

## QUANTITY DISCOUNT FOR INTEGRATED SUPPLY CHAIN MODEL WITH BACK ORDER AND CONTROLLABLE DETERIORATION RATE

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Received: October 2017 / Accepted: March 2018

**Abstract:** Due to uncertainty in economy, business players examine different ways to ensure the survival and growth in the competitive atmosphere. In this scenario, the use of effective promotional tool and co-ordination among players enhance supply chain profit. The proposed model deals with the effect of quantity discount on an integrated inventory system for constantly deteriorating items with fix life time. We use advertisement and quantity discount to accelerate stock dependent demand and further, the offered preservation technology for controlling deterioration rate. The model is validated numerically, and the sensitivity analysis for critical supply chain parameters is carried out. The results can be used in the decision making process of the supply chains associated with the supply of cosmetic, tinned food, drugs, and other FMCGs.

**Keywords:** Integrated Inventory, Advertise and Stock Dependent Demand, Constant Deterioration, Back Order, Quantity Discount, Preservative Technology.

**MSC:** 90B85, 90C26.

## 1. INTRODUCTION

A supply chain contains different business players like supplier, manufacturer, distributor, retailer, customer, who work together to improve sustainability. Goyal [11] developed the first integrated model for a single supplier and a single customer. Banerjee [1] jointly optimized ordering policy so that either both parties get benefit or, at least, no one incurs losses. Goyal and Gunasekaran [10] extended that model for deteriorating items. Rau *et al.* [18] extended the same model for a single supplier, single producer, and a single buyer. Crdenas-Barrn [2] solved vendor-buyer model with arithmetic and geometric inequalities. Sarkar *et al.* [22] formulated an integrated inventory model for defective items with payment delay scenario.

Break-even point of fixed and variable costs allows manufacturer to enjoy better profit on large lots. This large lots are offered to a retailer by offering quantity discount to accelerate overall demand. This gives a win-win situation both to manufacturer and retailer. A first model using quantity discount policy for increasing vendor's profit is developed by Monahan [15]. Chang [4] *et al.* extended the model for deteriorating items with price and stock dependent demand. Duan *et al.* [7] derived a model for fix life product and proved theoretically that after applying quantity discount, total cost was reduced. Zhang *et al.* [27], Ravithammal *et al.* [19], Ravithammal *et al.* [20], Pal and Chandra [17], Sarkar [21] extended that model by taking different assumptions to make it more realistic.

Ghare and Schrader [8] were the first who formulated a model for inventory that deteriorate exponentially. Murr and Morris [16] proved that lower temperature would increase storage time and decrease decay. So, as per this fact, preservation technology is used to reduce deterioration rate of items because higher rate of deterioration finally results into lower revenue generation. Hsu *et al.* [12] applied preservation technology on constantly deteriorating items to increase total profit. Chang [3] used preservation technology on non-instantaneous deteriorating items. Singh and Rathore [26] extended this model for shortages with the proposal of trade credit. Shah *et al.* [25] developed an integrated model by using preservation technology on time-varying deteriorating items when demand is time and price sensitive. Mishra *et al.* [14] applied preservation technology on seasonal deteriorating items in the presence of shortages.

In the classical EOQ models, demand is taken as constant. But researchers have always investigated parameters that affect demand as stock-level, time, price, advertisement, and trade credit. Khouja and Robbins [13], Shah and Pandey [23], Giri and Maiti [9], Chowdhury *et al.* [5], Shah [24], Chung and Crdenas-Barrn [6] *et al.* used different types of demand and developed their inventory models.

The proposed model works on single set-up multiple deliveries with just-in-time replenishment for deteriorating items that have a fix life time. Here, we develop two models: Model 1 (without quantity discount), and Model 2 (with quantity discount).

In the second model, a retailer agrees to change his/her order according to manufacturer's output. In response, the retailer gets benefit of quantity discount

from the manufacturer. Whereas there is no such an agreement, advertisement and stock dependent demand is considered to boost the demand. Preservation technology is used to reduce the rate of deterioration. Total inventory cost of supply chain is optimized for decision variables back order rate ( $k$ ) and preservation cost ( $\xi$ ). Both the models are optimized analytically and computational algorithms have been developed for the same. The obtained solutions are illustrated on a numerical example.

## 2. NOTATIONS AND ASSUMPTIONS

### 2.1. Notations

#### 2.1.1. Inventory parameters for a manufacturer

$A_m$	Set up costs(\$)
$m_1$	Manufacturer's order multiple in a without quantity discount system
$m_2$	Manufacturer's order multiple in a with quantity discount system
$h_m$	Holding cost / unit / annum
$k_1$	Back order rate(year) in a without quantity discount system
$k_2$	Back order rate(year) in a with quantity discount system
$\rho$	Capacity utilization
$P$	Production rate
$D$	Advertisement and stock dependent demand
$C_{io}$	Manufacturer's variable inspection cost per delivery
$C_{imu}$	Manufacturer's unit inspection cost (\$/unit time inspected)
$C_{imf}$	Manufacturer's fix inspection cost(\$/product lot)
$TC_{wm}$	Total cost for a manufacturer in a without quantity discount system
$TC_{qm}$	Total cost for a manufacturer in a with quantity discount system
$Q_m(t)$	Manufacturer's economic order quantity per cycle

$A$	Cost of advertisement
$\nu$	Frequency of advertisement
$TC_{wr}$	Total cost for a retailer in a without quantity discount system
$TC_{qr}$	Total cost for a retailer in a with quantity discount system
$TC_w$	Joint total cost for a without quantity discount integrated model
$TC_q$	Joint total cost for a with quantity discount integrated model
$Q_r(t)$	Retailer's economic order quantity per cycle
$\tau_p$	Resultant deterioration rate, $\theta - m(\xi)$
$B(\lambda)$	Discount given by manufacturer if the retailer placed the order each time

### 2.1.2. Inventory parameters for retailer

$A_r$	Ordering costs(\$)
$n$	Retailer's order multiple in the absence of any co-ordination
$\lambda$	Retailer's order multiple under co-ordination and $\lambda Q_r(t)$ as the retailer's new quantity
$h_r$	Holding cost / unit / annum
$\theta$	Constant deterioration
$\pi$	Retailer's back order cost
$L$	The maximum life time of a product(in year)
$\nu$	Rate of change of the advertisement frequency
$a$	Fix demand
$b$	Rate of change of demand
$\xi_1$	Preservative cost to reduce deterioration in a without quantity discount system
$\xi_2$	Preservative cost to reduce deterioration in a with quantity discount system
$m(\xi)$	Reduced deterioration rate

Necessary condition for different inventory parameters

$$\rho = \frac{D}{P}; \rho < 1; 0 < \theta < 1; \xi \geq 1$$

### 2.2. Assumptions

1. This model considers two-echelon form with a single manufacturer and a single retailer for items with expiry date  $L$ -years.
2. Manufacturer offers quantity discounts if a retailer agrees to change order quantity by the fix order quantity.
3. Demand is deterministic. Demand function  $D(A, Q)$  is defined as

$$D(A, Q) = A^\nu(a + bQ(t)); 0 \leq t \leq T \text{ where } a, b \geq 0 \text{ and } a \geq b$$

Where  $A$  = Cost of advertisement;  $\nu$  = Frequency of advertisement;  $a$  = Fix rate demand;  $b$  = Rate of change of the demand;  $Q$  = Instantaneous stock level For the convince, we use  $D$  for  $D(A, Q)$ .

4. Shortages are allowed and the backorder rate is assumed as a decision variable for a retailer.
5. Preservation technology is used to control the deterioration rate.
6. Three level inspections at the manufacturer's end assure no defective items.
7. Production rate is constant and the lead time is zero.
8. Items are subject to constant deterioration.

### 3. MODEL FORMULATION

In this section, we formulate models that follow a single-setup-multi-delivery (SSMD) policy with just-in-time (JIT) procurement. Here, a manufacturer produces in one set-up but ships through multiple deliveries after a fixed time. Two integrated models are proposed on the basis of agreement between manufacturer and retailer. Model 1 undertakes no quantity discount as this model assumes no agreement between manufacturer and retailer. Model 2 allows quantity discount as the retailer agrees to order as per the manufacturer production. Shortages are taken with back order rate ( $k$ ), and preservation technology cost ( $\xi$ ) is assumed in both of the models.

#### 3.1. Model 1: Without quantity discount

In this model, we use preservation technology to control constant deterioration rate. To control deterioration rate, as shown in Figure 1,  $m(\xi)$  is a function of preservation cost  $\xi$  so that,

$$m(\xi) = \theta(1 - \exp(-\eta\xi)); \quad \eta \geq 0$$

where  $\eta$  is the simulation coefficient, representing the percentage increase in  $m(\xi)$  per dollar increase in  $\xi$ . So  $m(\xi)$  is the increasing function which is bounded above by  $\theta$

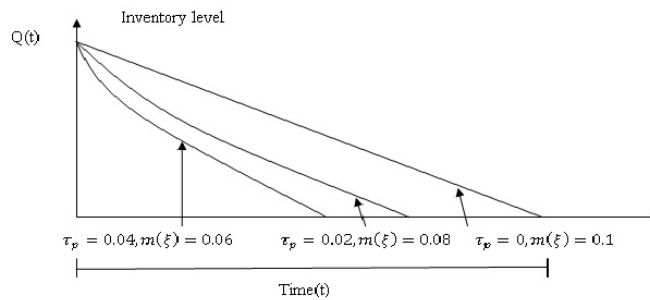


Figure 1: Inventory position for reduced deterioration rate

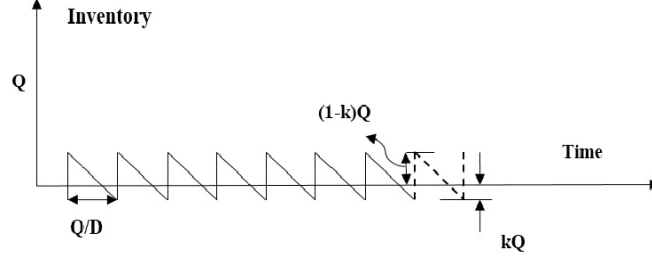


Figure 2: Inventory position for manufacturer

### 3.1.1. Manufacturer's total cost

Here production rate is constant. So, as shown in Figure 2, with constant supplement manufacturer on hand, inventory at any instant of time  $t$  is defined by differential equation.

$$\frac{dQ_m}{dt} + \tau_p Q_m = P; \quad 0 \leq t \leq T \quad (1)$$

Using boundary condition  $Q_m(0) = 0$ , we get a solution to differential equation (1)

$$Q_m(t) = \frac{P}{\theta - m(\xi)} + P e^{(\theta - m(\xi))(-t)} \quad (2)$$

At  $Q_m(T) = Q_m$ , we get a production lot size per cycle

$$Q_m = PT \quad (3)$$

The basic costs are

1. Setup cost: Constant set up cost

$$SC_m = A_m \quad (4)$$

2. Holding cost: For the final inventory level, for a manufacturer, it is the difference between the manufacturer's and the retailer's accumulated level. So, holding cost for a manufacturer is

$$HC_m = \frac{nQ_m \left[ \frac{Q_m}{P} + (m_1 - 1) \frac{Q_m}{D} \right] - \frac{m_1^2 Q_m^2}{2P} - \frac{Q_m^2 [1+2+\dots+(m_1-1)]}{2}}{m_1 Q_m} \quad (5)$$

$$HC_m = \frac{h_m [(m_1 - 1)(1 - \rho) + \rho]}{2} \left( \frac{PT}{\theta - m(\xi_1)} - \frac{P(1 - e^{\theta - m(\xi_1)T})}{\theta - m(\xi_1^2)} \right)$$

3. Inspection cost:

$$IC_m = \frac{a+b(a_1)}{m_1(a_1)} [m_1 C_{io} + m_1(a_1) C_{imu} + C_{imf}] \quad (6)$$

$$\text{Where } a_1 = \frac{P(1 - e^{\theta - m(\xi_1)T})}{\theta - m(\xi_1)}$$

Consequently, manufacturer's total cost is

$$TC_{wm}(m_1, \xi_1) = SC_m + HC_m + IC_m \tag{7}$$

Therefore, manufacturer's total cost can be written as

$$\begin{aligned} &MinTC_{wm}(m_1, \xi_1) \\ &subject\ to\ m_1 t \leq L; m_1 \geq 1; \xi_1 \geq 0 \end{aligned} \tag{8}$$

Where  $m_1 t \leq L$ , which shows that items are not overdue before they are sold up by the retailer.

3.1.2. Retailer's total cost

Retailer inventory depletes with demand rate  $D$  and resultant deterioration rate  $\tau_p$ . Then retailer's on hand inventory at any instant of time is shown in Figure 3 and is defined by the differential equation.

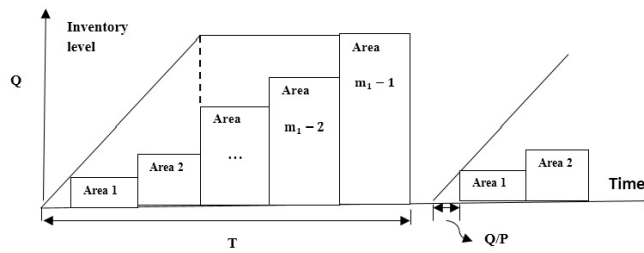


Figure 3: Inventory position for the retailer for the backorder

$$\frac{dQ_r}{dt} + \tau_p Q_r = -D; \quad 0 \leq t \leq 1 - k_1 \tag{9}$$

Using the boundary condition  $Q_r(1 - k_1) = 0$ , we get a solution to differential equation (9)

$$Q_r(t) = \frac{A^\nu a}{\theta - m(\xi_1) + A^\nu b} [e^{(\theta - m(\xi_1) + A^\nu b)(1 - k_1 - t)} - 1] \tag{10}$$

At  $t = 0$ , we get an initial quantity

$$Q_r = \frac{A^\nu a}{\theta - m(\xi_1) + A^\nu b} [e^{(\theta - m(\xi_1) + A^\nu b)(1 - k_1)} - 1] \tag{11}$$

The basic costs are

1. Ordering Cost: Constant set up cost

$$OC_r = nA_r \tag{12}$$

2. Holding cost: The retailer’s inventory level in the interval  $[0, 1 - k]$  is given by

$$HC_r = h_r[\int_0^{1-k} tQ_r(t) dt.]$$

$$HC_r = \frac{h_r A^\nu b}{a_2}[-(1 - k_1)(\frac{1}{a_2} + \frac{1-k_1}{2}) + \frac{1}{a_2}(e^{a_2(1-k_1)} - 1)] \tag{13}$$

Where  $a_2 = \theta - m(\xi_1) + A^\nu b$

3. Backorder Cost: The retailer’s inventory level in the interval  $[0, k]$  is given by

$$BC_r = \pi[\int_0^k tQ_r(t) dt.]$$

$$BC_r = \frac{\pi A^\nu a}{a_2}[(\frac{e^{a_2(1-2k_1)}}{a_2})(-k - \frac{1}{a_2}) + (\frac{e^{a_2(1-k_1)}}{a_2^2}) - (\frac{k_1^2}{2})] \tag{14}$$

So, the retailer total cost is

$$TC_{wr}(k_1, \xi_1) = OC_r + HC_r + BC_r \tag{15}$$

Therefore, the retailer total cost can be written as

$$MinTC_{wr}(k_1, \xi_1)$$

$$subject\ to\ k_1 \geq 0 ; \xi_1 \geq 0 \tag{16}$$

3.1.3. Joint total cost

$$TC_w = TC_{wm} + TC_{wr} \tag{17}$$

3.2. Model 2:With quantity discount

This model follows a strategy that the manufacturer requests the buyer to change his current order size by a factor  $\lambda (> 0)$ , offers to the retailer a quantity discount by a discount factor  $B(\lambda)$ , which the retailer accepts. Thus, the manufacturer’s and the retailer’s new order quantities are  $\lambda m_2 Q_m$  and  $\lambda Q_r$ , respectively.

3.2.1. Manufacturer’s total cost

Manufacturer offer quantity discount to retailer. Total cost for the manufacturer when quantity discount offered by to a retailer is

$$TC_{qm}(m_2, \xi_2) = A_m + \frac{h_m[(m_2-1)(1-\rho)+\rho]}{2}[\frac{PT}{b_1} + \frac{P(1-e^{b_1T})}{b_1^2}] +$$

$$[(\frac{a+b(\frac{P}{b_1}(1-e^{b_1T}))}{m_2\lambda(\frac{P}{b_1}(1-e^{b_1T}))})(m_2C_{io} + m_2(\frac{P}{b_1(1-e^{b_1T}))})C_{imu} + C_{imf})] + DB(\lambda) \tag{18}$$

Where  $b_1 = \theta - m(\xi_2)$

Thus, the problem can be formulated as



$$\begin{aligned}
 & \text{Min}TC_{qm}(m_2, \xi_2) \\
 & \text{Subject to } \lambda m_2 t \leq L; m_2 \geq 1; \xi_2 \geq 0 \\
 & \frac{\lambda h_r A^\nu a}{b_2} \left[ (1-k) \left( \frac{1}{b_2} - \frac{1-k}{2} \right) + \frac{(e^{b_2(1-k)} - 1)}{b_2^2} \right] + nA_r - TC_{wr}(k_1, \xi_1) + \\
 & \frac{\lambda \pi A^\nu a}{b_2} \left[ \frac{e^{b_2(1-2k)}}{b_2} \left( -k - \frac{1}{b_2} \right) - \frac{1}{b_2^2} - \frac{k^2}{2} \right] \leq DB(\lambda)
 \end{aligned} \tag{19}$$

Where  $b_2 = \theta - m(\xi_2) + A^\nu b$

In equation (19), the first constraint represents that items are not overdue before they are used, and the fourth constraint term  $DB(\lambda)$  represents compensation given by the manufacturer to the retailer.

### 3.2.2. Retailer's total cost

As per agreement, the retailer changes his order quantity, so according to new quantity and quantity discount, the retailer total cost is

$$\begin{aligned}
 TC_{qr}(k_2, \xi_2) &= \frac{\lambda h_r A^\nu a}{b_2} \left[ (1-k_2) \left( \frac{1}{b_2} - \frac{1-k_2}{2} \right) + \frac{(e^{b_2(1-k_2)} - 1)}{b_2^2} \right] + \\
 nA_r &+ \frac{\lambda \pi A^\nu a}{b_2} \left[ \frac{e^{b_2(1-2k_2)}}{b_2} \left( -k_2 - \frac{1}{b_2} \right) - \frac{1}{b_2^2} - \frac{k_2^2}{2} \right] + Q_m DB(\lambda)
 \end{aligned} \tag{20}$$

So, the problem is formulated as

$$\begin{aligned}
 & \text{Min}TC_{qr}(k_2, \xi_2) \\
 & \text{subject to } k_2 \geq 0; \xi_2 \geq 0
 \end{aligned} \tag{21}$$

### 3.2.3. Joint total cost

$$TC_q = TC_{qm} + TC_{qr} \tag{22}$$

## 4. COMPUTATIONAL ALGORITHM

1. Set  $m_1 = 1$  in without quantity discount model.
2. Optimize  $k_1$  and  $\xi_1$  simultaneously form  $\frac{\partial TC_{wj}}{\partial k_1}$  and  $\frac{\partial TC_{wj}}{\partial \xi_1}$
3. Take  $m_1 = m_1 + 1$
4. Repeat step 1 to 3 till  $TC_{wj}(m_1-1, k_1(m_1-1), \xi_1(m_1-1)) \geq TC_{wj}(m_1, k_1(m_1), \xi_1(m_1)) \leq TC_{wj}(m_1+1, k_1(m_1+1), \xi_1(m_1+1))$
5. Once optimal  $m_1^*, k_1^*, \xi_1^*$  are calculated, then optimal individual total cost for manufacturer, retailer, and the joint total cost for the without quantity discount model.
6. Repeat steps 1–5 for quantity discount model and obtain optimal  $m_2^*, k_2^*, \xi_2^*$
7. Using  $m_2^*, k_2^*, \xi_2^*$ , find the optimal individual total cost for manufacturer, retailer, and the joint total cost for the with quantity discount model.

**5. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS**

Consider an integrated inventory system with  $\theta = 0.2$ ,  $\rho = 0.4$ ,  $a = 400$ ,  $b = 0.6$ ,  $\alpha = 0.5$ ,  $C_{io} = 1\$/delivery$ ,  $C_{imu} = 0.02\$/unit$ ,  $C_{imf} = 0.2\$/productlot$ ,  $T = 0.7(year)$ ,  $\eta = 0.01$ ,  $\lambda = 0.5$ ,  $h_m = 0.02/unit/annum$ ,  $\nu = 1.35$ ,  $A = 3$ ,  $\pi = 10.5(\$)$ ,  $P = 50$ ,  $h_r = 0.02/unit/annum$

Models	Without quantity discount	With quantity discount
Optimal backorder rate(year)	0.000595	0.000582
Optimal preservation cost(\$)	326.7416926	54.37978617
Optimal number of order	2	4
Manufacturer(\$)	996.02	929.34
Retailer(\$)	1027.56	1027.38
System(\$)	2023.58	1956.72
PWCR(%)		3.43

Table 1: Comparison between with and without quantity discount models

Table 1 shows that for the model without quantity discount, joint total cost is 2023.58(\$), and for the model with quantity discount model, total cost is 1956.72(\$). Percentage of total cost reduction in case of quantity discount is 3.43 %. Optimality of backorder rate, preservation cost, and number of order are given below. Here, for the without quantity discount model, convexity of joint total cost mathematically and graphically (Figure 4) are shown below.

$$\left| \begin{matrix} \frac{\partial^2 TC_{wj}}{\partial \xi^2} & \frac{\partial^2 TC_{wj}}{\partial k \xi} \\ \frac{\partial^2 TC_{wj}}{\partial k \xi} & \frac{\partial^2 TC_{wj}}{\partial k^2} \end{matrix} \right| = 225.5392730 > 0 \text{ and } \frac{\partial^2 TC_{wj}}{\partial k^2} = 8.646586772 * 10^5 > 0$$

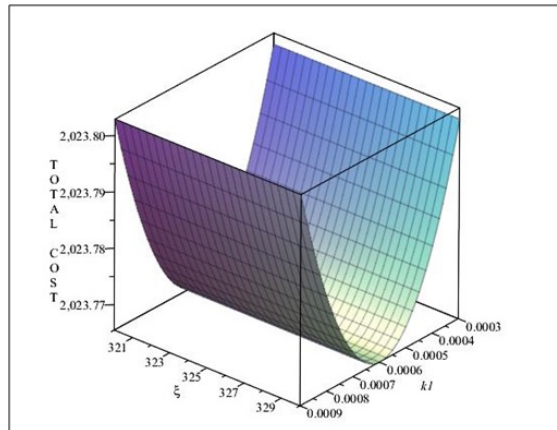


Figure 4: Optimal backorder and preservation cost in the without quantity discount model

And for the with quantity discount model, convexity of joint total cost mathematically and graphically (Figure 5) are shown below.

$$\left| \begin{array}{cc} \frac{\partial^2 TC_{qi}}{\partial \xi^2} & \frac{\partial^2 TC_{qi}}{\partial k \xi} \\ \frac{\partial^2 TC_{qi}}{\partial k \xi} & \frac{\partial^2 TC_{qi}}{\partial k^2} \end{array} \right| = 100.27597526 > 0 \text{ and } \frac{\partial^2 TC_{qi}}{\partial k^2} = 1.260239714 * 10^6 > 0$$

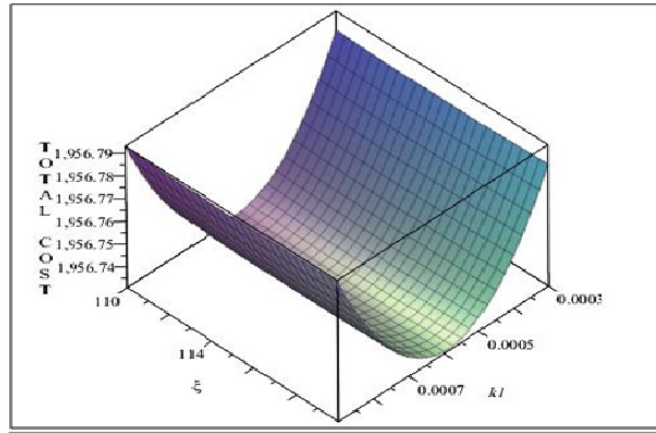


Figure 5: Optimal backorder and preservation cost in the with quantity discount model

As shown in Table 2, after order size 2, total cost is starting to increase in model 1, and in model 2, it increases after order size 4, so optimal order size for model 1 and model 2 are 2 and 4, respectively.

Model-1		Model-2	
Number of order	System total cost	Number of Order	System total cost
1	2023.2459713	1	1957.125145
2	2023.1245163	2	1957.025489
3	2023.5803716	3	1956.922208
		4	1956.548697
		5	1956.722208

Table 2: Optimal number of order

The results are shown in Table 3. Observe that increasing value of saving in

percentage (SIP) depends on whether the manufacturer shares the profit with the retailer or not. If he shares the profit, then SIP for manufacturer and retailer are as below.

$$SIP_{m_1} = \frac{100(1-\alpha)(TC_{wm}(m_1)-TC_{qm}(m_1))}{TC_{wm}(m_1)} \text{ and } SIP_r = \frac{100\alpha(TC_{wm}(m_1)-TC_{qm}(m_2))}{TC_{wr}(k_1)}$$

$$SIP_{q_i} = \frac{100(TC_{wm}(m_2)-TC_{wm}(m_1))}{TC_{qm}(m_2)} \text{ and } SIP_r = \frac{100\alpha(TC_{qm}(m_2)-TC_{wm}(m_1))}{TC_{qj}(k,m_2,\xi)}$$

$h_m$	$h_r$	$SIP_r$	$SIP_{m_1}$	$SIP_{m_2}$	$SIP_i$
0.017	0.02	3.20416	3.298710461	6.597420923	3.35937202
0.018	0.02	3.20657	3.300860369	6.601720738	3.368549173
0.019	0.02	3.20873	3.300923447	6.601846894	3.370903849
0.02	0.02	3.20981	3.300943427	6.601886854	3.370961628
0.02	0.021	3.18946	3.283565812	6.567131624	3.35107378
0.02	0.022	3.18742	3.280123658	6.561248956	3.348956274
0.02	0.023	3.18546	3.275984256	6.551245897	3.347585412
0.02	0.024	3.18235	3.272716685	6.545433369	3.34037204

Table 3: Saving in percentage values for with and quantity discount models

Table 4 shows the sensitivity for different parameters of the integrated supply chain.

Observations

- Table 1 concludes that back order rate, preservation cost individual, and joint total costs are decreasing on applying quantity discount policy.
- Optimal number of order in the without quantity discount model is 2, and in the with quantity discount model is 4, which is shown in Table 2
- Computational results from Table 3 show that with the increase of manufacturer’s holding cost, the retailer’s holding cost keeps constant increase of SIP, whereas the increase in the retailer’s holding cost keeps manufacturer’s holding cost constant decrease of SIP. When both are the same, SIP attain maximum. This is the major observation of the single set-up multiple delivery (SSMD). If it is a single set-up single delivery (SSSD), than we get the inverse result. So, according to requirements, the policy can be chosen.
- Results obtained for  $\theta$  in Table 4 show that as deterioration increases, preservation cost increases but as system attains optimal preservation cost, the total cost remains the same. It is clear from Table 4, as for the simulation coefficient  $\eta$ , the decrease in preservation cost but total cost remains the same. As shown from Table4, the advertisement frequency  $\nu$  is very sensitive. The changing effect of capacity utilization is observed from Table 4, the increase of preservation cost as well as of the total cost.

Parameters	without quantity discount model			with quantity discount model		
	$k$	$\xi$	system TC	$k$	$\xi$	system TC
$\theta$						
0.16	0.00059	304.4273371	2023.58	0.00058	93.16776	1956.72
0.18	0.00059	316.2057921	2023.58	0.00058	104.9461	1956.72
0.2	0.00059	326.7416926	2023.58	0.00058	115.4815	1956.72
0.22	0.00059	336.2727191	2023.58	0.00058	125.0131	1956.72
0.24	0.00059	344.9737985	2023.58	0.00058	133.7143	1956.72
$\eta$						
0.008	0.00059	408.4271153	2023.58	0.00058	144.3526	1956.72
0.009	0.00059	363.0463249	2023.58	0.00058	128.3134	1956.72
0.01	0.00059	326.7416926	2023.58	0.00058	115.4815	1956.72
0.011	0.00059	297.0379025	2023.58	0.00058	104.9837	1956.72
0.012	0.00059	272.2847441	2023.58	0.00058	96.2350	1956.72
$\nu$						
1.08	0.00065	Not feasible	–	0.00064	Not feasible	–
1.215	0.00064	Not feasible	–	0.00061	Not feasible	–
1.35	0.00059	326.7416926	2023.58	0.00058	115.4815	1956.72
1.485	Not feasible	Not feasible	–	Not feasible	Not feasible	–
1.62	Not feasible	Not feasible	–	Not feasible	Not feasible	–
$\rho$						
0.32	0.00059	326.196349	2023.42	0.00058	115.4447	1956.52
0.36	0.00059	326.4686515	2023.52	0.00058	115.4634	1956.61
0.4	0.00059	326.7416926	2023.58	0.00058	115.4815	1956.72
0.44	0.00059	327.0154764	2023.62	0.00058	115.5007	1957.02
0.46	0.00059	327.2900069	2023.69	0.00058	115.5194	1957.15

Table 4: Sensitivity analysis for different integrated inventory parameters

## 6. CONCLUSIONS

This model follows single set-up multiple delivery for just in time procurement. It works for items that deteriorate constantly but in a fix life time  $L$ . The effect of quantity discount when order quantity of retailer is changed is demonstrated in the model. The quantity discount policy reduces back order rate, preservation cost, and total cost for the individual as well as for the joint cost of the whole system. Preservation cost is optimized to minimize total cost of deterioration. Also, we show that frequency of advertisement plays important role in inventory control. Convexity of total cost function with respect to back order rate and preservation cost are studied, as they are the most significant parameters in this model. Our results can help a retailer to accept or reject the proposal of change in ordered quantity because we have shown that the appropriate investment in preservation decreases back-order and total cost hence, increases profit.

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