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# NUMERICAL SOLUTION OF THE AGE STRUCTURE OPTIMIZATION PROBLEM FOR BASIC TYPES OF POWER PLANTS

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**Abstract:** Our paper addresses an integral model of the large electric power system optimal development. The model takes into account the age structure of the main equipment, which is divided into several types regarding its technical characteristics. This mathematical model is a system of Volterra type integral equations with variable integration limits. The system describes the balance between the given demand for electricity, the commissioning of new equipment and the dismantling of obsolete equipment, as well as the shares of different types of power plants in the total composition of the electric power system equipment. Based on the developed model, we got numerical solution to the problem of finding the optimal strategy for replacing equipment with a minimum of the cost functional. The case study is the Unified Electric Power System of Russia. Calculations of the forecast for development of the electric power system of Russia until 2050 were made using real-life data.

**Keywords:** Integral Models of Developing Electric Power Systems, Lifetime, Optimization Problem, Numerical Solution.

**MSC:** 90C90, 65D30, 45D05.

## 1. INTRODUCTION

Important tasks of modern electric power industry are associated with high wear and tear of fixed production assets at low rates of upgrading it. At present, a significant part of the main equipment at power facilities is operated beyond the normative lifetime. This can lead to an increase in the costs of maintaining the main equipment in proper technical condition, a growth of technological limitations, which, in turn, affects the reliability of power supply. In this connection, analysis of the age structure of generating equipment is important for future strategies of its upgrading.

A useful simulation tool for management of obsolete equipment is vintage capital models [6, 7, 9, 11, 18, 19, 24]. They take into account the embodied technological changes and are described by nonlinear Volterra integral equations with variable upper and lower integration limits [8, 9, 10]. Such models are used for qualitative investigation of the aging equipment replacement [12, 13, 14, 15, 25].

Mathematical models of generating capacities development of electric power system are considered in the works [3, 16, 17, 21, 22]: models with different degrees of aggregation by types of power plants, estimation models (analysis of the consequences of the specified capacity upgrading strategy) and optimization ones (optimization of the capacities lifetime), models with describing the processes of prolonging the lifetime of the generating equipment (modernization) or without it.

A new integral model of the developing system was proposed in [2, 5]. It allows us to provide a detailed description of the technical and economic parameters of the generating power plants equipment, taking into account its age structure. The equipment is divided into several age groups with different indicators of the effectiveness of their functioning. The fundamentals of the theory of corresponding Volterra equations of the first kind are presented in [2, 4].

## 2. INTEGRAL MODEL OF THE EPS DEVELOPMENT

To model an electric power system (EPS), we use the integral model described in [5]. It is assumed that all equipment of the system has the same technical characteristics. In this article, we consider a vector model of Russia's EPS, in which the generating equipment is divided into three groups by the type of energy resources that they use: stations operating on fossil fuel (TPP), stations operating on nuclear fuel (NPP), and hydro power plants (HPP). Stations of the same type are divided into three age groups.

Introduce the following denotations for the mathematical description of the model:

$x(t) \equiv (x_1(t), x_2(t), x_3(t))$ is the commissioning of electric capacities (by types of power plants): $x_1(t)$ corresponds to TPPs, $x_2(t)$ corresponds to NPPs, $x_3(t)$ corresponds to HPPs; variable $t$ is located in the forecast period $[t_0, T]$;

$\beta_{ij}$ is the efficiency coefficient of the age group $j$ or a power plant of type $i$, in addition within the same age group the efficiency coefficient $\beta_{ij} = \text{const}$, $1 \geqslant \beta_{i1} \geqslant \beta_{i2} \geqslant \beta_{i3} \geqslant 0$, $i, j = \overline{1,3}$;

$y\,(t)$ is the total available capacity of the electric power system specified by the experts for the future;

$T_{ij}(t)$ is the upper age limit of group $j$ for a power plant of type $i$, $i, j = \overline{1,3}$, in addition $0 < T_{i1}(t) < T_{i2}(t) < T_{i3}(t)$, $T_{i3}(t)$ is the lifetime of the equipment of type $i$ (the age of the oldest equipment of type $i$ still in use at the moment $t$);

$x^0(t) \equiv (x_1^0(t), x_2^0(t), x_3^0(t))$ is the known dynamics of commissioning the capacities on the prehistory $[0, t_0)$ (by types of power plants);

$\alpha(t)$ is a given change in the share of the TPP capacity in the total composition of generating equipment;

$\gamma(t)$ is a given change in the share of the HPP capacity in the total composition of generating equipment.

The model of the EPS development has the form of the following system of equations:

$$\sum_{i=1}^{3}\left(\beta_{i1}\int\limits_{t-T_{i1}(t)}^{t} x_i(s)ds + \beta_{i2}\int\limits_{t-T_{i2}(t)}^{t-T_{i1}(t)} x_i(s)ds + \beta_{i3}\int\limits_{t-T_{i3}(t)}^{t-T_{i2}(t)} x_i(s)ds\right) = y(t), \quad t \in [t_0, T], \tag{1}$$

$$\int\limits_{t-T_{13}(t)}^{t} x_1(s)ds = \alpha(t)\left(\int\limits_{t-T_{13}(t)}^{t} x_1(s)ds + \int\limits_{t-T_{23}(t)}^{t} x_2(s)ds + \int\limits_{t-T_{33}(t)}^{t} x_3(s)ds\right), \tag{2}$$

$$\int\limits_{t-T_{33}(t)}^{t} x_3(s)ds = \gamma(t)\left(\int\limits_{t-T_{13}(t)}^{t} x_1(s)ds + \int\limits_{t-T_{23}(t)}^{t} x_2(s)ds + \int\limits_{t-T_{33}(t)}^{t} x_3(s)ds\right) \tag{3}$$

with the initial conditions on the prehistory

$$x(t) = x^0(t),\ t \in [0, t_0), \tag{4}$$

and the restrictions on the commissioning capacities

$$x_i(t) \geqslant 0,\ t \in [t_0, T],\ i = \overline{1,3}. \tag{5}$$

Here, equation (1) indicates the balance between the number of commissioning capacities of different types and the available power $y(t)$, given for the future. Equations (2), (3) specify the required ratio of different electric power types. In addition, it is assumed that the restoration of dismantled capacities is impossible:

$$T_{i3}'(t) \leqslant 1,\ t \in [t_0, T],\ i = \overline{1,3}. \tag{6}$$

In the case of one term, the problems of the existence and uniqueness of solution to (1) in the space $C_{[t_0,T]}$ are studied in detail in [1]. In the case of several terms, elements of the theory of corresponding Volterra equations of the first kind are presented in [2].

Since the solution to (1)–(3) can be explicitly set down only in some special cases, we turn to numerical solution. Note that (1)–(3) contain integrals with variables in both upper and lower integration limits, which means that we have to adapt the numerical procedures used to solve the classical first kind Volterra equations. The reader is referred to [1] for the theory and numerical methods for solving this equation. A numerical solution by the quadrature method is characterized by accumulation of the integral approximation error over the prehistory, which reduces its order of convergence by one. There exist some procedures of restoring the method order, some of which can be found in [1]. For example, to keep the order of convergence of quadrature method equal to the approximation order of the quadrature formulae, one can use a quadrature on the prehistory whose approximation order exceeds that of the basic one by 1.

It is possible to formulate various economic problems on the basis of the above model. For example, if the commissioning capacities $x(t)$ are the sought-for functions, and all other functions are known, then we obtain the problem of forecasting the EPS development.

You can set various optimization problems using the model (1)–(5).

## 3. OPTIMIZATION PROBLEM

As noted earlier, the growth in the part of obsolete equipment in the structure of Russia's power plants leads to many negative consequences. Along with the increase in operating costs at present, this will require a very large investment in the future for the inevitable replacement of a large number of generating capacities. In this regard, the problem of technical reequipment and dismantling of the main equipment of power plants is one of the most urgent tasks for the Russian electric power industry development.

We consider a search problem for dynamics of change in the equipment lifetime (i.e. parameter $T_3(t) \equiv (T_{13}(t), T_{23}(t), T_{33}(t)))$ that minimizes total costs of putting capacities into service and operation of capacities during the time $[t_0, T]$ for a given demand for electricity $y(t)$. Here $T_{i3}(t)$ is the moment of dismantling capacity of type $i$.

We take the cost functional as the objective functional:

$$I\big(x, T_3\big) = \sum_{i=1}^{3} \int_{t_0}^{T} a^{t-t_0} \left\{ \sum_{j=1}^{3} \beta_{ij} \int_{t-T_{ij}(t)}^{t-T_{i,j-1}(t)} u_1^i(t-s) u_2^i(s) x_i(s) ds \right\} dt + \\ + \sum_{i=1}^{3} \int_{t_0}^{T} a^{t-t_0} k_i(t) x_i(t) dt, \quad T_{i0}(t) = 0, \quad i = \overline{1,3}. \tag{7}$$

Here the first term corresponds to the operating costs; the second term corresponds to the costs of putting capacities into service.

The following functions are known in (7):

$u_1^i(t-s)$ are coefficients of increase in the costs of operating the capacities of type $i$ at time $t$ that are commissioned at time $s$;

$u_2^i(t)$ are the specific annual costs of operating the capacity of type $i$, commissioned at time $t$;

$k_i(t)$ are the specific capital costs of commissioning a capacity unit of type $i$ at time $t$;

$a^{t-t_0}$ is the costs discount coefficient, $0 < a < 1$.

The control parameter $T_3(t)$ belongs to the feasible set

$$U = \left\{T_3(t) : \underline{T_3} \leqslant T_3(t) \leqslant \overline{T_3},\ T'_{i3}(t) \leqslant 1,\ t \in [t_0, T],\ i = \overline{1,3}\right\}. \tag{8}$$

It is required to find

$$T_3^*(t) = arg \min_{T_3(t) \in U} I\big(x, T_3\big) \tag{9}$$

under the conditions (1)–(8).

The problem (1)–(9) is nonlinear, the components of the vector $T_3(t)$ are in the lower limits of integration in (7) and (1)–(3). In addition, there are restrictions on the phase variable (5). All these factors make the problem quite complex and challenging.

## 4. NUMERICAL SOLUTION OF OPTIMIZATION PROBLEM

To solve the optimal control problem, we use a heuristic algorithm based on the discretization of all the elements on a grid with step $h = 1$ (year) and replacement of the feasible set $U$ by the set $U_h$ of piecewise linear functions

$$T_{i3}(t) = \begin{cases} m_i, & t \in (t_0, T], & m_i \leqslant T_{i3}(t_0), \\ t - t_0 + T_{i3}(t_0), & t \in [t_0, t_0 - T_{i3}(t_0) + m_i), & m_i > T_{i3}(t_0), \\ m_i, & t \in [t_0 - T_{i3}(t_0) + m_i, T], & \end{cases} \tag{10}$$

where $m_i$ is desired integer constant lifetime, $i = \overline{1,3}$. Equation (10) means that the maximum lifetime can be decreased abruptly and increased only by a unit per year (according to (6)) until we reach the desired level (see Figure 1). The upper and lower limits for the lifetimes $\underline{T_3}$ and $\overline{T_3}$ were chosen basing on physical considerations.

On this feasible set we use enumerative technique among all possible $m_i$, $i = \overline{1,3}$. We introduce the grid of nodes with step $h$: $t_j = t_0 + j$, $j = \overline{1,N}$, $Nh = T - t_0$. We used the right rectangle rule with step $h = 1$ to approximate the integrals in (1)–(3), (7) on the forecast period. The midpoint rule is used to approximate the integrals on the prehistory. We employ the forecast values of economic indices provided by experts.

Let us describe the algorithm for solving the optimization problem.

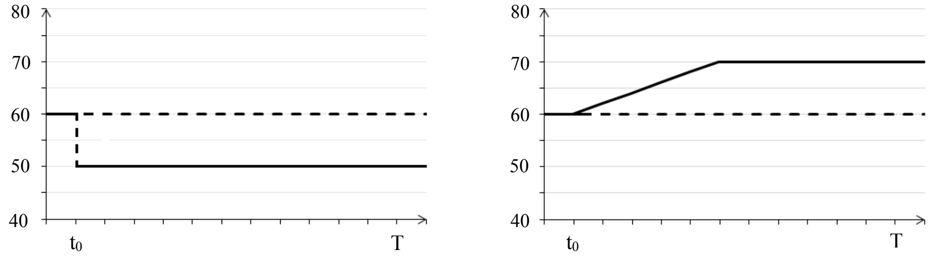


Figure 1: Example of two variants of transition to a constant lifetime.

*Step 1.* Choose the lifetime $T_{i3}(t) = m_i$, $i = \overline{1,3}$, from the feasible set (10).

*Step 2.* For the given $T_{i3}(t)$, solve numerically the system of equations (1)–(3) with respect to $x_i(t)$ in $t = t_0$.

*Step 3.* Check the inequality (5). If it is valid, then go to *Step 4*. If $x_i(t_0) < 0$, then set $x_i(t_0) = 0$ and correct $\alpha(t_0)$ and $\gamma(t_0)$ from (2)–(3). We actually get the inequality instead of the balance equation (1).

*Step 4.* Return to *Step 2* for the next point $t_j, j = \overline{1,N}$, until the forecast period ends.

*Step 5.* Substitute the obtained numerical solution $x(t)$, $t \in [t_0, T]$, into a discrete analogue (8), and calculate the value of the functional $I(T_3(t), x)$.

Looping over all possible $m_i$ from the feasible set, we find the solution to the problem.

The above algorithm is implemented for the scalar case as a software package in the MatLab environment [20]. The vector statement of the problem required some software package modifications.

## 5. REAL-LIFE DATA CALCULATIONS

Now we consider a solution to (1)–(9) as applied to the Unified Energy System (UES) of Russia. The year 1950 is taken as the beginning of modeling. The forecast period is $[t_0, T] = [2016, 2050]$. The upper age limits of the groups are $T_{11}(t) = T_{21}(t) = T_{31}(t) = 30$, $T_{12}(t) = T_{22}(t) = T_{32}(t) = 50$, $T_{13}(t) = T_{23}(t) = 60$, $T_{33}(t) = 101$. The efficiency coefficients are $\beta_{i1} = 1$, $\beta_{i2} = 0.97$, $\beta_{i3} = 0.9$, $i = \overline{1,3}$. Using the known data from the prehistory $[0, t_0) = [1950, 2016)$, we find the shares of the TPP and HPP capacities and assume them to be constant throughout the forecast period: $\alpha(t) = 0.69$, $\gamma(t) = 0.19$. The following variant was defined as a basic one: in 2015 $T_{13} = 60$, $T_{23} = 47$, $T_{33} = 66$ (according to real-life data), in 2050 $T_{13} = T_{23} = 60$, $T_{33} = 101$ (lifetime increases by 1 per year). The dynamics of commissioning capacities on the prehistory correspond to the real-life data [23].

The growth dynamics of the right-hand side at $[2016, 2050]$ provide for a low level of consumption (0.5% per year). The dynamics of commissioning capacities of the UES and the average age of the UES equipment for the basic variant (*Steps 2–4*) are shown in Fig. 2.

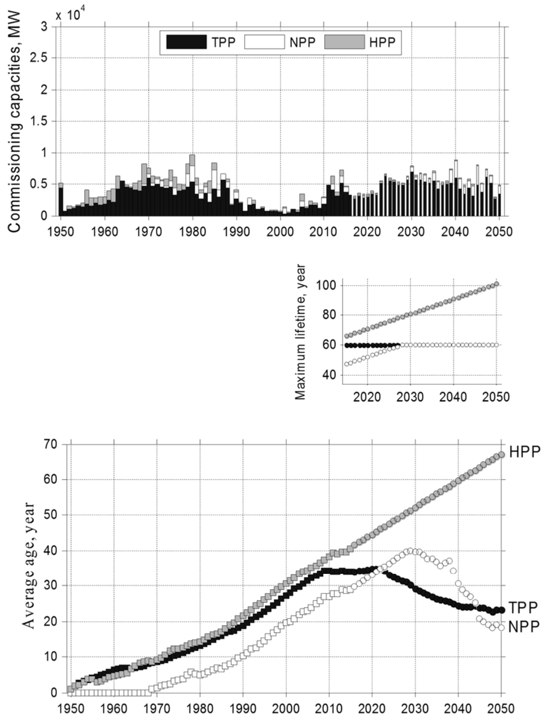


Figure 2: The basic variant.

The following data was used for the optimization problem. The functions of the specific growth of operating costs $u_1^i(t-s) \equiv u_1^i(\tau)$ are given as follows:

$$u_1^1(\tau) = u_1^3(\tau) = \begin{cases} 1, & \tau \leqslant 45, \\ 1.03^{\tau-45}, & \tau > 45, \end{cases} \quad u_1^2(\tau) = \begin{cases} 1, & \tau \leqslant 45, \\ 1.1^{\tau-45}, & \tau > 45, \end{cases}$$

(costs of operation capacity increase exponentially at the rate of 3% per year after 45 years of exploitation for TPPs and HPPs, and 10% per year after 45 years of exploitation for NPPs). The functions $k_i(t)$ and $u_2^i(t)$ were taken to be constant: $k_1(t) = 1300$ (USD/MW), $k_2(t) = 2500$ (USD/MW), $k_3(t) = 3000$ (USD/MW), $t \in [2016, 2050]$; $u_2^1(t) = 189$ (USD/MW), $u_2^2(t) = 170$ (USD/MW), $u_2^3(t) = 200$ (USD/MW), $t \in [1950, 2050]$. Optimization of lifetimes of the TPP and NPP equipment was carried out, assuming that the HPP equipment is not decommissioned for the forecast period.

The obtained optimal lifetimes, the corresponding commissioning of the UES capacities, and the average age of the UES equipment are shown in Fig. 3. The optimal lifetime for the equipment of TPPs is proposed to be reduced from 60 to 53 years. The optimal lifetime for the equipment of NPPs shifts from 47 to 50 years. Mass dismantling of equipment at the beginning of the forecast period requires a sharp increase (up to 27 GW) of commissioning capacities. Thus, by 2050 the obtained strategy will have given an economic benefit 2.31% with respect to the basic variant.

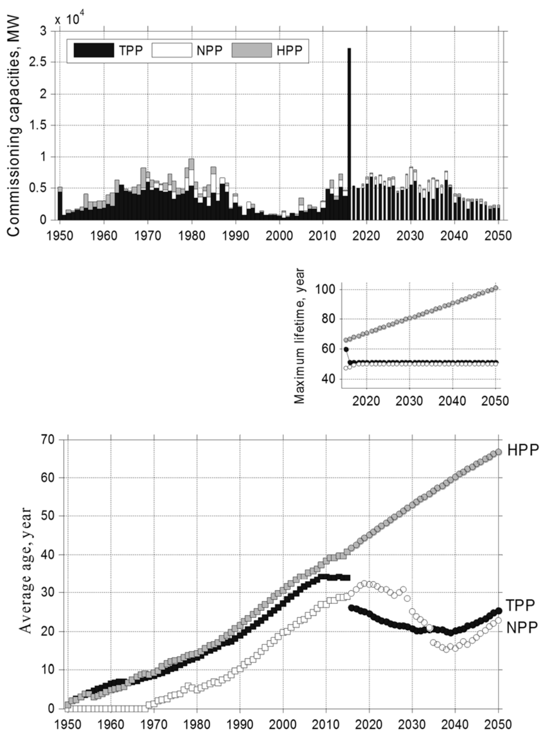


Figure 3: The dynamics of commissioning capacities, corresponding optimal lifetimes and the average age of generating capacities.

## 6. CORRECTED ALGORITHM

From the economical and technical points of view, a strategy with a mass commissioning of equipment is unacceptable since it is related to large one-time capital costs of commissioning of new capacities and limited technical capabilities. Therefore, it is natural to introduce additional restrictions on the phase variable into the statement (1)–(9):

$$\sum_{i=1}^{3} x_i(t) \leqslant \overline{x}(t), \quad t \in [t_0, T]. \tag{11}$$

It is necessary to add one step to the algorithm for solving the problem.

*Step 3.1.* Check the inequality (11). If it is valid, then go to *Step 4.* Overwise, return to *Step 1* and increase the desired $m_i$ by a unit for each type of power plant, until (11) is executed on *Step 3.1.* For the next moment of time, return to the original set $m$.

Note that the proposed way to account for the restriction (11) is not unique. It depends on how important it is to keep the ratio of different types of power plants $\alpha$ and $\gamma$.

To illustrate solution of the problem with the restriction on power inputs, we take as $\overline{x}(t)$ a linear function that is equal to the maximum input of powers on the prehistory at $t_0 = 2016$ and triples by the end of the forecast period:

$$\overline{x}(2016) = \max_{t \in [1950, 2015]} \sum_{i=1}^{3} x_i^0(t); \;\; \overline{x}(2050) = 3 \cdot \overline{x}(2016).$$

Figure 4 shows the obtained optimal lifetimes, the corresponding commissioning of the UES capacities, and the average age of the UES equipment taking into account the restrictions. The optimal lifetime for the TPP equipment implies gradual transition from 60 to 52 years by 2020. The optimal lifetime for the NPP equipment shifts from 47 to 50 years. The economic benefit with respect to the basic variant will have been 2.30% by 2050.

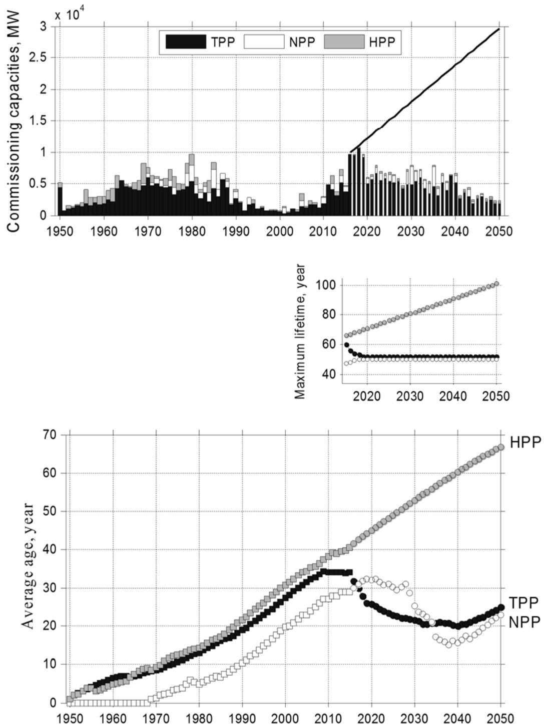


Figure 4: The dynamics of commissioning capacities, corresponding optimal lifetimes and the average age of generating capacities taking into account the restrictions.

## 7. CONCLUSIONS

In this paper we considered an integral model of the optimal development of a large electric power system using the example of the Unified Electric Power System of Russia. We proposed a numerical algorithm for finding an optimal strategy for replacing equipment with a minimum of the cost functional. Using real-life data, we made calculations for the case of constant costs for commissioning new capacities and operating generating capacities.

The results of the calculations show the efficiency of the accelerated renovation of the TPP equipment with a reduction in their lifetime from the presently accepted 60 years to 53 years for the problem without restrictions on the commissioning of new capacities, and to 52 years for the problem with restrictions. At the same time, it is proposed to increase the lifetime of the NPP equipment from 47 to 50 years, and the resulting benefits are 2.31% of the initial value of the functional for the problem without restrictions on commissioning capacities, and 2.30% for the problem with restrictions.

The problem of searching the optimal strategy for replacing equipment taking into account technological changes is the subject of further research.

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