M/M/c/N QUEUING SYSTEMS WITH ENCOURAGED ARRIVALS, RENEGING, RETENTION AND FEEDBACK CUSTOMERS

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Abstract: Customers often get attracted by lucrative deals and discounts offered by firms. These, attracted customers are termed as encouraged arrivals. In this paper, we developed a multi-server Feedback Markovian queuing model with encouraged arrivals, customer impatience, and retention of impatient customers. The stationary system size probabilities are obtained recursively. Also, we presented the necessary measures of performance and gave numerical illustrations. Some particular, and special cases of the model are discussed.

Keywords: Encouraged Arrivals, Stochastic Models, Queuing Theory, Reneging, Impatience, Feedback.

MSC: 60K25, 68M20, 90B22.

1. INTRODUCTION AND LITERATURE SURVEY

In today's era of cut throat competition, companies compete not only with local markets but also with the big global players. Moreover, they have constantly to follow technological advancements, and to deal with the customers' uncertain behavior regarding the access they have to global products. Customers often look for lucrative deals offered by various firms before buying any product. In order to ensure sustainable growth, firms release various offers and discounts to retain old customers and to engage new ones. The discounts and offers attract customers towards the particular firm. Those, attracted customers are termed as encouraged

arrivals in this paper. The phenomenon of encouraged arrivals can also be understood as contrary to discouraged arrivals, as discussed by Kumar and Sharma [14]. Som and Seth [4] discussed a single-server queuing model with encouraged arrivals and performed a cost-profit analysis of the model. Further, Som [2] incorporated a queuing model with encouraged arrivals in the health-care management system with encouraged arrivals and discussed a single-server queuing system with encouraged arrivals and customer impatience. He mentioned that patients get encouraged towards healthcare facilities that offer better service at affordable cost. Som and Seth [5] further developed a two-server queuing system with encouraged arrivals and performed its economic analysis. They discussed a two-server queuing system with heterogeneous service and encouraged arrivals with retention of impatient customers. Extending his work, Som [3] discussed a feedback queuing system with encouraged arrivals and retention of impatient customers. Som and Seth [6], also, studied a single server queuing system with encouraged arrivals and impatient customers for effectively managing business in uncertain environment.

Eventually, encouraged arrivals put the service facility under pressure, wich can make customers dissatisfied with the service. Such customers are termed as feedback customers in queuing literature. Those customers re-join the queue to complete their service. Feedback in queuing system is studied rigorously by researchers such as Takacs [17], who studied the queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time a customer spent in the system. Davignon and Disney [14] studied single server queues with state dependent feedback. Santhakumaran and Thangaraj [1] considered a single server feedback queue with impatient customers. They studied M/M/1 queueing model for queue length at arrival epochs and obtained results for stationary distribution, mean and variance of queue length. Thangaraj and Vanitha [25] obtained transient solution of M/M/1 feedback queue with catastrophes using continued fractions. The steady-state solution, moments under steady state, and busy period analysis were calculated. Encouraged arrivals also result in longer queues and higher waiting times. Due to this, customers become impatient and leave the system without getting service, which is termed as reneging. Roots of customer impatience, as traced in work of Barrer [9], can be that a customer is available for service only for a limited amount of time. Haight [11], [12] studied a queue with balking and continued by studying queues with reneging. Further, queues with balking and reneging are studied by Ancker and Gaffarain [7], [8], where they mentioned that every customer arrives with a threshold value of tolerance time. When that threshold value of time expires, the customer retires from the system without completion of service. A number of papers appeared on customer impatience in queuing theory afterwards. Reneging phenomenon of single channel queues is discussed in detail by Robert [10], where he considered a single channel queuing system in which nth arrival may renege if its service does not commence before an elapsed random time Z_n. Baccelli et. al. [13] considered single server queuing system in which a customer gives up whenever his waiting time is more than his patience time. They

studied stability conditions and the relation between actual and virtual waiting time distribution functions. Bae and Kim [15] considered a G/M/1 queue with constant patience time of the customers and derived the stationary distribution of the workload of the server, or the virtual waiting time. Wang et. al. [16] gave a detailed review on queuing systems with customer impatience. For oragnisations, it is not just about loosing a customer but also such customers hamper the brand image of the organisation by posting negative comments about the service quality of the firm on various social media platforms,too. Thus, reneging is a loss to business and goodwill of the company as well. Liao [18] developed a queuing model for estimating business loss using balking index and reneging rate. Hence companies employ various retention strategies, due to which a reneged customer may be retained with some probability. Kumar et. al. [19], [20], [21], [22], [23], [24] introduced the concept of retention of impatient customers and mentioned that a reneging customer may be retained with some probability by employing a retention strategy. They studied retention of reneged customers in single and multi server queues with balking and feedback as well.

In this paper, we discuss multi-server queuing system with encouraged arrivals, customer impatience, retention of impatient customers and feedback. Various particular cases of the model are discussed one by one by removing particular factors from the context. Some special cases of the model are also discussed. Steady-state probabilities of the models are derived recursively.

2. Model-1: An M/M/c/N FEEDBACK QUEUING SYSTEM WITH ENCOURAGED ARRIVALS, RENEGING AND RETENTION OF RENEGED CUSTOMERS

2.1. Assumptions of the Model

A multi-server finite capacity Markovian feedback queuing model with encouraged arrivals, reneging, and retention of reneged customers is formulated under the following assumptions:

(i) The arrivals occur one by one in accordance with Poisson process with parameter $\lambda(1 + \eta)$, where $'\eta'$ represents the percentage increase in the arrival rate of customers, calculated from past or observed data. For instance, if in past, an organization offered discounts and the percentage increase in number of customers was observed as 75 percent or 150 percent, then $\eta = 0.75$ or $\eta = 1.5$, respectively.

(ii) Service times are exponentially distributed with parameter μ.

(iii) Customers are serviced in the order of their arrival, i.e. the queue discipline is first come, first served.

(iv) There are c servers through which the service is provided.

(v) The capacity of the system is finite, say N.

(vi) The reneging times are exponentially distributed with parameter ξ.

(vii) A dissatisfied customer may rejoin the queue for completion of the service with probability q, and may leave the queue satisfactorily with probability $p = 1 - q$.

(viii) The probability of retention of a reneged customer is q', and the probability that customer is not retained is $p' = 1 - q'$.

2.2. Steady State equations of the model

$$\mu p P_1 = \lambda(1 + \eta)P_0; \qquad\qquad n = 0 \qquad (1)$$

$$(n + 1)\mu p P_{n+1} = \{\lambda(1 + \eta) + n\mu p\}P_n - \lambda(1 + \eta)P_{n-1}; 1 \leq n \leq c - 1 \qquad (2)$$

$$\{c\mu p + (n + 1 - c)\xi p'\}P_{n+1} = \{\lambda(1 + \eta) + c\mu p + (n - c)\xi p'\}P_n - \lambda(1 + \eta)P_{n-1}; c \leq n \leq N - 1 \qquad (3)$$

$$\{c\mu p + (N - c)\xi p'\}P_N = \lambda(1 + \eta)P_{N-1}; \qquad\qquad n = N \qquad (4)$$

Solving equations (1) to (4) recursively, we obtain:

$P_n = Pr\{\text{n customers in the system}\}$

$$= \begin{cases} \frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^n P_0, & 1 \leq n \leq c \\ \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \left[\prod_{i=c+1}^{n}\left(\frac{\lambda(1+\eta)}{c\mu p+(i-c)\xi p'}\right)\right]P_0, & c < n \leq N - 1 \end{cases} \qquad (5)$$

$P_N = Pr\{\text{system is full}\}$

$$= \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \left[\prod_{n=c+1}^{N}\left(\frac{\lambda(1+\eta)}{c\mu p+(n-c)\xi p'}\right)\right]P_0 \qquad (6)$$

Using condition of normality $\sum_{n=0}^{N} P_n = 1$, we get

$P_0 = Pr\{\text{system is empty}\}$

$$= \left[\sum_{n=0}^{c}\frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^n + \sum_{n=c+1}^{N}\left\{\frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \prod_{i=c+1}^{n}\left(\frac{\lambda(1+\eta)}{c\mu p+(i-c)\xi p'}\right)\right\}\right]^{-1} \qquad (7)$$

3. MEASURES OF PERFORMANCE

Having calculated probabilistic measures, the following measures of performance can be derived:

3.1. Expected System Size (L_s):

$L_s = \sum_{n=1}^{N} nP_n$

3.2. Expected queue length (L_q):

$$L_q = \sum_{n=c}^{N}(n-c)P_n$$

3.3. Expected waiting time in the system (W_s):

$$W_s = \frac{L_s}{\lambda(1+\eta)}$$

3.4. Expected waiting time in the queue (W_q):

$$W_q = \frac{L_q}{\lambda(1+\eta)}$$

3.5. Average rate of reneging (R_r):

$$R_r = \sum_{n=c}^{N}(n-c)\xi p' P_n$$

3.6. Average rate of retention (R_R):

$$R_R = \sum_{n=c}^{N}(n-c)\xi q' P_n$$

4. NUMERICAL ILLUSTRATIONS

In this section, we present numerical illustrations of the model to study the variations in various performance measures with respect to some particular parameters:

From table 1, it is evident that as the arrival rate increases, the expected size of the system, expected queue length, average waiting time in the system, waiting time in the queue, average reneging rate, as well as average retention rate increase. Thus, in order to keep the system size under control and to avoid reneging of customers in case of increased arrival rate, firm should employ some strategies, which could be either increasing the service rate or introducing an additional service channel.

From table 2, it is evident that with increase in average rate of service, expected queue length, waiting time in the queue, and average reneging rate all decrease. Thus, with the increased service rate, customers have to spend less time in queues, and there will be less chances of customers get impatient and leave the system without getting service, which is a desirable condition for any firm.

Table 1

Variation in measures of performance with respect to arrival rate.
Taking $\mu = 4, \eta = 0.75, q = 0.3, q' = 0.2, \xi = 0.2, c = 3$ and $N = 10$.

λ	L_s	L_q	W_s	W_q	R_r	R_R
2	1.34505	0.100965	0.384299	0.028847	0.016154	0.004039
2.2	1.51262	0.146355	0.392889	0.038014	0.023417	0.005854
2.4	1.69299	0.205581	0.403094	0.048948	0.032893	0.008223
2.6	1.88827	0.281092	0.415005	0.061778	0.044975	0.011244
2.8	2.10043	0.375302	0.428659	0.076592	0.060048	0.015012
3	2.33109	0.490397	0.444018	0.093409	0.078464	0.019616
3.2	2.58129	0.628091	0.460946	0.112159	0.100495	0.025124
3.4	2.85124	0.789365	0.479199	0.132666	0.126298	0.031575
3.6	3.14008	0.97423	0.498425	0.15464	0.155877	0.038969
3.8	3.44584	1.181566	0.518171	0.177679	0.189051	0.047263
4	3.76541	1.409076	0.537915	0.201297	0.225452	0.056363
4.2	4.09473	1.653383	0.557106	0.22495	0.264541	0.066135
4.4	4.42910	1.910274	0.575208	0.248088	0.305644	0.076411
4.6	4.76355	2.175034	0.591745	0.270191	0.348005	0.087001
4.8	5.09321	2.44283	0.606335	0.290813	0.390853	0.097713
5	5.41371	2.709074	0.61871	0.309608	0.433452	0.108363
5.2	5.72140	2.969708	0.628725	0.326342	0.475153	0.118788

From table 3, it is evident that the expected system size decreases while the average rate of reneging increases with the increase in reneging times. Thus, more customers leaving the system without getting service will result in loss of business, so firms need to employ strategies to minimize customers reneging either by increasing the service rate or by giving more lucrative offers to the customers.

5. SOME PARTICULAR CASES

5.1. Model-2: An M/M/c/N Feedback Queuing System with Encouraged Arrivals and Reneging (A particular case of Model -1, when there is no retention of reneged customers)

In this case, the probability of retention is $q' = 0$, i.e. $p' = 1 - q' = 1$. It is clear that by substituting $p' = 1$, Model-1 reduces to the following:

Table 2

Variation in measures of performance with respect to service rate.
Taking $\lambda = 5, \eta = 0.75, q = 0.3, q' = 0.2, \xi = 0.2, c = 3$ and $N = 10$.

μ	L_q	W_q	R_r
3	4.38249	0.500856	0.701198
3.2	4.049187	0.462764	0.64787
3.4	3.707288	0.42369	0.593166
3.6	3.364843	0.384553	0.538375
3.8	3.029774	0.34626	0.484764
4	2.709074	0.309608	0.433452
4.2	2.40823	0.275226	0.385317
4.4	2.130969	0.243539	0.340955
4.6	1.879277	0.214775	0.300684
4.8	1.653642	0.188988	0.264583
5	1.453387	0.166101	0.232542
5.2	1.277033	0.145947	0.204325
5.4	1.122618	0.128299	0.179619
5.6	0.987957	0.112909	0.158073
5.8	0.870822	0.099523	0.139332
6	0.769071	0.087894	0.123051
6.2	0.680717	0.077796	0.108915

5.1.1. Steady State equations of the model

$$\mu p P_1 = \lambda(1 + \eta)P_0; \qquad\qquad\qquad\qquad n = 0 \qquad\qquad (8)$$

$$(n + 1)\mu p P_{n+1} = \{\lambda(1 + \eta) + n\mu p\}P_n - \lambda(1 + \eta)P_{n-1}; 1 \le n \le c - 1 \qquad (9)$$

$$\{c\mu p + (n + 1 - c)\xi\}P_{n+1} = \{\lambda(1 + \eta) + c\mu p + (n - c)\xi\}P_n - \lambda(1 + \eta)P_{n-1}; c \le n \le N - 1$$
$$(10)$$

$$\{c\mu p + (N - c)\xi\}P_N = \lambda(1 + \eta)P_{N-1}; \qquad\qquad n = N \qquad\qquad (11)$$

Solving equations (8) to (11) recursively, we obtain:

$P_n = Pr\{$n customers in the system$\}$

$$= \begin{cases} \frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^n P_0, & 1 \le n \le c \\ \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu p}\right)^c \left[\prod_{i=c+1}^{n}\left(\frac{\lambda(1+\eta)}{c\mu p+(i-c)\xi}\right)\right]P_0, & c < n \le N - 1 \end{cases} \qquad (12)$$

$P_N = Pr\{$system is full$\}$

$$= \frac{1}{c!}\left(\frac{\lambda(1 + \eta)}{\mu p}\right)^c \left[\prod_{n=c+1}^{N}\left(\frac{\lambda(1 + \eta)}{c\mu p + (n - c)\xi}\right)\right]P_0 \qquad (13)$$

Table 3
Variation in measures of performance with respect to reneging rate.
Taking $\lambda = 5, \mu = 4, \eta = 0.75, q = 0.3, q' = 0.2, c = 3$ and $N = 10$.

ξ	L_s	R_r
0.1	2.85611	0.064732
0.15	2.83003	0.094629
0.2	2.80522	0.123051
0.25	2.78160	0.150111
0.3	2.75909	0.175911
0.35	2.73761	0.200543
0.4	2.71709	0.224092
0.45	2.69746	0.246634
0.5	2.67868	0.268238
0.55	2.66069	0.288968
0.6	2.64344	0.308883
0.65	2.62688	0.328035
0.7	2.61098	0.346474
0.75	2.59569	0.364243
0.8	2.58097	0.381384
0.85	2.56681	0.397936
0.9	2.55315	0.413932

Using the condition of normality $\sum_{n=0}^{N} P_n = 1$, we get

$P_0 = Pr\{\text{system is empty}\}$

$$= \left[\sum_{n=0}^{c} \frac{1}{n!} \left(\frac{\lambda(1+\eta)}{\mu p} \right)^n + \sum_{n=c+1}^{N} \left\{ \frac{1}{c!} \left(\frac{\lambda(1+\eta)}{\mu p} \right)^c \prod_{i=c+1}^{n} \left(\frac{\lambda(1+\eta)}{c\mu p + (i-c)\xi} \right) \right\} \right]^{-1} \quad (14)$$

5.2. Model-3: An M/M/c/N Queuing System with Encouraged Arrivals and Reneging (A particular case of Model -2, when all customers leave the system satisfactorily after completion of service)

In this case, the probability that a customer rejoins the queue, $q = 0$, i.e. $p = 1 - q = 1$. Substituting $p = 1$, Model-2 reduces to the following:

5.2.1. Steady State equations of the model

$$\mu P_1 = \lambda(1 + \eta)P_0; \qquad\qquad\qquad\qquad n = 0 \qquad\qquad (15)$$

$$(n + 1)\mu P_{n+1} = \{\lambda(1 + \eta) + n\mu\}P_n - \lambda(1 + \eta)P_{n-1}; 1 \le n \le c - 1 \qquad (16)$$

$$\{c\mu + (n + 1 - c)\xi\}P_{n+1} = \{\lambda(1 + \eta) + c\mu + (n - c)\xi\}P_n - \lambda(1 + \eta)P_{n-1}; c \le n \le N - 1$$
$$(17)$$

$$\{c\mu + (N - c)\xi\}P_N = \lambda(1 + \eta)P_{N-1}; \qquad\qquad n = N \qquad\qquad (18)$$

Solving equations (15) to (18) recursively, we obtain:

$P_n = Pr\{n \text{ customers in the system}\}$

$$= \begin{cases} \frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^n P_0, & 1 \le n \le c \\ \frac{1}{c!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^c \left[\prod_{i=c+1}^{n}\left(\frac{\lambda(1+\eta)}{c\mu+(i-c)\xi}\right)\right]P_0, & c < n \le N - 1 \end{cases} \qquad (19)$$

$P_N = Pr\{\text{system is full}\}$

$$= \frac{1}{c!}\left(\frac{\lambda(1 + \eta)}{\mu}\right)^c \left[\prod_{n=c+1}^{N}\left(\frac{\lambda(1 + \eta)}{c\mu + (n - c)\xi}\right)\right]P_0 \qquad (20)$$

Using the condition of normality $\sum_{n=0}^{N} P_n = 1$, we get

$P_0 = Pr\{\text{system is empty}\}$

$$= \left[\sum_{n=0}^{c}\frac{1}{n!}\left(\frac{\lambda(1 + \eta)}{\mu}\right)^n + \sum_{n=c+1}^{N}\left\{\frac{1}{c!}\left(\frac{\lambda(1 + \eta)}{\mu}\right)^c \prod_{i=c+1}^{n}\left(\frac{\lambda(1 + \eta)}{c\mu + (i - c)\xi}\right)\right\}\right]^{-1} \qquad (21)$$

5.3. Model-4: An M/M/c/N Queuing System with Encouraged Arrivals (A particular case of Model -3, when customers do not get impatient)

In this case, there is no reneging, i.e. $\xi = 0$. Thus, the queuing system in Model-3 reduces to the following model.

5.3.1. Steady State equations of the model

$$\mu P_1 = \lambda(1 + \eta)P_0; \qquad\qquad\qquad n = 0 \qquad\qquad (22)$$

$$(n + 1)\mu P_{n+1} = \{\lambda(1 + \eta) + n\mu\}P_n - \lambda(1 + \eta)P_{n-1}; \quad 1 \le n \le c - 1 \qquad (23)$$

$$c\mu P_{n+1} = \{\lambda(1 + \eta) + c\mu\}P_n - \lambda(1 + \eta)P_{n-1}; \qquad c \le n \le N - 1 \qquad (24)$$

$$c\mu P_N = \lambda(1 + \eta)P_{N-1}; \qquad\qquad\qquad n = N \qquad\qquad (25)$$

Solving equations (22) to (25) recursively, we obtain:

$P_n = Pr\{\text{n customers in the system}\}$

$$= \begin{cases} \frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^n P_0, & 1 \le n \le c \\ \frac{1}{c^{n-c}c!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^n P_0, & c < n \le N-1 \end{cases} \tag{26}$$

$P_N = Pr\{\text{system is full}\}$

$$= \frac{1}{c^{N-c}c!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^N P_0 \tag{27}$$

Using the condition of normality $\sum_{n=0}^{N} P_n = 1$, we get

$P_0 = Pr\{\text{system is empty}\}$

$$= \left[\sum_{n=0}^{c} \frac{1}{n!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^n + \sum_{n=c+1}^{N} \frac{1}{c^{n-c}c!}\left(\frac{\lambda(1+\eta)}{\mu}\right)^n\right]^{-1} \tag{28}$$

6. SOME SPECIAL CASES

6.1. A special case, when there is no encouragement in Model - 4

In this case, there is no increase in the arrival rate of customers, i.e. $\eta = 0$. Substituting $\eta = 0$ in stationary system size probabilities of Model-4, we get

$P_n = Pr\{\text{n customers in the system}\}$

$$= \begin{cases} \frac{\rho^n}{n!}P_0, & 1 \le n \le c \\ \frac{\rho^n}{c!}\left(\frac{\rho}{c}\right)^{n-c} P_0, & c < n \le N-1 \end{cases} \tag{29}$$

$P_N = Pr\{\text{system is full}\}$

$$= \frac{\rho^N}{c!}\left(\frac{\rho}{c}\right)^{N-c} P_0 \tag{30}$$

Using the condition of normality $\sum_{n=0}^{N} P_n = 1$, we get

$P_0 = Pr\{\text{system is empty}\}$

$$= \left[\sum_{n=0}^{c} \frac{\rho^n}{n!} + \sum_{n=c+1}^{N} \frac{\rho^n}{c!}\left(\frac{\rho}{c}\right)^{n-c}\right]^{-1} \tag{31}$$

where $\rho = \frac{\lambda}{\mu}$.

It is evident that when there is no encouragement, Model-1 reduces to the classical multi-server queuing model with finite capacity.

6.2. A special case, when there is a single server in case of Model - 3

In case of a single server, substituting $c = 1$ in Model - 3, it reduces to a queuing model with encouraged arrivals and impatient customers as discussed by Som and Seth, [6].

6.3. A special case, when there is a single server in case of Model - 4

In case of a single server, substituting $c = 1$ in Model - 4, it reduces to M/M/1/N queuing system with encouraged arrivals as discussed by Som and Seth, [4].

7. CONCLUSIONS AND SUGGESTIONS

In this paper, a multi-server queuing system with encouraged arrivals, reneging, retention, and feedback customers is studied. Steady-state probabilities are calculated, with which the desired measures of performances can be derived using classical queuing theory approach . Various particular and special cases of the above mentioned model are discussed. The model addresses practically valid contemporary challenges. Any system that is undergoing the above mentioned challenges can refer to its most suitable particular model, discussed in this paper, and can have measures of its performance for better governance.

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