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JOINT REPLENISHMENT AND CREDIT POLICIES UNDER TWO LEVELS OF TRADE CREDIT FINANCING WHEN DEMAND AND BAD-DEBT LOSS DEPENDS UPON CREDIT PERIOD

K.K. AGGARAWAL [a], Arun Kumar TYAGI [b,*]

Department of Operational Research, Faculty of Mathematical Sciences, University of Delhi, Delhi-110007, India
[a] kkaggarwal@gmail.com, [b]aruntyagi.du@gmail.com

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Abstract: In practice, a firm usually receives trade credit financing from its supplier on the purchase of inventory. Similarly, in order to meet competition and generate credit sales over and above cash sales, the firm also gives credit period to their customers. However, the decision of granting credit period may have a disintegrating effect on cash sales apart from generating new credit sales because some of the cash customers may switch to credit purchase. In addition, despite of the best credit granting policies and collection practices, the firm may incur some amount of bad debt losses because a certain fraction of buyers will undoubtedly be unable to pay off their debt obligations. In this paper, using discounted cash flow (DCF) approach, a mathematical model is developed to jointly determine optimal inventory and credit policies under two levels of trade credit financing when demand and bad-debt losses are dependent on credit period. The objective of the model is to maximize the present value of firm's net profit per unit time by jointly optimizing the replenishment interval and date-terms credit period. Numerical example and sensitivity analysis are presented to illustrate the effectiveness of the proposed model, and the results are discussed.

Keywords: Inventory, Trade Credit, Date-Terms Credit, Credit Linked Demand, Bad-Debt, Discounted Cash Flow, DCF.

MSC: 90B05.

1. INTRODUCTION

As the basic purpose of any firm is to maximize its own value, the appropriate inventory management is crucial. Over the years, several economic ordering quantity (EOQ) models have been developed as an aid to assist firms in optimal inventory control. However, the classical EOQ models ignore the impact of trade credit financing in inventory control decisions though trade credit financing plays an important role in the conduct of business operations. Due to this fact, many researchers have developed inventory models under one stage and two stage trade credit financing. Beranek [5] brought attention to trade credit financing in making inventory decisions and showed that ignoring credit terms leads to an infeasible replenishment policy. Haley and Higgins [16], Kingsman [22] ,Chapman et al. [10], Goyal [15], Dave [13], Daellenbach [12], Chand and Ward [8], Chung [11] investigated inventory policies under permissible delay in payments. Kingsman [22] pointed out that the two commonly used terms of payment are: (a) payment within a specified period after delivery of the order, e.g. one month; and,(b) payment by some specified time in the month following the month of delivery, e.g. by the 15^{th} day of the month following or the end of the month following. Carlson and Rousseau [7] called the Kingsman's 'type a' terms as 'day- terms', and 'type b' terms as 'date-terms' since in the former, payment is due a given number of days from a reference date such as the date of invoice, shipment or receipt; and in later, a future date is specified on which payment is due. Ventura [30] suggested for incorporating the customer's trade credit in EOQ models under permissible delay in payments. This viewpoint was considered by Biskup, Simons and Jahnke [6], and Huang [18] who developed inventory models under the two levels of trade credit financing. Since the publication of Goyal [15], many important articles on inventory modeling under one and two-stage trade credit can be found, for details see a review article by Chang et al. [9]. They classified the inventory lot-size models related to trade credits into five categories, (a) without deterioration, (b) with deterioration, (c) with allowable shortages, (d) linked to order quantity, and (e) with inflation. Seifert, Seifert and Sieke [26] also provided an integrative review of different streams of trade credit literature.

In general, every firm would sell on cash but in order to increase sales over and above cash sales, it also gives trade credit to its customers. Granting credit entails both benefits and costs. The benefit is the boost in sales and profit that would otherwise be lost if credit not was not extended. Besides that, there are many costs involved in it, e.g. the increase in sales volume leads to the increase in inventory carrying requirement, and the sales made on credit result in the creation of accounts receivables, whose accumulation would result in accounts receivable carrying cost. The costs associated with carrying accounts receivable are the cost of financing accounts receivable, administrative costs in running a credit department, delinquency or collection costs, and cost of default by the customers, i.e. bad debt losses. In addition, some of the cash customers, whom for credit period has value, would possibly purchase on credit. Therefore, the decision of granting trade credit may have a disintegrating effect on cash sales apart from stimulating new credit sales. This will bring further cost to the firm due to the delayed revenue realization. Perhaps the most significant cost is uncollectible accounts receivable or bad debt expense. In spite of the best credit granting and collection policies, a certain fraction of accounts receivable remains uncollectible and becomes bad debt losses. Therefore, the information about the potential bad debt losses should be considered in the decision making process. In general,

lengthening of credit period pushes up the sales accompanied by a larger investment in accounts receivables and a higher incidence of bad debt losses. The shortening of credit period would have opposite influences. It tends to lower sales, decrease investment in accounts receivable, and reduce the incidence of bad-debt loss.

Owning to the fact that credit period has influence on the demand, Aggarwal and Aggarwal [1] developed an EOQ model with credit linked demand when firm purchases items on cash in an inflationary condition. Jaggi, Aggarwal and Goel [19] expand on this theme and developed an EOQ model with trade credit linked demand under the two stage trade credit financing. Other relevant articles with credit linked demand are Su et al.[27], Jaggi, Goyal and Goel [20], Thangam and Uthayakumar [29], Maiti [24], Ho [17], Annadurai and Uthayakumar [4], Aggarwal and Tyagi [2], Giri and Maiti [14] and Aggarwal and Tyagi [3].

All the above mentioned articles ignore the impact of credit period on bad debt losses. But credit period has impact on demand as well as on the level of bad-debt expenses. The simultaneous consideration of the impact of credit period on demand as well as on bad debt losses has not received much attention in the literature. Recently, Teng and Lou [28], and Lou and Wang [23] developed the inventory models when credit period has impact on demand as well as on receiving the buyer's debt obligations. In these models, the sales are composed only of credit sales depending on day-terms credit, the accounts receivable carrying cost is not included, and average cost approach is used to develop the model. However, in most cases, sales of the firm consist of cash as well as credit sales. Moreover, in average cost approach, the time value of money is not explicitly taken into account, and no distinction between out-of-pocket holding cost and opportunity costs is made due to inventory investments.

Consequently, in this article, by using DCF approach, we developed a mathematical model to jointly determine inventory and credit policies when demand and bad debt losses are linked to date-terms credit period, so to reflect a more realistic consumer behavior. The firm purchases a single item, gets a fixed credit period from its supplier, keeps its inventory, and sells it on cash as well as on date-terms credit to its customers. The objective of the model is to maximize the present value of firm's net profit per unit time by jointly optimizing the date-terms credit period and replenishment interval. A hypothetical numerical example, sensitivity analysis and observations are presented to illustrate the effectiveness of the proposed model.

The rest of the article is organized as follows: Section 2 gives assumptions and notations used to develop the model. In section 3, mathematical model is developed and solution procedure is illustrated. Section 4 presents computational analyses where a hypothetical numerical example is solved, sensitivity analysis is done, and results are discussed. Finally, paper is concluded in section 5.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

The model has been developed under the following assumptions:

1. Inventory system involves one type of item.
2. The supplier of the firm gives a fixed credit period (M) to the firm for the purchase of items.
3. The firm sells on cash as well as on credit.
4. The firm gives credit period to its customers to generate extra demand over and above cash demand. This means, sales of the firm (i.e. demand) are composed of two parts (i) Cash Sales (ii) Credit Sales.
5. The credit period given to the customers depends upon their time of purchase and is equal to the difference between the maximum credit period offered by the firm and the time at which customer has purchased the goods. Thus, firm follows a date-terms credit policy of the type net $(N - t)$, where N is the maximum permissible credit period, and t is the time of purchase.
6. The effect of credit policy on demand is observed instantaneously without any delay.
7. Credit period is assumed to have a disintegrating effect on the existing cash demand apart from stimulating new sales. When credit period is offered, it is quite realistic that amongst the existing cash customers (i.e. demand in the absence of trade credit), for whom the credit period has some worth and utility, would now take the benefit of trade credit and purchase on credit Thus, credit demand composed of two components (i) New customers captured by credit policy, and (ii) Customers shifting from cash purchase to credit purchase due to disintegrating effect of credit policy.
8. A certain fraction of accounts receivable of the firm remains uncollectible, i.e. the firm incurs bad debt losses. Since bad debt losses increase as the credit period increases, therefore without the loss of generality the proportion of bad debt loss as a function of credit period is assumed to be
   $f(N) = 1 - e^{-hN}$
   where, $h \geq 0$ is a constant.
9. All the non-defaulting customers settle their account on the last day of the credit period.
10. During trade credit period, the firm deposits the generated sales revenue in an interest bearing account. At the end of credit period, the account is settled and the firm starts paying for the interest charges on the items in stock.
11. Replenishment is instantaneous.
12. Shortages are not allowed.
13. Lead time is negligible.

2.2. Notations

The notations used in this paper are as follows:

$Q =$	Ordering quantity
$T =$	Inventory cycle time
$N =$	Maximum permissible credit period given by the firm, such that, $N \le T$ or $N \ge T$
$M =$	Fixed credit period received by the firm from its supplier.
$d =$	Constant and uniform cash demand rate of the firm's product in the absence of any credit policy
$d_1(t) =$	Sales rate on cash in units per unit time at time $'t'$ in the presence of credit policy
$d_2(t) =$	Sales rate on credit in units per unit time at time $'t'$
$d_u =$	Maximum achievable new credit demand rate in units per unit time
$d(t) = d_1(t) + d_2(t) =$	Demand rate function
$O =$	Ordering cost per order
$C =$	Unit purchase cost
$P =$	Unit selling price
$k =$	Rate of interest or discount rate per unit time
$I =$	Out-of-pocket inventory carrying charge per unit per unit time (IC is the per unit out-of-pocket inventory carrying cost for unit time)
$R =$	Out-of-pocket receivable carrying charge per unit per unit time (RP is the per unit out-of-pocket receivable carrying cost for unit time)
$I_e =$	Rate of interest earned per unit time
$I_p =$	Rate of interest payable per unit time
$I(t) =$	Inventory level at any time $'t'$
$R(t) =$	Accounts receivable level at any time $'t'$
$Z(N,T) =$	Net profit per unit time as a function of decision variables $'N'$ and $'T'$
$ACP =$	Average collection period

Unit time is taken as one year

3. MATHEMATICAL MODELING

In this section, we develop an inventory-trade credit model on the basis of assumptions and notations described in section 2. We first describe the demand model as follows:

Cash Demand Rate Function:

Under the assumptions that the marginal effect of credit period on reducing cash demand rate is proportional to the existing (i.e. non-switched) cash demand rate, and the effect of credit period is observed instantaneously without any delay, the cash demand rate function for any credit period (say, x) can be represented by the following differential equation:

$$\frac{-d.d_1(t)}{dx} = a.d_1(t) \tag{1}$$

where, $a > 0$ is the constant of proportionality, and the negative sign indicates that the quantity of cash demand remains non switched to credit demand decreases as the credit period increases, i.e. as the credit period increases, the conversion of cash demand to credit demand also increases.

The solution of this differential equation, under the condition that when $x = 0,\ d_1(t) = d$ (i.e. cash demand rate in the absence of credit period), is given by

$$d_1(t) = de^{-ax} \tag{2}$$

For date - terms credit policy, we have,

$$x = (N - t) \tag{3}$$

Where, N is the maximum permissible credit period given by the firm and t is the time of purchase by the customers.

Substituting the value of 'x' given by (3) in (2), we get,

$$d_1(t) = de^{-ax} = de^{-a(N-t)}, \qquad 0 \le t \le N \tag{4}$$

Thus, the cash demand rate function is

$$d_1(t) = \begin{cases} d.e^{-a(N-t)}, & 0 \le t \le N \\ d, & N \le t \le T \end{cases} \tag{5}$$

Where, $a > 0,$ the constant of proportionality can be interpreted as a constant of disintegration (Aggarwal and Tyagi [3]).

Credit Demand Rate Function

Let $d_s(t) =$ Cash demand in units per unit time converted or switched to credit demand at any time t due to disintegrating effect of credit period, $d_n(t) =$ New credit demand in units per unit time at any time t due the influence of credit period

According to assumption (7) of section (2.1), the credit demand rate $d_2(t)$ is

$$d_2(t) = d_n(t) + d_s(t) \tag{6}$$

The cash demand in units per unit time converted or switched to credit demand at any time t due to disintegrating effect of credit period, $d_s(t)$, is

$$d_s(t) = d - d_1(t) = d(1 - e^{-a(N-t)}), \quad 0 \le t \le N \tag{7}$$

Further, under the assumption that the marginal effect of credit period on stimulating new credit demand $d_n(t)$ is proportional to the unrealized potential of new credit demand rate, the credit demand rate function $d_n(t)$ for any credit period (say, x) can be written as the following differential equation:

$$\frac{dd_n(t)}{dx} = b(d_u - d_n(t)), \tag{8}$$

Where, $b > 0$, the constant of proportionality is the rate of saturation of credit demand rate, which can be estimated from the past data, and d_u is the maximum achievable new credit demand rate or the upper limit of new credit demand rate.

Solving this equation under the condition that when credit period is zero, the credit demand rate is also zero, i.e. when $x = 0, \ d_n(t) = 0,$ we get,

$$d_n(t) = d_u(1 - e^{-bx}) \tag{9}$$

Since for a date term credit policy, we have,

$$x = (N - t), \tag{10}$$

Substituting the value of 'x' given by (10) in (9), we get,

$$d_n(t) = d_u(1 - e^{-b(N-t)}) \tag{11}$$

Using equations (6), (7) and (11), the credit demand rate function $d_2(t)$, is,

$$\begin{aligned} d_2(t) &= d_n(t) + d_s(t) \\ &= d_u(1 - e^{-b(N-t)}) + d(1 - e^{-a(N-t)}), \quad 0 \le t \le T \end{aligned} \tag{12}$$

The credit demand rate function represented by the equation (12) indicates that credit demand is more at the start of the credit cycle and then decreases as the time progresses, and becomes zero at *'N'*. Kalyanji [21] as well as Robb and Silver [25] have also observed this pattern of credit demand in real world.

Using the cash and credit demand rate functions represented by the equations (5) and (12), we now develop the decision model on the basis of assumptions and notations described in section 2. According to the assumptions about demand, the firm has to keep inventory in order to fulfill cash demand, as well as credit demand.

Let

Q_1 = Quantity require to fulfill cash demand during the inventory cycle length T

Q_2 = Quantity require to fulfill credit demand during the inventory cycle length T

From its sales, the firm will generate continuous revenue from cash sales from '0' to 'T', while revenue from the credit sales will be received all at once at $t = N$. Depending upon the values of credit period (N) and inventory cycle length (T), there are two potential cases viz.

1 $N \le T$
2 $N \ge T$

For the firm, the present value of its net profit per unit time $Z(N,T)$ can be expressed as:

$Z(N,T) =$ Revenue from cash sales + Revenue from credit sales after bad-debt loss - Ordering cost -Purchasing cost- Inventory carrying cost – Accounts receivable carrying cost +Interest Earned-Interest Payable (13)

We now develop the mathematical formulations.

3.1. Case 1. N ≤ T

At the start of the cycle, the inventory level is raised to Q units; afterwards as time progress, inventory decreases to fulfill cash, as well as credit demand up to ' N '. After ' N ', it decreases to satisfy cash demand and ultimately becomes zero at ' T '. Furthermore, at the start of the cycle, the level of accounts receivable is zero; afterwards as time progresses, it increases due to credit sales, and reaches maximum level at N, where these are settled by the non-defaulting customers. As the cash sale takes place from '0' to 'T' and credit sales from '0' to 'N', therefore, the demand function at any time 't' is:

$$d(t) = \begin{Bmatrix} \left\{d.e^{-a(N-t)}\right\} + \left\{d_u\left(1-e^{-b(N-t)}\right) + d\left(1-e^{-a(N-t)}\right)\right\}, & 0 \le t \le N \\ d, & N \le t \le T \end{Bmatrix} \quad (14)$$

Since replenishment is instantaneous and shortages are not allowed, so the initial inventory level, $I(0)$ (i.e. the order quantity, Q) is:

$$I(0) = Q = Q_1 + Q_2 = \int_0^T d_1(t)dt + \int_0^N d_2(t)dt$$

$$= \int_0^N d_1(t)dt + \int_N^T d_1(t)dt + \int_0^N d_2(t)dt$$

$$= \int_0^N \left(de^{-a(N-t)}\right)dt + \int_N^T ddt + \int_0^N \left(d_u(1-e^{-b(N-t)}) + d(1-e^{-a(N-t)})\right)dt$$

$$= \frac{d_u(e^{-bN} + bN - 1) + bdT}{b} \tag{15}$$

As a result of the demand function (eq. (14)), the inventory level $I(t)$ at any time t decreases due to cash sales as well as credit sales from $0 \le t \le N$, and due to only cash sales from $N \le t \le T$.

Let

$$I(t) = \begin{Bmatrix} I_1(t) = \text{inventory level at any time } t, \text{ for } 0 \le t \le N \\ I_2(t) = \text{inventory level at any time } t, \text{ for } N \le t \le T \end{Bmatrix}$$

The variation of inventory level with respect to time can be described by the following differential equations:

$$\frac{dI(t)}{dt} = \begin{Bmatrix} \dfrac{dI_1(t)}{dt} = -d - d_u(1 - e^{-b(N-t)}), & 0 \le t \le N \\ \dfrac{dI_2(t)}{dt} = -d, & N \le t \le T \end{Bmatrix} \tag{16}$$

with the boundary conditions:

$I(0) = I_1(0) = Q = \dfrac{d_u(e^{-bN} + bN - 1) + bdT}{b}$ & $I(T) = I_2(T) = 0$. Consequently, the solution of (16) is given by

$$I(t) = \begin{Bmatrix} I_1(t) = \dfrac{d_u(e^{-b(N-t)} + bN - bt - 1) + bd(T-t)}{b}, & 0 \le t \le N \\ I_2(t) = d(T-t), & N \le t \le T \end{Bmatrix} \tag{17}$$

Further, the accounts receivable level $R(t)$ at any time t increases due to credit sales from $0 \le t \le N$, becomes maximum at N, where these are also completely settled by the customers. Let

$$R(t) = \begin{Bmatrix} R_1(t) = \text{ accounts receivable level at any time } t, \text{ for } 0 \le t \le N \\ R_2(t) = \text{accounts receivable level at any time } t, \text{ for } N \le t \le T \end{Bmatrix}$$

The accounts receivable level at any time t during the cycle is

$$R(t) = \begin{Bmatrix} R_1(t) = 0 + \displaystyle\int_0^t \left(d_u(1 - e^{-b(N-t)}) + d(1 - e^{-a(N-t)}) \right) dt, & 0 \le t \le N \\ R_2(t) = 0, & T \le t \le N \end{Bmatrix}$$

$$\Rightarrow R(t) = \begin{Bmatrix} R_1(t) = \dfrac{\left(bde^{-aN}(1-e^{at}) + ad_u e^{-bN}(1-e^{bt}) + abt(d+d_u)\right)}{ab}, & 0 \le t \le N \\ R_2(t) = 0, & T \le t \le N \end{Bmatrix} \quad (18)$$

Equations (17) and (18) together represent the state of the system at any time t for the case $N \le T$. By using the discounted cash flow approach, the components of profit function are as follows:

The present value of revenue per unit time from cash sales

$$= \frac{P}{T}\int_0^T d_1(t)e^{-kt}dt = \frac{P}{T}\left(\int_0^N de^{-a(N-t)}e^{-kt}dt \;+\; \int_N^T de^{-kt}dt\right)$$

$$= \frac{Pd\left(ae^{-kN} - e^{-aN}k - e^{-kT}(a-k)\right)}{(a-k)kT} \quad (19)$$

The present value of revenue per unit time from credit sales

$$= \frac{\left(1-f(N)\right)Pe^{-kN}}{T}\int_0^N d_2(t)dt = \frac{Pe^{-kN}e^{-hN}}{T}\int_0^N d_2(t)dt$$

$$= \left(\frac{Pe^{-kN}e^{-hN}}{abT}\right)\left(bd(e^{-aN} + aN - 1) + ad_u(e^{-bN} + bN - 1)\right) \quad (20)$$

The present value of ordering cost per unit time $= \dfrac{O}{T}$ (21)

The present value of purchasing cost per unit time

$$= \frac{Ce^{-kM}Q}{T} = \frac{Ce^{-kM}\left(d_u(e^{-bN} + bN - 1) + bdT\right)}{bT} \quad (22)$$

The present value of inventory carrying cost per unit time

$$= \frac{IC}{T}\int_0^T I(t)e^{-kt}dt = \frac{IC}{T}\int_0^N I_1(t)e^{-kt}dt + \frac{IC}{T}\int_N^T I_2(t)e^{-kt}dt$$

$$= \frac{IC}{T}\left(\int_0^N\left(\frac{d_u(e^{-b(N-t)} + bN - bt - 1) + bd(T-t)}{b}\right)e^{-kt}dt + \int_N^T d\left(T-t\right)e^{-kt}dt\right)$$

$$= \frac{IC}{T}\left(\frac{b^2 d_u(e^{-kN} + kN - 1) + d_u k^2(1 - d_u e^{-bN} - bN)}{b(b-k)k^2} + \frac{d(e^{-kT} + kT - 1)}{k^2}\right) \quad (23)$$

The present value of accounts receivable carrying cost per unit time

$$= \frac{RP}{T}\int_0^N R(t)e^{-kt}dt$$

$$= \frac{RP}{T}\int_0^N \left(\frac{bde^{-aN}(1-e^{at}) + ad_u e^{-bN}(1-e^{bt}) + abt(d+d_u)}{ab}\right)e^{-kt}dt$$

$$= \frac{RP}{T}\left(\begin{array}{l}\frac{d(e^{-aN}-e^{-kN})}{a(a-k)} + \frac{d_u(e^{-bN}-e^{-kN})}{b(b-k)} + \frac{de^{-aN}(1-e^{-kN})}{ak} \\ + \frac{d_u e^{-bN}(1-e^{-kN})}{bk} + \frac{(d+d_u)(1-e^{-kN}-e^{-kN}kN)}{k^2}\end{array}\right) \quad (24)$$

The computation for interest earned and interest payable will depend upon the following sub-cases: Sub-Case 1.1: $M \le N \le T$, Sub-Case 1.2: $N \le M \le T$, Sub-Case 1.3: $N \le T \le M$. We now calculate the expressions for interest earned, interest payable, and corresponding profit expression for each sub-case.

Sub-Case 1.1: $M \le N \le T$

The present value of the interest earned per unit time

$$= \frac{I_e P}{T}\int_0^M d_1(t)te^{-kt}dt = \frac{I_e P}{T}\int_0^M de^{-a(N-t)}te^{-kt}dt$$

$$= \frac{I_e Pd\left(e^{-aN} + e^{aM-kM-aN}(aM-kM-1)\right)}{(a-k)^2 T} \quad (25)$$

The present value of interest payable per unit time

$$= \frac{I_p C}{T}\int_M^N I_1(t)\, e^{-kt}dt + \frac{I_p C}{T}\int_N^T I_2(t)\, e^{-kt}dt$$

$$= \frac{I_p C}{T}\left(\int_M^N \left(\frac{d_u(e^{-b(N-t)} + bN - bt - 1) + bd(T-t)}{b}\right)e^{-kt}dt + \int_N^T d(T-t)e^{-kt}dt\right)$$

$$= \frac{I_p C}{T}\left(\begin{array}{l}\left(\frac{d_u}{b(b-k)k^2}\right)\left(e^{-kM}\left(-b^2 + k^2(1-e^{bM-bN}) + bk(b-k)(N-M)\right) + b^2e^{-kN}\right) \\ + \left(\frac{d}{k^2}\right)\left(e^{-kM}(kT-kM-1) + e^{-kT}\right)\end{array}\right) \quad (26)$$

Using equations (19) to (26), the present worth of firm's net profit per unit time $Z_{11}(N,T)$ is,

$$Z_{11}(N,T) = \{(19)+(20)-(21)-(22)-(23)-(24)+(25)-(26)\} \tag{27}$$

Sub-Case 1.2: $N \le M \le T$

The present value of the interest earned per unit time

$$= \frac{I_e P}{T}\left(\int_0^N d_1(t)te^{-kt}dt + \int_N^M d_1(t)te^{-kt}dt + (1-f(N))\int_N^M Q_2 e^{-kt}dt\right)$$

$$= \frac{I_e P}{T}\left(\int_0^N de^{-a(N-t)}te^{-kt}dt + \int_N^M dte^{-kt}dt + \frac{e^{-hN}}{ab}\int_N^M \begin{pmatrix} bd(e^{-aN}-1)+ad_u(e^{-bN}-1) \\ +abN(d+d_u) \end{pmatrix} e^{-kt}dt\right)$$

$$= \frac{I_e P}{T}\left(\begin{array}{l} \dfrac{d\left(e^{-aN}+e^{-kN}(aN-kN-1)\right)}{(a-k)^2} + \dfrac{d\left(e^{-kN}(1+kN)-e^{-kM}(1+kM)\right)}{k^2} \\ + \dfrac{(e^{-kM}-e^{-kN})e^{-hN}\left(bd(1-e^{-aN}-aN)+ad_u(1-e^{-bN}-bN)\right)}{abk} \end{array}\right) \tag{28}$$

The present value of interest payable per unit time

$$= \frac{I_p C}{T}\int_M^T I(t)e^{-kt}dt = \frac{I_p C}{T}\int_M^T d(T-t)e^{-kt}dt$$

$$= \frac{I_p Cd\left(e^{-kM}(kT-kM-1)+e^{-kT}\right)}{k^2 T} \tag{29}$$

Using equations (19) to (24), (28) & (29), the present worth of firm's net profit per unit time $Z_{12}(N,T)$ is,

$$Z_{12}(N,T) = \{(19)+(20)-(21)-(22)-(23)-(24)+(28)-(29)\} \tag{30}$$

Sub-Case 1.3: $N \le T \le M$

The present value of the interest earned per unit time

$$
\begin{aligned}
&= \frac{I_e P}{T}\left(\int_0^N d_1(t)te^{-kt}dt + \int_N^T d_1(t)te^{-kt}dt + \int_T^M Q_1 e^{-kt}dt + (1-f(N))\int_N^M Q_2 e^{-kt}dt\right) \\
&= \frac{I_e P}{T}\left(\begin{array}{l} \int_0^N de^{-a(N-t)}te^{-kt}dt + \int_N^T dte^{-kt}dt + \int_T^M \frac{d(1-e^{-aN}-aN+aT)}{a}e^{-kt}dt \\ +e^{-hN}\int_N^M \frac{bd(e^{-aN}+aN-1)+ad_u(e^{-bN}+bN-1)}{ab}e^{-kt}dt \end{array}\right) \\
&= \frac{I_e P}{T}\left(\begin{array}{l} \frac{d}{(a-k)^2}\left(e^{-aN}+e^{-kN}(aN-kN-1)\right)+\frac{d}{k^2}\left(e^{-kN}(1+kN))-e^{-kT}(1+kT)\right) \\ +\frac{d}{ak}\left((1-e^{-aN}-aN+aT)(e^{-kT}-e^{-kM})\right)+\frac{e^{-hN}}{abk}\left(\begin{array}{l}(e^{-kM}-e^{-kN})(bd(1-e^{-aN}-aN) \\ +ad_u(1-e^{-bN}-bN))\end{array}\right) \end{array}\right)
\end{aligned} \tag{31}
$$

The present value of the interest payable per unit time $= 0$ (32)

Using equations (19) to (24), (31) & (32), the present worth of firm's net profit per unit time $Z_{13}(N,T)$ is,

$$Z_{13}(N,T) = \{(19)+(20)-(21)-(22)-(23)-(24)+(31)-(32)\} \tag{33}$$

3.2. Case 2. N ≥ T

At the start of the cycle, the inventory level is raised to Q; afterwards as time progresses, inventory level decreases to fulfill cash and credit demand up to time T, and also becomes zero at T. Furthermore, at the start of the cycle, the level of accounts receivable is zero; afterwards as time progresses, it increases due to credit sales, reaches maximum level at T and remains at this level up to N, where these are settled by the non-defaulting customers. As both cash and credit sales occur from 0 to T, therefore, the demand function at any time t is:

$$d(t) = \left(de^{-a(N-t)}\right)+\left(d_u(1-e^{-b(N-t)})+d(1-e^{-a(N-t)})\right),\ 0 \le t \le T \tag{34}$$

Since replenishment is instantaneous and shortages are not allowed, so the initial inventory level, $I(0)$ (i.e. the order quantity, Q) is:

$$
\begin{aligned}
I(0) = Q &= Q_1 + Q_2 = \int_0^T d_1(t)dt + \int_0^T d_2(t)dt \\
&= \int_0^T de^{-a(N-t)}dt + \int_0^T \left(d_u(1-e^{-b(N-t)})+d(1-e^{-a(N-t)})\right)dt \\
&= \frac{d_u e^{-bN}(1-e^{bT})+bT(d+d_u)}{b}
\end{aligned} \tag{35}
$$

As a result of the demand function (eq. (34)), the inventory level $I(t)$ at any time t decreases, due to cash sales as well as credit sales, from $0 \le t \le T$. The variation of

inventory level with respect to time can be described by the following differential equation:

$$\frac{dI(t)}{dt} = -d - d_u(1-e^{-b(N-t)}), \qquad 0 \le t \le T \tag{36}$$

with the boundary conditions: $I(0) = Q$ and $I(T) = 0$. Consequently, the solution of (36) is:

$$I(t) = \frac{d_u e^{-bN}(e^{bt} - e^{bT}) + b(d+d_u)(T-t)}{b}, \quad 0 \le t \le T \tag{37}$$

Further, the accounts receivable level $R(t)$ at any time t increases due to credit sales from $0 \le t \le T$, becomes maximum at T and remain constant at this level from $T \le t \le N$ .Let

$$R(t) = \begin{Bmatrix} R_1(t) = \text{ accounts receivable level at any time } t, \text{ for } 0 \le t \le T \\ R_2(t) = \text{accounts receivable level at any time } t, \text{ for } T \le t \le N \end{Bmatrix}$$

The accounts receivable level at any time *t* during the cycle is

$$R(t) = \begin{Bmatrix} R_1(t) = 0 + \int_0^t \left(d_u(1-e^{-b(N-t)}) + d(1-e^{-a(N-t)})\right)dt, & 0 \le t \le T \\ R_2(t) = \int_0^T \left(d_u(1-e^{-b(N-t)}) + d(1-e^{-a(N-t)})\right)dt, & T \le t \le N \end{Bmatrix} \tag{38}$$

$$\Rightarrow R(t) = \begin{Bmatrix} R_1(t) = \dfrac{bde^{-aN}(1-e^{at}) + ad_u e^{-bN}(1-e^{bt}) + abt(d+d_u)}{ab}, & 0 \le t \le T \\ R_2(t) = \dfrac{bde^{-aN}(1-e^{aT}) + ad_u e^{-bN}(1-e^{bT}) + abT(d+d_u)}{ab}, & T \le t \le N \end{Bmatrix} \tag{39}$$

Equations (37) and (39) together represent the state of the system at any time t for the case $N \ge T$. By using the discounted cash flow approach, the components of profit function are as follows:

The present value of revenue per unit time from cash sales

$$= \frac{P}{T}\int_0^T d_1(t)e^{-kt}dt = \frac{P}{T}\int_0^T de^{-a(N-t)}e^{-kt}dt = \frac{Pde^{-aN}(e^{aT-kT}-1)}{(a-k)T} \tag{40}$$

The present value of revenue per unit time from credit sales

$$= \frac{(1-f(N))Pe^{-kN}}{T}\int_0^T d_2(t)dt$$

$$= \frac{Pe^{-kN}e^{-hN}}{T}\int_0^T (d_u(1-e^{-b(N-t)}) + d(1-e^{-a(N-t)}))dt \tag{41}$$

$$= \frac{Pe^{-kN}e^{-hN}\left(bde^{-aN}(1-e^{aT}) + ad_u e^{-bN}(1-e^{bT}) + abT(d+d_u)\right)}{abT}$$

The present value of ordering cost per unit time $= \frac{O}{T}$ (42)

The present value of purchasing cost per unit time

$$= \frac{Ce^{-kM}Q}{T} = \frac{Ce^{-kM}}{bT}\left(d_u e^{-bN}(1-e^{bT}) + bT(d+d_u)\right) \tag{43}$$

The present value of inventory carrying cost per unit time

$$= \frac{IC}{T}\int_0^T I(t)e^{-kt}dt = \frac{IC}{T}\int_0^T \frac{d_u e^{-bN}(e^{bt}-e^{bT}) + b(d+d_u)(T-t)}{b}e^{-kt}dt$$

$$= \frac{IC}{T}\left(\frac{bd_u e^{-bN+bT}(1-e^{-kT}) + d_u e^{-bN}k(1-e^{bT})}{bk(k-b)} + \frac{(d+d_u)(e^{-kT}+kT-1)}{k^2}\right) \tag{44}$$

The present value of accounts receivable carrying cost per unit time

$$= \frac{RP}{T}\left(\int_0^T R_1(t)e^{-kt}dt + \int_T^N R_2(t)e^{-kt}dt\right)$$

$$= \frac{RP}{abT}\left(\int_0^T \begin{pmatrix} bde^{-aN}(1-e^{at}) + abt(d+d_u) \\ +ad_u e^{-bN}(1-e^{bt}) \end{pmatrix} e^{-kt}dt + \int_T^N \begin{pmatrix} bde^{-aN}(1-e^{aT}) + abT(d+d_u) \\ +ad_u e^{-bN}(1-e^{bT}) \end{pmatrix} e^{-kt}dt\right)$$

$$= \frac{RP}{T}\left(\begin{array}{l} \frac{(1-e^{-kT})(bde^{-aN}+ad_u e^{-bN})}{abk} + \frac{de^{-aN}(1-e^{aT-kT})}{a(a-k)} \\ +\frac{d_u e^{-bN}(1-e^{bT-kT})}{b(b-k)} + \frac{(d+d_u)(1-e^{-kT}-e^{-kT}kT)}{k^2} \\ \frac{\left((e^{-kN}-e^{-kT})\left((bde^{-aN}(e^{aT}-1)-abdT) + (ad_u e^{-bN}(e^{bT}-1)-abd_u T)\right)\right)}{abk} \end{array}\right) \tag{45}$$

The computation for interest earned and interest payable will depend upon the following sub-cases: Sub-Case 2.1: $M \le T \le N$, Sub-Case 2.2: $T \le M \le N$, Sub-Case 2.3: $T \le N \le M$. We now calculate the expressions for interest earned, interest payable, and the corresponding profit expression for each sub-case.

Sub-Case 2.1: $M \le T \le N$

The present value of the interest earned per unit time

$$= \frac{I_e P}{T}\int_0^M d_1(t)te^{-kt}dt = \frac{I_e P}{T}\int_0^M de^{-a(N-t)}t\ e^{-kt}dt$$
$$= \frac{I_e Pd\left(e^{aM-kM-aN}(aM-kM-1)+e^{-aN}\right)}{T(a-k)^2} \quad (46)$$

The present worth of interest payable per unit time

$$= \frac{I_p C}{T}\int_M^T I(t)\ e^{-kt}dt$$
$$= \frac{I_p C}{T}\int_M^T \left(\frac{d_u e^{-bN}(e^{bt}-e^{bT})+b(d+d_u)(T-t)}{b}\right)e^{-kt}dt \quad (47)$$
$$= \frac{I_p C}{T}\left(\begin{array}{l}\dfrac{bd_u e^{-bN+bT}(e^{-kT}-e^{-kM})+d_u e^{-kM-bN}k(e^{bT}-e^{bM})}{b(b-k)k} \\ +\dfrac{(d+d_u)(e^{-kM}(kT-kM-1)+e^{-kT})}{k^2}\end{array}\right)$$

Using equations (40) to (48), (46) & (47), the present worth of firm's net profit per unit time $Z_{21}(N,T)$ is,

$$Z_{21}(N,T) = \{(40)+(41)-(42)-(43)-(44)-(45)+(46)-(47)\} \quad (48)$$

Sub-Case 2.2: $T \le M \le N$

The present value of interest earned per unit time

$$= \frac{I_e P}{T}\int_0^T d_1(t)te^{-kt}dt + \frac{I_e P}{T}\int_T^M Q_1 e^{-kt}dt$$
$$= \frac{I_e P}{T}\left(\int_0^T de^{-a(N-t)}te^{-kt}dt + \int_T^M \frac{de^{-aN}(e^{aT}-1)}{a}e^{-kt}dt\right)$$
$$= \frac{I_e P}{T}\left(\frac{d(e^{-aN}+e^{-aN+aT-kT}(aT-kT-1))}{(a-k)^2}+\frac{d(1-e^{aT})(e^{-kM-aN}-e^{-aN-kT})}{ak}\right) \quad (49)$$

The present value of interest payable per unit time $= 0$ (50)

Using equations (40) to (45), (49) & (50), the present worth of firm's net profit per unit time $Z_{22}(N,T)$ is,

$$Z_{22}(N,T)=\{(40)+(41)-(42)-(43)-(44)-(45)+(49)-(50)\} \tag{51}$$

Sub-Case 2.3: $T \le N \le M$

The present value of interest earned per unit time

$$\begin{aligned}
&=\frac{I_e P}{T}\left(\int_0^T d_1(t)te^{-kt}dt+\int_T^M Q_1 e^{-kt}dt+(1-f(N))\int_N^M Q_2 e^{-kt}dt\right)\\
&=\frac{I_e P}{T}\left(\begin{array}{l}\int_0^T de^{-a(N-t)}te^{-kt}dt+\int_T^M \frac{de^{-aN}(e^{aT}-1)}{a}e^{-kt}dt\\ +e^{-hN}\int_N^M \frac{\left(bde^{-aN}(1-e^{aT})+ad_u e^{-bN}(1-e^{bT})+abT(d+d_u)\right)}{ab}e^{-kt}dt\end{array}\right)\\
&=\frac{I_e P}{T}\left(\begin{array}{l}\frac{d\left(e^{-aN}+e^{-aN+aT-kT}(aT-kT-1)\right)}{(a-k)^2}+\frac{d\left((1-e^{aT})(e^{-kM-aN}-e^{-aN-kT})\right)}{ak}\\ +\frac{e^{-hN}\left((e^{-kN}-e^{-kM})\left((bde^{-aN}(1-e^{aT})+abdT)+(ad_u e^{-bN}(1-e^{bT})+abd_u T)\right)\right)}{abk}\end{array}\right)
\end{aligned} \tag{52}$$

The present value of interest payable per unit time $=0$ (53)

Using equations (40) to (45), (52) & (53), the present worth of firm's net profit per unit time $Z_{23}(N,T)$ is,

$$Z_{23}(N,T)=\{(40)+(41)-(42)-(43)-(44)-(45)+(52)-(53)\} \tag{54}$$

Combining both cases, we get the firm's net profit per unit time, $Z(N,T)$ as:

$$Z(N,T)=\left\{\begin{array}{l}\left(\begin{array}{ll}Z_{11}(N,T), & M\le N\le T\\ Z_{12}(N,T), & N\le M\le T\\ Z_{13}(N,T), & N\le T\le M\end{array}\right)\\ \left(\begin{array}{ll}Z_{21}(N,T), & M\le T\le N\\ Z_{22}(N,T), & T\le M\le N\\ Z_{23}(N,T), & T\le N\le M\end{array}\right)\end{array}\right\} \tag{55}$$

Our problem is to find the values of N and T which maximizes $Z(N,T)$.

3.3. Solution procedure

To solve the model, we solve each of the six cases separately and then combine the results to obtain the optimal solution. Due to highly complex and non-linear form, the

model cannot be solved analytically in a closed form. However, the model can be solved numerically using LINGO as follows:

Maximize $Z_{11}(N,T)$, $Z_{12}(N,T)$, $Z_{13}(N,T)$, $Z_{21}(N,T)$, $Z_{22}(N,T)$, $Z_{23}(N,T)$ with respect to N and T so as to satisfy their respective conditions viz., $M \le N \le T$, $N \le M \le T$, $N \le T \le M$, $M \le T \le N$, $T \le M \le N$, and $T \le N \le M$, respectively.

Due to complex nature of each function, the optimality of the solution can only be checked graphically. Hence, to confirm optimality, plot surface graphs for each case or plot a combined surface graph of all cases.

Choose values of N and T corresponding to $Max\{Z_{11}, Z_{12}, Z_{13}, Z_{21}, Z_{22}, Z_{23}\}$.

The optimal value of Q can be calculated from the values of N and T, selected in step 3 by using eq. (15) when $N \le T$, and using eq. (35) when $N \ge T$.

4. COMPUTATIONAL ANALYSIS

In this section, a numerical example is given and solved by using LINGO, and sensitivity analysis is done to illustrate the effectiveness of the model. Since in a date-terms trade credit policy, each customer does not get equal credit period and the decision variable 'N' is only a maximum permissible credit period, so we have also calculated the average collection period for managerial purposes. The calculation for the average collection period is shown in the appendix.

4.1. Numerical example

In this section, a numerical example is presented to illustrate the model. The values of the model parameters are taken as follows: $d = 5000$ units/year, $d_u = 5000$ units/year, $a = 5$, $b = 10$, $O =$ \$1000/order, $C =$ \$200/unit, $P =$ \$250/unit, $I = 0.3$/unit/year, $R = 0.2$/unit/year, $k = 15\%$, $M = 30$ days i.e.30/365 years, $I_e = 12\%$, $I_p = 18\%$, $h = 0.2$

Solving the model according to step 1, 2, and 3, we get

$$Z_{11}^*(N,T) = \$292876.2,\ N_{11}^* = 30.645(days),\ T_{11}^* = 30.645(days)$$
$$Z_{12}^*(N,T) = \$292856.5,\ N_{12}^* = 30.000(days),\ T_{12}^* = 30.000(days)$$
$$Z_{13}^*(N,T) = \$292876.3,\ N_{13}^* = 29.391(days),\ T_{13}^* = 29.391(days)$$
$$Z_{21}^*(N,T) = \$308995.0,\ N_{21}^* = 44.519(days),\ T_{21}^* = 30.000(days)$$
$$Z_{22}^*(N,T) = \$320343.8,\ N_{22}^* = 38.077(days),\ T_{22}^* = 15.378(days)$$
$$Z_{23}^*(N,T) = \$316225.4,\ N_{23}^* = 30.000(days),\ T_{23}^* = 13.531(days)$$

Clearly, $Max\{Z_{11}^*, Z_{12}^*, Z_{13}^*, Z_{21}^*, Z_{22}^*, Z_{23}^*\}$ is Z_{22}^*. Therefore,

$N^* = N_{22}^* = 38.077(days)$, $T^* = T_{22}^* = 15.378(days)$, & $Z(N^*, T^*) = Z_{22}^* = \$320343.8$.

Following step 4, using equation (35), we get $Q^* = 329.016(units)$. Also, the average collection period (ACP) is:

$$ACP(days) = \frac{Net\ average\ accounts\ receivable}{Net\ credit\ sales} \times 365 = 30.848(days)$$

$Z(N^*,T^*)$ is optimal at (N^*,T^*), which can be checked by comparing the values of the function $Z(N,T)$ at any pair of points (N,T) around (N^*,T^*) such that $N < N^*$, or $N > N^*$ and $T < T^*$ or $T > T^*$. Therefore, we have performed a grid search for each case using MATLAB, and evaluated the corresponding difference "$Z(N^*,T^*) - Z(N,T)$" by taking $N = [0,1]$ years and $T = [0,1]$ years as the domain of search space, and a step size approximately equivalent to one day or half of a day for verifying the results. We got similar results with a very small error, which is only due to the approximations involved in specifying the step size. Since inventory and credit policy decisions are short-term decisions, usually of one year, so it is sufficient to check the global optimality of the solution within this domain for most of the practical applications. Also, unit time in the model is taken to be one year, so the optimal value of *T* would always be less than or equal to one year. In addition, we have generated surface graphs of the function using MATLAB for the parameters values taken in the numerical example. The surface graphs clearly show that at (N^*,T^*) the value of $Z(N^*,T^*)$ is maximum. Thus, for the given values of parameters in the numerical example, $N^* = 38.077(days)$ and $T^* = 15.378(days)$ is the optimal solution.

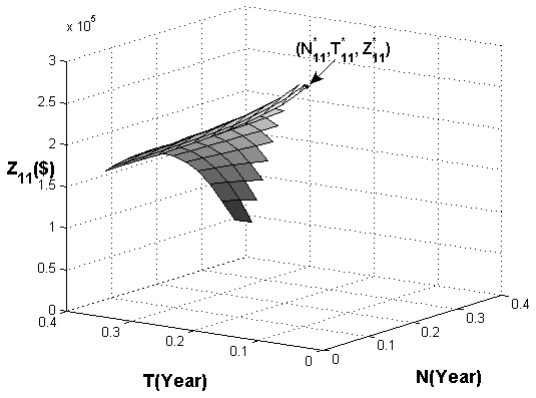

Figure 1: Graph of profit function when $M \le N \le T$.

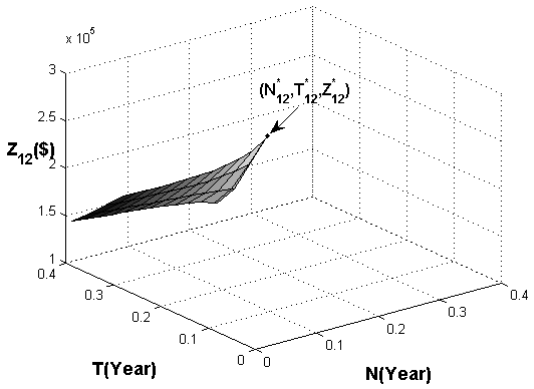

Figure 2: Graph of profit function when $N \le M \le T$.

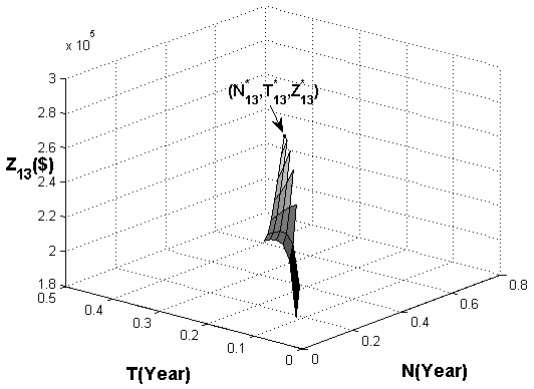

Figure 3: Graph of profit function when $N \le T \le M$.

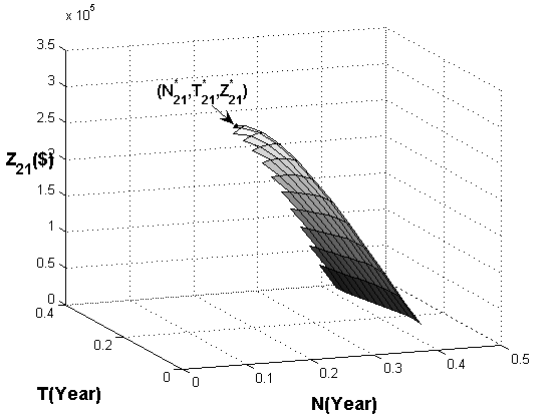

Figure 4: Graph of profit function when $M \le T \le N$.

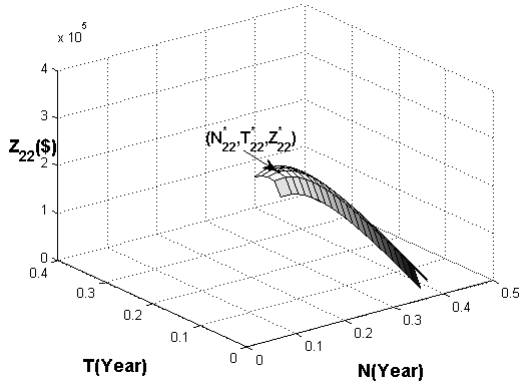

Figure 5: Graph of profit function when $T \le M \le N$.

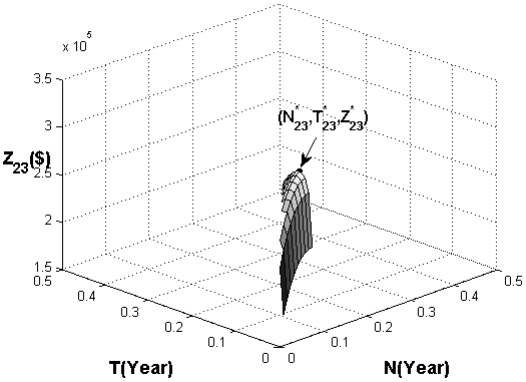

Figure 6: Graph of profit function when $T \le N \le M$.

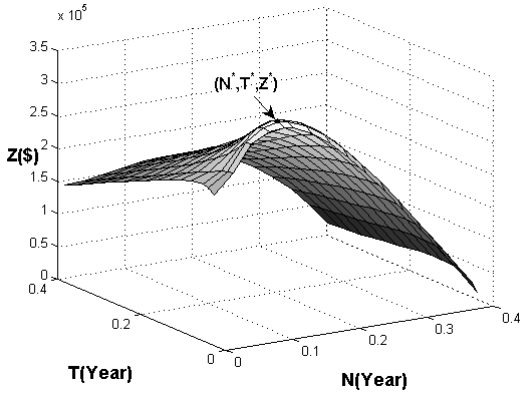

Figure 7: Graph of profit function (combined graph of all cases).

4.2. Sensitivity analysis

For sensitivity analysis, we considered the data as given in the numerical example, and studied the effects of changes in the values of input parameters 'I', 'R' and 'h' on the optimal solution.

Table 1: Effects of changing 'I' on the optimal solution

I	N*(days)	T*(days)	ACP*(days)	Q1*(units)	Q2*(units)	Q*(units)	Z(N*,T*) ($)	Optimal Case
0.0	40.673	19.503	31.647	175.432	243.015	418.447	331139.4	$T \le M \le N$
0.1	39.615	17.808	31.320	160.569	221.109	381.678	327250.2	$T \le M \le N$
0.2	38.771	16.468	31.061	148.788	203.842	352.630	323671.9	$T \le M \le N$
0.3	38.077	15.378	30.848	139.186	189.827	329.013	320343.8	$T \le M \le N$
0.4	37.492	14.472	30.665	131.195	178.193	309.388	317221.8	$T \le M \le N$
0.5	36.990	13.705	30.506	124.422	168.355	292.777	314273.2	$T \le M \le N$

Table 2: Effects of changing 'R' on the optimal solution

R	N*(days)	T*(days)	ACP*(days)	Q1*(units)	Q2*(units)	Q*(units)	Z(N*,T*) ($)	Optimal Case
0.0	48.996	15.022	41.766	116.78	228.277	345.057	344621.3	$T \le M \le N$
0.1	42.646	15.229	35.401	129.338	207.418	336.756	330948.6	$T \le M \le N$
0.2	38.077	15.378	30.848	139.186	189.827	329.013	320343.8	$T \le M \le N$
0.3	34.609	15.478	27.419	147.011	174.731	321.742	311838.8	$T \le M \le N$
0.4	31.873	15.538	24.742	153.282	161.615	314.897	304841.9	$T \le M \le N$
0.5	28.209	14.985	21.435	154.828	138.532	293.36	299268.2	$T \le N \le M$

Table 3: Effects of changing 'h' on the optimal solution

h	N*(days)	T*(days)	ACP*(days)	Q1*(units)	Q2*(units)	Q*(units)	Z(N*,T*)($)	Optimal Case
0.0	50.375	16.794	42.324	129.733	256.894	386.627	349535.6	$T \le M \le N$
0.1	43.236	15.951	35.663	135.069	218.121	353.19	333090.5	$T \le M \le N$
0.2	38.077	15.378	30.848	139.186	189.827	329.013	320343.8	$T \le M \le N$
0.5	27.248	14.016	20.887	145.732	127.314	273.046	295354.8	$T \le N \le M$
0.8	22.588	13.665	16.589	151.073	101.031	252.104	280874.8	$T \le N \le M$
1.0	20.39	13.538	14.592	154.106	88.373	242.479	273844.0	$T \le N \le M$
3.0	10.607	16.565	7.001	216.855	29.271	246.126	245972.4	$N \le T \le M$
5.0	6.834	18.945	4.527	255.272	12.49	267.762	241671.8	$N \le T \le M$

Following observations and managerial insights are got from the results of the numerical exercise, which appeared to be consistent with our expectations and economic rationality.

Table 1 shows that as the inventory carrying cost increases, the optimal value of credit period decreases. This is quite logical because in order to satisfy the credit demand generated as a result of offering credit period, the firm has to maintain inventory, and thus incurs inventory carrying cost. Therefore, when inventory carrying cost increases, the firm needs less inventory thus, the credit period given to the customers is reduced. Consequently, the demand would change causing a simultaneous change in optimal

inventory policy according to the structure, and parameters of the model. This shows that when demand is influenced by credit period, the decision on investment in accounts receivables (i.e. credit decisions) must take into account the inventory carrying cost. The results suggest that at high value of inventory carrying cost, the investment in accounts receivables should be lower. It can also be seen that ordering quantity and replenishment interval decrease as inventory carrying cost increases, which is in concordance with the characteristics of EOQ models. The optimal total profit is reduced when inventory carrying costs increases, which is obvious and confirms our expectations.

Table 2 shows that an increase in accounts receivable carrying cost causes a decrease in the optimal credit period. This is true due to the fact that at high value of accounts receivable carrying cost, the firm chose to carry smaller amount of accounts receivables and to give shorter credit period to its customers. Consequently, the demand would change in a manner in which credit demand decreases, while cash demand increases because of reduction in disintegrating effect of credit period on it. The change in demand results in a simultaneous change in optimal inventory policy according to the structure and parameters of the model. This shows that when demand is dependent upon credit period, the inventory decisions are sensitive to credit decisions and accounts receivable carrying cost. The results suggest that at high value of accounts receivable carrying cost, the firm should invest less in accounts receivable, and therefore should follow a stringent credit policy. This is in confirmation with economic rationality. The optimal total profit decreases as accounts receivable carrying cost increases, which is quite obvious.

A high value of 'h' signifies a higher amount of bad debt losses for a given credit period. Therefore, as expected, it can be seen from Table 3 that an increase in the value of 'h' results in the decrease of optimal credit period. This is quite logical because of the reason that if bad- debt losses are expected to be higher, due to given trade credit, then the firm would give shorter trade credit period. Consequently, the demand would change as per the structure and parameters of the model causing a simultaneous change in the optimal inventory policy. This shows that inventory decisions are sensitive to the amount of bad-debt losses. Therefore, in determining optimal inventory and credit decisions, the bad-debt losses should be taken into consideration by estimating them properly. Since bad-debt loss is an expression of customers' paying habits, the firm should appropriately evaluate the character and capacity of its credit customers and should invest in doing so. It is also observed that the optimal total profit decreases as the value of 'h' increases, which is quite obvious and confirms our expectation.

5. CONCLUSIONS

In this paper, we have developed a mathematical model to jointly determine optimal inventory and credit policies, taking into account the facts that: (1) firm receives and offers trade credit simultaneously, (2) the sales of the firm are divided into cash and credit, and credit period is assumed to have stimulating as well as disintegrating effect on demand, (3) despite the best credit granting and collection practices, there are some bad-debt losses to the firm, and the amount of bad-debt loss is an increasing function of credit period. DCF approach is used to establish the model. Subsequently, a solution procedure is given to find the optimal decision rules, and numerical example is presented. Finally, sensitivity analysis was done, and results were discussed to illustrate the effectiveness of the proposed model.

The results are consistent with economic rationality and provide valuable insights for managerial decision making. For example, the separation of inventory and receivable management within the organization must be dissolved and a centralized approach should be adopted. In this respect, this model provides a unified framework to coordinate inventory and credit decisions.

For further research, the proposed model can be extended in several ways. For instance, it can be extended to the situation when supplier's credit is linked to order quantity, by incorporating the effect of customer's delayed payment behavior and bad debt losses simultaneously. This approach can also be extended to the case of day-terms-credit linked demand, or can be generalized to economic production quantity (EPQ) framework by considering the finite rate of replenishment.

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The commercial identities mentioned in the paper such as LINGO and MATLAB software are used only for the numerical analysis of the model discussed in the paper which is only for the academic purpose. The authors of this paper do not have any direct financial relation with the commercial identities mentioned in the paper and there is no conflict of interests regarding financial gains for all the authors.

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APPENDIX

The calculations for and the average collection period (ACP) are as follows.

$$ACP(days) = \frac{Average\ receivable}{Net\ credit\ sales} * 365$$

$$Average\ receivable(N \le T)$$

$$= \frac{1}{T}\left(\int_0^N R(t)dt + \int_N^T 0.dt\right)$$

$$= \frac{1}{T}\left(\int_0^N \left(\frac{\left(bde^{-aN}(1-e^{at}) + ad_u e^{-bN}(1-e^{bt}) + abt(d+d_u)\right)}{ab}\right)dt + \int_N^T 0dt\right)$$

$$= \frac{de^{-aN}(1-e^{aN}+aN)}{a^2} + \frac{d_u e^{-bN}(1-e^{bN}+bN)}{b^2} + \frac{1}{2}(d+d_u)N^2$$

$$Average\ receivable(N \ge T)$$

$$= \frac{1}{T}\left(\int_0^T R_1(t)dt + \int_T^N R_2(t)dt\right)$$

$$= \frac{1}{T}\left[\begin{array}{l}\int_0^T \left(\frac{bde^{-aN}(1-e^{at}) + ad_u e^{-bN}(1-e^{bt}) + abt(d+d_u)}{ab}\right)dt \\ + \int_T^N \left(\frac{bde^{-aN}(1-e^{aT}) + ad_u e^{-bN}(1-e^{bT}) + abT(d+d_u)}{ab}\right)dt\end{array}\right]$$

$$= \frac{e^{-aN}(a^2e^{aN}T(6N(d+bN) - 3(d+2bN)T + 2bT^2) + 6d(1+aN+e^{aT}(a(T-N)-1)))}{6a^2T}$$

For the values of parameters taken in numerical example,

$$ACP = \frac{Average\ receivable(N \ge T).P}{\frac{1}{T}.P\int_0^T b(N-t) + d(1-e^{-a(N-t)}).dt} * 365 = 30.848(days)$$