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FUZZY STOCHASTIC INVENTORY MODEL FOR DETERIORATING ITEM

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Abstract: A multi item profit maximization inventory model is developed in fuzzy stochastic environment. Demand is taken as Stock dependent demand. Available storage space is assumed to be imprecise and vague in nature. Impreciseness has been expressed by linear membership function. Purchasing cost and investment constraint are considered to be random and their randomness is expressed by normal distribution. The model has been formulated as a fuzzy stochastic programming problem and reduced to corresponding equivalent fuzzy linear programming problem. The model has been solved by using fuzzy linear programming technique and illustrated numerically.

Keywords: Inventory Model, Stock Dependent Demand, Fuzzy-Stochastic Programming, Fuzzy Linear Programming.

MSC: 90B05.

1. INTRODUCTION

The classical inventory models were developed regarding the specific requirements of deterministic cost and demand without deterioration of the items in stock. Gradually, the concept of deterioration in inventory system is considered by inventory researchers so that now most of the inventory problems, include the effect of deterioration as a natural phenomenon. In general, deterioration is defined as damage, spoilage, dryness, vaporization, and so forth.It is deterioration that results in a decrease of the usefulness of the original item.Most of time parameters of inventory model may be uncertain in probabilistic sense, or imprecise. If the parameters are random in nature,stochastic inventory models has been developed by using probability theory. When parameters are of imprecise in nature, their impreciseness is represented by using fuzzy numbers.In such situation, fuzzy inventory models have been developed.

Recently, researchers have focused on situations in which inventory parameters are random as well as imprecise. Models developed in such situations are known as fuzzy stochastic inventory models. In such mixed environment,very few models have been developed. Das, Roy and Maiti [2004][2004] constructed multi item fuzzy stochastic inventory model in which demand and budgetary resources are assumed to be random and available storage space as well as total expenditure are considered as imprecise in nature. Panda and Kar [2005][2005] extended the model of Das, Roy and Maiti [2004][2004] by considering price as random variable. Das and Maiti[2011][2011] developed production inventory model by considering one constraint in fuzzy environment and the other in fuzzy, as well as random environment. Janna et. al [2014][2014] developed an inventory model by assuming time horizon as random variable with exponential distribution and deterioration rate, as well as available budget in fuzzy environment. Recently, Naserabadi [2014][2014] used triangular membership function to represent fuzzy parameters such as lead time and inflation rate where as weibull distribution is used to represent deterioration rate.

In the present paper, a multi item inventory model is developed in fuzzy-stochastic environment by considering stock dependent demand. Purchasing cost and investment goal are considered to be a random variable with normal distribution and profit as well as available storage space are assumed to be imprecise and vague. Impreciseness is expressed through linear membership function. The fuzzy-stochastic inventory problem is first converted into the equivalent fuzzy problem.Further fuzzy problem is converted into equivalent crisp problem using linear membership functions. Fuzzy linear programming technique is used to solve the crisp problem. The model is illustrated with some numerical values for inventory parameters.

2. MODEL AND ASSUMPTIONS

2.1. Notations:

c_i -Purchasing cost per unit i^{th} item.

p_i -Selling price per unit i^{th} item.

Q_i -Initial stock level of unit i^{th} item.

θ_i -Deteriorating rate of i^{th} item.

$Q_i(t)$ -Inventory level at time t of i^{th} item.

$D_i(t)$ -Demand rate of per unit of i^{th} item ,

$D_i(t)$ $-a_i + b_i Q_i(t)$

C_{hi} -Holding cost per unit of i^{th} item.

C_{di} -Deteriorating cost per unit of i^{th} item.

t_i -Time period for each cycle of i^{th} item

('∼' represents the fuzzification of the parameters and '∧' represents randomization of parameters).

2.2. Assumptions:

1. Replenishment is instantaneous.
2. Lead time is zero.
3. Selling price is known and constant.
4. Shortages are not allowed.

3. MATHEMATICAL ANALYSIS

Let $Q_i(t)$ be the stock level of i^{th} item at time t.Q_i is the initial stock level of i^{th} item.The inventory level decreases mainly due to demand, and partially due to deterioration. The stock reaches to zero level at $t = t_i$. The differential equation describing the state of inventory in the interval $(0,t_i)$ is given by

$$\frac{dQ_i(t)}{dt} + \theta_i t Q_i(t) = -(a_i + b_i Q_i(t)), 0 \le t \le t_i \tag{1}$$

Solving the above differential equation using boundary condition $Q_i(0) = Q_i$ we get,

$$Q_i(t) = (-a_i[t + \frac{\theta_i t^3}{6} + \frac{b_i t^2}{2}] + Q_i) \exp -(\frac{\theta_i t^2}{2} + b_i t), 0 \le t \le t_i \tag{2}$$

The above equation can be simplified by using series form of exponential term and ignoring second and higher order terms as follows

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3} + \cdots$$

$$Q_i(t) = (-a_i[t - \frac{2\theta_i t^3}{6} - \frac{b_i t^2}{2}] + Q_i[1 - \frac{\theta_i t^2}{2} - b_i t]), 0 \le t \le t_i \tag{3}$$

using boundary condition $Q_i(t_i) = 0$ we get,

$$(-a_i[t_i - \frac{b_i t_i^2}{2} - \frac{\theta_i t_i^3}{3}] + Q_i[1 - \frac{\theta_i t_i^2}{2} - b_i t_i]) = 0 \tag{4}$$

Holding cost over the time period (0,ti) is given by

$$c_{hi} \int_0^{t_i} Q_i(t)dt = c_{hi}\left(-a_i\left[\frac{t^2}{2} - \frac{b_i t_i^3}{6} - \frac{\theta_i t_i^4}{12}\right] + Q_i\left[t_i - \frac{\theta_i t_i^3}{6} - \frac{b_i t_i^2}{2}\right]\right) \tag{5}$$

Total deterioration cost is given by

$$c_{di} \int_0^{t_i} t_i \theta_i Q_i(t)dt = c_{di}\theta_i(-a_i[\frac{t_i^3}{3} - \frac{b_i t_i^4}{8} - \frac{\theta_i t_i^5}{15}] + Q_i[\frac{t_i^2}{2} - \frac{\theta_i t_i^4}{8} - \frac{b_i t_i^3}{3}]) \tag{6}$$

Then the total profit is given by

$$PF = \sum_{i=1}^{n}\left((p_i - c_i)Q_i - c_{hi}\int_0^{t_i} Q_i(t)dt - c_{di}\int_0^{t_i} t_i\theta_i Q_i(t)dt\right)$$

$$PF = \sum_{i=1}^{n}\left((p_i - c_i)Q_i - C_{hi}\left(-a_i\left[\frac{t_i^2}{2} - \frac{b_i t_i^3}{6} - \frac{\theta t_i^4}{12}\right] + Q_i\left[t_i - \frac{\theta_i t_i^3}{6} - \frac{b_i t_i^2}{2}\right]\right)\right.$$
$$\left. - C_{di}\theta_i\left(-a_i\left[\frac{t_i^3}{3} - \frac{b_i t_i^4}{8} - \frac{\theta t_i^5}{15}\right] + Q_i\left[\frac{t_i^2}{2} - \frac{\theta t_i^4}{8} - \frac{b_i t_i^3}{3}\right]\right)\right)$$

Hence the problem is to maximize profit subject to investments and shortage area. That is

Max PF=$\sum_{i=1}^{n}$ PF (Q_i)

Subject to

$\sum_{i=1}^{n} w_i Q_i \leq$ W

$\sum_{i=1}^{n} c_i Q_i \leq$ B

$(-a_i[t_i - \frac{b_i t_i^2}{2} - \frac{\theta_i t_i^3}{3}] + Q_i[1 - \frac{\theta_i t_i^2}{2} - b_i t_i]) = 0$

$Q_i \geq 0, i = 1, 2, ..., n$

Fuzzy-Probabilistic Model:

When c_i's and investment are probabilistic and storage area becomes fuzzy, the crisp model is transformed to a probabilistic model in fuzzy environment as

Max PF=$\sum_{i=1}^{n}$ PF (Q_i)

Subject to

$\sum_{i=1}^{n} w_i Q_i \leq \widetilde{W}$

$\sum_{i=1}^{n} \hat{c}_i Q_i \leq \widehat{B}$

$(-a_i[t_i - \frac{b_i t_i^2}{2} - \frac{\theta_i t_i^3}{3}] + Q_i[1 - \frac{\theta_i t_i^2}{2} - b_i t_i]) = 0$

$Q_i \geq 0, i = 1, 2, ..., n$

In fuzzy set theory, the fuzzy objective and fuzzy constraints are defined by their membership functions, which may be linear or non-linear. Here, we assume $\mu_{E_{PF}}, \mu_{V_{PF}}, \mu_W$ to be linear membership functions for two objectives and one constraint, respectively, and these are

$$\mu_{E_{PF}} = \begin{cases} 0, & E_{PF} \leq C_0 - P_{E_{PF}} \\ 1 - \frac{C_0 - E_{PF}}{P_{E_{PF}}} & C_0 - P_{E_{PF}} \leq E_{PF} \leq C_0 \\ 1 & E_{PF} \geq C_0 \end{cases}$$

$$\mu_{V_{PF}} = \begin{cases} 0, & E_{PF} \geq D_0 + P_{V_{PF}} \\ 1 - \frac{V_{PF} - D_0}{P_{V_{PF}}} & D_0 \leq V_{PF} \leq D_0 + P_{V_{PF}} \\ 1 & V_{PF} \leq D_0 \end{cases}$$

$$\mu_W = \begin{cases} 0 & \sum_{i=1}^{n} w_i Q_i \geq W + P_W \\ 1 - \dfrac{\sum_{i=1}^{n} w_i Q_i - W}{P_W} & W \leq \sum_{i=1}^{n} w_i Q_i \leq W + P_W \\ 1 & \sum_{i=1}^{n} w_i Q_i \leq W \end{cases}$$

$$\text{where} \qquad E_{PF} = \sum_{i=1}^{n} \Bigg((p_i - \bar{c}_i) Q_i - C_{hi} \left(-a_i \left[\frac{t_i^2}{2} - \frac{b_i t_i^3}{6} - \frac{\theta t_i^4}{12} \right] + Q_i \left[t_i - \frac{\theta_i t_i^3}{6} - \frac{b_i t_i^2}{2} \right] \right) - C_{di} \theta_i \left(-a_i \left[\frac{t_i^3}{3} - \frac{b_i t_i^4}{8} - \frac{\theta t_i^5}{15} \right] + Q_i \left[\frac{t_i^2}{2} - \frac{\theta t_i^4}{8} - \frac{b_i t_i^3}{3} \right] \right) \Bigg)$$

$$V_{PF} = \sum_{i=1}^{n} \sigma_{ci}^2 Q_i^2$$

The expected gain for total profit is C_0 with tolerance $P_{E_{PF}}$, while the standard deviation is D_0 with tolerance $P_{V_{PF}}$. For space constraint, the goal is W with tolerance P_W. Using fuzzy linear programming problem technique, the solution of fuzzy-stochastic inventory model is transformed to

Max= α

Subject to

$1-\frac{C_0-E_{PF}}{P_{E_{PF}}}\geq\alpha$

$1-\frac{V_{PF}-D_0}{P_{V_{PF}}}\geq\alpha$

$$1-\frac{\sum_{i=1}^{n}w_iQ_i-W}{P_W}\geq\alpha$$

$$\sum_{i=1}^{n}\bar{c}_iQ_i-\bar{B}-1.96\left[\sum_{i=1}^{n}\left(\sigma_{ci}^2Q_i^2\right)+\sigma_B^2\right]^{1/2}\leq 0$$

$$(-a_i[t_i-\frac{b_it_i^2}{2}-\frac{\theta_it_i^3}{3}]+Q_i[1-\frac{\theta_it_i^2}{2}-b_it_i])=0$$

4. NUMERICAL RESULT

4.1. Crisp Model:

Input: $C_1=7, C_2=10, p_1=p_2=10, c_{h1}=c_{h2}=2.2, a_1=100, a_2=110, b_1=b_2=0.5, B=1800, W=275, w_1=2, w_2=2.2, \theta_1=0.05, \theta_2=0.06, c_{d1}=c_{d2}=7, c_{21}=c_{22}=1, T=1$

Output: $Q_1=64.56616, Q_2=66.30349, PF=337.9477, t_1=0.5420381, t_1=0.5119359$

4.2. Fuzzy Stochastic Model:

Input: $\hat{c_1}\sim N(7,0.01), \hat{c_2}\sim N(6.75,0.015), \hat{B}\sim N(1800,100), p1=p2=10, a_1=100, a_2=110, b_1=b_2=0.5, B=1800, W=275, w_1=2, w_2=2.2, \theta_1=0.05, \theta_2=0.06, c_{d1}=c_{d2}=7, C_0=337.94, P_{E_{PF}}=40, D_0=10.3093, P_{V_{PF}}=2, P_W=30$

Output: $\alpha=0.9987954, E_{PF}=339.5160, V_{PF}=10.31171, Q_1=67.51629, Q_2=63.63798, t_1=0.5620287, t_1=0.4947807$

5. CONCLUSION

The inventory model is formulated in a fuzzy stochastic environment, where the purchasing cost and investment goals are considered random along with imprecise storage space.Till now, very few models have been developed in such a mixed environment.

Profit maximization inventory model developed in this paper is simple.The techniques illustrated in this paper can easily be applied to other inventory problems with partial shortages, discount, fixed time horizon, etc. These techniques are the appropriate to handle the real-life inventory problems in realistic environments

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