,4. & \end{array}\right\} \quad (24)$$

The optimum allocation of the above problem obtained by the optimization software LINGO [22] is $\underline{n}_1^* = (n_{11}^*, n_{12}^*, n_{13}^*, n_{14}^*) = (2,34,2,12)$ with corresponding value of the objective function $z_1^* = 0.0021838930$.

For *k=2*

$$\left.\begin{array}{ll} \textit{Maximize}\ z_1 = 0.0003949505n_1 + 0.0001974752n_2 + 0.0001184851n_3 + 0.0002764654n_4 & \\ \textit{subject to} \quad 10n_1 + 5n_2 + 3n_3 + 7n_4 \le 280 & \\ \qquad\qquad 2 \le n_1 \le 8,\ 2 \le n_2 \le 34,\ 2 \le n_3 \le 45,\ 2 \le n_4 \le 12 & \\ \text{and} \qquad n_h \text{ are integers; } h = 1,2,...,4. & \end{array}\right\} \quad (25)$$

The optimum allocation of the above problem obtained by the optimization software LINGO [22] is $\underline{n}_2^* = (n_{21}^*, n_{22}^*, n_{23}^*, n_{24}^*) = (8,22,2,12)$ with corresponding value of the objective function $z_2^* = 0.0110586100$.

Now, after following solution procedures given in sections (3.1) & (4), the preferred compromise solutions are obtained and summarized in table 2:

Table 2: Comparison of Allocations

S. No. (1)	Compromise allocations (2)	n_1 (3)	n_2 (4)	n_3 (5)	n_4 (6)	$(CV)_1$ (7)	$(CV)_2$ (8)	Trace (9)=(7)+(8)
1	IFGP (Linear membership function)	8	22	2	12	0.1284458	0.2210449	0.3494907
2	IFGP (Exponential membership function)	8	16	12	12	0.05048933	0.1096062	0.16009553
3	IFGP (Hyperbolic membership function)	8	16	12	12	0.05048933	0.1096062	0.16009553
4	Cochran's Compromise allocation	5	28	2	12	0.1281595	0.2172052	0.3453647
5	Chatterjee's Compromise allocation	6	27	2	11	0.1281665	0.2176830	0.3458495
6	Khowaja's Compromise allocation	3	32	2	12	0.1283588	0.2158083	0.3441671

6. DISCUSSION AND CONCLUSION

The aim of this paper is to minimize the coefficient of variation of multi-response sample survey problem by using the proposed Interactive Fuzzy Goal Programming (IFGP) approach. This is a powerful method for solving a Multi-Objective Programming Problem. IFGP approach is easy and simple to use, can be easily implemented in minimum number of steps, and provides an optimal compromise solution by updating both lower bounds and aspiration level of each objective function. An appropriate aspiration level of the objective functions is obtained by this approach. An algorithm with linear, exponential, and hyperbolic membership functions has been developed to obtain the preferred compromise allocation. Then, the comparison of proposed compromise allocation has been made with some existing compromise allocations such as Cochran's, Chatterjee's and Khowaja's compromise allocations using a farm survey data. From the computational results summarized in Table 2 and graphical representation in Figure 4, we conclude that the IFGP approach with non linear membership functions (i.e. exponential & hyperbolic) provides the best preferred compromise allocations.

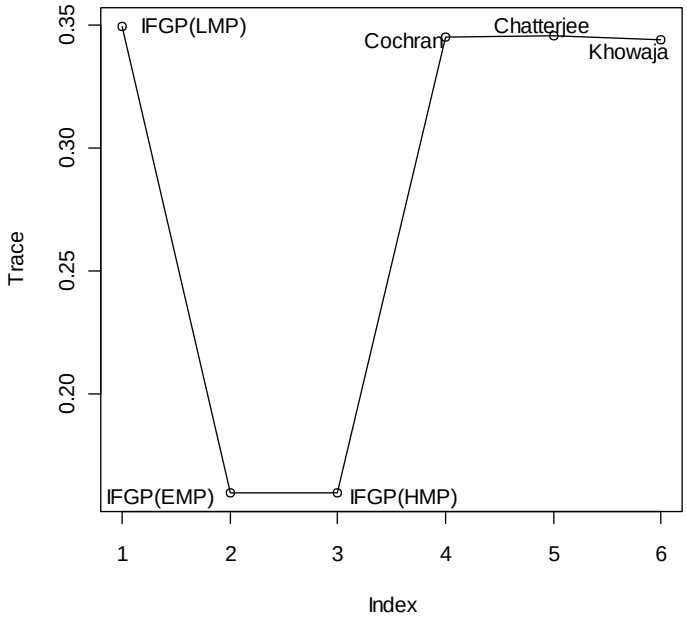

Figure 4: Graphical representation of results

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