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SOLVING A POSYNOMIAL GEOMETRIC PROGRAMMING PROBLEM WITH FULLY FUZZY APPROACH

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Abstract: In this paper we have investigated a class of geometric programming problems in which all the parameters are fuzzy numbers. In fact, due to impreciseness of the cost components and exponents in geometric programming with their inherently behavior as in economics and many other areas, we have used fuzzy parametric geometric programming. Transforming the primal problem of fuzzy geometric programming into its dual and using the Zadeh's extension principle, we convert the dual form into a pair of mathematical programs. By applying the α-cut on the objective function and r-cut on the constraints in dual form of geometric programming, we obtain an acceptable (α, r) optimal values. Then, we further calculate the lower and upper bounds of the fuzzy objective with emphasize on modification of a method presented in [14, 32]. Finaly, we illustrate the methodology of the approach with a numerical example to clarify the idea by drawing the different steps of LR representation of $Z_{\alpha r}$.

Keywords: Fuzzy logic, Posynomial, Geometric Programming, Optimization.

MSC: 90C30, 90C70, 86.

1. INTRODUCTION

The formulation of engineering design problems with specific types of non-linear optimization problems with flexible variables are known as geometric programming. Duffin et al. [9] proposed an excellent idea to solve application of engineering problems by developing basic theories of geometric programs. Since last few decades, we have seen a rapid development in geometric programming used in a variety of optimization problems involving digital circuit design [4, 5, 8], resource allocation in communication network systems [27], linear multi-objective geometric programming problems via reference point approach [2], and the problem of temperature-aware floor planning in which the parameters of the problem are often undetermined [36]. Therefore, in this paper, to clarify the subject, we consider geometric programming problems where the exponents of the variables, cost coefficients, and the constraint coefficients and their right-hand sides are all fuzzy numbers.

Due to uncertainty of the parameters of the real-world, Bellman and Zadeh investigated the problem of decision-making in a fuzzy environment and management science [3]. Fuzzy logic is a very powerful tool to handle the problem of system design in optimization of the solution of non-convex optimization problems in multiple-input multiple-output systems on using fuzzy predictive filters, which was investigated by Mendoça et al. [22]. A number of methods have been so far proposed to solve the fuzzy linear programming problems [1, 10, 12, 23], and proposing a new algorithm to solve fuzzy linear programming problems using the MOLP problem is a recent work done in [11]. Different models have been so far presented to deal with decision making problems where evaluations of alternatives are uncertain or affected by a fuzzy parameters [26]. A multi-objective problem with fuzzy parameters is being investigated by larbani [13] and Sakawa [30].

Ojha and Das [24] developed a solution procedure using geometric programming technique by splitting the coefficients and exponents with the help of binary numbers. Multi-objective geometric programming problem is worked out by Ojha et al. [25], in which they have proposed ε-constraint method that has been applied to find the non-inferior solution. In view of Rajgopal et al. [28], the problem of posynomial geometric programming has been studied via generalized linear program.

A lot of research works have been done in the area of risk management, inventory management and planning [29, 33]. Mahapatra and Mandal have discussed parametric functional form of an interval number and then solved the problem by geometric programming technique [17, 21]. They got optimal solution of the objective function directly without solving the equivalent transformed problem. They have also presented production inventory model with fuzzy coefficients using parametric geometric programming approach [18].

Mahapatra and Mahapatra [15] used fuzzy parametric geometric programming with cost constraint to find optimal reliability, and they have considered reliability series system with limited system cost as a constraint function [16]. Mahapatra et al. [20, 19] investigated and developed the problem of economic production quantity model with demand dependent unit production cost under fuzzy environment.

Sen and Pal [31] solved linear multi-objective fuzzy goal programming problem with interval weights. Chen and Tsai [7] studied different methodologies to derive weights or priorities of fuzzy goal programming. An essential book about fuzzy geometric programming is written by Cao in [6]. Yang and Cao [35] presented an outline of the applications of fuzzy geometric programming. Global optimization of signomial geometric programming problems is investigated by Xu [34].

Our aim is to calculate a lower bound and an upper bound for the objective function by applying (α, r)-cut on both fuzzy parameters of the objective function and the constraints which is based on Zadeh's extension principle [37].

Here, we present (α, r) optimum value for fully fuzzy geometric programming problems in which the exponents of the variables, cost coefficients, and the constraint coefficients and the resources are all fuzzy numbers. This paper is organized as follows. We first introduce the fuzzy geometric programming problem and next we calculate the lower and upper bounds of the objective value at different (α, r)-levels. We draw the graph of the membership function of fuzzy objective value, and finally, the implementation of our proposed model is illustrated by a numerical example. A brief summary is presented in the conclusion.

2. MATHEMATICAL MODELLING

The general form of posynomial geometric programming problem is as follows

$$\begin{aligned} & Z = \min_{t} \sum_{k=1}^{l_0} c_{0k} \prod_{j=1}^{n} t_j^{a_{0kj}} \\ & Subject\,to \\ & \qquad \sum_{k=1}^{l_i} c_{ik} \prod_{j=1}^{n} t_j^{a_{ikj}} \leq b_i \quad i = 1, ..., m \\ & \qquad t_j > 0 \quad j = 1, ..., n \end{aligned} \tag{1}$$

By the definition of posynomial all b_i, $i = 1, 2, \ldots, m$, are positive real numbers and the exponents $a_{ikj} \in R$, $i = 0, 1, \ldots, m$, $j = 1, 2, \cdots, n$ and all the coefficients c_{ik}, $i = 0, 1, \ldots, m$, $k = 1, 2, \cdots, l_i$, are positive. If at least one of the parameters a_{0kj}, a_{ikj}, b_i, c_{0k} or c_{ik} is fuzzy, then the objective value will be fuzzy as well. Let c_{0k}, c_{ik}, b_i, a_{0kj} and a_{ikj} be fuzzy numbers of the corresponding posynomial geometric program given by Model (1) that can be replaced by the convex fuzzy sets $\tilde{C}_{0k}$, $\tilde{C}_{ik}$, $\tilde{B}_i$, $\tilde{A}_{0kj}$ and A_{ikj} respectively. Therefore (1) can be reformulated as the following fuzzy geometric programming problem.

$$
\begin{aligned}
&\tilde{Z} = \min_{t} \sum_{k=1}^{l_0} \tilde{C}_{0k} \prod_{j=1}^{n} t_j^{\tilde{A}_{0kj}} \\
&\textit{Subject to} \\
&\qquad \sum_{k=1}^{l_i} \tilde{C}_{ik} \prod_{j=1}^{n} t_j^{\tilde{A}_{ikj}} \le \tilde{B}_i \qquad i = 1, \dots, m \\
&\qquad t_j > 0 \quad j = 1, \dots, n
\end{aligned} \tag{2}
$$

Since geometric programms are solved via their duals, so (2) can be written in the form of its dual as:

$$
\begin{aligned}
&\tilde{Z} = \max_{\lambda} \prod_{k=1}^{l_0} \left(\frac{\tilde{C}_{0k}}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{m} (\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)^{\left(\sum_{k=1}^{l_i} \lambda_{ik}\right)} \prod_{k=1}^{l_i} \left(\frac{\tilde{C}_{ik}}{\tilde{B}_i \, \lambda_{ik}} \right)^{\lambda_{ik}}) \\
&\textit{Subject to} \\
&\qquad \sum_{k=1}^{l_0} \lambda_{0k} = 1 \qquad\qquad (\textit{Normal Condition}) \\
&\qquad \sum_{i=0}^{m} \sum_{k=1}^{l_i} \tilde{A}_{ikj} \, \lambda_{ik} = 0 \qquad j = 1, ..., n \quad (\textit{Orthogonal Conditions}) \\
&\qquad \lambda_{ik} \ge 0 \quad \forall i, k
\end{aligned} \tag{3}
$$

Let $\mu_{\tilde{C}_{0k}}, \ \mu_{\tilde{C}_{ik}}, \ \mu_{\tilde{B}_i}, \ \mu_{\tilde{A}_{0kj}}$ and $\tilde{A}_{ikj}$ be membership functions of $\tilde{C}_{0k}, \ \tilde{C}_{ik}, \ \tilde{B}_i, \ \tilde{A}_{0kj}$ and $\tilde{A}_{ikj} \ \ \forall i, j, k$ respectively. Without loss of generality, all $\tilde{C}_{0k}, \ \tilde{C}_{ik}, \ \tilde{B}_i, \ \tilde{A}_{0kj}$ and $\tilde{A}_{ikj} \ \forall i, j, k$ in (3) are assumed to be convex fuzzy numbers. Therefore, the objective value $\tilde{Z}$ will be fuzzy as well. On applying the α-cuts ($\alpha \in [0, 1]$) of $\tilde{C}_{0k}, \ \tilde{C}_{ik}, \ \tilde{B}_i$, and r-cuts ($r \in [0, 1]$) of $\tilde{A}_{0kj}$, $\tilde{A}_{ikj} \ \ \forall i, j, k$ and denoting them by $\tilde{C}_{0k}, \ \tilde{C}_{ik}, \tilde{B}_i, \ \tilde{A}_{0kj}$ and $\tilde{A}_{ikj} \ \ \forall i, j, k$ respectively and further, using Zadeh's extension principle [37], we define the membership function $\mu_{\tilde{Z}}$ as follow

$$
\begin{aligned}
\mu_{\tilde{Z}}(z) = \sup_{a,b,c} \ \min \{ \ & (C_{0k})_\alpha^L \le c_{0k} \le (C_{0k})_\alpha^U , (C_{ik})_\alpha^L \le c_{ik} \le (C_{ik})_\alpha^U , \\
& (B_i)_\alpha^L \le b_i \le (B_i)_\alpha^U , (A_{0kj})_r^L \le a_{0kj} \le (A_{0kj})_r^U , \\
& (A_{ikj})_r^L \le a_{ikj} \le (A_{ikj})_r^U , \quad \forall \, i, j, k \}
\end{aligned} \tag{4}
$$

Since a fuzzy number is uniquely represented by its α-cut, which is a closed interval for all α, this enables us to define arithmetic operations on fuzzy number in term of their (α, r)-cuts.

$$Z_{\alpha r} = \max_{\lambda} \prod_{k=1}^{l_0} \left(\frac{[(C_{0k})_\alpha^L, (C_{0k})_\alpha^U]}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{m} (\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)^{\left(\sum_{k=1}^{l_i} \lambda_{ik}\right)}$$
$$\prod_{k=1}^{l_i} \left(\frac{[(C_{ik})_\alpha^L, (C_{ik})_\alpha^U]}{[(B_i)_\alpha^L, (B_i)_\alpha^U]\ \lambda_{ik}} \right)^{\lambda_{ik}})$$

Subject to

$$\begin{aligned}
&\sum_{k=1}^{l_0} \lambda_{0k} = 1 && (Normal\ Condition)\\
&\sum_{i=0}^{m} \sum_{k=1}^{l_i} a_{ikj}\, \lambda_{ik} = 0 \qquad j = 1, ..., n && (Orthogonal\ Conditions)\\
&(A_{ikj})_r^L \leq a_{ikj} \leq (A_{ikj})_r^U &&\\
&\quad \lambda_{ik} \geq 0 \quad \forall i, k, j && (5)
\end{aligned}$$

In fact, calculation of $\mu_{\tilde{Z}}$ of the form (4) is difficult. To obtain the membership function of objective value, we need to find the left shape and right shape functions of $\mu_{\tilde{Z}}$, which is equivalent to finding the upper and lower bounds of objective value $\tilde{Z}$ at different (α, r) level possibility.

3. SOLUTION METHODOLOGY

For fuzzy numbers, $\tilde{A} = [A_\alpha^L, A_\alpha^U]$ and $\tilde{B} = [B_\alpha^L, B_\alpha^U]$ in which $\tilde{A} \in F(R^{\geq 0})$ and $\tilde{B} \in F(R^{>0})$, we have:

$$\left(\frac{\tilde{A}}{\tilde{B}} \right)_\alpha = \left[\frac{A_\alpha^L}{B_\alpha^U}, \frac{A_\alpha^U}{B_\alpha^L} \right] \qquad \forall \alpha \in [0, 1].$$

Therefore, to find the lower bound of the objective value, we choose $(C_{0k})_\alpha^L$ as the lower bound of the interval $[(C_{0k})_\alpha^L, (C_{0k})_\alpha^U]$ and in the same manner we choose $\left[\frac{(C_{ik})_\alpha^l}{(B_i)_\alpha^U} \right]$ as the lower bound of $\frac{\left[(C_{ik})_\alpha^l\ ,\ (C_{ik})_\alpha^U\right]}{[(B_i)_\alpha^L\ ,\ (B_i)_\alpha^L]}$, which converts (5) in the form of (6) as below:

$$Z^L_{\alpha r} = \max_{\lambda} \prod_{k=1}^{l_0} \left(\frac{(C_{0k})^L_\alpha}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{m} (\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)^{\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)} \prod_{k=1}^{l_i} \left(\frac{(C_{ik})^L_\alpha}{(B_i)^U_\alpha \ \lambda_{ik}} \right)^{\lambda_{ik}}$$

Subject to

$$\begin{aligned} &\sum_{k=1}^{l_0} \lambda_{0k} = 1 \\ &\sum_{i=0}^{m} \sum_{k=1}^{l_i} a_{ikj} \, \lambda_{ik} = 0 \qquad j = 1, ..., n \\ &(A_{ikj})^L_r \leq \ a_{ikj} \ \leq (A_{ikj})^U_r \\ &\qquad \lambda_{ik} \geq 0 \quad \forall i, k, j \end{aligned} \tag{6}$$

Also, to obtain the upper bound of the objective value, we choose $(C_{0k})^U_\alpha$ as the upper bound of the interval $[(C_{0k})^L_\alpha, (C_{0k})^U_\alpha]$ and in the same manner $\left[\frac{(C_{ik})^U_\alpha}{(B_i)^L_\alpha} \right]$ as the upper bound of $\frac{[(C_{ik})^l_\alpha \ , \ (C_{ik})^U_\alpha]}{[(B_i)^L_\alpha \ , \ (B_i)^L_\alpha]}$ through which (5) can be reformulated as (7).

$$Z^U_{\alpha} = \max_{\lambda} \prod_{k=1}^{l_0} \left(\frac{(C_{0k})^U_\alpha}{\lambda_{0k}} \right)^{\lambda_{0k}} \prod_{i=1}^{m} (\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)^{\left(\sum_{k=1}^{l_i} \lambda_{ik} \right)} \prod_{k=1}^{l_i} \left(\frac{(C_{ik})^U_\alpha}{(B_i)^L_\alpha \ \lambda_{ik}} \right)^{\lambda_{ik}}$$

Subject to

$$\begin{aligned} &\sum_{k=1}^{l_0} \lambda_{0k} = 1 \\ &\sum_{i=0}^{m} \sum_{k=1}^{l_i} a_{ikj} \, \lambda_{ik} = 0 \qquad j = 1, ..., n \\ &(A_{ikj})^L_\alpha \leq \ a_{ikj} \ \leq (A_{ikj})^U_\alpha \\ &\qquad \lambda_{ik} \geq 0 \quad \forall i, \, k, \, j \end{aligned} \tag{7}$$

From the (α, r) acceptable value of $\tilde{Z}$ for different values of r, we can obtain the crisp interval $[Z^L_{\alpha r} \ , \ Z^U_{\alpha r}]$ from (6) and (7) respectively.
The feasible regoins defined by α_1 in (6) and (7) are smaller than those defined by α_2 with regards to $0 \leq \alpha_2 < \alpha_1 \leq 1$ for two possibility levels α_1 and α_2 which results $Z^L_{\alpha_1 r} \geqslant Z^L_{\alpha_2 r}$ and $Z^U_{\alpha_1 r} \leqslant Z^U_{\alpha_2 r}$.
According to nondecreasing left shape function $L(Z) = [Z^L_{\alpha r}]^{-1}$ and nonincreasing

right shape function $R(Z) = [Z^{U}_{\alpha r}]^{-1}$, the membership function $\mu_{\tilde{Z}}$ for $L(Z)$ and $R(Z)$ is constructed as:

$$\mu_{\tilde{Z}} = \begin{cases} L(Z) & Z^{L}_{(\alpha=0)r} \leq z \leq Z^{L}_{(\alpha=1)r} \\ 1 & Z^{L}_{(\alpha=1)r} \leq z \leq Z^{U}_{(\alpha=1)r} \\ R(Z) & Z^{U}_{(\alpha=1)r} \leq z \leq Z^{U}_{(\alpha=0)r} \end{cases}$$

4. NUMERICAL EXAMPLE

Consider the following geometric programming problem with fuzzy exponents in the objective function and the constraints.

$$\begin{aligned} &\min_{t}\ (36,\,40,\,42)\,t_1^{-1}\,t_2^{(-0.6,\,-0.5,\,-0.4)}\,t_3^{-1} + 20\ t_1\,t_2\,t_4 \\ &\textit{Subject to} \\ &\quad t_1^3\,t_2^{(0.7,\,0.75,\,0.8)}\,t_3 + (3,\,4,\,5)\,t_2^{0.5}\,t_4^{(-2.2,\,-2,\,-1.8)} \leq (2,\,3,\,5) \\ &\quad 8\,t_1^{(-1.2,\,-1,\,-0.8)}\,t_2^{-1}\,t_3\,t_4 \leq 1 \\ &\quad t_j > 0 \quad j = 1, ..., 4 \end{aligned} \tag{8}$$

The dual form of (8) is as followes:

$$\tilde{Z} = \max_{\lambda} \left(\frac{(36,\,40,\,42)}{\lambda_{01}}\right)^{\lambda_{01}} \left(\frac{20}{\lambda_{02}}\right)^{\lambda_{02}} \left(\frac{1}{(2,3,5)\lambda_{11}}\right)^{\lambda_{11}} \left(\frac{(3,4,5)}{(2,3,5)\lambda_{12}}\right)^{\lambda_{12}}$$

$$(8)^{\lambda_{21}}\ (\lambda_{11} + \lambda_{12})^{(\lambda_{11}+\lambda_{12})}$$

Subject to

$$-\lambda_{01} + \lambda_{02} + 3\,\lambda_{11} + (-1.2,\ -1,\ -0.8)\,\lambda_{21} = 0$$

$$(-0.6,\,-0.5,\,-0.4)\,\lambda_{01} + \lambda_{02} + (0.7,\,0.75,\,0.8)\,\lambda_{11} + 0.5\,\lambda_{12} - \lambda_{21} = 0$$

$$-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$$

$$\lambda_{02} + (-2.2,\,-2,\,-1.8)\,\lambda_{12} + \lambda_{21} = 0$$

$$\lambda_{01} + \lambda_{02} = 1$$

$$\lambda_{ik} \geq 0 \quad \forall i,\,k$$

The $Z^L_{\alpha r}$ can be calculated by performing the Model (6) and $Z^U_{\alpha r}$ by the Model (7) as follows

$$Z^L_{\alpha} = \max_{\lambda} \left(\frac{(36+4\alpha)}{\lambda_{01}}\right)^{\lambda_{01}} \left(\frac{20}{\lambda_{02}}\right)^{\lambda_{02}} \left(\frac{1}{(5-2\alpha)\lambda_{11}}\right)^{\lambda_{11}} \left(\frac{(3+\alpha)}{(5-2\alpha)\lambda_{12}}\right)^{\lambda_{12}}$$
$$(8)^{\lambda_{21}} \ (\lambda_{11}+\lambda_{12})^{(\lambda_{11}+\lambda_{12})}$$

Subject to

$$-\lambda_{01} + \lambda_{02} + 3\,\lambda_{11} + a_{211}\,\lambda_{21} = 0$$

$$a_{012}\,\lambda_{01} + \lambda_{02} + a_{112}\,\lambda_{11} + 0.5\,\lambda_{12} - \lambda_{21} = 0$$

$$-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$$

$$\lambda_{02} + a_{124}\,\lambda_{12} + \lambda_{21} = 0$$

$$\lambda_{01} + \lambda_{02} = 1$$

$$(-1.2 + 0.2r) \le a_{211} \le (-0.8 - 0.2r)$$
$$(-0.6 + 0.1r) \le a_{012} \le (-0.4 - 0.1r)$$
$$(0.7 + 0.05r) \le a_{112} \le (0.8 - 0.05r)$$
$$(-2.2 + 0.2r) \le a_{124} \le (-1.8 - 0.2r)$$
$$\lambda_{ik} \ge 0 \quad \forall i,\, k$$

Table 1: Lower bounds of the optimal value $Z^L_{\alpha r}$

	$\downarrow r$ / $\alpha \rightarrow$	0.00	0.25	0.50	0.75	1.00
state (1)	0.00	105.6194	117.3167	130.9025	147.0518	166.8195
	0.25	107.1390	119.2289	133.2972	150.0532	170.6070
	0.50	108.6877	121.1890	135.7644	153.1601	174.5459
	0.75	110.2633	123.1957	138.3044	156.3750	178.6415
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(2)	0.00	105.7386	119.2822	135.2639	154.5846	178.6717
	0.25	107.1984	120.6836	136.5628	155.7162	179.5354
	0.50	108.7035	122.1429	137.9365	156.9454	180.5287
	0.75	110.2572	123.6631	139.3870	158.2728	181.6501
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(3)	0.00	105.9267	117.7360	131.4705	147.8196	167.8621
	0.25	107.3523	119.5248	133.7033	150.6084	171.3686
	0.50	108.8167	121.3716	136.0194	153.5136	175.0364
	0.75	110.3201	123.2785	138.4225	156.5417	178.8761
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(4)	0.00	105.9698	119.5997	135.7014	155.1904	179.5181
	0.25	107.3642	120.9166	136.8890	156.1726	180.1775
	0.50	108.8090	122.2949	138.1527	157.2511	180.9616
	0.75	110.3075	123.7375	139.4946	158.4264	181.8691
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state (5)	0.00	114.3829	126.7323	140.9667	157.7490	178.1094
	0.25	114.2248	126.8822	141.5279	158.8661	179.9943
	0.50	113.7688	126.7052	141.7319	159.5948	181.4597
	0.75	112.9915	126.1725	141.5433	159.8913	182.4510
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(6)	0.00	114.7284	129.2709	146.3344	166.8364	192.2238
	0.25	114.4116	128.6669	145.3769	165.4336	190.2420
	0.50	113.8421	127.8104	144.1726	163.7975	188.0527
	0.75	113.0002	126.6791	142.6957	161.8978	185.6189
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(7)	0.00	115.2110	127.7051	142.1192	159.1294	179.7869
	0.25	114.7814	127.5464	142.3277	159.8402	181.1987
	050	114.0739	127.0784	142.1927	160.1700	182.1883
	0.75	113.0987	126.3101	141.7211	160.1229	182.7563
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989

	↓ r / α →	0.00	0.25	0.50	0.75	1.00
state(8)	0.00	115.5574	130.2476	147.4976	168.2408	193.9493
	0.25	114.9536	129.3140	146.1584	166.3907	191.4352
	0.50	114.1338	128.1669	144.6135	164.3502	188.7574
	0.75	113.1022	126.8101	142.8654	162.1197	185.9130
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(9)	0.00	104.9160	117.0065	131.1632	148.1343	169.0964
	0.25	106.9428	119.3579	133.8905	151.3073	172.8137
	0.50	108.7790	121.5175	136.4268	154.2929	176.3517
	0.75	110.4205	123.4818	138.7696	157.0905	179.7126
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(10)	0.00	104.3995	118.0956	134.3624	154.1623	179.0272
	0.25	106.6384	120.3176	136.5065	156.1367	180.6869
	0.50	108.6298	122.2499	138.3111	157.7123	181.8758
	0.75	110.3721	123.8932	139.7803	158.8977	182.6088
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(11)	0.00	103.7664	115.8436	130.0133	147.0359	168.1092
	0.25	106.3929	118.8251	133.3977	150.8872	172.5165
	0.50	108.5921	121.3571	136.3098	154.2431	176.4052
	0.75	110.4038	123.4847	138.8011	157.1632	179.8456
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(12)	0.00	102.9904	116.5854	132.7592	152.4804	177.2931
	0.25	105.9644	119.6202	135.8004	155.4447	180.0453
	0.50	108.3970	122.0292	138.1167	157.5649	181.8072
	0.75	110.3461	123.8840	139.7965	158.9513	182.7183
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
state(13)	0.00	118.7248	132.0202	147.4587	165.8024	188.2415
	0.25	116.9060	130.2101	145.6895	164.1214	186.7215
	0.50	115.1586	128.4802	144.0114	162.5453	185.3248
	0.75	113.4788	126.8270	142.4214	161.0720	184.0508
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
sate(14)	0.00	118.4241	133.7704	151.8835	173.7824	201.0800
	0.25	116.7268	131.5398	148.9852	170.0278	196.1930
	0.50	115.0668	129.3764	146.1939	166.4348	191.5437
	0.75	113.4453	127.2793	143.5057	162.9947	187.1171
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989

	$\downarrow r$ / $\alpha \rightarrow$	0.00	0.25	0.50	0.75	1.00
	0.00	118.8878	132.2963	147.8887	166.4430	189.1763
	0.25	117.0325	130.4188	146.0103	164.5962	187.4120
state(15)	0.50	115.2464	128.6210	144.2249	162.8588	185.7788
	0.75	113.5247	126.8986	142.5282	161.2275	184.2749
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989
	0.00	118.4135	133.8283	152.0444	174.0958	201.6211
	0.25	116.7652	131.6372	149.1684	170.3351	196.6821
state(16)	0.50	115.1193	129.4724	146.3517	166.6804	191.9162
	0.75	113.4832	127.3407	143.5999	163.1347	187.3226
	1.00	111.8630	125.2472	140.9166	159.6996	182.8989

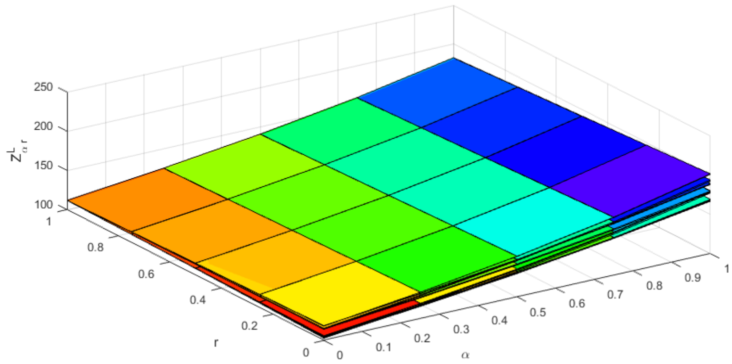

Figure 1: Lower bounds $Z^L_{\alpha r}$ for the objective value

$$Z^U_{\alpha r} = \max_{\lambda} \left(\frac{(42-2\alpha)}{\lambda_{01}}\right)^{\lambda_{01}} \left(\frac{20}{\lambda_{02}}\right)^{\lambda_{02}} \left(\frac{1}{(2+\alpha)\lambda_{11}}\right)^{\lambda_{11}} \left(\frac{(5-\alpha)}{(2+\alpha)\lambda_{12}}\right)^{\lambda_{12}}$$

$$(8)^{\lambda_{21}} \; (\lambda_{11}+\lambda_{12})^{(\lambda_{11}+\lambda_{12})}$$

subject to

$$-\lambda_{01} + \lambda_{02} + 3\,\lambda_{11} + a_{211}\,\lambda_{21} = 0$$

$$a_{012}\,\lambda_{01} + \lambda_{02} + a_{112}\,\lambda_{11} + 0.5\,\lambda_{12} - \lambda_{21} = 0$$

$$-\lambda_{01} + \lambda_{11} + \lambda_{21} = 0$$

$$\lambda_{02} + a_{124}\,\lambda_{12} + \lambda_{21} = 0$$

$$\lambda_{01} + \lambda_{02} = 1$$

$$(-1.2 + 0.2r) \le a_{211} \le (-0.8 - 0.2r)$$
$$(-0.6 + 0.1r) \le a_{012} \le (-0.4 - 0.1r)$$
$$(0.7 + 0.05r) \le a_{112} \le (0.8 - 0.05r)$$
$$(-2.2 + 0.2r) \le a_{124} \le (-1.8 - 0.2r)$$
$$\lambda_{ik} \ge 0 \quad \forall i,\, k$$

Table 2: Upper bounds of the optimal value $Z^U_{\alpha r}$

	$\downarrow r$ / $\alpha \rightarrow$	0.00	0.25	0.50	0.75	1.00
state(1)	0.00	234.4367	213.6795	195.9357	180.4844	166.8195
	0.25	241.2236	219.5022	200.9633	184.8432	170.6070
	0.50	248.3574	225.6060	206.2196	189.3884	174.5459
	0.75	255.8590	232.0064	211.7160	194.1280	178.6415
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(2)	0.00	264.5794	237.7494	215.1239	195.6702	178.6717
	0.25	263.9667	237.6592	215.4331	196.2899	179.5354
	0.50	263.6334	237.7969	215.9301	197.0652	180.5287
	0.75	263.5654	238.1530	216.6086	197.9927	181.6501
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(3)	0.00	236.4063	215.3328	197.3399	181.6888	167.8621
	0.25	242.6814	220.7212	201.9949	185.7254	171.3686
	0.50	249.3105	226.3996	206.8886	189.9584	175.0364
	0.75	256.3229	232.3907	212.0384	194.4016	178.8761
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(4)	0.00	266.2284	239.1141	216.2713	196.6489	179.5181
	0.25	265.2308	238.7038	216.3094	197.0351	180.1775
	0.50	264.4936	238.5068	216.5245	197.5693	180.9616
	0.75	264.0040	238.5146	216.9108	198.2484	181.8691
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(5)	0.00	247.7874	226.5879	208.3391	192.3436	178.1094
	0.25	252.6213	230.4286	211.3893	194.7536	179.9943
	0.50	256.9593	233.7884	213.9771	196.7221	181.4597
	0.75	260.7087	236.5864	216.0320	198.1871	182.4510
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(6)	0.00	283.0472	254.8707	230.9833	210.3430	192.2238
	0.25	278.3573	251.0478	227.8776	207.8424	190.2420
	0.50	273.6233	247.1202	224.6223	205.1588	188.0527
	0.75	268.7782	243.0312	221.1689	202.2502	185.6189
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(7)	0.00	250.4670	228.9409	210.4255	194.2082	179.7869
	0.25	254.6041	232.1547	212.9077	196.1009	181.1987
	0.50	258.2074	234.8628	214.9125	197.5442	182.1883
	0.75	261.2633	237.0561	216.4347	198.5361	182.7563
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989

	$\downarrow r$ / $\alpha \rightarrow$	0.00	0.25	0.50	0.75	1.00
	0.00	285.8771	257.3336	233.1513	212.2695	193.9493
	0.25	280.3570	252.7772	229.3912	209.1805	191.4352
state(8)	0.50	274.8447	248.1666	225.5301	205.9549	188.7574
	0.75	269.3164	243.4856	221.5575	202.5866	185.9130
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	240.7453	218.5521	199.7133	183.4169	169.0964
	0.25	246.6418	223.7829	204.3729	187.5774	172.8137
state(9)	0.50	252.4309	228.8802	208.8796	191.5700	176.3517
	0.75	258.1283	233.8553	213.2407	195.3992	179.7126
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	267.4016	239.6046	216.2949	196.3590	179.0272
	0.25	267.5081	240.3064	217.4246	197.7975	180.6869
state(10)	0.50	266.9079	240.3716	217.9796	198.7161	181.8758
	0.75	265.6431	239.8330	217.9852	199.1345	182.6088
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	240.1346	217.7751	198.8280	182.4652	168.1092
	0.25	246.7571	223.7357	204.2109	187.3349	172.5165
state(11)	0.50	252.8307	229.1515	209.0559	191.6757	176.4052
	0.75	258.4634	234.1180	213.4485	195.5647	179.8456
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	265.4099	237.6435	214.3930	194.5348	177.2931
	0.25	267.0008	239.7216	216.7982	197.1545	180.0453
state(12)	0.50	267.0929	240.4556	217.9929	198.6805	181.8072
	0.75	265.9384	240.0606	218.1621	199.2730	182.7183
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	264.9264	241.4013	221.2802	203.7503	188.2415
	0.25	264.3186	240.4605	220.0902	202.3724	186.7215
state(13)	0.50	263.9157	239.6973	219.0563	201.1328	185.3248
	0.75	263.7241	239.1156	218.1801	200.0316	184.0508
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
	0.00	298.3973	268.0098	242.3793	220.3392	201.0800
	0.25	288.9065	260.0235	235.6173	214.5939	196.1930
state(14)	0.50	280.0005	252.5039	229.2290	209.1476	191.5437
	0.75	271.6300	245.4138	223.1864	203.9792	187.1171
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989

	↓ r / α →	0.00	0.25	0.50	0.75	1.00
state(15)	0.00	266.8524	242.9851	222.5966	204.8548	189.1763
	0.25	265.7362	241.6263	221.0599	203.1869	187.4120
	0.50	264.8435	240.4606	219.6916	201.6672	185.7788
	0.75	264.1795	239.4904	218.4925	200.2948	184.2749
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989
state(16)	0.00	299.6849	269.0237	243.1893	220.9960	201.6211
	0.25	289.9987	260.8982	236.3279	215.1793	196.6821
	0.50	280.7968	253.1492	229.7594	209.5891	191.9162
	0.75	272.0555	245.7617	223.4748	204.2212	187.3226
	1.00	263.7508	238.7196	217.4640	199.0698	182.8989

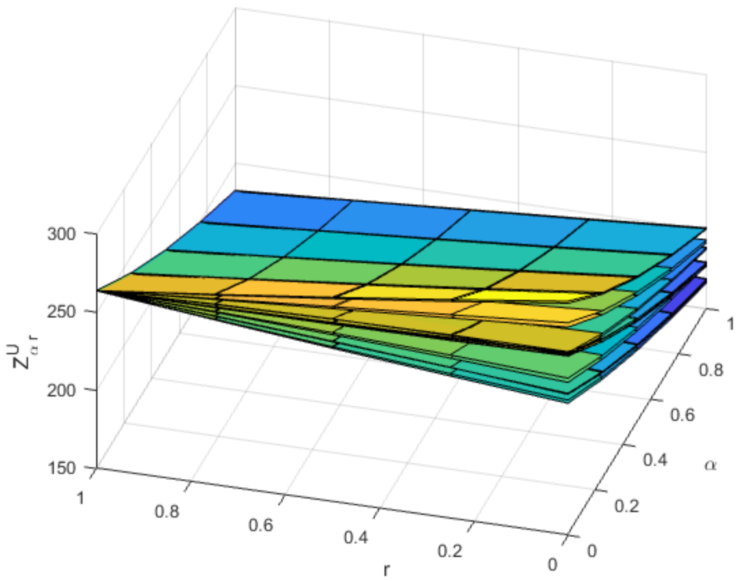

Figure 2: Upper bounds $Z^U_{\alpha r}$ for the objective value

The upper and lower bounds of the objective value for different levels of (α, r)-values are obtained and illustrated in the Figure 3.

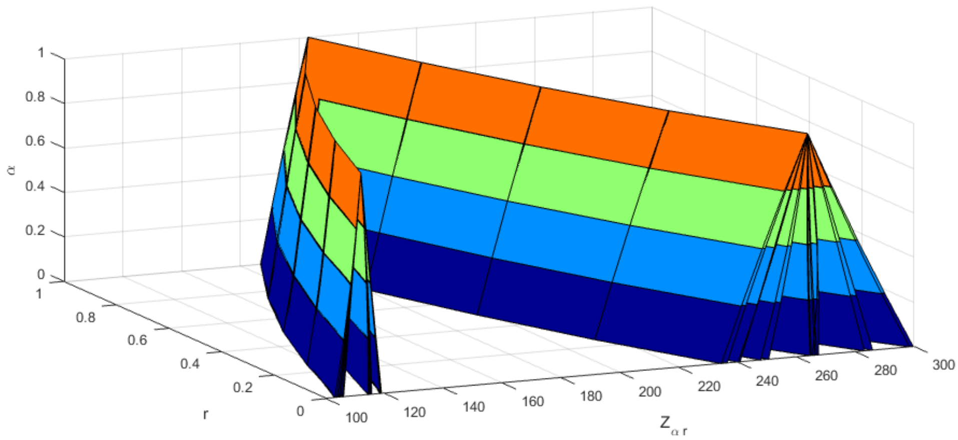

Figure 3: General graphical representation of $Z_{\alpha r}$ for the objective value

The Figures 4 and 5 represent the membership function of $Z^L_{\alpha r}$ and $Z^U_{\alpha r}$.

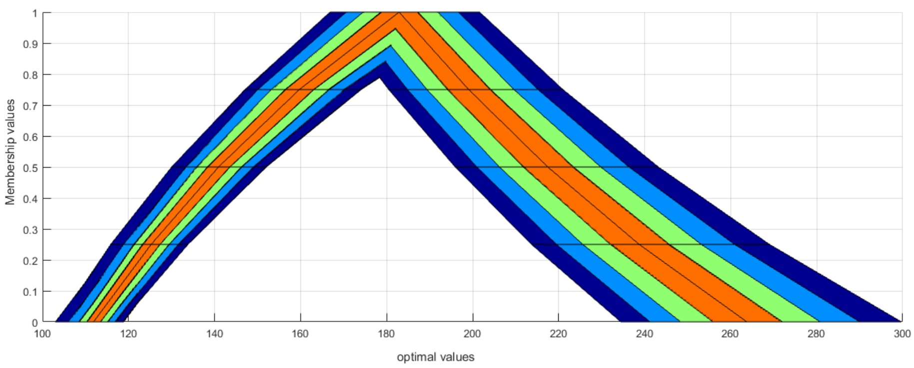

Figure 4: General form of different steps of LR representation of $Z_{\alpha r}$

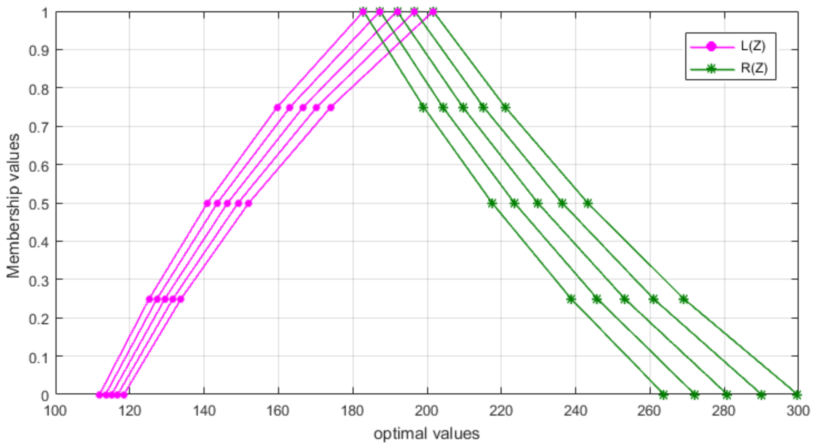

Figure 5: Different steps of LR representation of $Z_{\alpha r}$ of state (16) for the tables 1 and 2.

5. CONCLUSION

Due to uncertainty of design parameters and the closeness of fuzzy logic concept to such problems, which have many applications in engineering design, economics and management, we decided to study geometric programming with full fuzziness in exponents and coefficients of objective function and constraints as well.
In fact, the full fuzziness in geometric programming helps us to get the result that is much closer to the real optimal solution of the problem due to uncertainty of the parameters in the real physical world.
A very clear representation of fuzzy behavior of the objective function and membership values is given for different steps of LR fuzzy types in Figures 1 to 4. We compared our results with (Liu 2007) and got much more accurate result for optimum value of the problem. The extension of this problem can be applied to interval valued geometric programming and fractional geometric programming, too.

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