

MULTI-OBJECTIVE STOCHASTIC TRANSPORTATION PROBLEM INVOLVING THREE-PARAMETER EXTREME VALUE DISTRIBUTION

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Abstract: In this paper, we considered a multi-objective stochastic transportation problem where the supply and demand parameters follow extreme value distribution having three-parameters. The proposed mathematical model for stochastic transportation problem cannot be solved directly by mathematical approaches. Therefore, we converted it to an equivalent deterministic multi-objective mathematical programming problem. For solving the deterministic multi-objective mathematical programming problem, we used an ε -constraint method. A case study is provided to illustrate the methodology.

Keywords: Multi-Objective Programming, Stochastic Programming Problem, Transportation Problem, Extreme Value Distribution, ε -Constraint Method.

MSC: 90B06, 90C15, 90C29.

1. INTRODUCTION

Mathematical models for the transportation problem are concerned about the optimal way by which an item produced at various industrial facilities can be transported to various stockrooms. To solve a transportation problem means to fulfill the destination requirements within the limitation requirements of operating production capacity constraints at the minimum cost. However, in real life situation, we may face multi-objective transportation problems.

Multi-objective transportation problem(MOTP) is one of the vector minimum linear programming problems from the various classes, in which constraints are imbalanced and every objective is non-commensurable and conflicts with each other.

The parameters of the MOTP are available amounts at the supply focuses, and amounts required at the request focuses. In general, in real life situation, these parameters are unstable or unknown exactly. Lack of exactness is known as randomness or vagueness. Randomness is addressed by probability theory and vagueness by fuzzy set theory. When a mathematical programming problem deals with uncertainty, it is known as a stochastic mathematical model. One of the popular ways to address randomness is via a distribution function depending on regression.

In the real world decision making circumstances, we regularly need to settle on a choice under questionable information or data. In many concrete situations, it is difficult to present the mathematical models using random parameters. So, true issues have been demonstrated by thinking about probabilistic vulnerability parameters. Stochastic programming (SP) is a standout amongst the most critical methodologies to handle the uncertainty.

Stochastic programming(SP) problem is one of the mathematical programming problems that involves randomness. At the point when uncertainty occurs on the market demands for a commodity, the issue of booking shipments from supply points to demand points is called a stochastic transportation problem [27].

In general, the coefficient of the MOTP are described by unverifiable parameters, for example, by random, fuzzy, and multi-choice parameters. Different researchers have been considering different indicators containing random variables such as normal, log-normal, exponential, Cauchy, Weibull and others for the source and destination parameters of MOTP in SP model. Therefore, in our work, attention has been given to solving the above problem as a multi-objective stochastic transportation problem(MOSTP) where the supply and demand parameters follow three-parameter extreme value distribution.

This paper is organized as follows. Following the introduction section, in Section 2, the literature survey is stated. Basic preliminary is included in Section 3. In Section 4 the mathematical model is defined and its deterministic equivalent

form is derived. Case study, its discussion, and results are included in Section 5 and Section 6, respectively. The paper is concluded in Section 7.

2. LITERATURE SURVEY

In mathematics and economics, the study of optimal transportation and allocation of resources is known as the transportation theory. A French mathematician, Gaspard Monge [12], formalized the problem in 1781, but major advances in the field of transportation were made by Leonid Kantorovich, a Soviet, Russian mathematician and economist, during Second World War. Due to the founder of the problem, it is sometimes referred to as the Monge-Kantorovich transportation problem. Hitchcock presented transportation problem in 1941, and Koopmans developed it in detail in 1960 [14, 12, 2, 13, 19]. Dantzig in 1963 primarily developed an efficient method for transportation problem derived from the simplex algorithm [28, 19].

Many researchers studied MOTP and its solution methods. Zangiabadi and Maleki [30] applied fuzzy goal programming to determine an optimal compromise solution for the MOTP by assuming that each objective function has a fuzzy goal. Mousa *et al.* [16] presented an efficient evolutionary algorithm for solving MOTP by integrating the merits of both genetic algorithm (GA) and local search (LS). Zaki *et al.* [29] proposed an efficient genetic algorithm for solving MOTP and the algorithms that integrated the merits of both GA and LS scheme. Yeola and Jahav [28] proposed a method parallel to New Row Maxima Method and used a fuzzy programming technique with fuzzy linear membership function for different costs to solve MOTP. In real life situations, we face the parameters of MOTP which are not steady, not known exactly, and stochastic in nature. In this regard, different researchers have discussed a stochastic programming (SP) problem.

Dantzig [6] formulated the stochastic programming (SP) model in 1955. The first optimization problem with disjoint chance constraints was defined by Charnes *et al.* [4]. Various models have been suggested by several researchers on stochastic linear programming [15, 9, 7, 8, 1, 3, 24].

Many researchers studied MOSTP and its solution methods. Mahapatra *et al.* [14] discussed the solution procedure of multi-objective minimization type problems (i.e. non-commensurable and conflicting in nature) where supplies and demands are normal random variables. They applied a fuzzy programming technique for solving the deterministic MOTP. Roy and Mahapatra [22] concentrated on MOSTP, involving an inequality type constraints in which all parameters are log-normal random variables and the coefficients of the objectives are interval numbers. They used weighted sum method for solving the equivalent deterministic problem. Again, Roy *et al.* [21], presented a stochastic transportation problem where supply and demand parameters follow exponential distribution and cost coefficients of objective function are multi-choice. Biswal and Samal [2] presented both single objective and multi-objective stochastic transportation problems with Cauchy random variables, or multi-choice type and their deterministic

equivalents. They used goal programming method to solve the equivalent deterministic problem by taking cost coefficients of the objective function, associated with transportation problem as multi-choice type. Roy[19] worked on multi-choice stochastic transportation problem where the supply and demand parameters of the constraints follow Weibull distribution. Quddoos *et al.*[18] developed on an multi-choice stochastic transportation problem and obtained a summed up equivalent deterministic model of it, where accessibility and request parameters follow general form of distribution. Roy *et al.* [23] investigated the study of multi-choice MOTP under the light of conic scalarizing function. Chávez *et al.* [5], taking genuine contextual investigation, broadened a stochastic multi-objective minimum cost flow model for perishable agrarian items by comprising street transportation. As of late, Roy *et al.* [20] examined MOTP under intuitionistic fuzzy environment.

Also, from the literature survey, we found papers of Quddoos *et al.*[18] and Mahapatra *et al.*[13]. Quddoos *et al.*[18] concentrated on multi-choice stochastic transportation problem where the supply and demand parameters of the constraints follow general form of distribution. In a similar manner, Mahapatra *et al.*[13] concentrated on multi-choice stochastic transportation problem where the supply and demand parameters of the constraints follow extreme value distribution. Both papers considered cost coefficients of the objective function associated with transportation problem are to be multi-choice type for a single objective stochastic transportation problem. In addition, the supply and demand parameters of the constraints follow two-parameter form of distribution. In this paper, a novel strategy is developed for multi-objective stochastic transportation problem(MOSTP) involving three-parameter extreme value distribution. The main difference between this paper and the above two papers is listed as follows. First, we are concentrated on multi-objective but not multi-choice. Second, the parameters involved in our model are three-parameter and not two-parameter as in both of the above two papers.

To set up the solution procedures for the above problem, it was changed into an equivalent deterministic model. Then, a standard mathematical programming technique is used to tackle it. The derived model is discussed in Section 4, and implemented on a case study in Section 5.

3. BASIC PRELIMINARIES

A historical survey by Kotz and Nadarajah [10] shows that the extreme value distribution has curious and fascinating number of variety of applications involving natural phenomena such as rainfall, floods, wind blasts, wind speeds, air contamination, corrosion, and delicate advanced mathematical results on point forms and consistently varying functions. Extreme value distributions are used to describe the limiting distribution of the minimum or maximum of n observations selected from an exponential family of distributions such as normal, gamma, and exponential. It is also used to model the distributions of breaking strength of metals, capacitor breakdown voltage, and gust velocities encountered by airplanes [11].

Further, Parsons and Lal [17] in their paper, found that the extreme value distribution better fits to data than three-parameter Weibull [11]. Kotz and Nadarajah [10] usually considered extreme value distributions to compromise the following three families:

Type 1, (Gumbel-type distribution) Probability density function of a random variable X is given by:

$$f(x) = \frac{1}{\theta} e^{\frac{x-\mu}{\theta}} \exp[-e^{\frac{x-\mu}{\theta}}], \quad -\infty < x < \infty, \theta > 0. \quad (1)$$

where μ is location parameter and θ is scale parameter.

Type 2, (Frechet-type distribution) Probability density function of a random variable X is given by:

$$f(x) = \begin{cases} 0, & x < \mu, \\ \frac{\alpha}{\theta} \left(\frac{x-\mu}{\theta}\right)^{-\alpha-1} e^{-\left(\frac{x-\mu}{\theta}\right)^{-\alpha}} & x \geq \mu. \end{cases} \quad (2)$$

where μ is location parameter and $\theta > 0$ is scale parameter and $\alpha > 0$ shape parameter.

Type 3, (Weibull-type distribution) Probability density function of a random variable X is given by:

$$f(x) = \begin{cases} \frac{\alpha}{\theta} \left(\frac{\mu-x}{\theta}\right)^{\alpha-1} e^{-\left(\frac{\mu-x}{\theta}\right)^{\alpha}} & x \leq \mu, \\ 0, & x > \mu. \end{cases} \quad (3)$$

where μ is location parameter and $\theta > 0$ is scale parameter and $\alpha > 0$ shape parameter.

The three types of distribution are presented as members of a single family, called generalized extreme value distributions [26, 10] with a probability density function of a random variable X given as follows.

$$f(x) = \frac{1}{\theta} \left[1 + \alpha \left(\frac{x-\mu}{\theta} \right) \right]^{-\frac{1}{\alpha}-1} e^{-\left[1 + \alpha \left(\frac{x-\mu}{\theta} \right) \right]^{-\frac{1}{\alpha}}} \quad (4)$$

where μ represents location parameter, $\theta > 0$ represents scale parameter, and $\alpha \neq 0$ shape parameter. The range of X depends on the value of α : it is bounded by $\mu + \left(\frac{\theta}{\alpha}\right)$ from above for $\alpha > 0$, i.e., $-\infty < x < \mu + \left(\frac{\theta}{\alpha}\right)$; and it is bounded from below for $\alpha < 0$ i.e., $\mu + \left(\frac{\theta}{\alpha}\right) < x < \infty$. The shape parameter α determines which extreme value distribution is represented.

So, in this paper, we present a solution procedure for MOSTP involving random variables which follow generalized extreme value distribution with known parameters. In the whole section of the paper we use extreme value distribution, or generalized extreme value distribution. To establish the solution procedures of the proposed problem, we transform the problem into an equivalent deterministic multi-objective model.

4. MULTI-OBJECTIVE STOCHASTIC TRANSPORTATION PROBLEM(MOSTP) INVOLVING THREE-PARAMETER EXTREME VALUE DISTRIBUTION

Mathematically, MOSTP where randomness is considered in the right-hand-side constraints is expressed as:

Model-I

$$\min: Z^k(x) = \sum_{u=1}^m \sum_{v=1}^n c_{uv}^k x_{uv}, \quad k = 1, 2, \dots, K. \quad (5)$$

Subject to:

$$P \left(\sum_{v=1}^n x_{uv} \leq a_u \right) \geq 1 - \gamma_u, \quad u = 1, 2, \dots, m. \quad (6)$$

$$P \left(\sum_{v=1}^n x_{uv} \geq b_v \right) \geq 1 - \delta_v, \quad v = 1, 2, \dots, n. \quad (7)$$

$$x_{uv} \geq 0, \quad \forall u \text{ and } v. \quad (8)$$

where $0 < \gamma_u < 1, \forall u$ and $0 < \delta_v < 1, \forall v$. $a_u (u = 1, 2, \dots, m)$ and $b_v (v = 1, 2, \dots, n)$ are considered as extreme value random variables, and $c_{uv}^k (u = 1, 2, \dots, m; v = 1, 2, \dots, n; k = 1, 2, \dots, K)$ are the cost coefficients associated with the decision variables in the objective function, P represents probability, γ_u and δ_v are the admissible violates in probability of the u^{th} and v^{th} constraints.

4.1. Transformation technique

Due to the presence of probabilistic parameters in the constraints, the proposed mathematical model can not be solved directly using mathematical methods. Therefore, the proposed problem is transformed to its equivalent multi-objective deterministic mathematical programming problem. In three different cases, the probabilistic constraints will be transformed to their equivalent deterministic constraints.

1. **Only $a_u (u = 1, 2, \dots, m)$ follow three-parameter extreme value distribution:** Here $a_u (u = 1, 2, \dots, m)$ are assumed independent RVs that follow extreme value distribution with location, scale, and shape parameters, represented as μ_u, θ_u , and α_u respectively, with aspiration level $0 < \gamma_u < 1$. The probability density function (pdf) of the u^{th} constraint a_u is given in (9).

$$f(a_u) = \frac{1}{\theta_u} \left(1 + \alpha_u \left(\frac{a_u - \mu_u}{\theta_u} \right) \right)^{\frac{-1}{\alpha_u} - 1} e^{-(1 + \alpha_u \left(\frac{a_u - \mu_u}{\theta_u} \right))^{\frac{-1}{\alpha_u}}}, \quad u = 1, 2, \dots, m.$$

(9)

where $\theta_u > 0, \alpha_u \neq 0$.

To solve the problems in (5) to (8), we establish the deterministic form of the problem. From chance constraints in (6), we have

$$\begin{aligned} P\left(\sum_{v=1}^n x_{uv} \leq a_u\right) &\geq 1 - \gamma_u \\ \implies P\left(a_u \geq \sum_{v=1}^n x_{uv}\right) &\geq 1 - \gamma_u \\ \implies \int_{\sum_{v=1}^n x_{uv}}^{\infty} f(a_u) da_u &\geq 1 - \gamma_u \\ \implies \int_{\sum_{v=1}^n x_{uv}}^{\infty} \frac{1}{\theta_u} \left(1 + \alpha_u \left(\frac{a_u - \mu_u}{\theta_u}\right)\right)^{\frac{-1}{\alpha_u} - 1} e^{-(1 + \alpha_u \left(\frac{a_u - \mu_u}{\theta_u}\right))^{\frac{-1}{\alpha_u}}} da_u &\geq 1 - \gamma_u \end{aligned}$$

Applying simple integration technique and rearranging the above equation, we get the result in (10).

$$\sum_{v=1}^n x_{uv} \leq \mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln\left(\frac{1}{\gamma_u}\right)}\right)^{\alpha_u} - 1 \right], \quad u = 1, 2, \dots, m. \tag{10}$$

Using (10) in (6), we establish an equivalent deterministic model of **Model-I** as expressed in **Model-II**.

Model-II

$$\text{min: } Z^k(x) = \sum_{u=1}^m \sum_{v=1}^n c_{uv}^k x_{uv}, \quad k = 1, 2, \dots, K. \tag{11}$$

Subject to:

$$\sum_{v=1}^n x_{uv} \leq \mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln\left(\frac{1}{\gamma_u}\right)}\right)^{\alpha_u} - 1 \right], \quad u = 1, 2, \dots, m. \tag{12}$$

$$\sum_{u=1}^m x_{uv} \geq b_v, \quad v = 1, 2, \dots, n. \tag{13}$$

$$x_v \geq 0, \quad v = 1, 2, \dots, n. \tag{14}$$

where

$$\sum_{u=1}^m \left(\mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln\left(\frac{1}{\gamma_u}\right)}\right)^{\alpha_u} - 1 \right] \right) \geq \sum_{v=1}^n b_v \text{ (feasibility condition)}$$

2. **Only $b_v(v = 1, 2, \dots, n)$ follow three parameter extreme value distribution:** Let $b_v(v = 1, 2, \dots, n)$ be independent RVs that follow extreme value distribution with location, scale, and shape parameters represented by $\mu'_v, \theta'_v,$ and α'_v respectively, with aspiration level $0 < \delta_v < 1$. The probability density function (pdf) of the v^{th} constraint b_v is given in (15).

$$f(b_v) = \frac{1}{\theta'_v} \left(1 + \alpha'_v \left(\frac{b'_v - \mu'_v}{\theta'_v} \right) \right)^{\frac{-1}{\alpha'_v} - 1} e^{-\left(1 + \alpha'_v \left(\frac{b_v - \mu'_v}{\theta'_v} \right) \right)^{\frac{-1}{\alpha'_v}}}, \quad v = 1, 2, \dots, n. \tag{15}$$

where $\theta'_v > 0, \alpha'_v \neq 0$.

To solve the problems in (5) to (8), we establish the deterministic form of the problem. From chance constraints in (7), we have

$$\begin{aligned} P \left(\sum_{u=1}^m x_{uv} \geq b_v \right) &\geq 1 - \delta_v \\ \implies P \left(b_v \leq \sum_{u=1}^m x_{uv} \right) &\geq 1 - \delta_v \\ \implies 1 - P \left(\sum_{u=1}^m x_{uv} \leq b_v \right) &\geq 1 - \delta_v \\ \implies \int_{\sum_{u=1}^m x_{uv}}^{\infty} f(b_v) db_v &\leq \delta_v \\ \implies \int_{\sum_{u=1}^m x_{uv}}^{\infty} \frac{1}{\theta'_v} \left(1 + \alpha'_v \left(\frac{b'_v - \mu'_v}{\theta'_v} \right) \right)^{\frac{-1}{\alpha'_v} - 1} e^{-\left(1 + \alpha'_v \left(\frac{b_v - \mu'_v}{\theta'_v} \right) \right)^{\frac{-1}{\alpha'_v}}} db_v &\leq \delta_v \end{aligned}$$

Applying simple integration technique and rearranging the above equation, we get the result in (16).

$$\sum_{u=1}^m x_{uv} \geq \mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1 - \delta_v} \right)} \right)^{\alpha'_v} - 1 \right], \quad v = 1, 2, \dots, n. \tag{16}$$

Using (16) in (7), we establish an equivalent deterministic model of **Model-I** as expressed in **Model-III**.

Model-III

$$\min: Z^k(x) = \sum_{u=1}^m \sum_{v=1}^n c_{uv}^k x_{uv}, \quad k = 1, 2, \dots, K. \tag{17}$$

Subject to:

$$\sum_{u=1}^m x_{uv} \geq \mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right], \quad v = 1, 2, \dots, n. \quad (18)$$

$$\sum_{v=1}^n x_{uv} \leq a_u, \quad u = 1, 2, \dots, m. \quad (19)$$

$$x_v \geq 0, \quad v = 1, 2, \dots, n. \quad (20)$$

where

$$\sum_{u=1}^m \left(\mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right] \right) \geq \sum_{v=1}^n b_v \text{ (feasibility condition)}$$

3. **Both** $a_u (u = 1, 2, \dots, m)$ **and** $b_v (v = 1, 2, \dots, n)$ **follow three parameter extreme value distribution:** Assume that both $a_u (u = 1, 2, \dots, m)$ and $b_v (v = 1, 2, \dots, n)$ are independent RVs that follow extreme value distribution. Using (12) in **Model-II**, we have

$$\sum_{v=1}^n x_{uv} \leq \mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right], \quad u = 1, 2, \dots, m.$$

and using (16) in **Model-III**, we have

$$\sum_{u=1}^m x_{uv} \geq \mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right], \quad v = 1, 2, \dots, n.$$

Using the above two equations, we establish an equivalent deterministic model of (5) to (8), expressed in **Model-IV**.

Model-IV

$$\min: Z^k(x) = \sum_{u=1}^m \sum_{v=1}^n c_{uv}^k x_{uv}, \quad k = 1, 2, \dots, K. \quad (21)$$

Subject to:

$$\sum_{v=1}^n x_{uv} \leq \mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right], \quad u = 1, 2, \dots, m. \quad (22)$$

$$\sum_{u=1}^m x_{uv} \geq \mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right], \quad v = 1, 2, \dots, n. \quad (23)$$

$$x_{uv} \geq 0, \forall u \text{ and } v. \quad (24)$$

where

$$\sum_{u=1}^m \left(\mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right] \right) \geq \sum_{v=1}^n \left(\mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right] \right)$$

(feasibility condition)

4.2. ε -Constraint method as a solution method for deterministic MOTP

Algorithm:

Step-1: Select $Z^k(x)$, (for $k = 1$) and solve it as a single objective mathematical programming problem subject to the constraints.

$$\min: Z^1(x) = \sum_{u=1}^m \sum_{v=1}^n c_{uv} x_{uv}$$

Subject to:

$$\sum_{v=1}^n x_{uv} \leq \mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right], \quad u = 1, 2, \dots, m.$$

$$\sum_{u=1}^m x_{uv} \geq \mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right], \quad v = 1, 2, \dots, n.$$

$$x_{uv} \geq 0, \forall u \text{ and } v.$$

where

$$\sum_{u=1}^m \left(\mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right] \right) \geq \sum_{v=1}^n \left(\mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right] \right)$$

(feasibility condition)

Let $x^{(1)}$ be the ideal solution. Then identify the ideal solutions $x^{(2)}, x^{(3)}, \dots, x^{(K)}$ for the second, third, ..., K^{th} different objective functions respectively.

Table 1: *Pay-Off Matrix*

	$Z^1(x)$	$Z^2(x)$...	$Z^K(x)$
$x^{(1)}$	$Z^{11}(x)$	$Z^{12}(x)$...	$Z^{1K}(x)$
$x^{(2)}$	$Z^{21}(x)$	$Z^{22}(x)$...	$Z^{2K}(x)$
\vdots	\vdots	\vdots	...	\vdots
$x^{(K)}$	$Z^{K1}(x)$	$Z^{K2}(x)$...	$Z^{KK}(x)$

Step-2: Formulate a pay-off matrix by evaluating all of the objective functions individually.

Step-3: Determine the bounds for k^{th} objective function $Z^k(x)$ as the best lower bound L_b^k and worst upper bound U_w^k , $k = 1, 2, \dots, K$. The bounds of ε_k , $k = 1, 2, \dots, K$, which is the point in the range of $Z^k(x)$, is also obtained as $L_b^k \leq \varepsilon_k \leq U_w^k$.

Step-4: Using ε_k , $k = 1, 2, \dots, K$, define K number of single objective different problems as follows.

- (i) For $k = 1$, $Z^k(x)$ is solved as a single objective mathematical programming problem subject to the original constraints and new constraint as $Z^k(x) \leq \varepsilon_k$, $k = 2, 3, \dots, K$ ($k \neq 1$).
- (ii) For $k = 2$, $Z^k(x)$ is solved as a single objective mathematical programming problem subject to the original constraints and new constraint as $Z^k(x) \leq \varepsilon_k$, $k = 1, 3, \dots, K$ ($k \neq 2$).
- (iii) Continue the process K times for K different objective functions.

Step-5: Solve the deterministic model by using an appropriate mathematical programming method to find an optimal compromise solution for different values of ε_k , $k = 1, 2, \dots, K$.

5. CASE STUDY

The “AAgfresh.com” (name changed) is an online vegetable and fruits marketplace to buy fresh vegetables and organic fruits in Cuttack and Bhubaneswar of Odisha, India. Currently, they do home delivery of fresh vegetables and fruits to their consumers in Bhubaneswar, Cuttack, Angul, Koraput of Odisha, India within 12 hours from the time of buying online. The main purpose is to maximize the profit by minimizing the transportation cost, transportation time or delivery time and loss during transportation through a given route. The total transportation cost of carrying per unit, transportation time per unit and loss during transportation per unit from sources to destinations along with availability and demands are

unit in this measurement is 100KG

represented by the matrix in Tables 2 to 4 with specified probability levels(SPL) respectively. However, the supply and demand may not be known previously. This is due to different unpredictable factors such as weather, season, different social activities, and so forth. After discussing with managers of the company and analyzing the past data, it is finalized that supply and demand parameters must follow extreme value distribution with known parameters. To manage the issue emerging due to previously mentioned cases, a MOSTP approach has been considered. Let x_{uv} represent the allocations(or amounts) which are nonnegative real variables. Let $A_u(u = 1, 2)$ represent sources where fruits and vegetables are accessible, $A_1 =$ Bhubaneswar, $A_2 =$ Cuttack, and $R_v(v = 1, 2, 3, 4)$ represent request where fruits and vegetables are required, $R_1 =$ Bhubaneswar, $R_2 =$ Cuttack, $R_3 =$ Angul, $R_4 =$ Koraput.

Table 2: *Transportation Cost per unit(in Rupees)*

	R_1	R_2	R_3	R_4	SPL of Supply
A_1	12	15	17	19	0.01
A_2	15	12	16	18	0.02
SPL of Demand	0.04	0.05	0.06	0.07	

Table 3: *Transportation time per unit(in hours)*

	R_1	R_2	R_3	R_4	SPL of Supply
A_1	0.5	1	1.4	1.75	0.01
A_2	1	0.5	1.3	1.7	0.02
SPL of Demand	0.04	0.05	0.06	0.07	

Table 4: *Loss during transportation per unit(in Rupees)*

	R_1	R_2	R_3	R_4	SPL of Supply
A_1	2	5	7	9	0.01
A_2	5	2	6	8	0.02
SPL of Demand	0.04	0.05	0.06	0.07	

By using the information provided in the Tables 2 to 4, one can formulate the mathematical stochastic transportation programming model as expressed in (25)-(29).

$$\min Z^1(x) = 12x_{11} + 15x_{12} + 17x_{13} + 19x_{14} + 15x_{21} + 12x_{22} + 16x_{23} + 18x_{24} \quad (25)$$

$$\min Z^2(x) = 0.5x_{11} + x_{12} + 1.4x_{13} + 1.75x_{14} + x_{21} + 0.5x_{22} + 1.3x_{23} + 1.7x_{24} \quad (26)$$

$$\min Z^3(x) = 2x_{11} + 5x_{12} + 7x_{13} + 9x_{14} + 5x_{21} + 2x_{22} + 6x_{23} + 8x_{24} \quad (27)$$

Subject to:

$$P\left(\sum_{v=1}^4 x_{uv} \leq A_u\right) \geq 1 - \gamma_u, \quad u = 1, 2. \quad (28)$$

$$P\left(\sum_{u=1}^2 x_{uv} \geq R_v\right) \geq 1 - \delta_v, \quad v = 1, 2, 3, 4. \quad (29)$$

$x_{uv} \geq 0, u = 1, 2, 3; v = 1, 2, 3, 4$ and $0 < \gamma_u < 1, 0 < \delta_v < 1, \forall u, v$.
 where Z^1 =Total transportation cost per unit(in Rupees), Z^2 =Total delivery time(in hours), and Z^3 =Total loss during transportation per unit(in Rupees).
 With specified probability level(SPL) of supplies, i.e., A_u for $u = 1, 2$, and demands, i.e., R_v for $v = 1, 2, 3, 4$, one can represent in Tables 5 and 6 respectively by taking three known parameters of extreme value distribution.

Table 5: Location, scale and shape parameters for supplies A_u with SPL values

Location parameter	Scale parameter	Shape parameter	SPL (γ_u)
$\mu_1 = 36.5$	$\theta_1 = 5.8$	$\alpha_1 = 9$	$\gamma_1 = 0.01$
$\mu_2 = 37$	$\theta_2 = 6.4$	$\alpha_2 = 10$	$\gamma_2 = 0.02$

Table 6: Location, scale and shape parameters for demands R_u with SPL values

Location parameter	Scale parameter	Shape parameter	SPL (δ_v)
$\mu'_1 = 25.45$	$\theta'_1 = 6.2$	$\alpha'_1 = 7$	$\delta'_1 = 0.04$
$\mu'_2 = 24.15$	$\theta'_2 = 6.9$	$\alpha'_2 = 6$	$\delta'_2 = 0.05$
$\mu'_3 = 13.5$	$\theta'_3 = 7.3$	$\alpha'_3 = 5$	$\delta'_3 = 0.06$
$\mu'_4 = 11.5$	$\theta'_4 = 7.8$	$\alpha'_4 = 4$	$\delta'_4 = 0.07$

Using the information provided in Tables 5 and 6, we developed the deterministic MOTP as expressed in (30) to (38).

$$\min Z^1(x) = 12x_{11} + 15x_{12} + 17x_{13} + 19x_{14} + 15x_{21} + 12x_{22} + 16x_{23} + 18x_{24} \quad (30)$$

$$\min Z^2(x) = 0.5x_{11} + x_{12} + 1.4x_{13} + 1.75x_{14} + x_{21} + 0.5x_{22} + 1.3x_{23} + 1.7x_{24} \quad (31)$$

$$\min Z^3(x) = 2x_{11} + 5x_{12} + 7x_{13} + 9x_{14} + 5x_{21} + 2x_{22} + 6x_{23} + 8x_{24} \quad (32)$$

Subject to:

$$\sum_{v=1}^4 x_{1v} \leq 35.8555563 \quad (33)$$

$$\sum_{v=1}^4 x_{2v} \leq 36.3600008 \quad (34)$$

$$\sum_{u=1}^2 x_{u1} \geq 24.98612715 \quad (35)$$

$$\sum_{u=1}^2 x_{u2} \geq 24.980376691 \quad (36)$$

$$\sum_{u=1}^2 x_{u3} \geq 12.0384627 \quad (37)$$

$$\sum_{u=1}^2 x_{u4} \geq 9.57421155 \quad (38)$$

$$x_{uv} \geq 0, \quad u = 1, 2; v = 1, 2, 3, 4$$

One can check that the feasibility condition is satisfied, i.e.,

$$\sum_{u=1}^2 \left(\mu_u + \frac{\theta_u}{\alpha_u} \left[\left(\frac{1}{\ln \left(\frac{1}{\gamma_u} \right)} \right)^{\alpha_u} - 1 \right] \right) = 72.2155571$$

$$\sum_{v=1}^4 \left(\mu'_v + \frac{\theta'_v}{\alpha'_v} \left[\left(\frac{1}{\ln \left(\frac{1}{1-\delta_v} \right)} \right)^{\alpha'_v} - 1 \right] \right) = 71.57917809$$

The deterministic MOTP expressed in (30) to (38) is transformed to a single objective transportation problem and solved using ε -constraint method. Applying

the steps of the ε -constraint method and LINGO [25] software, three ideal solutions are obtained as:

$$\begin{aligned} X^{(1)} &= (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24})^T \\ &= (24.98613, 0.0, 10.23305, 0.0, 0.0, 24.98038, 1.805413, 9.574212)^T \\ X^{(2)} &= (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24})^T \\ &= (24.98613, 0.0, 0.6588386, 9.574212, 0.0, 24.98038, 11.37962, 0.0)^T \\ X^{(3)} &= (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24})^T \\ &= (24.98613, 0.0, 10.23305, 0.0, 0.0, 24.98038, 1.805413, 9.574212)^T \end{aligned}$$

where the value of $Z^1 = 974.78239$, $Z^2 = 57.45401$, and $Z^3 = 258.99054$. A pay-off matrix is formulated in Table 7 using the three ideal solutions.

Table 7: *Pay-Off Matrix*

	$Z^1(x)$	$Z^2(x)$	$Z^3(x)$
$X^{(1)}$	974.78239	57.93272	258.99054
$X^{(2)}$	974.78232	57.45401	258.990526
$X^{(3)}$	974.78239	57.93272	258.99054

From the pay-off matrix, we obtained the upper and lower bounds of the objective functions. From the bounds of the objective functions, one can determine $\varepsilon_k, k = 1, 2, 3$. Here in our case, all the objective functions are minimization type and so $\varepsilon_k, k = 1, 2, 3$ lie between the best lower bound and worst upper bounds of the objective functions, i.e., for objective functions 1, 2, and 3 respectively, we represent it as follows.

$$\begin{aligned} 974.78232 &< \varepsilon_1 < 974.78239 \\ 57.45401 &< \varepsilon_2 < 57.93272 \\ 258.990526 &< \varepsilon_3 < 258.99054. \end{aligned}$$

Hence, the deterministic problem formulated in (30) to (38) is transformed as a single objective by taking first, second, and third objective functions separately as a single objective function in (39) to (47), (48) to (56) and (57) to (65) respectively, and taking the other constraints and one of the objective functions as a new constraint.

$$\min Z^1(x) = 12x_{11} + 15x_{12} + 17x_{13} + 19x_{14} + 15x_{21} + 12x_{22} + 16x_{23} + 18x_{24} \quad (39)$$

Subject to:

$$0.5x_{11} + x_{12} + 1.4x_{13} + 1.75x_{14} + x_{21} + 0.5x_{22} + 1.3x_{23} + 1.7x_{24} \leq \varepsilon_2 \quad (40)$$

$$2x_{11} + 5x_{12} + 7x_{13} + 9x_{14} + 5x_{21} + 2x_{22} + 6x_{23} + 8x_{24} \leq \varepsilon_3 \quad (41)$$

$$\sum_{v=1}^4 x_{1v} \leq 35.8555563 \quad (42)$$

$$\sum_{v=1}^4 x_{2v} \leq 36.3600008 \quad (43)$$

$$\sum_{u=1}^2 x_{u1} \geq 24.98612715 \quad (44)$$

$$\sum_{u=1}^2 x_{u2} \geq 24.980376691 \quad (45)$$

$$\sum_{u=1}^2 x_{u3} \geq 12.0384627 \quad (46)$$

$$\sum_{u=1}^2 x_{u4} \geq 9.57421155 \quad (47)$$

$$x_{uv} \geq 0, \quad u = 1, 2; v = 1, 2, 3, 4$$

$$\min Z^2(x) = 0.5x_{11} + x_{12} + 1.4x_{13} + 1.75x_{14} + x_{21} + 0.5x_{22} + 1.3x_{23} + 1.7x_{24} \quad (48)$$

Subject to:

$$12x_{11} + 15x_{12} + 17x_{13} + 19x_{14} + 15x_{21} + 12x_{22} + 16x_{23} + 18x_{24} \leq \varepsilon_1 \quad (49)$$

$$2x_{11} + 5x_{12} + 7x_{13} + 9x_{14} + 5x_{21} + 2x_{22} + 6x_{23} + 8x_{24} \leq \varepsilon_3 \quad (50)$$

$$\sum_{v=1}^4 x_{1v} \leq 35.8555563 \quad (51)$$

$$\sum_{v=1}^4 x_{2v} \leq 36.3600008 \quad (52)$$

$$\sum_{u=1}^2 x_{u1} \geq 24.98612715 \quad (53)$$

$$\sum_{u=1}^2 x_{u2} \geq 24.980376691 \quad (54)$$

$$\sum_{u=1}^2 x_{u3} \geq 12.0384627 \quad (55)$$

$$\sum_{u=1}^2 x_{u4} \geq 9.57421155 \quad (56)$$

$$x_{uv} \geq 0, \quad u = 1, 2; v = 1, 2, 3, 4$$

$$\min: Z^3(x) = 2x_{11} + 5x_{12} + 7x_{13} + 9x_{14} + 5x_{21} + 2x_{22} + 6x_{23} + 8x_{24} \quad (57)$$

Subject to:

$$12x_{11} + 15x_{12} + 17x_{13} + 19x_{14} + 15x_{21} + 12x_{22} + 16x_{23} + 18x_{24} \leq \varepsilon_1 \quad (58)$$

$$0.5x_{11} + x_{12} + 1.4x_{13} + 1.75x_{14} + x_{21} + 0.5x_{22} + 1.3x_{23} + 1.7x_{24} \leq \varepsilon_2 \quad (59)$$

$$\sum_{v=1}^4 x_{1v} \leq 35.8555563 \quad (60)$$

$$\sum_{v=1}^4 x_{2v} \leq 36.3600008 \quad (61)$$

$$\sum_{u=1}^2 x_{u1} \geq 24.98612715 \quad (62)$$

$$\sum_{u=1}^2 x_{u2} \geq 24.980376691 \quad (63)$$

$$\sum_{u=1}^2 x_{u3} \geq 12.0384627 \quad (64)$$

$$\sum_{u=1}^2 x_{u4} \geq 9.57421155 \quad (65)$$

$$x_{uv} \geq 0, \quad u = 1, 2; v = 1, 2, 3, 4$$

Taking six different values for ε_1 , ε_2 , and ε_3 in the intervals [974.78232, 974.78239], [57.45401, 57.93272] and [258.990526, 258.99054], respectively for the objective functions Z^1 , Z^2 , and Z^3 , we can find the optimal solutions of first, second, and third objective functions in Tables 8 to 10.

From Tables 8 to 10 one can observe that an efficient solution for Z^1 , Z^2 and Z^3 is obtained for a value of $X^*(1)$, $X^*(2)$ and $X^*(3)$ respectively as listed below.

$$\begin{aligned} X^*(1) &= (24.98613, 0.0, 0.6588885, 9.574162, 0.0, 24.98038, 11.37957, 0.49958E - 04)^T \\ X^*(2) &= (24.98613, 0.0, 0.6588386, 9.574212, 0.0, 24.98038, 11.37962, 0.0)^T \\ X^*(3) &= (24.98613, 0.0, 0.6588885, 9.574162, 0.0, 24.98038, 11.37957, 0.49958E - 043)^T \end{aligned}$$

From those values, the best optimal value for $Z^1 = 974.782322$ at $X^*(1)$, $X^*(3)$, $Z^2 = 57.454006$ at $X^*(2)$ and $Z^3 = 258.990517$ at $X^*(1)$, $X^*(3)$. Hence, the best compromise solution is

$$\begin{aligned} X^* &= (x_{11}, x_{12}, x_{13}, x_{14}, x_{21}, x_{22}, x_{23}, x_{24})^T \\ &= (24.98613, 0.0, 0.6588885, 9.574162, 0.0, 24.98038, 11.37957, 0.49958E - 04)^T \end{aligned}$$

Table 8: Solution of Z^1 for 6 different values of ε_2 and ε_3

ε_2	ε_3	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	Z^1
57.45401	258.990526	24.98613	0.0	0.6588885	9.574162	0.0	24.98038	11.37957	0.49958E-04	974.78232
57.549752	258.9905288	24.98613	0.0	2.573729	7.659322	0.0	24.98038	9.464734	1.91489	974.782395
57.645494	258.9905316	24.98613	0.0	4.488569	5.744482	0.0	24.98038	7.549894	3.82973	974.782395
57.741236	258.9905344	24.98613	0.0	6.403409	3.829642	0.0	24.98038	5.635054	5.74457	974.782395
57.836978	258.9905372	24.98613	0.0	8.318249	1.914802	0.0	24.98038	3.720214	7.65941	974.782395
57.93272	258.99054	24.98613	0.0	10.23305	0.0	0.0	24.98038	1.805413	9.574212	974.782394

Table 9: Solution of Z^2 for 6 different values of ε_1 and ε_3

ε_1	ε_3	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	Z^2
974.78232	258.990526	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401
974.782334	258.9905288	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401
974.782348	258.9905316	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401
974.782362	258.9905344	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401
974.782376	258.9905372	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401
974.78239	258.99054	24.98613	0.0	0.6588386	9.574212	0.0	24.98038	11.37962	0.0	57.45401

Table 10: Solution of Z^3 for 6 different values of ε_1 and ε_2

ε_1	ε_2	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	Z^3
974.78232	57.45401	24.98613	0.0	0.6588885	9.574162	0.0	24.98038	11.37957	0.49958E-04	258.99052
974.782334	57.549752	24.98613	0.0	2.573729	7.659322	0.0	24.98038	9.464734	1.91489	258.990545
974.782348	57.645494	24.98613	0.0	4.488569	5.744482	0.0	24.98038	7.549894	3.82973	258.990545
974.782362	57.741236	24.98613	0.0	6.403409	3.829642	0.0	24.98038	5.635054	5.74457	258.990545
974.782376	57.836978	24.98613	0.0	8.318249	1.914802	0.0	24.98038	3.720214	7.65941	258.990545
974.78239	57.93272	24.98613	0.0	10.23305	0.0	0.0	24.98038	1.805413	9.574212	258.990544

Again the values of the objective functions are

$$\begin{aligned} Z^1(X^*(1)) &= 974.782322, & Z^1(X^*(2)) &= 974.782324, & Z^1(X^*(3)) &= 974.782322, \\ Z^2(X^*(1)) &= 57.454008, & Z^2(X^*(2)) &= 57.454006, & Z^2(X^*(3)) &= 57.454008, \\ Z^3(X^*(1)) &= 258.990517, & Z^3(X^*(2)) &= 258.990518, & Z^3(X^*(3)) &= 258.990517. \end{aligned}$$

6. RESULT and DISCUSSION

From the tabular results shown above, Tables (8 to 10), it is found that the minimum transportation cost per unit is Rs 974.782322, the minimum delivery time per unit is 57.454006 hours, and the minimum loss cost per unit is Rs 258.990517. It is observed that amount of fruits and vegetables sold per unit from Bhubaneswar and Cuttack are 24.98613 and 24.98038 respectively. So, fruits and vegetables sold from Bhubaneswar to their customers and from Cuttack to their customers are almost same. Next to this, the amount per unit sold from Cuttack to customers to Angul is 11.37957 and the amount per unit sold from Bhubaneswar to customers in Koraput is 9.574162. The amount per unit sold from Bhubaneswar to customers in Angul is 0.6588885. The least amount per unit sold from Cuttack to customers in Koraput is 0.49958E-04. Nothing was sold from Bhubaneswar to customers in Cuttack and vice-versa.

In general, for the online market, it is necessary to minimize distance and maximize the number of customers. But here, for fruits and vegetables to maintain profitability, there is a need for minimum cost loss of the transportation time. Therefore, the proposed model is developed keeping into account the quantity of vegetables and fruits sold from source to destinations where loss of transportation time is minimized. So, the proposed method gives minimum aggregate time and minimum total cost.

7. CONCLUSION

In this study, we proposed a solution procedure for solving multi-objective stochastic transportation problems (MOSTP) by considering random variables as the supply and demand points that follows three-parameter extreme value distribution. We gave three different cases, (i) only the supply points follow three-parameter extreme value distribution and the others are deterministic, (ii) only the demand points follow three-parameter extreme value distribution and the others are deterministic, and lastly both supply and demand points follow three-parameter extreme value distribution and all other parameters are assumed to be deterministic. We have derived the deterministic equivalent model of MOSTP and the feasibility condition for all of the three cases. The multi-objective deterministic nonlinear programming problem was solved using ε -constraint technique and LINGO 14.0 package. For clarification, a case study on the online market is provided to illustrate the methodology when both the supply and demand points follow three parameter extreme value distribution. However, other multi-objective techniques, namely, goal programming, weighting method, fuzzy programming method, etc. could be used.

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