PRODUCTION INVENTORY MODEL WITH DISRUPTION CONSIDERING SHORTAGE AND TIME PROPORTIONAL DEMAND

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Abstract: The disruption in a production system occurs due to labor problem, machines breakdown, strikes, political issue, and weather disturbance, etc. This leads to delay in the supply of the products, resulting customer to approach other dealers for the products. This paper is an attempt to develop an economic production quantity model using optimization method for deteriorating items with production disruption. We obtained optimal production time before and after the system gets disrupted. We have also devised the model for optimizing the shortage of the products. This research is useful to determine the time for start and stop of the production when system gets disrupted. The optimal production and inventory plan are provided, so that the manufacturer can reduce the loss occurred due to disruption. Finally a graph based simulation study has been given to illustrate the proposed model.

1. INTRODUCTION

The production system can always be affected by labor problem, political crises, machine breakdown, strike, political issue, and undetermined weather. If the production disruption appears, this leads us to a big loss because we are unable to fulfill the demand and new orders are still being received from the costumers. Other loss is loss of credibility of the firm that affects the goodwill, and the costumer may turn to another supplier/seller or product. So, the analytical study is necessary to manage the production system.

In real life, the effect of deterioration is very important in every inventory systems. Generally, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, loss of utility, or loss of marginal value of commodity that results in decreasing usefulness. A continuous production control inventory model for deteriorating items with shortage is developed by Samanta and Roy (2004) and the optimal average system costs, stock level, backlog level and production cycle time are formulated when the deterioration rate is very small. Roy and Chaudhuri (2011) introduced an economic production lot size model, where production rate depends on stock and selling price per unit. In this model deterioration is assumed as a constant fraction and shortages are not allowed. Rosenblatt and Lee (1986) studied the effect of an imperfect production process on the optimal production cycle time by assuming that system gets deteriorate during the production process and produces some defective items. Chandel and Khedlekar (2013) presented an integrated inventory model to optimize the total expenditure of warehouse set-up. Moon *et al.* (2005) developed an inventory model by considering both amelioration and deterioration over a finite planning horizon with time varying demand.

Benhadid *et al.* (2008) developed production inventory model for deteriorating item and dynamic costs. Shukla *et al.* (2012) presented an inventory model for deteriorating items by assuming that there exists an optimal number of price setting for obtaining maximum profit. Khedlekar and Shukla (2012) applied the concept for logarithmic demand and simulated the results for various businesses. The outcomes of the study is that β is the most significant parameter that affected optimal profit and respective number of price setting. Widyadana and Wee (2010) designed an EPQ models for deteriorating items by considering stochastic machine unavailability time and price dependent demand. In this model lost sales will occur when machine unavailability time is longer than the non production time. They used Genetic Algorithm to solve the model. The price rate is the more sensitive parameter than the machine unavailability time and the lost sales cost. Balkhi and Bakry (2009) considered a dynamic inventory model for deteriorating items in which each of the production, demand, and the deterioration rate, as well as costs parameters are assumed to be a general function of time. Both inflation and time value of money are taken into account. Wee (1993) devised an economic production quantity model for deteriorating items with partial back-ordering. There are numerous studies on inventory models for deteriorating items under different conditions, such as Chung and Huang (2007), Ouyang *et al.* (2005), Khedlekar and

Namdeo (2015), Shukla and Khedlekar (2015), Choudhuri and Mukherjee (2011), Giri *et al.* (2003), Kumara and Sharma (2012 *a, b, c*) etc.

Priority of any manufacturing firm, retailer, and storekeeper should be preventing the commodity from deterioration. For this purpose, we may apply the preservation technology to reduce deterioration rate. The investment in preservation technology includes an additional cost that we have to bear. You and Huang (2013) developed a model for deteriorating seasonal product whose deterioration rate could be controlled by investing in preservation efforts. Zhang *et al.* (2014) developed an inventory model in which demand is dependent on both selling price and time; also, deterioration could be controlled by preservation technology. Khedlekar *et al.* (2016) devised a deteriorating inventory model for linear declining demand where preservation technology is applied to preserve the commodity, and they shown that the profit is a concave function of optimal selling price, replenishment time, and preservation cost parameter. Mishra (2013) devised a model for Weibull distribution deteriorating seasonal product by considering constant demand rate, shortage and salvage value; also, the deterioration rate is reduced by applying the preservation technology. Khedlekar *et al.* (2016) extended his model [Khedlekar and Shukla (2013)] by incorporating exponential declining demand in which a part of inventory was prevented from deterioration by preservation technology.

At the beginning of each cycle, the manufacturer should decide the optimal production time so that the production quantity meet both demand and deterioration, and all quantity should be sold out in each cycle, that is, at the end of each cycle, the inventory level should reach to zero. However, after the plan is implemented, the production run is often disrupted by some emergent events, such as supply disruptions, machine breakdowns, financial crisis, political event, and policy change. Production disruption will lead us to a hard decision in production and inventory plan. In this paper we incorporated shortage at the end of time horizon because after the planning horizon, there is a possibility of some disruption before starting the next production run. But as new orders are still receiving, we have to cop-up this shortage before starting the next planning horizon. Recently, there is a growing literature on production disruptions. He and He (2010) proposed a production-inventory model for a deteriorating item with production disruption. In this study, an extension is made to consider the fact that some products may deteriorate during their storage. Chen and Zhang (2010) considered a model of three-echelon supply chain system which consists of suppliers, manufacturer, and customers under demand disruptions. Furthermore, an improved Analytical Hierarchy Process (AHP) is studied to select the best supplier based on quantitative factors such as the optimal long-term total cost obtained through the simulated annealing method under demand disruptions. The objective is to minimize the total cost under different demand disruption scenarios. Khedlekar *et al.* (2014) formulated a production inventory model for deteriorating item with production disruption and analyzed the system under different situations. Sarkar and Moon (2011) considered a classical EPQ model with stochastic demand under the effect of inflation. The model is described by considering a general distribu-

tion function. Benjaafar and ElHafsi (2006) considered the optimal production and inventory control of an assemble-to-order system with m components, one end product, and n costumer classes.

Therefore, in this model, we devised a production inventory model for deteriorating items with production disruption, shortage occurs once at the end of the time horizon, and preservation technology is applied for reducing deterioration. Once the production rate is disrupted, our object is to find the answer to following questions:

- Whether to replenish from spot market or not ?

- How to adjust the production plan if the production system can still satisfy the demand ?

- When to replenish from spot market if the new production system no longer satisfy the demand ?

- How long and how much quantity we have to replenish if the shortage occurs at the end of the cycle ?

2. ASSUMPTION AND NOTATION

In this model we consider time proportional demand rate, which is deterministic but not constant. The normal production rate is always greater than the demand rate, therefore $p - at > 0$. Suppose that constant deterioration exists in the system. Shortage is allowed at the end of the finite cycle. To reduce deterioration, we incorporate preservation technology. The relation between deterioration rate and preservation technology investment parameter satisfies $\frac{\partial \lambda(\alpha)}{\partial \alpha} < 0$, and $\frac{\partial^2 \lambda(\alpha)}{\partial \alpha^2} > 0$.

Hence, in this paper we assumed that $\lambda(\alpha) = \lambda_0 e^{-\alpha \delta}$. Here $\lambda(\alpha)$ is the deterioration rate after investing preservation technology, λ_0 is the deterioration rate without preservation technology investment, and δ is the sensitive parameter of investment to the deterioration rate. In this model the basic parameters are as follow:

p: normal production rate,

at: demand rate, such that $p > at$, $a > 0$,

$\lambda(\alpha)$: deterioration rate,

α: cost of preservation technology investment per unit time,

H: normal time horizon,

T_o: time horizon including shortage,

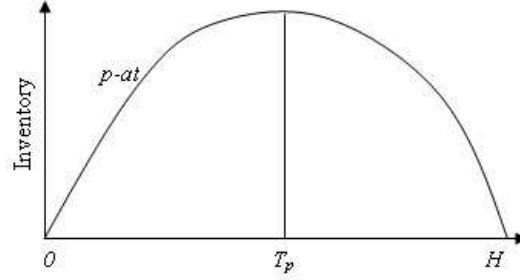T_p: production time without disruption,

T_d: production disruption time,

Figure 1: *Production system without disruption*

T_p^d: production period with disruption,

T_r: replenishment time,

Q_r: replenishment quantity at time T_r,

T_s: shortage time,

Q_s: shortage quantity.

3. MODEL WITHOUT DISRUPTION

Suppose a manufacturer produces a kind of product and sells it in market. Since the production rate is $p > 0$, and demand rate is $D(t) = at$ ($p > at > 0$), thus inventory is accumulated at rate $(p - at)$. Inventory management need to stop production at time T_p and there after, inventory is depicted due to demand rate (at) and deterioration $\lambda(\alpha)$ (See fig. 1). Now, it is assumed that the inventory is sufficient to fulfill the demand till time H. The inventory level $I(t)$ at any time $t \in [0, H]$ is obtained by the following differential equations (3.1) and (3.2).

$$\frac{\partial I_1(t)}{\partial t} + \lambda(\alpha)I_1(t) = p - at, \quad 0 \leq t \leq T_p \tag{3.1}$$

$$\frac{\partial I_2(t)}{\partial t} + \lambda(\alpha)I_2(t) = -at, \quad T_p \leq t \leq H \tag{3.2}$$

using the boundary condition $I_1(0) = 0$, and $I_2(H) = 0$, the solution of these differential equations are

$$I_1(t) = \left(\frac{p}{\lambda(\alpha)} + \frac{a}{\lambda(\alpha)^2} \right) \left(1 - e^{-\lambda(\alpha)t} \right) - \frac{at}{\lambda(\alpha)} \tag{3.3}$$

$$I_2(t) = \frac{a}{\lambda(\alpha)} \left(He^{\lambda(\alpha)H - \lambda(\alpha)t} - t \right) - \frac{a}{\lambda(\alpha)^2} \left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)t} \right) \tag{3.4}$$

The condition $I_1(T_p) = I_2(T_p)$ yields,

$$\left(\frac{p}{\lambda(\alpha)} + \frac{a}{\lambda(\alpha)^2}\right)\left(1 - e^{-\lambda(\alpha)T_p}\right) - \frac{aT_p}{\lambda(\alpha)} =$$

$$\frac{a}{\lambda(\alpha)}\left(He^{\lambda(\alpha)H - \lambda(\alpha)T_p} - T_p\right) - \frac{a}{\lambda(\alpha)^2}\left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)T_p}\right) \qquad (3.5)$$

If $\lambda(\alpha) << 1$, then expanding the exponential function and neglecting the second and higher power of $\lambda(\alpha)$,

$$T_p = \frac{\frac{2aH}{\lambda(\alpha)} + aH^2}{a + p + \frac{2a}{\lambda(\alpha)}} \qquad (3.6)$$

$$\frac{\partial T_p}{\partial \lambda(\alpha)} = \frac{\frac{2a}{\lambda(\alpha)^2}\left(aH(H-1) - pH\right)}{a + p + \frac{2a}{\lambda(\alpha)}} > 0, \qquad (3.7)$$

Now, we can get the following corollary.

Corollary 3.1. If $\lambda(\alpha) <<1$, then T_p is increasing in $\lambda(\alpha)$. That implies that the manufacturer has to produce longer in according with the deterioration rate increase. Hence, decreasing deterioration rate is most important to reduce the production cost.

4. THE PRODUCTION INVENTORY MODEL WITH PRODUCTION DISRUPTION

Proposition 4.1

If

$$\Delta p \geq \frac{-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}) - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH\lambda(\alpha))}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

then the manufacturer can still satisfy the demand after system get disrupted. Otherwise,

$$-p \leq \Delta p < \frac{-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}) - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH\lambda(\alpha))}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

then there will be shortage due to the production disruption, therefore the production rate decreases deeply.

Proof: Let the new disrupted production rate is $p + \Delta p$, where $\Delta p < 0$, if production rate decreases and $\Delta p > 0$ if production rate increases. Suppose that the production disruption time is T_d. Then, the differential equations in this situation are:

$$I_1(t) = \left(\frac{p}{\lambda(\alpha)} + \frac{a}{\lambda(\alpha)^2}\right)\left(1 - e^{\lambda(\alpha)t}\right) - \frac{at}{\lambda(\alpha)}, \quad 0 \leq t \leq T_d. \qquad (4.1)$$

The boundary condition $I_1(T_d) = I_2(T_d)$, yields

$$I_2(t) = \frac{p}{\lambda(\alpha)} \left(1 - e^{-\lambda(\alpha)t} + e^{\lambda(\alpha)T_d - \lambda(\alpha)t}\right)$$

$$+ \frac{\Delta p}{\lambda(\alpha)} \left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)t}\right) + \frac{a}{\lambda(\alpha)^2} \left(1 - e^{-\lambda(\alpha)t}\right) - \frac{at}{\lambda(\alpha)} \quad (4.2)$$

Hence,

$$I_2(H) = \frac{p}{\lambda(\alpha)} \left(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)$$

$$+ \frac{\Delta p}{\lambda(\alpha)} \left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right) + \frac{a}{\lambda(\alpha)^2} \left(1 - e^{-\lambda(\alpha)H}\right) - \frac{aH}{\lambda(\alpha)} \quad (4.3)$$

If $I_2(H) \geq 0$, then

$$\Delta p \geq \frac{-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}) - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH\lambda(\alpha))}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

This means that the manufacturer can still satisfy the demand after system get disrupted.
If $I_2(H) < 0$, then

$$-p \leq \Delta p < \frac{-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}) - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH\lambda(\alpha))}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

Therefore, the manufacturer will face the shortage since the production rate decrease deeply.

Proposition 4.2
If

$$\Delta p \geq \frac{-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}) - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH\lambda(\alpha))}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

then, the manufacturer's production time with production disruption is

$$T_p^d = \frac{1}{\lambda(\alpha)} \ln \frac{p - (p - \Delta p)e^{\lambda(\alpha)T_d} + (H + \frac{1}{\lambda(\alpha)})ae^{\lambda(\alpha)H} + \frac{a}{\lambda(\alpha)}}{p + \Delta p}$$

Proof: In this situation the differential equations are

$$\frac{\partial I_2(t)}{\partial t} + \lambda(\alpha)I_2(t) = p + \Delta p - at, \quad T_d \leq t \leq T_p^d$$

$$\frac{\partial I_3(t)}{\partial t} + \lambda(\alpha)I_3(t) = -at, \quad T_p^d \leq t \leq H$$

using the condition $I_1(T_d) = I_2(T_d)$ and $I_3(H) = 0$, we have

$$I_2(t) = \frac{p}{\lambda(\alpha)}\left(1 - e^{-\lambda(\alpha)t} + e^{\lambda(\alpha)T_d - \lambda(\alpha)t}\right)$$

$$+\frac{\Delta p}{\lambda(\alpha)}\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)t}\right) + \frac{a}{\lambda(\alpha)^2}\left(1 - e^{-\lambda(\alpha)t}\right) - \frac{at}{\lambda(\alpha)} \qquad (4.4)$$

and

$$I_3(t) = \frac{-at}{\lambda(\alpha)} + \frac{a}{\lambda(\alpha)^2} + \left(\frac{aH}{\lambda(\alpha)} - \frac{a}{\lambda(\alpha)^2}\right)e^{\lambda(\alpha)H - \lambda(\alpha)t} \qquad (4.5)$$

since, $I_2(T_p^d) = I_3(T_p^d)$, the production time after disruption is

$$T_p^d = \frac{1}{\lambda(\alpha)}\ln\frac{p - (p - \Delta p)e^{\lambda(\alpha)T_d} + (H + \frac{1}{\lambda(\alpha)})ae^{\lambda(\alpha)H} + \frac{a}{\lambda(\alpha)}}{p + \Delta p} \qquad (4.6)$$

Thats the proof of the proposition.
Now, if $\lambda(\alpha) << 1$, then

$$I_2(t) = \frac{p}{\lambda(\alpha)}(1 + \lambda(\alpha)T_d) + \Delta p(t - T_d)$$

and

$$I_3(t) = aH^2 - aHt$$

So, $I_2(T_p^d) = I_3(T_p^d)$, reveals that

$$T_p^d = \frac{aH^2 - \frac{p}{\lambda(\alpha)}(1 + \lambda(\alpha)T_d) + \Delta pT_d}{\Delta p + aH}$$

Now, differentiate this with respect to T_d, we can get

$$\frac{\partial T_p^d}{\partial T_d} = \frac{-p + \Delta p}{\Delta p + aH}$$

Now, we can get the following corollary.
Corollary 4.2.1 If $\Delta p < 0$, then T_p^d is decreasing in T_d, and if $\Delta p > p > 0$, then T_p^d is increasing in T_d.
Similarly, we have

$$\frac{\partial T_p^d}{\partial \lambda(\alpha)} = \frac{p}{\lambda(\alpha)^2(\Delta p + aH)} > 0,$$

Corollary 4.2.2 For $\lambda(\alpha) << 1$, T_p^d is increasing in $\lambda(\alpha)$.
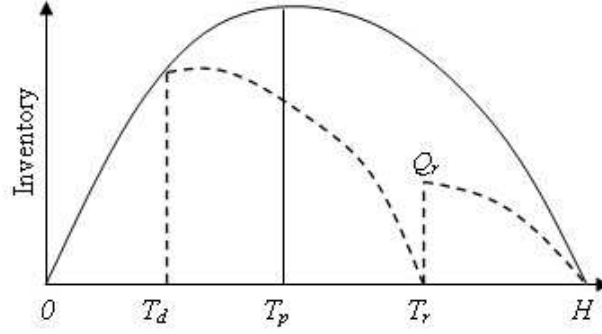
Figure 2: *Production system after disruption*

Proposition 4.3

If $I_2(H) < 0$, then the production system does not fulfill the time proportional demand. The replenishment time T_r and the order quantity Q_r are

$$e^{-\lambda(\alpha)T_r}\left((p - \Delta p)e^{\lambda(\alpha)T_d} - \left(p + \frac{a}{\lambda(\alpha)}\right)\right) - aT_r + \left((p + \Delta p) + \frac{a}{\lambda(\alpha)}\right) = 0$$

and

$$Q_r = \frac{(p + \Delta p)}{\lambda(\alpha)}\left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right)$$

$$-\frac{a}{\lambda(\alpha)}\left(T_r - He^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right) + \frac{a}{\lambda(\alpha)^2}\left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right) \tag{4.7}$$

Proof:

Suppose T_r and Q_r are time of placing an order and order quantity respectively (See fig. 2), then $I_2(T_r) = 0$.

equation (4.3) leads to

$$e^{-\lambda(\alpha)T_r}\left((p - \Delta p)e^{\lambda(\alpha)T_d} - \left(p + \frac{a}{\lambda(\alpha)}\right)\right) - aT_r + p + \Delta p + \frac{a}{\lambda(\alpha)} = 0 \tag{4.8}$$

The formulation of differential equation in this situation is

$$\frac{\partial I_3(t)}{\partial t} + \lambda(\alpha)I_3(t) = p + \Delta p - at, \quad T_r \le t \le H$$

Boundary condition $I_3(H) = 0$, yields

$$I_3(t) = \frac{(p + \Delta p)}{\lambda(\alpha)}\left(1 - e^{-\lambda(\alpha)H - \lambda(\alpha)t}\right)$$

$$-\frac{a}{\lambda(\alpha)}\left(t - He^{\lambda(\alpha)H - \lambda(\alpha)t}\right) + \frac{a}{\lambda(\alpha)^2}\left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)t}\right)$$

Hence, the order quantity $Q_r = I_3(T_r)$ will be

$$Q_r = \frac{(p + \Delta p)}{\lambda(\alpha)} \left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right)$$

$$- \frac{a}{\lambda(\alpha)} \left(T_r - He^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right) + \frac{a}{\lambda(\alpha)^2} \left(1 - e^{\lambda(\alpha)H - \lambda(\alpha)T_r}\right) \tag{4.9}$$

The proposition is proved.

If $I_2(H) < 0$, and $\lambda(\alpha) << 1$, then

$$\frac{\partial T_r}{\partial T_d} = \frac{1 - \lambda(\alpha)T_r}{\lambda(\alpha)T_d} < 0,$$

and

$$\frac{\partial Q_r}{\partial T_d} = (p + \Delta p - aH)\frac{\partial T_r}{\partial T_d} > 0,$$

Now, we can get the following corollary.

Corollary 4.3.1 If $I_2(H) < 0$, and $\lambda(\alpha) << 1$, then T_r is decreasing in T_d while Q_r is increasing in T_d.

Proposition 4.4

Suppose there is an additional disruption occurs between two successive production cycle and order are still receiving, so the shortage could exist at time H till time T_s, and then the production starts to cop-up this shortage and the production grows at the beginning of next cycle time T_o. Then shortage time T_s and shortage quantity Q_s are

$$e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s} (aT_o - a - p) + \frac{a\lambda(\alpha)T_s^2}{2} - aT_s + p + a - \frac{aH^2}{2} = 0$$

and

$$Q_s = \frac{p}{\lambda(\alpha)} \left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right)$$

$$- \frac{a}{\lambda(\alpha)} \left(T_s - T_o e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right) + \frac{a}{\lambda(\alpha)^2} \left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right).$$

Proof:

If shortage occurs at time H, and continues till time T_s thereafter production will start at time T_s, (See fig. 3) and continues till time T_o. If shortage quantity Q_s is at time T_s then, the differential equation will be

$$\frac{\partial I_4(t)}{\partial t} = at, \quad H \leq t \leq T_s, \tag{4.10}$$

and

$$\frac{\partial I_5(t)}{\partial t} + \lambda(\alpha)I_5(t) = p - at \tag{4.11}$$
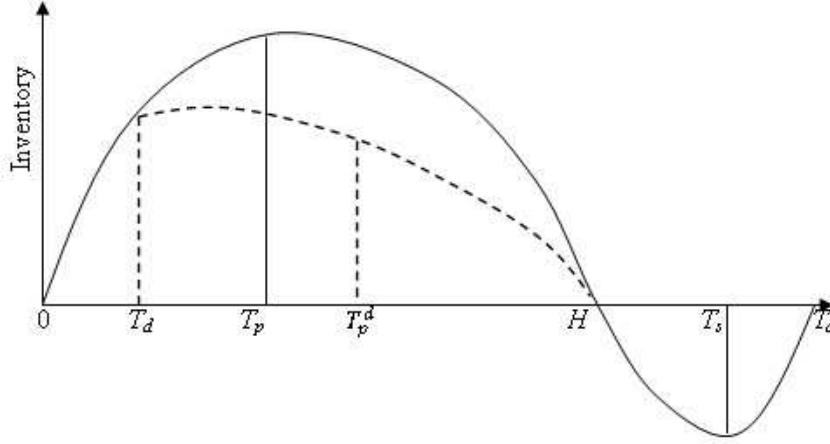
Figure 3: *Production system after disruption with shortage*

where $T_s \leq t \leq T_o$ and $I_4(T_s) = I_5(T_s)$, and $I_5(T_o) = 0$.
Equation (4.10) yields

$$I_4(t) = \frac{a}{2}\left(t^2 - H^2\right)$$

Boundary condition $I_5(T_o) = 0$ yields

$$I_5(t) = \frac{p}{\lambda(\alpha)}\left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)t}\right)$$

$$-\frac{a}{\lambda(\alpha)}\left(t - T_o e^{\lambda(\alpha)T_o - \lambda(\alpha)t}\right) + \frac{a}{\lambda(\alpha)^2}\left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)t}\right). \tag{4.12}$$

The condition $I_4(T_s) = I_5(T_s)$, reveals that

$$e^{\lambda(\alpha)(T_o - T_s)}\left(aT_o - a - p\right) + \frac{a\lambda(\alpha)T_s^2}{2} - aT_s + p + a - \frac{aH^2}{2} = 0 \tag{4.13}$$
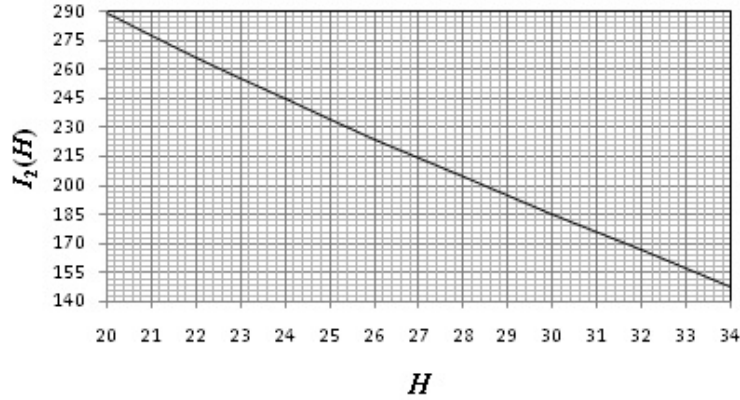
the ordering quantity $Q_s = I_5(T_s)$ will be

$$Q_s = \frac{p}{\lambda(\alpha)}\left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right)$$

$$-\frac{a}{\lambda(\alpha)}\left(T_s - T_o e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right) + \frac{a}{\lambda(\alpha)^2}\left(1 - e^{\lambda(\alpha)T_o - \lambda(\alpha)T_s}\right) \tag{4.14}$$

The proposition is proved.
If $\lambda(\alpha) << 1$, then

$$\frac{\partial T_s}{\partial T_o} = \frac{aT_o - a - p}{a(T_o - T_s) + \frac{a}{\lambda(\alpha)} - a - p} > 0,$$

Figure 4: $I_2(H)$ with respect to H

and

$$\frac{\partial Q_s}{\partial T_o} = (aT_o - p)(1 - \frac{\partial T_s}{\partial T_o}) + a(T_o - T_s) > 0,$$

for $0 \leq \frac{\partial T_s}{\partial T_o} \leq 1$.

Now, we can get the following corollary.

Corollary 4.4.1 For $\lambda(\alpha) << 1$, T_s is increasing in T_o while Q_s is increasing in T_o for $0 \leq \frac{\partial T_s}{\partial T_o} \leq 1$.

5. APPLICATION AND SENSITIVE ANALYSIS

In this section we assume that production rate $p = 200$ unit/day, $a = 10$ disruption occur in production system at rate $\Delta p = -100$, rate of deterioration $\lambda(\alpha) = 0.22$, time horizon $H = 20$ days, production system disrupted after $T_d = 2$ days, if $I_2(H) = 289.5 > 0$, then production time after disruption is longer than the production time before disruption, so the management need to maintain more stock to consume for next 7.5 hours.

Case I
If $I_2(H) > 0$,
or

$$\Delta p \geq \frac{-p\left(1 - e^{\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right) - \frac{a(1 - e^{-\lambda(\alpha)H}}{\lambda(\alpha)} + aH}{\left(1 - e^{(\lambda(\alpha)T_d - \lambda(\alpha)H)}\right)}$$
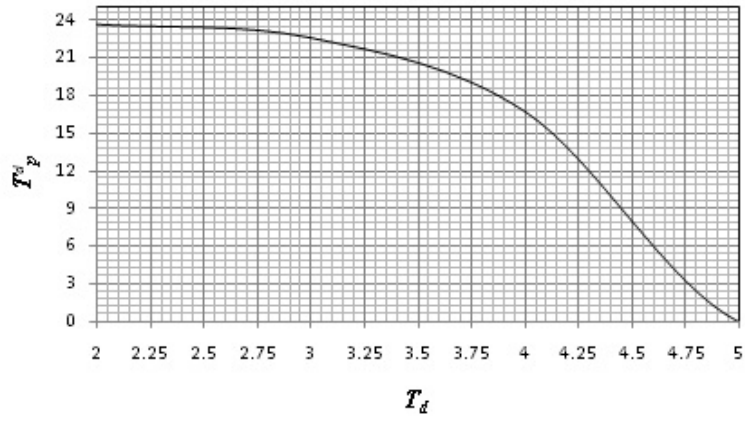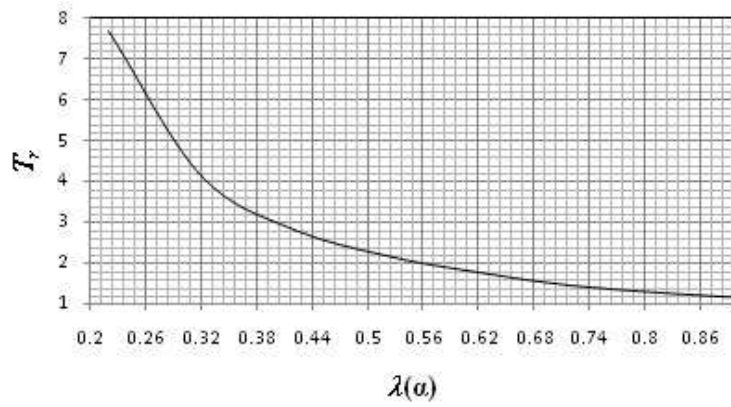
Figure 5: T_p^d with respect to T_d

Figure 6: T_r with respect to $\lambda(\alpha)$

If $I_2(H) = 0$, and the orders are still being received, then there could exist the shortage. Hence, the shortage time is $T_s = 3.15$ days, and the shortage quantity $Q_s = 435$ unit. Next, we observe how $I_2(H)$, T_p^d, T_d, T_r would change as H, T_d, T_r, and $\lambda(\alpha)$, respectively. Figure 4 shows that $I_2(H)$ is decreasing in H, therefore the on hand inventory $I_2(H)$ will decrease if time horizon H is large. So the advice to manufacturer/retailer is to keep the time horizon as small as possible.

Case II
If $I_2(H) \le 0$, that is

$$-p \le \Delta p < \frac{\left(-p(1 - e^{-\lambda(\alpha)H} + e^{\lambda(\alpha)T_d - \lambda(\alpha)H} - \frac{a}{\lambda(\alpha)}(1 - e^{-\lambda(\alpha)H} + aH)\right)}{\left(1 - e^{\lambda(\alpha)T_d - \lambda(\alpha)H}\right)}$$

From figure 5, we can find that T_p^d is decreasing in T_d when $2 \le T_d \le 5$. For $0 \le T_d \le 2$, the manufacturer will have to replenish inventory from spot market. From figure 6, we can see that T_r is decreasing in $\lambda(\alpha)$ when $\lambda(\alpha) > 0.2$.

6. CONCLUSION

In this paper, inventory production model has been developed considering time proportional demand for deteriorating items. The shortage has been incorporated at the end of the time cycle. We have calculated and graphically simulated the time of production disruption and the quantity of production after disruption. This paper suggest the production management to keep short time span to produce the product in a small lot and to keep minimum deterioration when the replenishment time occurs shorten. For this, management can use preservation technology to reduce deterioration rate. In case of early disruption, it is difficult for management to manage with time proportional demand. We have considered constant deterioration rate in this study, but future research in this field may consider variable deterioration rate, one can consider variable deterioration with stochastic demand. Also, one can formulate the model in fuzzy environments.

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