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PRODUCTION INVENTORY MODELS FOR DETEIORATIVE ITEMS WITH THREE LEVELS OF PRODUCTION AND SHORTAGES

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Abstract: In this paper, three level production inventory models for deteriorative items are considered under the variation in production rate. Namely, it is possible that production started at one rate, after some time, switches to another rate. Such a situation is desirable in the sense that by starting at a low rate of production, a large quantum stock of manufacturing items at the initial stage are avoided, leading to reduction in the holding cost. The variation in production rate results in consumer satisfaction and potential profit. Two levels of production inventory models are developed, and the optimum lot size quantity and total cost are derived when the production inventory model without shortages is studied first and a production inventory model with shortages next. An optimal production lot size, which minimizes the total cost, is developed. The optimal solution is derived and a numerical example is provided. The validation of the results in this model was coded in Microsoft Visual Basic 6.0.

Keywords: EPQ, Deteriorative Items, Cycle Time, Demand, Three Levels of Production, Optimality.

MSC: 90B05.

1. INTRODUCTION

To be cost competitive and to acquire decent profit in the market, means that a firm needs good inventory management. Inventory management has been developing for decades both in the academic fields and in real practice to achieve these objectives. The problem of deteriorating inventory has received considerable attention in recent years. This is a realistic trend since most products such as medicine, dairy products, and chemicals start to deteriorate once they are produced. The economic order quantity (EOQ) model, introduced by Harris [1], was the first mathematical model to assist corporations in minimizing total inventory costs. It balances inventory holding and setup costs and derives the optimal order quantity. Regardless of its simplicity, the EOQ model is still applied in industry. Schrader and et. [2] concluded that the consumption of deteriorating items was closely relative to a negative exponential function of time. They proposed the following deteriorating items inventory model: $\frac{dI(t)}{dt}+\theta I(t)=-f(t)$. In the function, θ stands for the deteriorating rate of an item, I (t) refers to the inventory level at time t, and f (t) is the demand rate at time t. This inventory model laid foundations for the follow-up study. Sharma [3] developed a deterministic inventory model for a single deteriorating item which is stored in two different warehouses, and optimal stock level for the beginning of the period is found. The model is in accordance with the order level model for non deteriorating items with a single storage facility. Linn (4) derived a production model for the lot-size, order level inventory system with finite production rate, taking into consideration the effect of decay. The objective is to minimize total cost by selecting the optimal lot size and order level, using a search algorithm to obtain the optimal lot size and order level. Achary (5) developed a deterministic inventory model for deteriorating items with two warehouses when the replenishment rate is finite, the demand is at a uniform rate, and shortages are allowed. Wee [6] studied an inventory management of deteriorating items with decreasing demand rate and the system allows shortages alone. Benkherouf [7] presented a method for finding the optimal replenishment schedule for the production lot size model with deteriorating items, where demand and production are allowed to vary with time in an arbitrary way, and the shortages are allowed. Balan [8] described an inventory model in which the demand is considered as a composite function consisting of a constant component and a variable component, which is proportional to the inventory level in the periods when there is a positive inventory buildup, and the rate of production is considered finite while the decay rate is exponential. Yang [9] assumed that the demand function is positive and fluctuating with time (which is more general than increasing, decreasing, and log-concave demand patterns), and he developed the model with deteriorating items and shortages. Papachristos [10] studied a continuous review inventory model with five costs considered as significant-deterioration; holding, shortage, and the opportunity cost due to the lost sales, and the replenishment cost per replenishment, which is linear dependent on the lot size. Wee [11] developed an integrated two-stage production-inventory deteriorating model for the buyer and the supplier with stock-dependent selling rate, considering imperfect items and JIT multiple deliveries as well, deriving the optimal number of inspection optimal deliveries and the optimal delivery-time interval. Cardenas-Barron [12] presented a simple derivation of the

two inventory policies proposed by [Jamal, A.A.M., Sarker, B.R., & Mondal, S.(2004), Optimal manufacturing batch size with rework process at a single-stage production system, Computers and Industrial Engineering, 47(1), 77-89]. In order to find the optimal solutions for both policies, they used differential calculus. Their simple derivation is based on an algebraic derivation, and the final results are simple and easy to compute manually and results are equivalent. Wang [13] studied the inventory model for deteriorating items with trapezoidal type demand rate (the demand rate is a piecewise linearly function), and he proposed an inventory replenishment policy for this type of inventory model. Cardenas-Barron [14] developed an EPQ type inventory model with planned backorders for deteriorating the economic production quantity for a single product, which is manufactured in a single-stage manufacturing system that generates imperfect quality products, reworked in the same cycle. Cardenas-Barron (2009) corrected some mathematical expressions in the work of Sarkar, B.R., Jamal, A.M.M., Chern [15]. He proposed a partial backlogging inventory lot-size model for deteriorating items with stock-dependent demand and showed that not only the optimal replenishment schedule exists uniquely, but also that the total profit, associated with the inventory system, is a concave function of the number of replenishments. Wang [16] studied the inventory model for time-dependent deteriorating items with trapezoidal type demand rate and partial backlogging that is, the demand rate is a pricewise time-dependent function and an optimal replenishment policy of inventory model is proposed. Wee (2011) a deteriorating inventory problem with and without backorders is developed and this study is one of the first attempts by researchers to solve a deteriorating inventory problem with a simplified approach. The optimal solutions are compared with the classical methods for solving deteriorating inventory model, and the total cost of the simplified model is almost identical to the original model. Bozorgi [17] developed location of distribution centers with inventory or transportation decision, which plays an important role in optimizing supply chain management, by using a genetic algorithm. Hsu [18] developed an inventory model for vendor-buyer coordination under an imperfect production process and the proportion of defective items in each production lot is assumed to be stochastic and follows a known probability density function. Cardenas-Barron [19] presented an alternative approach to solve a finite horizon production lot sizing model with backorders using Cauchy-Bunyakovsky-Schwarz Inequality. The optimal batch size is derived from a sequence number of batches and that a constant batch size policy with one fill rate is proved to be better than the variable batch sizes with variable fill rates. Finally, a practically approach is proposed to find the optimal solutions for a discrete planning horizon and discrete batch sizes. Cardenas-Barron [20] revisited the work by Cardenas-Barron [Cardenas-Barron (2009), Economic production quantity with rework process at a single-stage manufacturing system with planned backorders, Computers and Industrial Engineering, 57(3), 1105-1113]. The optimal solution condition is analyzed using the production time and the time to eliminate backorders as decision variables instead of the classical decisions variables of lot and backorder quantities. The new approach leads to an alternative inventory policy for imperfect quality items when the optimal production is less than the optimal time. Hsu [21] developed a mathematical model to determine an integrated vendor-buyer inventory policy, where the vendor's production process is imperfect and produces a certain number of defective items with a known probability density function. Sivashankari and Panayappan [22] developed a production inventory model with planned backorders for

determining the optimum quantity for a single product manufactured in a single stage manufacturing system that generates imperfect quality products where a proportion of the defective products are reworked into a same cycle. Sivashankari and Panayappan [23] integrated a cost reduction delivery policy into a production inventory model with defective items in which three different rates of production are considered. Sivashankari and Panayappan [24] introduced a multi-delivery policy into a production inventory model with defective items in which two different rates of production are considered. Kianfar [25] developed a production planning and marketing model in unreliable flexible manufacturing systems with inconstant demand rate such that its rate depends on the level of advertisement on that product; the proposed model is more realistic and more useful from a practical point of view. Sadegheih [26] proposed an integrated inventory management model within a multi-item, multi-echelon supply chain; he developed three inventory models with respect to different layers of supply chain in an integrated manner, seeking to optimize total cost of the whole supply chain. Aalikar [27] modeled a seasonal multi-product multi-period inventory control problem in which the inventory costs are obtained under inflation and all-unit discount policy; furthermore, the products are delivered in boxes of known number of items and in case of shortage, a fraction of demand is considered so as backorder and a fraction lost sale. Besides, the total storage space and total available budget are limited. The objective is to find the optimal number of boxes of the products in different periods to minimize the total inventory cost (including ordering, holding, shortage and purchasing costs). Sivashankari and Panayappan [28] introduced the rate of growth; the rate of growth in the production period is $D(1+i)^n$ and the consumption period is $D(1-i)^n$. The relevant model is built, solved and closed formulas are obtained. In this paper, a production inventory model for deteriorating items in which three levels of production are considered and the possibility that production started at one rate, after some time, may be switched to another rate. Such a situation is desirable in the sense that by starting at a low rate of production, a large quantum stock of manufactured item at the initial stage is avoided, which leads to reduction in the holding cost. Two models are developed considering shortages, with and with out shortages, and the model with shortages is discussed in detail. The remainder of the paper is organized as follows. Section 2 presents the assumptions and notations. Section 3 is devoted to mathematical modeling and numerical examples. Finally, the paper summarizes and concludes in section 4.

2. ASSUMPTIONS AND NOTATIONS

a) Assumptions: the assumptions of an inventory model are as follows:

The production rate is known and constant.
The demand rate is known, constant and non negative.
Items are produced and added to the inventory.
Three rates of production are considered.
The item is a single product; it does not interact with any other inventory items.
The production rate is always greater than or equal to the sum of the demand rate.
The inventory system involves only one item and the lead time is zero.

Shortages are allowed and there is sufficient capacity and capital to procure the desired lot size.

b) Notations:

P – Production rate in units time
D – Demand rate in units per unit time
θ – deterioration rate is constant
Q_1 – on hand inventory level at time T_1
Q_2 – on hand inventory level at time T_2
Q_3 – on hand inventory level at time T_3
B – Maximum shortage level
Q^* – production lot size considered as a decision variable
C_p – Production Cost per unit
C_h – Holding cost per unit/ per unit time
C_0 – Setup cost per production cycle at $T = 0$
Cs – Shortage cost per unit/per unit time
T – length of the inventory cycle
T_i – unit time in periods $i\,(i = 1, 2, 3, 4, 5)$
TC – Total cost

3. MATHEMATICAL MODELS

3.1. Production inventory model for three levels of production

The changes in inventory level against time are represented in Figure 1. The first production setup starts with zero inventory at $t = 0$. During time T_1, the inventory level increases due to production less demand and deterioration until the maximum inventory level at $t = T_1$ is reached

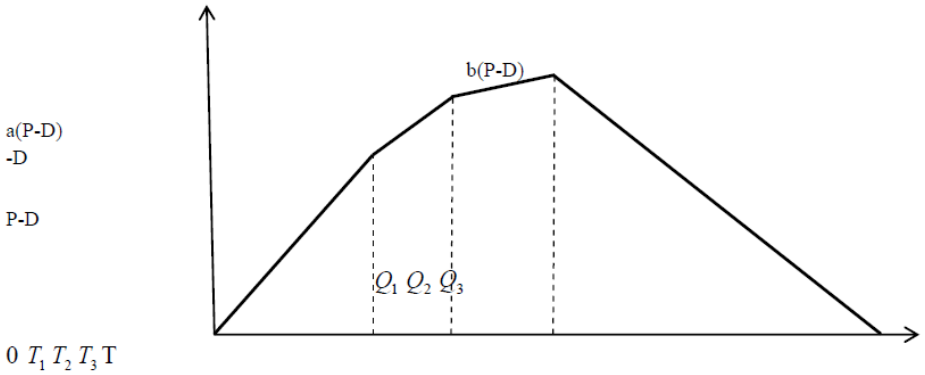

Figure 1: On hand inventory of three levels of production with deteriorative items

Therefore, the maximum inventory level equal to $(P-D)T_1$. During time T_2, Production and Demand increases at the rate of "a" time of P-D i.e. a (P-D) where "a" is a constant. Therefore, the maximum inventory level equal to $a(P-D)T_2$. During time T_3, Production and Demand increases at the rate of "b" time of P-D i.e. $b(P-D)$ where "b" is a constant. Therefore, the maximum inventory level equal to $b(P-D)T_3$. During decline time, the inventory level starts to decrease due to demand at a rate D up to time T. Let $I(t)$ denote the inventory level of the system at time T. The differential equations describing the system in the interval (0,T) given by

$$\frac{dI(t)}{dt}+\theta I(t)=P-D;\ 0\le t\le T_1 \tag{1}$$

$$\frac{dI(t)}{dt}+\theta I(t)=a(P-D);\ T_1\le t\le T_2 \tag{2}$$

$$\frac{dI(t)}{dt}+\theta I(t)=b(P-D);\ T_2\le t\le T_3 \tag{3}$$

$$\frac{dI(t)}{dt}+\theta I(t)=-D;\ T_3\le t\le T \tag{4}$$

The boundary conditions are

$$I(0)=0, I(T_1)=Q_1; I(T_2)=Q_2,\ I(T_3)=Q_3,\ I(T)=0 \tag{5}$$

The first order differential equations can be solved by using the bound conditions are

From the equation (1), $I,(t)=\frac{P-D}{\theta}\left[1-e^{-\theta t}\right];\ 0\le t\le T_1$ (6)

From the equation (2), $I(t)=\frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)$ (7)

From the equation (3), $I(t) = \frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)$ (8)

From the equation (4), $I(t) = \frac{D}{\theta}\left(e^{\theta(T-t)}-1\right)$ (9)

Maximum inventory Q_1: The maximum inventory during time T_1 is calculated as follows. From equations (5) and (6), $I(T_1) = Q_1 \Rightarrow \frac{P-D}{\theta}\left(1-e^{-\theta T_1}\right) = Q_1$

In order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

Therefore, $Q_1 = (P-D)T_1$ (10)

Maximum inventory Q_2: The maximum inventory during time T_2 is calculated as follows. From the equations (5) and (7), $I(T_2) = Q_2 \Rightarrow \frac{a(P-D)}{\theta}\left(1-e^{-\theta T_2}\right) = Q_2$

Again, in order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ.

Therefore, $Q_2 = a(P-D)T_2$ (11)

Maximum inventory Q_3: The maximum inventory during time T_3 is calculated as follows. From equations (5) and (8), $I(T_3) = Q_3 \Rightarrow \frac{P-D}{\theta}\left(1-e^{-\theta t_3}\right) = Q_3$

In order to facilitate analysis, we do an asymptotic analysis for $I(t)$. Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

Therefore, $Q_3 = b(P-D)T_3$ (12)

Total Cost: The total cost comprises of the sum of the Production cost, ordering cost, holding cost, and deteriorating cost. They are grouped together after evaluating the above cost individually.

(i)Production Cost $= DC_P$ (13)

(ii) Setup cost per set = $\frac{C_0}{T}$ (14)

(iii) Holding Cost per unit time: =

$$\frac{C_h}{T}\left[\int_0^{T_1} I(t)dt+\int_{T_1}^{T_2} I(t)dt+\int_{T_2}^{T_3} I(t)dt+\int_{T_3}^{T} I(t)dt\right]$$

=

$$\frac{C_h}{T}\left(\int_0^{T_1}\frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_1}^{T_2}\frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_2}^{T_3}\frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_3}^{T}\frac{D}{\theta}\left(e^{\theta(T-t)}-1\right)dt\right)$$

=

$$\frac{C_h}{T}\left[\frac{P-D}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_0^{T_1}+\frac{a(P-D)}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_{T_1}^{T_2}+\frac{b(P-D)}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_{T_2}^{T_3}+\frac{D}{\theta}\left(\frac{e^{\theta(T_3-t)}}{-\theta}-t\right)_{T_3}^{T}\right]$$

=

$$\frac{C_h}{T}\left[\begin{array}{l}\frac{P-D}{\theta^2}\left(\theta T_1+e^{-\theta T_1}-1\right)+\frac{a(P-D)}{\theta^2}\left(\theta(T_2-T_1)+e^{-\theta T_2}-e^{-\theta T_1}\right)\\+\frac{b\left(P-D\right)}{\theta^2}\left(\theta(T_3-T_2)+e^{-\theta T_3}-e^{-\theta T_2}\right)-\frac{D}{\theta^2}\left(1-e^{\theta(T-T_3)}+\theta(T-T_3)\right)\end{array}\right]$$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

$$= \frac{C_h}{T}\left[\begin{array}{l}\frac{P-D}{\theta^2}\left\{\frac{\theta^2T_1^2}{2}\right\}+\frac{a(P-D)}{\theta^2}\left(\frac{\theta^2}{2}(T_2^2-T_1^2\right)\\+\frac{b(P-D)}{\theta^2}\left(\frac{\theta^2(T_3^2-T_2^2)}{2}\right)+\frac{D}{\theta^2}\left\{\frac{\theta^2(T-T_3)^2}{2}\right\}\end{array}\right]$$

$$= \frac{C_h}{2T}\left[(P-D)T_1^2+a(P-D)(T_2^2-T_1^2)+b(P-D)(T_3^2-T_2^2)+D(T-T_3)^2\right] \quad (15)$$

(iv) Deteriorating Cost per unit time: Deteriorating cost

$$= \frac{\theta C_d}{T}\left[\int_0^{T_1} I(t)dt+\int_{T_1}^{T_2} I(t)dt+\int_{T_2}^{T_3} I(t)dt+\int_{T_3}^{T} I(t)dt\right]=$$

$$\frac{\theta C_d}{T}\left[\int_0^{T_1}\frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_1}^{T_2}\frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_2}^{T_3}\frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt+\int_{T_3}^{T}\frac{D}{\theta}\left(e^{\theta(T-t)}-1\right)dt\right]$$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ.

$$= \frac{\theta C_d}{2T}\left[(P-D)T_1^2+a(P-D)(T_2^2-T_1^2)+b(P-D)(T_3^2-T_2^2)+D(T-T_3)^2\right] \quad (16)$$

TC = Production Cost + Ordering Cost + (Holding Cost + Deteriorating Cost)

$$= DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_d)}{2T}\begin{bmatrix}(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2)\\ +b(P-D)(T_3^2 - T_2^2) + D(T-T_3)^2\end{bmatrix} \quad (17)$$

Let $T_1 = \alpha T_3$ and $T_2 = \beta T_3$ (18)

Therefore, the total cost

$$= DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_d)}{2T}\begin{bmatrix}(P-D)\alpha^2 T_3^2 + a(P-D)(\beta^2 - \alpha^2)T_3^2\\ +b(P-D)(1-\beta^2)T_3^2 + D(T-T_3)^2\end{bmatrix} \quad (19)$$

Partially differentiate the equation (19) with respect to T_3,

$$\frac{\partial}{\partial T_3}(TC) = \frac{C_h + \theta C_d}{T}\left[(P-D)\alpha^2 T_3 + a(P-D)(\beta^2 - \alpha^2)T_3 + b(P-D)(1-\beta^2)T_3 - D(T-T_3)\right] = 0$$

$$\frac{\partial^2}{\partial T_3^2}(TC) = \frac{C_h + \theta C_d}{T}\left[(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) + b(P-D)(1-\beta^2) + DT_3\right] > 0$$

Therefore, $T_3 = \dfrac{DT}{(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) + b(P-D)(1-\beta^2) + D}$

$$T_3 = \frac{DT}{D + (P-D)\left(\alpha^2 + a(\beta^2 - \alpha^2) + b(1-\beta^2)\right)} \quad (20)$$

Partially differentiate the equation (19) with respect to T

$$\frac{\partial}{\partial T} = \frac{-C_0}{T^2} - \frac{(C_h + \theta C_d)}{2T^2}T_3^2\begin{bmatrix}(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2)\\ +b(P-D)(1-\beta^2)\end{bmatrix} + \frac{D(C_h + \theta C_d)(T^2 - T_3^2)}{2T^2} =$$

0

$$\frac{\partial^2}{\partial T^2} = \frac{2C_0}{T^3} + \frac{2(C_h + \theta C_d)}{2T^3}T_3^2\begin{bmatrix}(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2)\\ +b(P-D)(1-\beta^2)\end{bmatrix} + \frac{D(C_h + \theta C_d)}{T} > 0$$

$$D(C_h + \theta C_d)(T^2 - T_3^2) = 2C_0 + (C_h + \theta C_d)T_3^2\left[(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) + b(P-D)(1-\beta^2)\right]$$

$$T^2\left[D(C_h + \theta C_d) - \frac{D^2(C_h + \theta C_d)}{D + (P-D)\left(\alpha^2 + a(\beta^2 - \alpha^2) + b(1-\beta^2)\right)}\right] = 2C_0$$

$$(C_h + \theta C_d)DT^2 = 2C_0 + \frac{(C_h + \theta C_d)D^2T^2}{D + (P-D)\left(\alpha^2 + a(\beta^2 - \alpha^2) + b(1-\beta^2)\right)}$$

$$T^2(C_h + \theta C_d)\left[\frac{D^2 + D(P-D)\left(\alpha^2 + a(\beta^2 - \alpha^2) + b(1-\beta^2) - D^2\right)}{D(P-D)\left(\alpha^2 + a(\beta^2 - \alpha^2) + b(1-\beta^2)\right)}\right]$$

$$T^2 = \frac{2C_0\left[D+(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)\right]}{(C_h+\theta C_d)D(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)},$$

$$\text{Therefore, } T = \sqrt{\frac{2C_0\left[D+(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)\right]}{(C_h+\theta C_d)D(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)}} \quad (21)$$

Note: When $T = \frac{Q}{D}$ then

$$Q = \sqrt{\frac{2DC_0\left[D+(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)\right]}{(C_h+\theta C_d)D(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)}} \quad (22)$$

Numerical Example

Let us consider the cost parameters P = 5000 units, D = 4500 units, C_h =10, C_p = 100, C_0 =100, θ = 0.01 to 0.10, C_d =100, a = 2, b= 3, $\alpha = 0.8$, $\beta = 0.9$

Optimum solution

From the equations (21), (10), (11), (12), (22), (13), (14), (15) and (16) Cycle Times: T = 0.1658; T_1 = 0.1132; T_2 = 0.1273; T_3 = 0.1415; Optimum Quantity Q* = 746.25, Q_1 = 56.59; Q_2 = 63.66; Q_3 = 70.73;

Production cost =450,000, Setup cost = 603.01, Holding cost = 548.19, Deteriorating cost = 54.82, Total cost = 451206.03

Table 1: Variation of Rate of Deteriorating Items with inventory and total Cost

θ	T	Q	Production Cost	Setup Cost	Holding Cost	Deteriorating Cost	Total Cost
0.01	0.1658	746.25	450000	603.01	548.19	54.82	451206.03
0.02	0.1588	714.48	450000	629.83	524.86	104.97	451259.65
0.03	0.1525	686.45	450000	655.55	504.27	151.28	451311.09
0.04	0.1470	661.48	450000	680.29	485.92	194.37	451360.58
0.05	0.1420	639.05	450000	704.17	469.45	234.72	451408.34
0.06	0.1375	618.76	450000	727.26	454.54	272.72	451454.52
0.07	0.1334	600.28	450000	749.64	440.97	308.68	451499.29
0.08	0.1296	583.37	450000	771.38	428.54	342.83	451542.76
0.09	0.1262	567.81	450000	792.52	417.11	375.40	451585.03

From the above table, a study of rate of deteriorative items with production time (T_1), and cycle time T is given and conclud that when the rate of deteriorative items increases, then the optimum quantity and cycle time decrease; also a study of rate of deteriorative item with setup cost, holding cost, deteriorative cost and total cost is given and conclud that when the rate of deteriorative items increases, then the holding cost decreases, but setup cost, deteriorative cost and Total cost increas.

The total cost functions are the real solution in which the model parameters are assumed to be a static value. It is reasonable to study the sensitivity, i.e. the effect of making chances in the model parameters over a given optimum solution. It is important to find the effects on different system performance measures, such as cost function, inventory system, etc. For this purpose, sensitivity analysis of various system parameters for the models of this research are required to observe whether the current solutions remain unchanged, or the current solutions become infeasible, etc.

Table 2: Effect of Demand and Cost parameters on optimal policies

Parameters		Optimum values								Total Cost
		T	Q	T_1	T_2	T_3	Q_1	Q_2	Q_3	
θ	0.01	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	0.02	0.1588	714.48	0.1084	0.1219	0.1354	54.18	60.95	67.72	451259.65
	0.03	0.1525	686.45	0.1041	0.1171	0.1301	52.05	58.56	65.07	451311.09
	0.04	0.1470	661.48	0.1003	0.1129	0.1254	50.16	56.43	62.70	451360.58
	0.05	0.1420	639.05	0.0969	0.1090	0.1211	48.46	54.52	60.57	451408.34
C_0	80	0.1483	667.47	0.1012	0.1139	0.1265	50.61	56.94	63.26	451078.70
	90	0.1573	707.96	0.1074	0.1208	0.1342	53.68	60.39	67.11	451144.14
	100	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	110	0.1739	782.68	0.1187	0.1335	0.1484	59.34	66.77	74.19	451264.89
	120	0.1817	817.48	0.1240	0.1395	0.1550	62.00	69.74	77.49	451321.14
C_h	8	0.1833	825.01	1251	0.1408	0.1564	62.56	70.38	78.20	451090.89
	9	0.1739	782.67	0.1187	0.1335	0.1484	59.39	66.77	74.19	451149.90
	10	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	11	0.1588	714.48	0.1084	0.1219	0.1354	54.18	60.95	67.72	451259.65
	12	0.1525	686.45	0.1041	0.1171	0.1301	52.05	58.56	65.07	451311.09
C_P	80	0.1674	753.13	0.1142	0.1285	0.1428	57.11	64.25	71.39	361195.01
	90	0.1666	749.67	0.1137	0.1279	0.1421	56.85	63.95	71.06	406200.53
	100	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	110	0.1651	742.88	0.1127	0.1267	0.1408	56.33	63.37	70.42	496211.50
	120	0.1643	739.56	0.1122	0.1262	0.1402	56.08	63.09	70.10	541216.94
a	1	0.1743	784.48	0.1209	0.1360	0.1511	60.46	68.02	75.58	451147.25
	2	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	3	0.1587	714.09	0.1066	0.1199	0.1332	53.29	59.95	66.61	451260.33
	4	0.1526	686.61	0.1009	0.1135	0.1261	50.44	56.74	63.05	451310.79
	5	0.1473	662.78	0.0959	0.1079	0.1199	47.94	53.93	59.93	451357.92
b	1	0.1874	843.32	0.1327	0.1493	0.1658	66.34	74.63	82.92	451067.21
	2	0.1754	789.47	0.1219	0.1372	0.1524	60.96	68.58	76.20	451140.01
	3	0.1658	746.25	0.1132	0.1273	0.1415	56.59	63.66	70.73	451206.03
	4	0.1579	710.64	0.1059	0.1191	0.1323	52.93	52.94	59.55	451266.46
	5	0.1513	680.70	0.0996	0.1121	0.1246	49.82	56.05	62.28	451322.17

Observations:

With the increase in rate of deteriorating items (θ), total cost increases but cycle time, optimum quantity, Cycles times (T,T_1,T_2,T_3) and optimum quantity and maximum inventory $Q_1,Q_2,Q_3)$ decreases.

With the increase in setup cost per unit (C_0), optimum quantity (Q*), maximum inventory Q_1, Q_2 and Q_3, Cycle times (T,T_1,T_2,T_3) and total cost increase.

With the increase in holding cost per unit (C_h), optimum quantity (Q*), maximum inventory Q_1, Q_2 and Q_3, cycle times (T,T_1,T_2,T_3) decreases but total cost increase.

Similarly, other parameters, deteriorating cost, a and b can also be observed from the Table 2.

Special Cases: If the production system is considered to be ideal, that is no deteriorative are produced, i.e. the value of θ is set to zero. In that case, equations (21) and (22) reduce to the classical economic production quantity model as follows

$$T = \sqrt{\frac{2C_0\left[D+(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)\right]}{C_h D(P-D)\left(\alpha^2+a(\beta^2-\alpha^2)+b(1-\beta^2)\right)}}$$

4. PRODUCTION INVENTORY MODEL FOR THREE LEVELS OF PRODUCTION AND SHORTAGES

During time T_1, inventory is increasing at the rate of P and simultaneously decreasing at the rate of D. Thus inventory accumulates at the rate of P - D units. Therefore, the maximum inventory level shall be equal to $(P-D)t_1$. During time T_2, Production and Demand increases at the rate of "a" time of P-D i.e. a(P-D) where "a" is a constant. During time T_3, Production and Demand increases at the rate of "b" time of P-D i.e. b(P-D) where "b" is a constant. During decline time, the inventory level starts to decrease due to demand at a rate D up to time T_5. In shortage period, shortages start to accumulate at a rate of B, the inventory level is zero at time T_5 but shortages accumulate at a rate of D up to time T_5. Therefore, time T_5 need to build-up B units of times. The production restarts again at time T at a rate of P-D to recover both the previous shortages in the period T_5 and to satisfy demand in the period T. Time T need to consume all units Q at demand rate. The process is repeated. The variation of the underlying inventory system for one cycle is shown in figure 2.

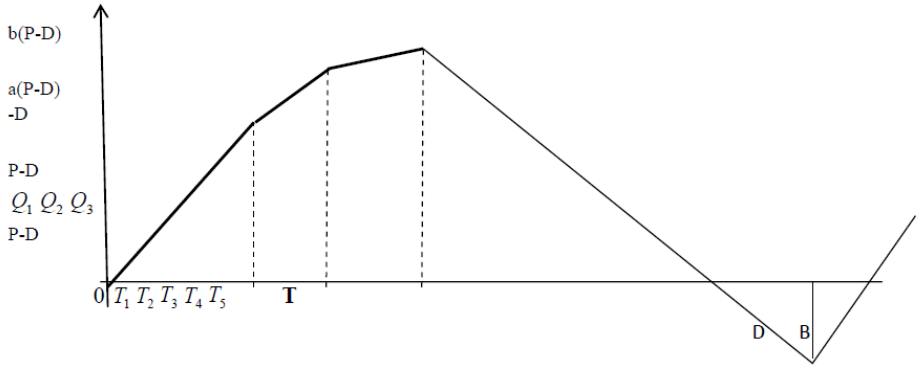

Figure 2: On-hand inventories of three levels of production with Deteriorating Items and shortages

Let I (t) denote the inventory level of the system at time T. The differential equation describing the system in the interval (0,T) are given by

$$\frac{dI(t)}{dt}+\theta I(t) = P-D\ ;\ 0 \le t \le T_1 \qquad (23)$$

$$\frac{dI(t)}{dt} + \theta I(t) = a(P-D)\,;\ T_1 \le t \le T_2 \tag{24}$$

$$\frac{dI(t)}{dt} + \theta I(t) = b(P-D)\,;\ T_2 \le t \le T_3 \tag{25}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D\,;\ T_3 \le t \le T_4 \tag{26}$$

$$\frac{dI(t)}{dt} = \text{-D}\,;\ T_4 \le t \le T_5 \tag{27}$$

$$\frac{dI(t)}{dt} = \text{(P-D)}\,;\ T_5 \le t \le T \tag{28}$$

The boundary conditions are

$$I(0) = 0, I(T_1) = Q_1; I(T_2) = Q_2, I(T_3 = Q_3); I(T_4) = 0; I(T_5) = B \text{ and } I(T) = 0 \tag{29}$$

The solutions of the above equations are

From the equation (23), I(t) = $\frac{P-D}{\theta}\left[1-e^{-\theta t}\right]\,;\ 0 \le t \le T_1$ (30)

From the equation (24), $I(t) = \frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)$ (31)

From the equation (25), $I(t) = \frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)$ (32)

From the equation (26), $I(t) = \frac{D}{\theta}\left(e^{\theta(T_4-t)}-1\right)$ (33)

From the equation (27), $I(t) = -D(T_4 - t)$ (34)

From the equation (28), $I(t) = (P-D)(T-t)$ (35)

Maximum inventory Q_1: The maximum inventory during time T_1 is calculated as follows. From equations (29) and (30), $I(T_1) = Q_1 \Rightarrow \frac{P-D}{\theta}\left(1-e^{-\theta T_1}\right) = Q_1 I$

In order to facilitate analysis, we do an asymptotic analysis for I(t). Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

Therefore, $Q_1 = (P-D)T_1$ (36)

Maximum inventory Q_2: The maximum inventory during time T_2 is calculated as follows. From the equations (29) and (31), $I(T_2) = Q_2 \Rightarrow \frac{a(P-D)}{\theta}\left(1-e^{-\theta T_2}\right) = Q_2$

In order to facilitate analysis, we do an asymptotic analysis for I(t). Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

Therefore, $Q_2 = a(P-D)T_2$ (37)

Maximum inventory Q_3 : The maximum inventory during time T_3 is calculated as follows. From equations (29) and (32), $I(T_3) = Q_3 \Rightarrow \frac{P-D}{\theta}\left(1-e^{-\theta t_3}\right) = Q_3$

In order to facilitate analysis, we do an asymptotic analysis for I(t). Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

Therefore, $Q_3 = (P-D)T_3$ (38)

Total Cost: The total cost comprises of the sum of the Production cost, ordering cost, holding cost, and Deteriorating cost. They are grouped together after evaluating the above cost individually.

Production Cost per unit time $= DC_P$ (39)

Setup cost per set = $\frac{C_0}{T}$ (40)

(i) Holding Cost per unit time :

$$= \frac{C_h}{T}\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt\right]$$

$$= \frac{C_h}{T}\left[\int_0^{T_1} \frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_1}^{T_2} \frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_2}^{T_3} \frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_3}^{T_4} \frac{D}{\theta}\left(e^{\theta(T-t)}-1\right)dt\right]$$

$$= \frac{C_h}{T}\left[\frac{P-D}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_0^{T_1} + \frac{a(P-D)}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_{T_1}^{T_2} + \frac{b(P-D)}{\theta}\left(t-\frac{e^{-\theta t}}{-\theta}\right)_{T_2}^{T_3} + \frac{D}{\theta}\left(\frac{e^{\theta(T_4-t)}}{-\theta}-t\right)_{T_3}^{T_4}\right]$$

$$= \frac{C_h}{T}\left[\begin{array}{l}\frac{P-D}{\theta^2}\left(\theta T_1 + e^{-\theta T_1} - 1\right) + \frac{a(P-D)}{\theta^2}\left(\theta(T_2-T_1) + e^{-\theta T_2} - e^{-\theta T_1}\right) \\ + \frac{b(P-D)}{\theta^2}\left(\theta(T_3-T_2) + e^{-\theta T_3} - e^{-\theta T_2}\right) - \frac{D}{\theta^2}\left(1-e^{\theta(T_4-T_3)} + \theta(T_4-T_3)\right)\end{array}\right]$$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ

$$= \frac{C_h}{T}\left[\begin{array}{l}\frac{P-D}{\theta^2}\left\{\frac{\theta^2 T_1^2}{2}\right\} + \frac{a(P-D)}{\theta^2}\left(\frac{\theta^2}{2}(T_2^2 - T_1^2)\right) \\ + \frac{b(P-D)}{\theta^2}\left(\frac{\theta^2(T_3^2 - T_2^2)}{2}\right) + \frac{D}{\theta^2}\left\{\frac{\theta^2(T_4-T_3)^2}{2}\right\}\end{array}\right]$$

$$= \frac{C_h}{2T}\left[(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) + b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2\right] \quad (41)$$

(i) Deteriorating Cost per unit time: Deteriorating cost

$$= \frac{\theta C_P}{T}\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T_2} I(t)dt + \int_{T_2}^{T_3} I(t)dt + \int_{T_3}^{T_4} I(t)dt\right]$$

$$= \frac{\theta C_P}{T}\left[\int_0^{T_1} \frac{P-D}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_1}^{T_2} \frac{a(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_2}^{T_3} \frac{b(P-D)}{\theta}\left(1-e^{-\theta t}\right)dt + \int_{T_3}^{T_4} \frac{D}{\theta}\left(e^{\theta(T-t)} - 1\right)dt\right]$$

Expanding the exponential functions and neglecting second and higher power of θ for small value of θ.

$$= \frac{\theta C_P}{2T}\left[(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) + b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2\right] \quad (42)$$

(ii) Shortage Cost : $\frac{C_S}{T}\left[\int_{T_4}^{T_5} I(t)dt + \int_{T_5}^{T} I(t)dt\right]$

$$= \frac{C_S}{T}\left[\int_{T_4}^{T_5} D(t-T_4)dt + \int_{T_5}^{T} (P-D)(T-t)dt\right]$$

$$= \frac{C_S}{2T}\left[D(T_5 - T_4)^2 + (P-D)(T-T_5)^2\right]$$

$$= \frac{C_S}{2T}\left[D\left(\frac{P-D}{P}T - \frac{P-D}{P}T_4\right)^2 + \frac{D(P-D)}{P}(T-T_4)^2\right] \quad (43)$$

$$= \frac{C_S}{2T}\left[\frac{D(P-D)}{P}(T-T_4)^2 + \frac{D(P-D)}{P}(T-T_4)^2\right]$$

$$= \frac{D(P-D)C_S}{TP}(T-T_4)^2$$

From the equations (34) and (35),

$I(T_5) = B \Rightarrow -D(T_4 - T_5) = \text{B}$ that is $D(T_5 - T_4) = B$

$I(T_5) = B \Rightarrow (P-D)(T-T_5) = B$ that is $(P-D)(T-T_5) = B$

$(P-D)(T-T_5) = D(T_5 - T_4)$

$$\text{Therefore, } T = \frac{P}{P-D}T_5 - \frac{D}{P-D}T_4 \text{ and } T_5 = \frac{P-D}{P}T + \frac{D}{P}T_4 \quad (44)$$

TC = Production Cost + Ordering Cost + (Holding Cost + Deteriorating Cost)

$$= DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_P)}{2T}\begin{bmatrix}(P-D)T_1^2 + a(P-D)(T_2^2 - T_1^2) \\ +b(P-D)(T_3^2 - T_2^2) + D(T_4 - T_3)^2\end{bmatrix} + \frac{D(P-D)C_S}{TP}(T - T_4)^2$$

Let $T_1 = \alpha T_3$; $T_2 = \beta T_3$ and $T_3 = \gamma T_4$ (45)

Therefore, the total cost

$$= DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_P)}{2T}\begin{bmatrix}(P-D)\alpha^2 T_4^2 + a(P-D)(\beta^2 - \alpha^2)T_4^2 \\ +b(P-D)(\gamma^2 - \beta^2)T_4^2 + D(1-\gamma)^2 T_4^2\end{bmatrix} + \frac{D(P-D)C_S}{TP}(T - T_4)^2$$

Partially differentiate the equation (24) with respect to T_4,

$$\frac{\partial}{\partial T_4}(TC) = \frac{(C_h + \theta C_P)T_4}{T}\begin{bmatrix}(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) \\ +b(P-D)(\gamma^2 - \beta^2) + D(1-\gamma)^2\end{bmatrix} - \frac{2D(P-D)C_s}{TP}(T - T_4) = 0$$

$$\frac{\partial^2}{\partial T_3^2}(TC) = \frac{C_h + \theta C_P}{T}\begin{bmatrix}(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) \\ +b(P-D)(\gamma^2 - \beta^2) + D(1-\gamma)\end{bmatrix} + \frac{2D(P-D)C_S}{TP} > 0$$

On simplification,

$$T_4 = \frac{2D(P-D)C_S T}{P(C_h + \theta C_P)\begin{Bmatrix}(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) \\ +b(P-D)(\gamma^2 - \beta^2) + D(1-\gamma)^2\end{Bmatrix} + 2D(P-D)C_S} \quad (46)$$

Let us assume A = $(P-D)\alpha^2 + a(P-D)(\beta^2 - \alpha^2) + b(P-D)(\gamma^2 - \beta^2) + D(1-\gamma)^2$

Therefore, $T_4 = \frac{2D(P-D)C_S T}{P(C_h + \theta C_P)A + 2D(P-D)C_S}$ and

Total cost = $DC_P + \frac{C_0}{T} + \frac{(C_h + \theta C_P)}{2T}A + \frac{D(P-D)C_S}{TP}(T - T_4)^2$

Partially differentiate the equation (46) with respect to T

$$\frac{\partial}{\partial T} = \frac{-C_0}{T^2} - \frac{(C_h + \theta C_P)T_4^2 A}{2T^2} + \frac{D(P-D)C_S(T^2 - T_4^2)}{PT^2} = 0$$

$$\frac{\partial^2}{\partial T^2} = \frac{2C_0}{T^3} + \frac{(C_h + \theta C_P)}{T^3}T_4^2 + \frac{D(P-D)C_S(T^2 - T_4^2)}{T} > 0$$

$$2D(P-D)C_S T^2 = 2PC_0 + P(C_h + \theta C_P)AT_4^2 + 2D(P-D)C_S T_4^2$$

$$T^2\left[2D(P-D)C_S - \frac{4D^2(P-D)^2 C_S^2}{P(C_h + \theta C_P)A + 2D(P-D)C_S}\right] = 2PC_0$$

$$T^2 = \frac{C_0\left[2D(P-D)C_S + P(C_h + \theta C_P)A\right]}{(C_h + \theta C_P)D(P-D)C_S A},$$

Therefore, $$T = \sqrt{\frac{C_0\left[2D(P-D)C_S + P(C_h+\theta C_P)A\right]}{(C_h+\theta C_P)D(P-D)C_S A}} \quad (47)$$

Note: When $T = \frac{Q}{D}$ then Q = TD

Numerical Example

Let us consider the cost parameters

P = 5000 units, D = 4500 units, C_h =10, C_p = 100, C_0 =100, θ = 0.01 to 0.10,

a = 2, b= 3, $\alpha = 0.8$, $\beta = 0.9$, γ =0.9

Optimum solution

Cycle Times: T = 0.2200; T_1 = 0.0832; T_2 = 0.0951; T_3 = 0.1070; T_4 = 0.1189, T_5 = 0.1290,

Optimum Quantity Q* = 989.83, Q_1 = 41.62; Q_2 = 95.15; Q_3 = 160.56; B = 45.46,

Production cost =450,000, Setup cost = 454.62, Holding cost = 223.47, Shortage Cost=208.81,

Deteriorating cost = 22.35, Total cost = 450909.25

Table 3: Variation of Rate of Deteriorating Items with inventory and total Cost

θ	T	Q	Production Cost	Setup Cost	Holding Cost	Deteriorating Cost	Shortage Cost	Total Cost
0.01	0.2200	989.83	450000	454.62	223.47	22.35	208.81	450909.25
0.02	0.2149	967.27	450000	465.23	201.22	40.24	223.76	450930.45
0.03	0.2106	947.76	450000	474.80	182.26	54.68	237.86	450949.60
0.04	0.2068	930.72	450000	483.50	165.95	66.38	251.17	450966.99
0.05	0.2035	915.69	450000	491.43	151.79	75.90	263.74	450982.86
0.06	0.2005	902.34	450000	498.71	139.42	83.65	275.63	450997.41
0.07	0.1979	890.38	450000	505.40	128.54	89.98	286.87	451010.80
0.08	0.1955	879.63	450000	511.58	118.92	95.13	297.53	451023.16
0.09	0.1933	869.89	450000	517.31	110.35	99.32	307.64	451034.61

From the above table, a study of rate of deteriorative items and optimum quantity and cycle time T, where it can be concluded that when the rate of deteriorative items increases, then the optimum quantity and cycle time decrease; the table gives also a study of rate of deteriorative item with Setup cost, Holding cost, Deteriorative Cost, Shortage cost and Total cost, where it can be concluded that when the rate of deteriorative items increases, then the Holding cost decreases but setup cost, deteriorative cost, shortage cost and Total cost increases.

Sensitivity Analysis:

Table 4: Effect of Demand and Cost parameters on optimal policies

Para meters		Optimum values									Total Cost
		T	Q	T_1	T_2	T_3	Q_1	Q_2	Q_3	B	
θ	0.01	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	0.02	0.2149	967.27	0.0781	0.0893	0.1004	39.05	89.25	150.61	46.52	450930.46
	0.03	0.2106	947.76	0.0736	0.0841	0.0946	36.79	84.08	141.89	47.48	450949.60
	0.04	0.2068	930.72	0.0696	0.0795	0.0894	34.78	79.51	134.17	48.35	450966.99
	0.05	0.2035	915.69	0.0659	0.0754	0.0848	33.00	75.42	127.28	49.14	450982.86
C_0	80	0.1967	885.33	0.0745	0.0851	0.0957	37.23	85.10	143.61	40.66	450813.26
	90	0.2087	939.03	0.0789	0.0903	0.1015	39.49	90.26	152.32	43.13	450862.59
	100	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	110	0.2307	1038.14	0.0873	0.0998	0.1123	43.66	99.79	168.40	47.68	450953.63
	120	0.2409	1084.30	0.0912	0.1042	0.1173	45.60	104.23	175.89	49.80	450996.03
C_h	8	0.2328	1047.61	0.0961	0.1099	0.1236	48.07	109.88	185.42	42.96	450859.10
	9	0.2258	1016.23	0.0892	0.1019	0.1147	44.60	101.94	172.03	44.28	450885.62
	10	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	11	0.2149	967.27	0.0781	0.0891	0.1004	39.05	89.25	150.61	46.52	450930.45
	12	0.2106	947.76	0.0735	0.0841	0.0946	36.79	84.08	141.89	47.48	450949.60
C_P	80	0.2211	994.77	0.0844	0.0964	0.1085	42.19	96.43	162.72	45.24	360904.73
	90	0.2205	992.28	0.0838	0.0958	0.1078	41.90	95.78	161.63	45.35	405907.00
	100	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	110	0.2194	987.41	0.0827	0.0945	0.1063	41.35	94.52	159.50	45.57	495911.47
	120	0.2189	985.04	0.0822	0.0939	0.1056	41.08	93.90	158.64	45.68	540913.67
C_S	8	0.2322	1045.11	0.0788	0.0901	0.1014	39.42	90.11	152.07	53.82	450861.15
	9	0.2255	1014.77	0.0121	0.0928	0.1044	40.60	92.81	156.61	49.27	450886.90
	10	0.2200	989.83	0.0832	0.0951	0.1070	41.63	95.15	160.56	45.46	450909.25
	11	0.2153	968.94	0.0850	0.0972	0.1093	42.52	97.20	164.02	42.22	450928.85
	12	0.2114	951.19	0.0866	0.0990	0.1114	43.32	99.01	167.08	39.42	450946.19

Observations:

1. With the increase in rate of deteriorating items (θ), total cost increases but cycle time, optimum quantity, Cycles times (T,T_1,T_2,T_3) and optimum quantity, buffer stock and maximum inventory (Q_1,Q_2,Q_3) decrease.
2. With the increase in setup cost per unit (C_0), optimum quantity (Q*), maximum inventory Q_1, Q_2 and Q_3, Cycle times (T,T_1,T_2,T_3), Buffer stock and total cost increase.
3. With the increase in holding cost per unit (C_h), optimum quantity (Q*), maximum inventory Q_1, Q_2, and Q_3, cycle times (T,T_1,T_2,T_3) decreases but total cost increase.
4. Similarly, other cost parameters, production cost, shortage cost can also be observed from Table 4.

Special Cases:

If the production system is considered to be ideal,no deteriorative are produced, the value of θ is set to zero. In that case, equations (35) and (36) reduce to the classical economic production quantity model as follows

Therefore, $T = \sqrt{\frac{C_0\left[2D(P-D)C_S + PC_hA\right]}{C_hD(P-D)C_SA}}$

Optimum solution

Cycle Times: T = 0.2258; T_1 = 0.0892; T_2 = 0.1019; T_3 = 0.1147; T_4 = 0.1274, T_5 = 0.1373,

Optimum Quantity Q* = 1016.10, Q_1 = 44.60; Q_2 = 101.94; Q_3 = 172.03; B = 44.28,

Production cost =450,000, Setup cost = 442.81, Holding cost = 249.86,

Shortage Cost=192.95, Total cost = 450885.62

5. CONCLUSION

In general, inventory models are based on the assumption that products generated have indefinitely long lives, but almost all items deteriorate over time. Often, the rate of deterioration is low and there is little need to consider the deterioration in the determination of economic lot size. In this paper, a dynamic inventory model is considered with deteriorating production in which each of the production, the demand and the deterioration rates, as well as all cost parameters are assumed to be general functions of time. The objective is to cycle time and optimal production lot size, which minimize total costs. The relevant model is built and solved. Illustrative examples are provided. The validation of the results in this model was coded in Microsoft Visual Basic 6.0.

This research can be extended as follows:

Most of the production systems today are multi-stage systems and in a multi-stage system the defective items and scrap can be produced in each stage. Again, the defectives and scrap proportion for a multi-stage system can differ in different stages. Taking these factors into consideration, this research can be extended for a multi-stage production process.

Traditionally, inspection procedures incurring cost is an important factor to identify the defectives and scrap and to remove them for the finished goods inventory. For better production, the placement and effectiveness of inspection procedures are required which is ignored in this research, so inspection cost can be included in developing future models.

The demand of a product may decrease with time owing to the introduction of a new product which is either technically superior or more attractive and cheaper than the old

one. On the other hand, the demand of a new product will increase. Thus, demand rate can be varied with time, so variable demand rate can be used to develop the model.

The proposed model can assist the manufacturer and retailer in accurately determining the optimal quantity, cycle time, and inventory total cost. Moreover, the proposed inventory model can be used in inventory control of certain items such as food items, fashionable commodities, stationary stores and others.

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