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AN ECONOMIC ORDER QUANTITY MODEL WITH RAMP TYPE DEMAND RATE, CONSTANT DETERIORATION RATE AND UNIT PRODUCTION COST

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Abstract: We have developed an order level inventory system for deteriorating items with demand rate as a ramp type function of time. The finite production rate is proportional to the demand rate and the deterioration rate is independent of time. The unit production cost is inversely proportional to the demand rate. The model with no shortages case is discussed considering that: (a) the demand rate is stabilized after the production stopping time and (b) the demand is stabilized before the production stopping time. Optimal costs are determined for two different cases.

Keywords: Ramp Type Demand, Constant Deterioration, Unit Production Cost, Without Shortage.

MSC: 90B05.

1. INTRODUCTION

It is observed that the life cycle of many seasonal products, over the entire time horizon, can be portrayed as a short period of growth, followed by a short period of relative level demand, finishing with a short period of decline. So, researchers commonly use a time-varying demand pattern to reflect sales in different phases of product life cycle. Resh et al. [19], and Donaldson [7] are the first researchers who considered an inventory model with a linear trend in demand. Thereafter, numerous research works have been carried out incorporating time-varying demand patterns into inventory models.

The time dependent demand patterns, mainly, used in the literature are: (i) linearly time dependent, (ii) exponentially time dependent (Dave and Patel [4]; Goyal [10]; Hariga [12]; Hariga and Benkherouf [13]; Yang et al. [28]; Sicilia et al. [21]). The time dependent demand patterns reported above are unidirectional, i.e., increase continuously or decrease continuously. Hill [15] proposed a time dependent demand pattern by considering it as the combination of two different types of demand in two successive time periods over the entire time horizon and termed it as "ramp-type" time dependent demand pattern. This type of demand pattern usually appears in the case of a new brand of consumer goods coming to the market. The inventory models with ramp type demand rate also studied by Wu et al.([26], [27]), Wu and Ouyang [25], Wu [24], Giri et al. [9], Deng [5], Chen et al. [1], Cheng and Wang [2], He et al. [14], Skouri et al. [22]. In these papers, the determination of the optimal replenishment policy requires the determination of the time point, when the inventory level falls to zero. So, the following two cases should be examined: (1) the time point occurs before the point where the demand is stabilized, and (2) the time point occurs after the point where the demand is stabilized. Almost all of the researchers examine only the first case. Deng et al. [6] considered the inventory model of Wu and Ouyang [25] and studied it by exploring these two cases. Skouri et al. [23] extended the work of Deng et al. [6] by introducing a general ramp type demand rate and considering Weibull distributed deterioration rate.

The assumption that the goods in inventory always preserve their physical characteristics is not true in general. There are some items, which are subject to risks of breakage, deterioration, evaporation, obsolescence, etc. Food items, pharmaceuticals, photographic film, chemicals and radioactive substances, to name only a few items in which appreciable deterioration can take place during the normal storage of the units. The study of deteriorating inventory problems has received much attention over the past few decades. Inventory problems for deteriorating item have been studied extensively by many researchers from time to time. Ghare and Schrader [8] developed an Economic Order Quantity (EOQ) model for items with an exponentially decaying inventory. They observed that certain commodities shrink with time by a portion which can be approximated by a negative exponential function of time. Covert and Philip [3] devised an EOQ model for items with deterioration patterns explained by the Weibull distribution. Thereafter, a great deal of research efforts have been devoted to inventory models of deteriorating items, the details can be found in the review articles by Raafat [18], Goyal and Giri [11], Manna and Chaudhuri [17] and Ruxian et al. [20].

Manna and Chaudhuri [17] studied a production-inventory system for time-dependent deteriorating items. They assumed the demand rate to be a ramp type function of time. The demand rate for such items increases with time up to a certain point and then ultimately stabilizes and becomes constant. The system is first studied by allowing no shortages in inventory and then it is extended to cover shortages. For these two models, Manna and Chaudhuri [17] considered that the time point at which the demand rate is stabilized occurs before the production stopping time. Present work is the extension of Manna and Chaudhuri's [17] without shortage model where (a) the demand rate is stabilized after the production stopping time and before the time when inventory level reaches zero and (b) Deterioration rate is constant.

The remainder of this paper is organized as follow. Section 2 recalls the assumptions and notations. In Section 3, the mathematical formulation of the model without shortages and its optimal replenishment policy is studied. Lastly, optimal costs are determined for two different cases.

2. ASSUMPTIONS AND NOTATIONS

The proposed inventory model is developed under the following assumptions and notations :

(i) Lead time is zero.

(ii) c_1 is the holding cost per unit per unit of time.

(iii) c_3 is the deterioration cost per unit per unit of time.

(iv) Demand rate $R = f(t)$ is assumed to be a ramp type function of time $f(t) = D_0[t - (t - \mu)H(t - \mu)], D_0 > 0$ where $H(t - \mu)$ is the Heaviside's function defined as follows:

$$H(t-\mu) = \begin{cases} 1 & \text{if } t \geq \mu \\ 0 & \text{if } t < \mu \end{cases}$$

(v) K= $\beta f(t)$ is the production rate where β (> 1) is a constant.

(vi) θ $(0 < \theta < 1)$ is the constant deterioration rate.

(vii) C is the total average cost for a production cycle.

The unit production cost $\nu = \alpha_1 R^{-\gamma}$, where $\alpha_1 > 0, \gamma > 0$ and $\gamma \neq 2$. α_1 is obviously positive since ν and R are both non-negative. Also, higher demands result in lower unit cost of production. This implies that ν and R are inversely

related and, hence, γ must be positive.
Now,

$$\frac{dv}{dR} = -\alpha_1 \gamma R^{-(\gamma+1)} < 0.$$
$$\frac{d^2 v}{dR^2} = \alpha_1 \gamma(\gamma+1) R^{-(\gamma+2)} > 0.$$

Thus marginal unit cost of production is an increasing function of R. These results imply that, as the demand rate increases, the unit cost of production decreases at an increasing rate. For this reason, the manufacture is encouraged to produce more as the demand for the item increases. The necessity of restriction $\gamma \neq 2$ arises from the nature of the solution of the problem.

3. MATHEMATICAL FORMULATION AND SOLUTION

Case I ($\mu \leq t_1 \leq t_2$)

The production starts with zero stock level at time $t = 0$. The production stops at time t_1 when the stock attains a level S. Due to reasons of market demand and deterioration of items, the inventory level gradually diminishes during the period $[t_1, t_2]$, and ultimately falls to zero at time $t = t_2$. After the scheduling period t_2, the cycle repeats itself.

Let $Q(t)$ be the inventory level of the system at any time t ($0 \leq t \leq t_2$). The differential equations governing the system in the interval $[0, t_2]$ are given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = K - f(t), \quad 0 \leq t \leq \mu \tag{1}$$

with the condition Q(0) = 0;

$$\frac{dQ(t)}{dt} + \theta Q(t) = K - f(t), \quad \mu \leq t \leq t_1 \tag{2}$$

with the condition Q(t_1) = S;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -f(t), \quad t_1 \leq t \leq t_2 \tag{3}$$

with the condition Q(t_1) = S, Q(t_2) = 0.

Using ramp type function $f(t)$, equations (1), (2) and (3) become respectively

$$\frac{dQ(t)}{dt} + \theta Q(t) = (\beta - 1) D_0 t, \quad 0 \leq t \leq \mu \tag{4}$$

with the condition Q(0) = 0;

$$\frac{dQ(t)}{dt} + \theta Q(t) = (\beta - 1)D_0\mu, \quad \mu \le t \le t_1 \tag{5}$$

with the condition Q(t_1) = S;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D_0\mu, \quad t_1 \le t \le t_2 \tag{6}$$

with the condition Q(t_1) = S, Q(t_2) = 0.

(4), (5) and (6) are first order linear differential equations.

For the solution of equation (4), we have

$$e^{\theta t}Q(t) = \frac{(\beta - 1)D_0 e^{\theta t}}{\theta^2}[\theta t - 1] + C^{'} \tag{7}$$

By using the initial condition Q(0) = 0 in equation (7), we get

$$C^{'} \quad = \quad \frac{(\beta - 1)D_0}{\theta^2} \tag{8}$$

Therefore, the solution of equation (4) is given by

$$Q(t) = \frac{(\beta - 1)D_0}{\theta^2}(\theta t + e^{-\theta t} - 1), \quad 0 \le t \le \mu \tag{9}$$

For the solution of equation (5), we have

$$\int_{\mu}^{t} d[e^{\theta t}Q(t)] = (\beta - 1)D_0\mu \int_{\mu}^{t} e^{\theta t}dt$$

which gives

$$Q(t) = \frac{(\beta - 1)D_0}{\theta^2}[\theta\mu + e^{-\theta t} - e^{\theta(\mu - t)}], \quad \mu \le t \le t_1 \tag{10}$$

The solution of equation (6) is given by

$$Q(t) = -\frac{D_0\mu}{\theta} + Se^{\theta(t_1 - t)} + \frac{D_0\mu}{\theta}e^{\theta(t_1 - t)} \tag{11}$$

By using initial condition $Q(t_2) = 0$ in equation (11), we get

$$S = \frac{D_0\mu}{\theta}[e^{\theta(t_2 - t_1)} - 1] \tag{12}$$

Substituting S in (11), the solution of equation (6) is

$$\begin{aligned} Q(t) &= -\frac{D_0\mu}{\theta} + \frac{D_0\mu}{\theta}[e^{\theta(t_2 - t_1)} - 1]e^{\theta(t_1 - t)} + \frac{D_0\mu}{\theta}e^{\theta(t_1 - t)} \\ &= -\frac{D_0\mu}{\theta} + \frac{D_0\mu}{\theta}e^{\theta(t_2 - t)} \\ &= \frac{D_0\mu}{\theta}[e^{\theta(t_2 - t)} - 1], \quad t_1 \le t \le t_2 \end{aligned} \tag{13}$$

$$\text{The total inventory in } [0, t_2] \text{ is } = \int_0^{t_2} Q(t)dt$$
$$= \int_0^{\mu} Q(t)dt + \int_{\mu}^{t_1} Q(t)dt + \int_{t_1}^{t_2} Q(t)dt$$

Where

$$\int_0^{\mu} Q(t)dt = \frac{(\beta-1)D_0}{\theta^2}[\frac{\theta\mu^2}{2} - \frac{e^{-\theta\mu}}{\theta} - \mu + \frac{1}{\theta}]$$
$$\int_{\mu}^{t_1} Q(t)dt = \frac{(\beta-1)D_0}{\theta^2}[\theta\mu t_1 - \frac{e^{-\theta t_1}}{\theta} + \frac{e^{\theta(\mu-t_1)}}{\theta} - \theta\mu^2 + \frac{e^{-\theta\mu}}{\theta} - \frac{1}{\theta}]$$
$$\int_{t_1}^{t_2} Q(t)dt = -\frac{D_0\mu}{\theta}[\frac{1}{\theta} + t_2 - \frac{e^{\theta(t_2-t_1)}}{\theta} - t_1]$$

Therefore, the total inventory in $[0, t_2]$ is given by

$$\int_0^{t_2} Q(t)dt = \frac{(\beta-1)D_0\mu}{\theta}(t_1 - \frac{\mu}{2}) + \frac{(\beta-1)D_0}{\theta^3}e^{-\theta t_1}(e^{\theta\mu} - 1) - \frac{D_0\mu}{\theta^2}[\theta(t_2 - t_1) + \beta - e^{\theta(t_2-t_1)}] \quad (14)$$

Total number of deteriorated items in $[0, t_2]$ is given by
Production in $[0, \mu]$+ Production in $[\mu, t_1]$ - Demand in $[0, \mu]$ - Demand in $[\mu, t_2]$

$$= \beta\int_0^{\mu} D_0 t\, dt + \beta\int_{\mu}^{t_1} D_0\mu\, dt - \int_0^{\mu} D_0 t\, dt - \int_{\mu}^{t_2} D_0\mu\, dt$$
$$= \beta D_0\frac{\mu^2}{2} + \beta D_0\mu(t_1 - \mu) - D_0\frac{\mu^2}{2} - D_0\mu(t_2 - \mu)$$
$$= \frac{1}{2}D_0\beta\mu(2t_1 + \mu - 2\mu) - \frac{1}{2}D_0\mu(\mu + 2t_2 - 2\mu)$$
$$= \frac{1}{2}D_0\beta\mu(2t_1 - \mu) - \frac{1}{2}D_0\mu(2t_2 - \mu) \quad (15)$$

The cost of production in $[u, u + du]$ is

$$Kvdu = \beta f(t)\alpha_1 R^{-\gamma}du$$
$$= \beta R\alpha_1 R^{-\gamma}du$$
$$= \frac{\alpha_1\beta}{R^{(\gamma-1)}} \quad (16)$$

Hence the production cost in $[0, t_1]$ is

$$\begin{aligned}
&\int_0^{t_1} Kv du \\
= &\int_0^{\mu} Kv du + \int_{\mu}^{t_1} Kv du \\
= &\int_0^{\mu} \frac{\alpha_1 \beta}{R^{(\gamma-1)}} du + \int_{\mu}^{t_1} \frac{\alpha_1 \beta}{R^{(\gamma-1)}} du \\
= &\frac{\alpha_1 \beta D_0^{1-\gamma}}{(2-\gamma)} [(\gamma-1)\mu^{2-\gamma} + (2-\gamma)\mu^{1-\gamma} t_1], \qquad \gamma \neq 2 \qquad (17)
\end{aligned}$$

The total average inventory cost C is given by
C = Inventory cost + Deterioration cost + Production cost

$$\begin{aligned}
= &\frac{1}{t_2}[\frac{(\beta-1)D_0\mu c_1}{\theta}(t_1 - \frac{\mu}{2}) + \frac{(\beta-1)D_0 c_1}{\theta^3} e^{-\theta t_1}(e^{\theta\mu} - 1) \\
&- \frac{D_0 \mu c_1}{\theta^2}(\theta(t_2 - t_1) + \beta - e^{\theta(t_2 - t_1)}) + \frac{1}{2} D_0 \beta \mu c_3 (2t_1 - \mu) \\
&- \frac{1}{2} D_0 \mu c_3 (2t_2 - \mu) + \frac{\alpha_1 \beta D_0^{1-\gamma}}{(2-\gamma)}((\gamma-1)\mu^{2-\gamma} + (2-\gamma)\mu^{1-\gamma} t_1)] \qquad (18)
\end{aligned}$$

Optimum values of t_1 and t_2 for minimum average cost C are the solutions of the equations

$$\frac{\partial C}{\partial t_1} = 0 \text{ and } \frac{\partial C}{\partial t_2} = 0$$

provided they satisfy the sufficient conditions

$$\frac{\partial^2 C}{\partial t_1^2} > 0, \ \frac{\partial^2 C}{\partial t_2^2} > 0 \text{ and } \frac{\partial^2 C}{\partial t_1^2}\frac{\partial^2 C}{\partial t_2^2} - (\frac{\partial^2 C}{\partial t_1 \partial t_2})^2 > 0.$$

$$\frac{\partial C}{\partial t_1} = 0 \text{ and } \frac{\partial C}{\partial t_2} = 0 \quad \text{gives}$$

$$\begin{aligned}
&D_0 \mu c_1 \theta(\beta - e^{\theta(t_2 - t_1)}) - (\beta - 1) D_0 c_1 e^{-\theta t_1}(e^{\theta\mu} - 1) \\
&\qquad + D_0 \theta^2 \beta \mu c_3 + \alpha_1 \theta^2 \beta D_0^{1-\gamma} \mu^{1-\gamma} = 0 \qquad (19)
\end{aligned}$$

and

$$D_0 \mu (c_1 (e^{\theta(t_2 - t_1)} - 1) - \theta c_3) + \theta C = 0 \text{ respectively.} \qquad (20)$$

Case II ($t_1 \leq \mu \leq t_2$)

The stock level initially is zero. Production begins just after $t = 0$, continues upto $t = t_1$ and stops as soon as the stock level becomes P at $t = t_2$. Then the inventory level decreases due to both demand and deterioration till it becomes again zero at $t = t_2$. Then, the cycle repeats.

Let $Q(t)$ be the inventory level of the system at any time $t(0 \leq t \leq t_2)$. The differential equations governing the system in the interval $[0, t_2]$ are given by

$$\frac{dQ(t)}{dt} + \theta Q(t) = K - f(t), \;\; 0 \leq t \leq t_1 \tag{21}$$

with the condition Q(0) = 0, Q(t_1)=P;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -f(t), \;\; t_1 \leq t \leq \mu \tag{22}$$

with the condition Q(t_1) = P;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -f(t), \;\; \mu \leq t \leq t_2 \tag{23}$$

with the condition Q(t_2) = 0.

Using ramp type function $f(t)$, equations (21), (22) and (23) become respectively

$$\frac{dQ(t)}{dt} + \theta Q(t) = (\beta - 1)D_0 t, \;\; 0 \leq t \leq t_1 \tag{24}$$

with the condition Q(0) = 0, Q(t_1) = P;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D_0 t, \;\; t_1 \leq t \leq \mu \tag{25}$$

with the condition Q(t_1) = P;

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D_0 \mu, \;\; \mu \leq t \leq t_2 \tag{26}$$

with the condition Q(t_2) = 0.

The solution of equation (24) is given by the expression (9) and we have

$$Q(t) = \frac{(\beta - 1)D_0}{\theta^2}(\theta t + e^{-\theta t} - 1), \;\; 0 \leq t \leq t_1 \tag{27}$$

Using boundary condition $Q(t_1) = P$ in (27), we get

$$P = \frac{(\beta - 1)D_0}{\theta^2}(\theta t_1 + e^{-\theta t_1} - 1) \tag{28}$$

Therefore, the solution of equation (25) is

$$\begin{aligned} Q(t) &= \frac{D_0}{\theta^2}(1-\theta t) + Pe^{\theta(t_1-t)} + \frac{D_0}{\theta^2}(\theta t_1 - 1)e^{\theta(t_1-t)} \\ &= \frac{D_0}{\theta^2}[1-\theta t + (\beta-1)e^{-\theta t}] \\ &\quad + \frac{\beta D_0}{\theta^2}e^{\theta(t_1-t)}[\theta t_1 - 1], \qquad t_1 \le t \le \mu \end{aligned} \tag{29}$$

Using boundary condition $Q(t_2) = 0$, the solution of equation (26) is given by

$$Q(t) = \frac{D_0\mu}{\theta}[e^{\theta(t_2-t)} - 1], \quad \mu \le t \le t_2 \tag{30}$$

Total inventory in $[0, t_2]$ is

$$\begin{aligned} &= \int_0^{t_2} Q(t)dt \\ &= \int_0^{t_1} Q(t)dt + \int_{t_1}^{\mu} Q(t)dt + \int_{\mu}^{t_2} Q(t)dt \\ &= \frac{(\beta-1)D_0}{\theta^2}[\frac{\theta t_1^2}{2} - \frac{e^{-\theta t_1}}{\theta} - t_1 + \frac{1}{\theta}] \\ &\quad + \frac{D_0}{\theta^2}[(\mu - \frac{\theta\mu^2}{2} - \frac{(\beta-1)}{\theta}e^{-\theta\mu} - t_1 + \frac{\theta t_1^2}{2} \\ &\quad + \frac{(\beta-1)}{\theta}e^{-\theta t_1}) - \beta(\theta t_1 - 1)(\frac{e^{\theta(t_1-\mu)}}{\theta} - \frac{1}{\theta})] \\ &\quad + \frac{D_0\mu}{\theta}[-\frac{1}{\theta} - t_2 + \frac{e^{\theta(t_2-\mu)}}{\theta} + \mu] \\ &= \frac{\beta D_0 t_1^2}{2\theta} - \frac{D_0}{\theta^3} + \frac{D_0\mu^2}{2\theta} - \frac{(\beta-1)D_0}{\theta^3}e^{-\theta\mu} \\ &\quad - \frac{\beta D_0}{\theta^3}(\theta t_1 - 1)e^{\theta(t_1-\mu)} \\ &\quad + \frac{D_0\mu}{\theta}[\frac{e^{\theta(t_2-\mu)}}{\theta} - t_2] \end{aligned} \tag{31}$$

Total number of deteriorated items in $[0, t_2]$ is given by
Production in $[0, t_1]$ - Demand in $[0, \mu]$ - Demand in $[\mu, t_2]$

$$\begin{aligned} &= \beta D_0 \int_0^{t_1} t\, dt - D_0 \int_0^{\mu} t\, dt - D_0\mu \int_{\mu}^{t_2} dt \\ &= \frac{\beta D_0 t_1^2}{2} - \frac{D_0\mu^2}{2} - D_0\mu(t_2 - \mu) \\ &= \frac{\beta D_0 t_1^2}{2} - D_0\mu t_2 + \frac{D_0\mu^2}{2} \\ &= \frac{\beta D_0 t_1^2}{2} + \frac{D_0\mu}{2}(\mu - 2t_2) \end{aligned} \tag{32}$$

Hence, the production cost in $[0, t_1]$ is given by

$$\begin{aligned} & \int_0^{t_1} Kvdu \\ = & \int_0^{t_1} \frac{\alpha_1 \beta}{R^{\gamma-1}} du \qquad \text{[using (16)]} \\ = & \int_0^{t_1} \frac{\alpha_1 \beta}{D_0^{\gamma-1} u^{\gamma-1}} du \\ = & \frac{\alpha_1 \beta D_0^{1-\gamma}}{(2-\gamma)} t_1^{2-\gamma}, \qquad \gamma \neq 2 \end{aligned} \tag{33}$$

From (31), (32), and (33), the total average inventory cost C of the system is

$$\begin{aligned} C = & \frac{1}{t_2}[\frac{\beta D_0 c_1 t_1^2}{2\theta} - \frac{D_0 c_1}{\theta^3} + \frac{D_0 c_1 \mu^2}{2\theta} - \frac{(\beta-1) D_0 c_1}{\theta^3} e^{-\theta\mu} \\ & - \frac{\beta D_0 c_1}{\theta^3} (\theta t_1 - 1) e^{\theta(t_1-\mu)} \\ & + \frac{D_0 \mu c_1}{\theta} (\frac{e^{\theta(t_2-\mu)}}{\theta} - t_2) + \frac{\beta D_0 c_3 t_1^2}{2} \\ & + \frac{D_0 \mu c_3}{2} (\mu - 2t_2) + \frac{\alpha_1 \beta D_0^{1-\gamma}}{(2-\gamma)} t_1^{2-\gamma}] \end{aligned} \tag{34}$$

Optimum values of t_1 and t_2 for minimum average cost are obtained as in Case I which gives

$$D_0 t_1 (\theta c_3 + c_1 (1 - e^{\theta(t_1-\mu)})) + \theta \alpha_1 D_0^{1-\gamma} t_1^{1-\gamma} = 0 \tag{35}$$

and

$$D_0 \mu c_1 (e^{\theta(t_2-\mu)} - 1) - D_0 \mu \theta c_3 - \theta C = 0 \tag{36}$$

4. CONCLUDING REMARKS

Equations (19) and (20) are non-linear equations in t_1 and t_2. These simultaneous non-linear equations can be solved by Newton-Rapson method for suitable choice of the parameters c_1, c_3, α_1, β, μ, D_0 and $\gamma(\neq 2)$. If t_1^* and t_2^* are the solution of (19) and (20) for Case I, the corresponding minimum cost $C^*(t_1, t_2)$ can be obtained from (18). It is very difficult to show analytically whether the cost function $C(t_1, t_2)$ is convex. That is why, $C(t_1, t_2)$ may not be global minimum. If $C(t_1, t_2)$ is not convex, then $C(t_1, t_2)$ will be local minimum.

Similarly, solution of equations (35) and (36) for Case II can be obtained by Newton-Raphson method and corresponding minimum cost $C(t_1, t_2)$ can be obtained from (34)

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