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QUANTITATIVE ANALYSIS FOR MEASURING AND SUPPRESSING BULLWHIP EFFECT

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Abstract: The increasing competition in the market generally leads to fluctuations in the products demand. Such fluctuations pose a serious concern for the decision maker at each stage of the supply chain. Moreover, the capacity constraint at any level of the supply chain would make the situation more critical by elevating the bullwhip effect. The present article introduces a new allocation mechanism, i.e. Iterative Proportional Allocation (IPA), which instead of elevating, discourages the bullwhip effect. A comparative analysis of the proposed allocation mechanism with the policies defined in Jaggi et. al(2010) has been provided to explain the bottlenecks of existing policies. It has been established numerically, that application of IPA is beneficial for both retailers as well as suppliers, as the combined profit (loss) of all the retailers increases (decreases) and subsequently, minimizes the bullwhip effect of the supplier. We have incorporated the concept of Product Fill Rate (PFR) through which it is shown that IPA gives better results as compared to other allocation mechanisms.

Keywords: Supply Chain, Bullwhip Effect, Allocation Mechanism, Product Fill Rate (PFR) .

MSC: 90B85, 90C26.

1. INTRODUCTION

Supply chain dynamics has been studied for more than half a century. In general, a supply chain includes raw materials, suppliers, manufacturers, wholesalers, retailers and end customers. In business, supply chain includes the stages, built to satisfy the demand of the all the downstream members, namely, retailers and end customers. Under this mechanism, orders from downstream members serve as a valuable informational input to upstream production and inventory decisions. This paper deals with the problem in supply chain management of how scarce resources can be efficiently allocated among retailers;e.g. in case of seat booking of the air lines or trains, where seating capacity is always limited and airline or railways allocates seats to different agencies corresponding to their demand. We present a formal model of allocation mechanisms with limited (production) capacity. The basic problem in this type of situation is that the information transferred in the form of "orders" tend to be distorted and can misguide upstream members in their inventory and production decisions. With an upstream move the distortion tend to increase. This phenomenon of variation in demand is known as "Bullwhip Effect". Many authors, like Forrester and Kaplan started research on these topics in 1960s, but story remained unexplored for long time. In late 1990s, Cachon G. and Lariviere, M. did lot of work on it, details of which are explained in literature review. The main objective of this article is to find optimal allocation of capacity which maximizes the total supply chain profit along with customer satisfaction, which can be measured in terms of PFR (Product Fill Rate). The PFR as defined by [6] is the fraction of product demand fulfilled from inventory. According to [17], the PFR is a measure of supply chains β-service level, defined as the proportion of incoming order quantities that can be fulfilled from inventory on hand, taking into account the extent to which orders cannot be fulfilled. In our model, we measure the PFR achieved by the supplier.

2. LITERATURE REVIEW

Forrester [10] discovered the fluctuation and amplification of demand from downstream to upstream of the supply chain. After that, a considerable amount of literature had explored this phenomenon. Nahmias [15] considers an inventory system in which stock is maintained to meet both high and low priority demands. When the stock level reaches some specified point, all low priority demands are backordered and high priority demands are continued to be filled. Kaplan [12] discussed the use of reserve levels, i.e. the stock levels at which a supplier should stop, in response to lower priority demand, filling the higher priority demand. Lee [13] and [14] explained the reasons of bullwhip effect, demonstrating that allocating capacity in proportion to orders induces strategic behavior but suggesting no remedy to that problem. Cachon and Lariviere[1] suggested a remedy. They study the properties of capacity allocation mechanisms for the market where a single supplier, who enjoys local monopoly,such that not whole capacity is allocated to the retailers and the supplier is left with some inventory. Deshpande and Schwarz[9] applied a mechanism design approach to obtain the optimal capacity allocation rule and pricing mechanism for the supplier but without guarantee of maximizing the supply chain profit. There are several articles related to the causes of bullwhip effect. Dejonckheere et. al.[8] analyzed the bullwhip effect induced by forecasting algorithms in order-up-to policies and suggested a new general replenishment rule that can reduce variance amplification significantly. Cachon et.al [3] shown that an industry exhibits the bullwhip effect if the variance of the inflow of material to the industry is greater than the variance of the industrys sales. The allocation mechanism of Deshpande and Schwarz were further explored by Jaggi et.al. [11], where they extended the allocations by providing reallocation mechanism. In this case, a decision is constrained on how many retailers, the supplier needs to fulfill the demand completely. Chen and Lee[5] developed a simple set of formulas that describes the traditional bullwhip measure as a combined outcome of several important drivers, such as finite capacity, batch-ordering, and seasonality. Chatfield & Pritchard [4] claim that permitting returns significantly increases the bullwhip effect. Nemtajela and Mbohwa [16] addressed relationship between inventory management and uncertain demand in Fast Moving Consumer Goods (FMCG). Jianhua Dai et.al.[7] identified the reasons of bullwhip effect and analyzed how usage of an advanced inventory management strategy can reduce bullwhip effect. They proved it in the light of McDonalds case study.

3. PROBLEM DESCRIPTION

Considering the same situation as has been taken by the authors in [1],[2] , and [11], a new allocation mechanism is presented in a single decision variable in contrast to aforesaid articles, where the model was developed as a two variable problem. In fact, Cachon and Lariviere in their papers [1]and[2] could not allocate whole capacity of supplier to the retailers and supplier is left with some inventory.Eventually, on one hand, a supplier is dealing with inventory carrying cost whereas on the other hand, the retailers are facing the problem of shortages, which was addressed by Jaggi et.al in [11] . Although they could take care of left over inventory by applying reallocation algorithm, they could not achieve the same in one go. Having these shortcomings in mind, a new Iterative Proportional Allocation (IPA) has been proposed to take care of both the bottlenecks of literature, i.e. there are neither reallocation nor the decision on how many retailers, the supplier needs to fulfill the demand completely, which makes the decision makers job easier. Furthermore, the proposed allocation model discourages the bullwhip effect unlike linear and uniform allocation. The supplier publicly announces his allocation policy. In case of linear allocation model, retailers know that high demand customers would be given priority, and there may be a situation that the customer with least demand would not get any supply. So, in order to get some supply, the customers with lower demand may inflate their demand. In case of uniform allocation, the scenario is different. Here priority is given to low demand customers and there may be a case that the customer who is demanding maximum will not get any unit at all. So, he may deflate his demand to ensure at least some supply. However, in case of the proposed allocation model, inflation and deflation of demand are loss for retailers. If a retailer deflates the demand, he will get lesser than his requirement is, and in case of inflation, he might get more than his actual demand. Hence, the proposed algorithm promotes truth inducing mechanism instead of manipulable mechanism. The proposed allocation model never allocates zero to any retailer as linear and uniform allocation do. It also overcomes the problem of deciding about the number of retailers who will get their demand satisfied at priority. The optimality of allocation can also be measured by evaluating Product Fill Rate(PFR) for all the algorithms under consideration. A comparative analysis between existing and the proposed algorithm is done. It has been shown numerically that the new algorithm dominates over the existing algorithms. Also, it is easier to apply and simple to understand.

3.1. NOTATIONS AND ASSUMPTIONS

Following notations are used for the development of the model:

N	*Number of retailers*
M_i	*Order quantity of retailer i*
$A_i(.)$	*Allocated quantity to retailer i*
c_s	*Purchasing Cost per unit of the supplier*
c_r	*Cost per unit at the retailer side which is also the selling price of the supplier*
p	*Selling price of the retailer*
h_s	*Holding cost per unit per cycle for supplier*
h_r	*Holding cost per unit per cycle for retailer*
S_s	*Shortage cost per unit for supplier*
S_r	*Shortage cost per unit for retailer*
P_s	*Profit for the supplier*
P_i	*Profit for the retailer i*
C	*capacity of the supplier*

The model is developed on the basis of following assumptions:

- The capacity (C) of a supplier is finite and constant during the period under review.
- The supplier has announced publicly the used allocation mechanism if total retailer orders exceed available capacity.
- Retailers submit their orders independently and the orders are the only communication between the retailers and the supplier.
- No retailer can share his private information with the other retailers.
- The supplier cannot deliver more than the retailer orders.

4. ALLOCATION GAME ANALYSIS

Consider a supply chain in a monopolistic environment with a single supplier selling goods to N downstream retailers. The supplier has limited capacity and he publicly announces the allocation policy. The retailers are privately informed of their optimal stocking levels. If total quantity ordered by retailers exceeds available capacity, the supplier had to do rationing, for which many allocation policies exist in literature, such as linear and uniform allocation mechanism. In this paper, a new allocation model is developed to satisfy the demand of retailers called "Iterative Proportional Allocation" (IPA). In this procedure, suppliers capacity is proportionally allocated iteratively starting from the least demand customer. We have developed a C++ program to find the allocation among the retailers using following logic: Index the retailer in increasing order of their orders and allocate the retailer as . Set i=1, j=N Repeat

$$\begin{aligned} A_i(C) &= min\Big(M_i, \Big[\frac{C}{j}\Big]\Big) \\ C &= C - A_i(C) \\ i &= i+1 \\ j &= j-1 \end{aligned} \tag{1}$$

Till i= N.

After allocating the capacity among the retailers, we can obtain the retailers profit by Jaggi et. al. [11]. They defined two models namely, linear allocation (LA) and uniform allocation (UA) models, respectively as

$$A_i(M,n) = \begin{cases} M_i - \frac{1}{n}max\Big(0, \sum_{j=1}^{n} M_j - C\Big) & i \le n \\ 0 & i > n \end{cases} \tag{2}$$

$$A_i(M,n) = \begin{cases} \frac{1}{n}\Big(C - \sum_{j=n+1}^{N} M_j\Big) & i \le n \\ M_i & i > n \end{cases} \tag{3}$$

Where n is the greatest integer less than or equal to N such that $A_i(M,n) \geq 0$ for linear allocation and $A_i(M,n) \leq M_i$ for uniform allocation.

After fulfilling the demand, if the supplier is left with some inventory, during re-allocation preference would be given to high demand retailers in case of linear allocation whereas in case of Uniform allocation, low demand retailers served first. The retailer's profit and the supplier's profit is calculated as (4) and (5) respectively:

$$P_i = (p - c_r)A_i(M,n) - h_r A_i(M,n) - s_r(M_i - A_i(M,n)) \tag{4}$$

$$P_s = c_r \sum_{i=1}^{n} A_i - c_s C - h_s(C - \sum_{i=1}^{n} A_i) - S_s(\sum_{i=1}^{n} M_i - C) \tag{5}$$

Here 'n' is a decision variable and one has to compute the allocation of units for all 'n'. The proposed algorithm provides a model independent of 'n'. The objective of this paper is to find optimal allocation of capacity. The allocation would be optimal if it satisfies the customer's demand up to maximum extent, which can be evaluated by Product Fill Rate (PFR). The PFR is a quantitative analysis used to find the percentage of demand satisfied, corresponding to each customer. For i^{th} customer, it is computed as

$$PFR_i = \frac{A_i}{M_i} * 100 \tag{6}$$

Now days, the market is customer oriented, so PFR is a better measure to evaluate the customer's satisfaction level.

5. COMPARATIVE NUMERICAL ANALYSIS

The existing algorithms, i.e. linear allocation and uniform allocation provide the allocation of units, but they fail to provide the value of decision variable 'n'. As a result, even after tedious calculations and bulky tables, results will depend on choice of 'n', whereas, the proposed algorithm provides a single solution for the same. The proposed algorithm has been compared with the two existing algorithms defined by Jaggi et al [11] and illustrated on with the help of following numerical examples. In Example 1, the values of the parameters are same as in [11].

Example 1. The demand (M_i) for 10 retailers is given in Table 1 and c_r =\$50, c_s =\$30, p =\$90, h_s=\$6, h_r =\$7, s_s =\$8 , s_r=\$10, C =150 units. The results of Table 1 - Table 4 are obtained by the authors[11] using algorithms for LA (equation (2)and equation (4)) and UA (equation(3)and equation (5)) respectively.

Table 1: Demand Allocation- Linear Allocation

Retailer	Demand	**After reallocation**						
	M_i	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	34	34	34	34	34	34	34	34
R_2	26	26	26	26	26	26	26	25
R_3	25	25	25	25	25	24	24	22
R_4	21	21	21	21	21	19	18	18
R_5	18	18	18	18	18	16	15	15
R_6	15	15	15	15	15	13	12	12
R_7	12	11	11	11	11	10	9	9
R_8	10	0	0	0	0	8	7	7
R_9	8	0	0	0	0	0	5	5
R_{10}	6	0	0	0	0	0	0	3
Sum	175	150	150	150	150	150	150	150

Table 2: Demand Allocation- Uniform Allocation

Retailer	Demand	After reallocation						
	M_i	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	34	20	19	19	18	17	16	25
R_2	26	20	19	19	18	18	19	20
R_3	25	20	22	22	24	25	25	25
R_4	21	21	21	21	21	21	21	21
R_5	18	18	18	18	18	18	18	18
R_6	15	15	15	15	15	15	15	15
R_7	12	12	12	12	12	12	12	12
R_8	10	10	10	10	10	10	10	10
R_9	8	8	8	8	8	8	8	8
R_{10}	6	6	6	6	6	6	6	6
Sum	175	150	150	150	150	150	150	150

Table 3: Profit for retailers (Linear Allocation)

Retailer	After reallocation						
	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	1122	1122	1122	1122	1122	1122	1122
R_2	858	858	858	858	858	858	815
R_3	825	825	825	825	782	782	696
R_4	693	693	693	693	607	564	564
R_5	594	594	594	594	508	465	465
R_6	495	495	495	495	409	366	366
R_7	353	353	353	353	310	267	267
R_8	-100	-100	-100	-100	244	201	201
R_9	-80	-80	-80	-80	-80	135	135
R_{10}	-60	-60	-60	-60	-60	-60	69
Sum	4700	4700	4700	4700	4700	4700	4700

In case of linear allocation, inflating demand and in case of uniform allocation, deflating demand will increase the variability of demand at supplier end. This implies that these two allocations favor manipulable mechanism, which in turn causes bullwhip effect.

Table 5 shows demand allocation and profit for retailers through the proposed Iterative Proportional Allocation (IPA)(using equation (1)).It is evident from the Table 5 that no matter the retailer inflates or deflates his demand , he will always get the same share. This shows that through proposed IPA, the variability between demand and sales reduces because the retailers reveal their actual demand information, which reduces bullwhip effect eventually.

Table 4: Profit for retailers (Uniform Allocation)

Retailer	After reallocation						
	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	520	477	477	434	391	348	305
R_2	600	557	557	514	514	557	600
R_3	610	696	696	782	825	825	825
R_4	693	693	693	693	693	693	693
R_5	594	594	594	594	594	594	594
R_6	495	495	495	495	495	495	495
R_7	396	396	396	396	396	396	396
R_8	330	330	330	330	330	330	330
R_9	264	264	264	264	264	264	264
R_{10}	198	198	198	198	198	198	198
Sum	4700	4700	4700	4700	4700	4700	4700

Now, if a low demand retailer inflates his demand, he may get more than his actual needs, are increasing his inventory carrying cost, and if a high demand retailer deflates his demand, he will get lesser than he needs, leading to shortage cost.Moreover, false information of demand floats in the market, which increases the variability . By using IPA, a supplier can promote retailers to reveal their actual demand information which will reduce bullwhip effect. Hence, instead of Manipulable Mechanism, Truth Inducing Mechanism is beneficial in suppressing the bullwhip Effect.

Table 5: Iterative Proportional Allocation

Retailer	M_i	A_i	P_i
R_1	34	21	563
R_2	26	20	600
R_3	25	20	610
R_4	21	20	650
R_5	18	18	594
R_6	15	15	495
R_7	12	12	396
R_8	10	10	330
R_9	8	8	264
R_{10}	6	6	198
Sum	175	150	4700

Again, one major drawback of the two existing algorithms is to decide an optimal 'n' for which the individual profits of the retailers can be obtained. It is evident from Table 5 that using IPA, all the capacity is allocated at one go and there is no need to decide the value of 'n' ,i.e. no need to decide about the number of customers to whom the manufacturer will supply with priority, for which the profit will be maximum. Therefore, IPA helps in eliminating 'n' unlike LA and UA. Moreover, there is no need of reallocation as well. Also, IPA never allocates zero units to any retailer. However, the total profit of supply chain is the same in all three allocation models. The supplier is allocating all the produced quantity at the same selling price to all the retailers, hence there is no change in supplier's profit due to choice of allocation mechanism. The different allocation mechanisms are affecting profit of individual retailers only. A comparative analysis is provided to prove that IPA is better than LA/UA mechanism.

Table 6 depicts the percentage change in profits of various retailers due to IPA, w.r.t different values of 'n' of linear allocation model.

Table 6: % change in profits of IPA w.r.t different 'n' of Linear Allocation

Retailer	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	-99.29	-99.29	-99.29	-99.29	-99.29	-99.29	-99.29
R_2	-43.00	-43.00	-43.00	-43.00	-43.00	-43.00	-35.83
R_3	-35.25	-35.25	-35.25	-35.25	-28.20	-28.20	-14.10
R_4	-6.62	-6.62	-6.62	-6.62	6.62	13.23	13.23
R_5	0.00	0.00	0.00	0.00	14.48	21.72	21.72
R_6	0.00	0.00	0.00	0.00	17.37	26.06	26.06
R_7	10.86	10.86	10.86	10.86	21.72	32.58	32.58
R_8	130.30	130.30	130.30	130.30	26.06	39.09	3.09
R_9	130.30	130.30	130.30	130.30	130.30	48.86	48.86
R_{10}	130.30	130.30	130.30	130.30	130.30	130.30	65.15
Sum	217.62	217.62	217.62	217.62	176.36	141.36	97.47

The negative values show that change in profit is negative, which means profit in case of IPA is less than LA or UA,but the sum of all changes are positive , which expresses that in totality values are positive for each n. The results summarized in Table 6 prove that for every value of 'n', IPA is better than LA. This analysis also helps in deciding that out of different 'n', n=10 is better than the rest of the values, as the percentage change in profits is minimum, corresponding to n=10, which cannot be determined in case of LA. Similar analysis is done for IPA vs. UA, which is shown in Table 7.

Table 7 shows that IPA is better than UA for every n, and in case of UA, n=4 is better than the rest of values of n. Apart from this, a pictorial representation of Product Fill Rate (PFR) using equation (6) for all three allocation models has been given in Figure 1. For LA, PFR ranges from 50% to 100%, whereas it is 44% to 100% for UA, and 62% to 100% for IPA. Though LA favors high demand retailers, yet it is giving 100% PFR for just one retailer. But in case of UA and IPA, more than 50% of retailers are getting 100% PFR . Even IPA is better than

UA as it not only satisfies higher percentage of retailers, but also it gives higher range of PFR.
Now, one can think that whether inflating or deflating orders affect the individual profits of the retailers. To study this, we did an analysis where the retailer"s demands were slightly changed,hence, their relative positions got changed,too.

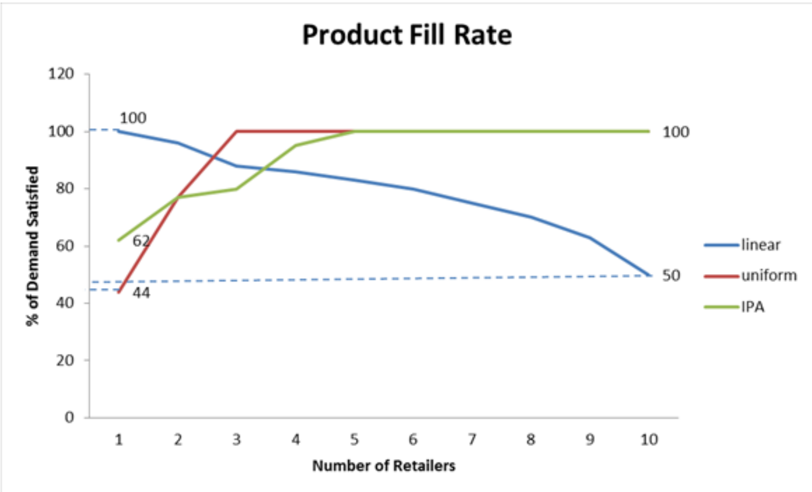

Figure 1: Product Fill Rate

Table 7: % change in profits of IPA w.r.t different 'n' of Uniform Allocation Model

Retailer	n=4	n=5	n=6	n=7	n=8	n=9	n=10
R_1	7.64	15.28	15.28	22.91	30.55	38.19	45.83
R_2	0.00	7.17	7.17	14.33	14.33	7.17	0.00
R_3	0.00	-14.10	-14.10	-28.20	-35.14	-35.14	-35.14
R_4	-6.62	-6.62	-6.62	-6.62	-6.62	-6.62	-6.62
R_5	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_6	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_7	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_8	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_9	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R_{10}	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Sum	1.02	1.73	1.73	2.43	3.02	3.49	3.96

Example 2. A new set of retailer's demand (Mi) for 10 retailers is given in Table 8.

Table 8: Comparative Analysis between IPA, LA and UA.

Retailer	M_i	IPA	**LA('n'=10)**	**UA('n'=4)**
			After Reallocation	After Reallocation
R_1	26	21	26	20
R_2	25	20	25	20
R_3	22	20	22	20
R_4	21	20	20	21
R_5	18	18	16	18
R_6	15	15	13	15
R_7	12	12	10	12
R_8	10	10	8	10
R_9	8	8	6	8
R_{10}	6	6	4	6
Sum	163	150	150	150

In example 1, through Table 6 and Table 7, we have shown that for Linear Allocation, n=10 and for Uniform Allocation, n=4 is better than other values of 'n'. Hence, in Table 8 the comparison is shown corresponding to best of LA and UA. It is evident that IPA is better than both allocation mechanisms and provides the remedy to their major drawback, that is reallocation and to decide for how many retailers demand must be satisfied completely (to evaluate the decision variable 'n'). As the allocation mechanism is already declared by the supplier, therefore in case of IPA, every retailer, who is ordering less than his proportionate share, will get his demand satisfied. Those who are ordering more or inflating their demand to get the allocation close to their original demand, may not be able to get their demand satisfied fully. The retailers would be most benefited by truth inducing mechanism rather than manipulable mechanism (MMi). It can further be proved by inducing manipulation in Example 2. Let us suppose that R4 has manipulated his demand to get more quantity .He demands for 23 units instead of 21 units. Table 9 highlights the changes in comparison of other two algorithms.

Table 9 explains clearly that if any retailer manipulates his demand because of declared allocation mechanism of supplier, he may get that increased demand because of change of relative position, as happened with R_4. His actual demand was 21, but according to LA, he gets 20. As a result, he inflated his demand to 23. In this case he is getting 22 i.e. 1 unit more than his requirement. Whereas in case of IPA, R_4 is getting the same amount as he was getting in case of true demand. This example shows that IPA supports truth-inducing mechanism.Similar type of comparison is done between IPA and UA through Table 9.

Table 9: Comparison between IPA,UA and LA

Retailer	MM_i	IPA	UA('n'=4)	LA('n'=10)
R_1	26	21	20	26
R_2	25	20	20	25
$\mathbf{R_4}$	**23**	20	20	22
$\mathbf{R_3}$	**22**	20	21	20
R_5	18	18	18	16
R_6	15	15	15	13
R_7	12	12	12	10
R_8	10	10	10	8
R_9	8	8	8	6
R_{10}	6	6	6	4
Sum	165	150	150	150

Consider that some retailer deflates his demand to get better level of satisfaction, say R_2 deflates his demand from 25 units to 21 units. Now, when he had given his true demand, i.e. 25 units, he was getting 20 units, which means he had to bear the shortages of 5 units(as explained in Table 8), but after manipulation he is getting 21 units, i.e. he is short of 4 units only. It means that manipulation can favor him whereas in case of IPA, R_2 is getting the same share as he was getting before manipulation. Hence, neither inflation nor deflation is helpful in case of IPA. Therefore, the best policy is to follow the Truth-Inducing-Mechanism, which will help in reducing bullwhip effect.

In Example 1 and Example 2, all retailers have the same parameters,so the total profit of all retailers would remain the same,i.e. \$4700, though the distribution of profit among the retailers would change. To explore the situation further, one more example is presented where retailers have different values of parameters like selling price, shortage cost, and holding cost.

Example 3. The demand (M_i) for 15 retailers along with their selling prices, shortage cost, and holding cost are given in Table 10. Rest of the parameters are: C_r =\$50, C =750 units, c_s=\$30.

The allocation and profits corresponding to existing allocation models, i.e, LA & UA are exhibited in Tables 11 & 12 and Tables 13 & 14, respectively. In both allocation techniques, i.e. LA and UA, 'n' is a decision variable and profit for each value of 'n' has to be calculated, whereas the proposed algorithm, IPA is independent of 'n', which is shown in Table 15.

Table 10: Data for example 3

Retailer	Demand	Selling Price	Holding cost	shortage cost
	M_i	P_i	h_i	S_i
R_1	140	60	1	1.5
R_2	130	60	1	1.5
R_3	120	60	1	1.5
R_4	115	61	0.85	1.35
R_5	110	61	0.85	1.35
R_6	105	62	0.75	1.25
R_7	100	62	0.75	1.25
R_8	98	63	0.65	1.15
R_9	95	63	0.65	1.15
R_{10}	92	64	0.6	1.1
R_{11}	85	64	0.6	1.1
R_{12}	78	65	0.55	1.05
R_{13}	70	66	0.55	1.05
R_{14}	65	66	0.55	1.05
R_{15}	65	67	0.5	1

It is clearly visible from Table 11 that Linear allocation is giving zero allocation to least demand retailer , which is not the case with Uniform and IPA. Corresponding results for IPA are expressed in Table 15.

Table 11: Demand Allocation- Linear Allocation

Retailer	Demand	**Allocation A_i**								
	M_i	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	140	140	140	130	128	130	128	124	122	122
R_2	120	120	115	110	106	103	102	102	102	102
R_3	110	110	105	100	96	93	92	92	92	92
R_4	100	100	95	90	86	83	82	82	82	82
R_5	95	95	90	85	81	78	77	77	77	77
R_6	85	85	80	75	71	68	67	67	67	67
R_7	70	70	65	60	56	53	52	52	52	52
R_8	65	30	60	55	51	48	47	47	47	47
R_9	55	0	0	45	41	38	37	37	37	37
R_{10}	48	0	0	0	34	31	30	30	30	30
R_{11}	42	0	0	0	07	25	24	24	24	24
R_{12}	30	0	0	0	0	0	12	12	12	12
R_{13}	22	0	0	0	0	0	0	4	4	4
R_{14}	20	0	0	0	0	0	0	0	2	2
R_{15}	18	0	0	0	0	0	0	0	0	0
SUM	1020	750	750	750	750	750	750	750	750	750

Table 12: Profits for Retailers - Linear Allocation

Retailer	**Profits**								
	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	1260	1260	1155	1134	1155	1134	1092	1071	1071
R_2	1080	1027.5	975	933	901.5	891	891	891	891
R_3	990	937.5	885	843	811.5	801	801	801	801
R_4	1015	957.5	900	854	819.5	808	808	808	808
R_5	964.25	906.75	849.25	803.25	768.75	757.25	757.25	757.25	757.25
R_6	956.25	893.75	831.25	781.25	743.75	731.25	731.25	731.25	731.25
R_7	787.5	725	662.5	612.5	575	562.5	562.5	562.5	562.5
R_8	330.25	735.25	667.75	613.75	573.25	559.75	559.75	559.75	559.75
R_9	-63.25	-63.25	544.25	490.25	449.75	436.25	436.25	436.25	436.25
R_{10}	-52.8	-52.8	-52.8	440.2	396.7	382.2	382.2	382.2	382.2
R_{11}	-46.2	-46.2	-46.2	-46.2	316.3	301.8	301.8	301.8	301.8
R_{12}	-31.5	-31.5	-31.5	-31.5	-31.5	154.5	154.5	154.5	154.5
R_{13}	-23.1	-23.1	-23.1	-23.1	-23.1	-23.1	42.9	42.9	42.9
R_{14}	-21	-21	-21	-21	-21	-21	-21	12	12
R_{15}	-18	-18	-18	-18	-18	-18	-18	-18	-18
Sum	7127.4	7187.4	7277.4	7365.4	7417.4	7457.4	7481.4	7493.4	7493.4

Table 13: Demand Allocation- Uniform Allocation

Retailer	Demand	**Allocation A_i**								
	M_i	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	140	64	64	63	61	60	57	57	52	50
R_2	120	64	64	63	61	60	57	57	52	50
R_3	110	64	64	63	61	60	57	57	52	50
R_4	100	64	64	63	61	60	57	57	52	50
R_5	95	64	64	63	61	60	67	67	87	95
R_6	85	64	64	65	75	80	85	85	85	85
R_7	70	66	66	70	70	70	70	70	70	70
R_8	65	65	65	65	65	65	65	65	65	65
R_9	55	55	55	55	55	55	55	55	55	55
R_{10}	48	48	48	48	48	48	48	48	48	48
R_{11}	42	42	42	42	42	42	42	42	42	42
R_{12}	30	30	30	30	30	30	30	30	30	30
R_{13}	22	22	22	22	22	22	22	22	22	22
R_{14}	20	20	20	20	20	20	20	20	20	20
R_{15}	18	18	18	18	18	18	18	18	18	18
SUM	1020	750	750	750	750	750	750	750	750	750

Table 14: Profit for retailers- Uniform Allocation

Retailer	**Profits**								
	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	462	462	451.5	430.5	420	388.5	388.5	336	315
R_2	492	492	481.5	460.5	450	418.5	418.5	366	345
R_3	507	507	496.5	475.5	465	433.5	433.5	381	360
R_4	601	601	589.5	566.5	555	520.5	520.5	463	440
R_5	607.75	607.75	596.25	573.25	561.75	642.25	642.25	872.25	964.25
R_6	693.75	693.75	706.25	831.25	893.75	956.25	956.25	956.25	956.25
R_7	737.5	737.5	787.5	787.5	787.5	787.5	787.5	787.5	787.5
R_8	802.75	802.75	802.75	802.75	802.75	802.75	802.75	802.75	802.75
R_9	679.25	679.25	679.25	679.25	679.25	679.25	679.25	679.25	679.25
R_{10}	643.2	643.2	643.2	643.2	643.2	643.2	643.2	643.2	643.2
R_{11}	562.8	562.8	562.8	562.8	562.8	562.8	562.8	562.8	562.8
R_{12}	433.5	433.5	433.5	433.5	433.5	433.5	433.5	433.5	433.5
R_{13}	339.9	339.9	339.9	339.9	339.9	339.9	339.9	339.9	339.9
R_{14}	309	309	309	309	309	309	309	309	309
R_{15}	297	297	297	297	297	297	297	297	297
Sum	8168.4	8168.4	8176.4	8192.4	8200.4	8214.4	8214.4	8229.4	8235.4

Table 15: Allocation and Profit for retailers- IPA

Retailer	Demand	Allocation	Profits
R_1	140	65	472.5
R_2	120	65	502.5
R_3	110	65	517.5
R_4	100	64	601
R_5	95	64	607.75
R_6	85	64	693.75
R_7	70	64	712.5
R_8	65	64	789.25
R_9	55	55	679.25
R_{10}	48	48	643.2
R_{11}	42	42	562.8
R_{12}	30	30	433.5
R_{13}	22	22	339.9
R_{14}	20	20	309
R_{15}	18	18	297
Sum	1020	750	8161.4

Table 16 depicts the percentage change in profits of various retailers due to IPA with respect to different values of 'n' of LA . Respective values for UA are expressed in Table 17.Table 12, Table 14 and Table 15 infer that in case of different parameters total profit for UA is little higher as compared to IPA, but PFR is low, which is explained in Figure 2. It shows that customer satisfaction rate is low in UA. Moreover the appearing high profit may be false information because of manipulable mechanism.

Table 16: % change in profits of IPA w.r.t different 'n' of Linear Allocation

Retailer	**% change in Profits**								
	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	-166.7	-166.7	-144.4	-140.0	-144.4	-140.0	-131.1	-126.7	-126.7
R_2	-114.9	-104.5	-94.0	-85.7	-79.4	-77.3	-77.3	-77.3	-77.3
R_3	-91.3	-81.2	-71.0	-62.9	-56.8	-54.8	-54.8	-54.8	-54.8
R_4	-68.9	-59.3	-49.8	-42.1	-36.4	-34.4	-34.4	-34.4	-34.4
R_5	-58.7	-49.2	-39.7	-32.2	-26.5	-24.6	-24.6	-24.6	-24.6
R_6	-37.8	-28.8	-19.8	-12.6	-7.2	-5.4	-5.4	-5.4	-5.4
R_7	-10.5	-1.8	7.0	14.0	19.3	21.1	21.1	21.1	21.1
R_8	58.2	6.8	15.4	22.2	27.4	29.1	29.1	29.1	29.1
R_9	109.3	109.3	19.9	27.8	33.8	35.8	35.8	35.8	35.8
R_{10}	108.2	108.2	108.2	31.6	38.3	40.6	40.6	40.6	40.6
R_{11}	108.2	108.2	108.2	108.2	43.8	46.4	46.4	46.4	46.4
R_{12}	107.3	107.3	107.3	107.3	107.3	64.4	64.4	64.4	64.4
R_{13}	106.8	106.8	106.8	106.8	106.8	106.8	87.4	87.4	87.4
R_{14}	106.8	106.8	106.8	106.8	106.8	106.8	106.8	96.1	96.1
R_{15}	106.1	106.1	106.1	106.1	106.1	106.1	106.1	106.1	106.1
Sum	262.0	268.1	266.8	255.3	238.8	220.3	209.8	203.6	203.6

Table 17: % change in profits of IPA w.r.t different 'n' of Uniform Allocation

Retailer	**% change in Profits**								
	n=7	n=8	n=9	n=10	n=11	n=12	n=13	n=14	n=15
R_1	2.2	2.2	4.4	8.9	11.1	17.8	17.8	28.9	33.3
R_2	2.1	2.1	4.2	8.4	10.4	16.7	16.7	27.2	31.3
R_3	2.0	2.0	4.1	8.1	10.1	16.2	16.2	26.4	30.4
R_4	0.0	0.0	1.9	5.7	7.7	13.4	13.4	23.0	26.8
R_5	0.0	0.0	1.9	5.7	7.6	-5.7	-5.7	-43.5	-58.7
R_6	0.0	0.0	-1.8	-19.8	-28.8	-37.8	-37.8	-37.8	-37.8
R_7	-3.5	-3.5	-10.5	-10.5	-10.5	-10.5	-10.5	-10.5	-10.5
R_8	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7	-1.7
R_9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{10}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{11}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{12}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{13}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{14}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
R_{15}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Sum	1.1	1.1	2.4	4.7	5.9	8.4	8.4	11.8	13.2

Now, through Table 16 and Table 17 , it is evident that total % change in profits is positive for IPA as compared to LA and UA irrespective of value of n.

This analysis shows that though the profit for IPA seems to be little lesser than UA, but it might not be a real situation. The reason for this claim is that LA and UA are giving manipulated information in market. For getting better share in monopolistic environment, they are generating false demand, so the corresponding profit is also false. Whereas IPA is promoting only truth inducing mechanism, so whatever profit appears is achievable. Moreover IPA is providing much better PFR, which can be seen in figure 2.

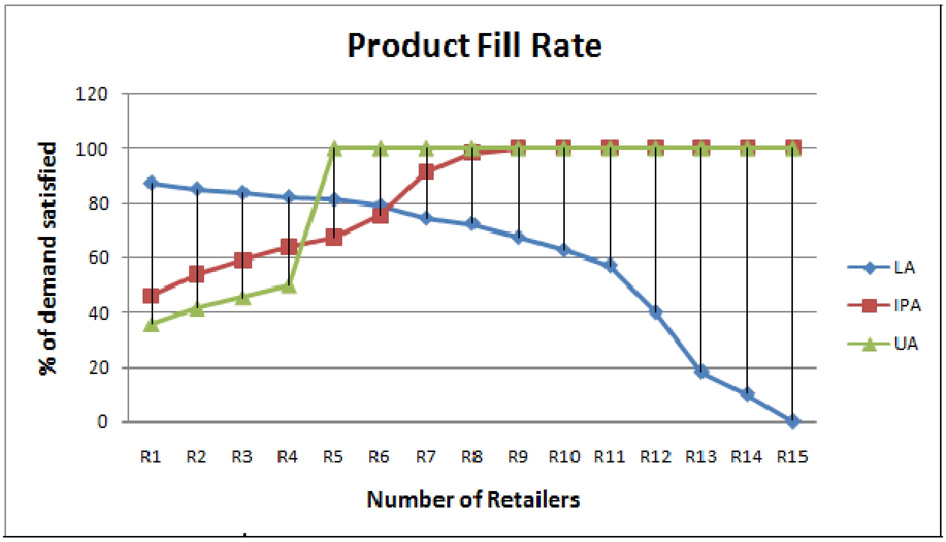

Figure 2: Product Fill Rate

Through above analysis we have shown that IPA is better than two existing algorithms in literature.

6. CONCLUSIONS and SUGGESTIONS

Present paper introduces an allocation algorithm for rationing of limited capacity among retailers in order to measure and suppress bullwhip effect. The proposed IPA algorithm , which is coded in C++, deals with two main bottlenecks of existing mechanism in literature i.e, LA and UA to take a decision for number of customers who will get their demand satisfied with priority (n) and to avoid reallocation. Further, it also promotes truth inducing mechanism, which eventually suppresses bullwhip effect. Through a numerical example, it has been established that IPA promotes truth inducing mechanism, which suggests that a retailer should reveal his actual demand without making any manipulation. Finally, a comparative analysis is presented between IPA,and LA and UA considering profits and product fill rate.

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