AN ALGORITHM FOR FUZZY SOFT SET BASED DECISION MAKING APPROACH

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Abstract: The notion of soft set theory was initiated as a general mathematical tool for handling ambiguities. Decision making is viewed as a cognitive-based human activity for selecting the best alternative. In the present time, decision making techniques based on fuzzy soft sets have gained enormous attentions. On this development, this paper proposes a new algorithm for decision making in fuzzy soft set environment by hybridizing some existing techniques. The first novelty is the idea of absolute scores. The second concerns the concept of priority table in group decision making problems. The advantages of our approach herein are stronger power of objects discrimination and a well-determined inference.

1. INTRODUCTION AND PRELIMINARIES

The real world is filled with uncertainty, vagueness and imprecision. The notions we meet in everyday life are vague rather than precise. In recent time, researchers have taken keen interest in modelling vagueness due to the fact that many practical problems within fields such as biology, economics, engineering, environmental sciences, medical sciences involve data containing various forms of uncertainty. To handle the complexity of vagueness, one cannot successfully

employ classical mathematical methods due to the presence of different kinds of incomplete knowledge, typical for these mix-ups. Earlier in the literature, there were four known theories for dealing with imperfect knowledge, namely, Probability Theory (PT), Fuzzy Set Theory (FST) [18] and Rough Set Theory (RST) [14]. All the aforementioned tools require pre-assignment of some parameters; for example, membership function in FST, probability density function in PT and equivalent relation in RST. Such pre-specifications, viewed in the backdrop of incomplete knowledge, give rise to every day problems. With this concern, Molodstov [13] initiated the concept of Soft Set Theory (SST) with the aim of handling phenomena and notions of ambiguous, undefined and imprecise environments. Hence, SST does not need the pre-specifications of a parameter, rather, it accommodates approximate descriptions of objects. In other words, one can use any suitable parametrization tool with the help of words, sentences, real numbers, mappings, and so on; thereby, making SST an adequate formalism for approximate reasoning. In the pioneer work of Molodstov [13], several potential applications of SST were pointed out in the areas of Riemann integration, smoothness of functions, theory of probability, theory of measurement, game theory and operation research. Interestingly, Molodstov [13] emphasizes that the models by fuzzy sets and soft sets are interrelated. Yang et al [17] emphasized that SST needed to be expanded in different directions to extend its applications to other fields. Currently, the concept of soft sets has been extended by several researchers. By combining the ideas of soft sets and fuzzy sets, Maji et al [9] initiated the notion of fuzzy soft sets and discussed its various properties. Wang et al. [16] initiated hesitant fuzzy soft sets which combined the views of hesitancy with the latter idea. Han et al. [6] and Zou and Xiao [19] studied incomplete soft sets appearing due to errors in data measurement. Feng et al. [5] established choice value soft sets to extend Cagman and Enginoglu's [2] decsion making approach. Recent researches [7, 9, 17] have shown that both theories of FST and SST can be combined to have a more flexible and expressive framework for modelling and processing data and information possessing nonstatistical uncertainties.

Due to the subjective nature of everyday problems, up to date, there is no universally adopted techniques for decision making using fuzzy soft sets or other hybrid models. The earliest fuzzy soft set theoretic approach to decision making was formulated by Roy and Maji [12]. They propose a solution for an object recognition problem where the technique depends on multi-observer input parameter data set. Unfortunately, some drawbacks of the techniques in [12] were noted thereafter. First, Roy and Maji [12] proposed to start with an aggregation procedure that produces a single resultant fuzzy soft set from preliminary multi-source information. By way of a counter example, Alcantud [1, Example 3] showed that approach in [12] might result in a loss of information and eventually lead to uncertainty and hence, defeating the primary objective of soft set theory. As a result, an alternative approach was suggested in [1]. Further, Roy and Maji [12] proposed a procedure that allows the computation of scores for the alternatives. This technique was fine-tuned with a different idea by Kong et al. [7]. On this matter, Feng et al [3] observed that the discrepancy of views between [7] and [12] was due to

wether the criterion for making a decision should use scores or fuzzy choice values. In this argument, hereafter, we agree with Feng et al's. [3] point of view that the approach by scores in Roy and Maji's [12] is more appropriate for decision making. To remedy the problem of possible ambiguity associated with Roy and Maji's [12] use of "AND" (as minimum) operator, Alcantud [1] formulated an information fusion procedure by replacing the "AND-minimum" operator by a particular t-norm in multi-valued logic, namely, the product operator. In line with Roy and Maji's [12] use of scores, Alcantud [1] produced a comparison table that abnegated the use of unsuitable "crisp" values at the core of the definition of Roy and Maji's [12] comparison table.

In this paper, we are concerned with three fundamental issues emerging from earlier literature. First, we propose the idea of absolute scores in object recognition problems. For this, we improve the formula for the computation of scores proposed by Roy and Maji [12], which is also adopted by Alcantud [1] and some previous articles in agreement with [12]. Secondly, we propose a more general and simple technique for solving the problems of ties or draw of objects which is one of the drawbacks of Roy and Maji's [12] approach. In this case, an idea of Alcantud [1] is adopted by replacing the "AND-minimum" operator used in [12] with the "AND-product" operator. A sharp difference between our approach and that of Alcantud [1] in this regard is in the method of computing scores. Finally, in certain decision making problems , decision makers may impose different thresholds on different decision parameters based on their impacts. In same vein, it is well-known from the concepts of weighted fuzzy soft sets that parameters may be unequally important. With this in mind, we put forward the notion of standard priority table which specifies the value of each parameter according to a consensus of a team of decision makers. Thus, the latter idea is in compliance with the concept of membership function for fuzzy soft set introduced in [15] and the theory of W-soft sets (or weighted soft sets) introduced by Lin [8].

1.1. Soft sets and fuzzy soft sets: Basic definitions and examples

In this subsection, some basic concepts and examples of soft sets and fuzzy soft sets are recalled. Let E be a parameter set, $A \subseteq E$ and $P(X)$ represents the power set of an initial universe of discourse X. Molodstov [13] established the concept of soft sets with the following definition.

Definition 1. [13] A pair (F, A) is called a soft set over X under E, where $A \subseteq E$ and F is a mapping given by $F : A \longrightarrow P(X)$.

In other words, a soft set over X is a parameterized family of subsets of X. For each $e \in E$, $F(e)$ is considered as the set of e-approximate elements of (F, A).

Example 2. Suppose the following:
X- is the universal set of all students at a certain university,
E- is the set of parameters, given as:

$$E = \{intelligent, \ hardworking, \ dull, \ hardworking \ and \ intelligent\}.$$

Assume that they are one hundred students at the university X given as

$$X = \{x_1, x_2, x_3, x_4, x_5 \cdots x_{100}\}, \quad and \quad E = \{e_1, e_2, e_3, e_4\},$$

where

$e_1 =$ *intelligent,* $e_2 =$ *hardworking,*

$e_3 =$ *dull,* $e_4 =$ *hardworking and intelligent.*

Then $F : E \longrightarrow P(X)$ defined by $F(e_1) = \{x_1, x_2, \cdots x_{10}\}$ means that $x_1, x_2, \cdots x_{10}$ are intelligent , $F(e_2) = \{x_{11}, x_{12}, \cdots x_{30}\}$ means that $x_{11}, x_{12}, \cdots x_{30}$ are hardworking, $F(e_3) = \emptyset$ means that there is no dull student in the university in question, $F(e_4) = \{x_{15}, x_{81}\}$ means that the students x_{15} and x_{81} are both intelligent and hardworking. Then we can view the soft set (F, E) describing the "kind of students" as the following approximations:

$$(F, E) = \Big\{ (\text{intelligent students}, \{x_1, x_2, \cdots x_{10}\}), (\text{hardworking students},$$
$$\{x_{11}, x_{12}, \cdots x_{30}\})$$
$$(\text{dull}, \emptyset), (\text{intelligent and hardworking students}, \{x_{15}, x_{81}\}) \Big\}.$$

Many researchers carried out formal studies of these basic ideas of soft sets and related notions . For example, Maji etal [11] developed these notions and established other concepts such as soft subsets and supersets, intersections and unions, soft equalities and so on. For soft set-based decision making approach, the interested reader may consult Cagman and Enginoglu [2], Feng and Zhou [4] and Maji et al [10].

In order to model more general scenarios, Maji et al [9] defined the notion of fuzzy soft sets in the following manner.

Definition 3. *[9] A pair (F, A) is a fuzzy soft set over X when $A \subseteq E$ and $F : A \longrightarrow I^X$, where I^X denotes the set of all fuzzy sets in X.*

Clearly, every soft set can be thought of as a fuzzy soft set. Following Example 2, fuzzy soft sets allow the investigation of some more intriguing properties such as "how much time each student works" in which case partial memberships are indispensable. A soft set or fuzzy soft set can be considered as an information system or an information table. For the soft set, each entry in this table is 1 or 0 decided on whether an object belongs to the range of a parameter or not. For the fuzzy soft set, every entry belongs to the interval [0, 1] and is determined by the membership degree of an object on a parameter.

Example 4. *Consider Example 2. The fuzzy soft set (F, E) describing the "kind of students" under fuzzy circumstances may be given as*

(i) $F(e_1) = \{x_1/0.5, x_2/0.1, x_4/0.7\}$, $F(e_2) = \{x_3/0.6, x_9/0.7, x_{20}/0.1\}$

(ii) $F(e_3) = \{x_{19}/0.8, x_{25}/0.1, x_4/0.7\}$, $F(e_4) = \{x_{13}/0.4, x_{91}/0.3, x_{94}/0.1, x_{97}/0.3\}$.

Definition 5. *[9] For two fuzzy soft sets (F_1, L) and (F_2, M) over a common universe X, (F_1, L) is a fuzzy -soft subset of (F_2, M) if*

(i) $L \subset M$, and

(ii) for all $e \in L$, $F_1(e)$ is a fuzzy subset of $F_2(e)$.

If $(i) - (ii)$ holds, then we write $(F_1, L)\tilde{\subset}(F_2, M)$.

Definition 6. *[9] If (F_1, L) and (F_2, M) are two fuzzy soft sets, then "(F_1, L) AND (F_2, M)" is a fuzzy soft set denoted by $(F_1, L) \wedge (F_2, M)$, and is defined by*

$$(F_1, L) \wedge (F_2, M) = (F_3, L \times M),$$

where $F_3(\alpha, \beta) = F_1(\alpha)\tilde{\cap}F_2(\beta)$, for all $\alpha \in L$ and $\beta \in M$, where $\tilde{\cap}$ denotes the operation of fuzzy intersection of two fuzzy soft sets.

2. DECISION MAKING TECHNIQUES USING FUZZY SOFT SETS

Most problems in real-life cannot be effectively resolved by a single decision-maker. So, it becomes needful to gather multi-decision makers with different experience and knowledge structures. In [1], it is noted that there are two main stages in fuzzy soft set based decision making problems. In the first place, an aggregation procedure that produces a single resultant fuzzy soft set from the original information (because we have multi-observer data in terms of various sets of parameters in the problem of object recognition) is employed. In the second stage, one makes the final decision using the overall information, without regard to wether it is produced as a resultant fuzzy soft set or not.

In what follows, we formulate a group decision making technique using fuzzy soft sets. In particular, our procedure is a hybridization of the ideas of Alcantud [1], Maji et al [12] and Tripathy [15], First, an algorithm is presented in which the AND Operator is used as in [12, Algorithm 3.1] is replaced with the $PRODUCT$ Operator as used in [1, Algorithm 2]. Also, using the idea of Priority rank table in [15], we developed the notion of Standard priority table. For detail analysis of the advantage of replacing the AND operator with a suitable definition or operator, or some counter examples showing the drawbacks in [12], the interested reader may consult [1, 4, 15] and the references therein.

Following [1, 12], the problem here is to choose an object from the set of available objects with respect to a set of choice parameters P. First, we give the following requisite definitions and formulae.

Definition 7. *[12] Comparison table is a square table in which the number of rows and columns are equal, rows and columns are both labelled by the object names $x_1, x_2, \cdots, x_n$ of the universe of discourse X, and the entries a_{ij} $i, j = 1, 2, \cdots, n$, are given by $a_{ij} =$ the number of parameters for which the membership value of x_i exceeds or equal to the membership valued of x_j. Obviously, $0 \leq a_{ij} \leq m$, and $a_{ii} = m$, for all i, j where m is the number of parameters in a fuzzy soft sets.*

The row sum of an object x_i is represented by r_i and is calculated by the formula

$$r_i = \sum_{j=1}^{n} a_{ij}. \tag{1}$$

Similarly, the column sum of an object x_j, written as c_j , is given by the formula

$$c_j = \sum_{i=1}^{n} a_{i,j}. \tag{2}$$

Using formulae (1) and (2), Roy and Maji [12] proposed that the score of an object x_i may be computed by the formula

$$S_i = r_i - c_i. \tag{3}$$

Formula (3), as used in [12] and adopted in [1] as well as in other articles, results in alternative positive and negative scores. To circumvent this irregular pattern of scores, we introduce the concept of absolute score by modifying Formula 3 as follows:

$$S_i = \left(\frac{r_i - c_i}{R_i} \right) \times (\pm 1), \tag{4}$$

where $R_1, R_2, \cdots R_q$ denote row positions of objects $x_1, x_2, \cdots x_q$, respectively, such that $R_1 = 1, R_2 = 2, \cdots R_q = q$, with

$$S_i = \left(\frac{r_i - c_i}{R_i} \right) \times (+1), \text{ if } r_i - c_i \geq 0$$

and

$$S_i = \left(\frac{r_i - c_i}{R_i} \right) \times (-1), \text{ if } r_i - c_i < 0.$$

2.1. ALGORITHM

Now, we formulate a novel algorithm which harmonizes the techniques of [1, 12, 15] as follows.

(i) Create a standard priority table. This is the table based on consensus of a team of decision makers from which values in the priority table of each observer are measured.

(ii) Input the fuzzy soft sets (F_1, L) , (F_2, M) and (F_3, Q) as provided by each observer.

(iii) Construct the priority table for each observer. This can be computed by multiplying priority values in observer's fuzzy soft set with the corresponding parameter value in the standard priority table.

(iv) Compute the resultant fuzzy soft set (R, P) from the priority table of each observer and display it in a tabular form, using the $PRODUCT$ as AND operator.

(v) Construct the Comparison-table of (R, P) and calculate r_i and c_i for all x_i.

(vi) Calculate the absolute score of x_i, for all i, using Formula 4 and display in a decision table (A decision table is a table from which final inference is drawn).

(vii) The decision is any object x_k that maximizes the absolute score; that is, any x_k such that $S_k = \max_i S_i$.

The progress in the topic of prioritizing fuzzy soft sets is based on discussions about the performance of solutions through examples. In this respect, we illustrate a particular application of Algorithm 2.1 in Example 8.

Example 8. *Assume that certain number of applicants/candidates applied for a job. Out of these candidates, the organization selects a particular number of them to attend the final interview based on certain criteria. In this case, the criteria set by the organization are named standard parameters. First, the panel of judges unanimously assigns values to the standard parameters based on their impact. Then, the performance of each candidate at the interview is analyzed by the panel of judges. The evaluation of the candidates by each member of the panel is further weighted using the standard parameters. This phenomenon is analyzed as follows:*

Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the applicants' universe of discourse. Let the universe of parameter set be given by

$$
E = \Big\{ \text{knowledge}(e_1) \text{ , communication}(e_2), \text{ response}(e_3), \text{ presentation}(e_4),
$$
$$
\text{extracurricular activities}(e_5) ,
$$
$$
\text{class of degree}(e_6), \text{ professional qualification}(e_7), \text{ foreign certificate}(e_8),
$$
$$
\text{local certificate}(e_9) \text{ , almamater}(e_{10}),
$$
$$
\text{years of graduation}(e_{11}), \text{ working experince}(e_{12}) \Big\}
$$

Let L, M, Q denote three subsets of E, where $L = \{e_1, e_2, e_3, e_4\}$, $M = \{e_5, e_6, e_7, e_8\}$ and $Q = \{e_9, e_{10}, e_{11}, e_{12}\}$. Let J_1, J_2 and J_3 be three judges who evaluate the applicants. The panel of judges assigns priority values to each parameter based upon the impact of the parameter. This gives a standard priority table. The judges assign parameter value to each applicant based on the evaluation.

The standard priority table provided by the panel of judges is as shown in Table1 .

Parameter	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
Parameter value	0.6	0.4	0.1	0.3	0.01	0.7	0.5	0.02	0.2	0.03	0.08	0.8

Table 1: Standard priority table

Let (F_1, L), (F_2, M) and (F_3, Q) be fuzzy soft sets provided by the judges J_1 J_2, and J_3, respectively. These fuzzy soft sets and their corresponding priority tables are as shown in tables 2, 4, 6, 3, 5 and 7.

	e_1	e_2	e_3	e_4
x_1	0.7	0.6	0.4	0.1
x_2	0.7	0.1	0.7	0.5
x_3	0.6	0.5	0.2	0.3
x_4	0.2	0.8	0.6	0.2
x_5	0.3	0.7	0.4	0.5
x_6	0.1	0.8	0.6	0.7

Table 2: Fuzzy soft set (F_1, L) by Judge J_1

	e_1	e_2	e_3	e_4
x_1	0.42	0.24	0.04	0.03
x_2	0.42	0.04	0.07	0.15
x_3	0.36	0.2	0.02	0.09
x_4	0.12	0.32	0.06	0.06
x_5	0.18	0.28	0.04	0.15
x_6	0.06	0.32	0.06	0.21

Table 3: Priority table for J_1

	e_5	e_6	e_7	e_8
x_1	0.6	0.8	0.2	0.4
x_2	0.2	0.4	0.7	0.9
x_3	0.4	0.6	0.6	0.9
x_4	0.1	0.2	0.8	0.3
x_5	0.8	0.9	0.1	0.2
x_6	0.7	0.8	0.2	0.4

Table 4: Fuzzy soft set (F_2, M) by Judge J_2

	e_5	e_6	e_7	e_8
x_1	0.006	0.56	0.1	0.008
x_2	0.002	0.28	0.35	0.018
x_3	0.004	0.42	0.3	0.018
x_4	0.001	0.14	0.4	0.006
x_5	0.008	0.63	0.05	0.004
x_6	0.007	0.56	0.1	0.008

Table 5: Priority table for J_2

	e_9	e_{10}	e_{11}	e_{12}
x_1	0.7	0.6	0.9	0.1
x_2	0.4	0.5	0.6	0.4
x_3	0.5	0.4	0.7	0.7
x_4	0.3	0.2	0.4	0.6
x_5	0.4	0.1	0.5	0.1
x_6	0.2	0.3	0.3	0.8

Table 6: Fuzzy soft set (F_3, Q) by Judge J_3

	e_9	e_{10}	e_{11}	e_{12}
x_1	0.14	0.018	0.072	0.08
x_2	0.08	0.015	0.048	0.32
x_3	0.1	0.012	0.056	0.56
x_4	0.06	0.006	0.032	0.48
x_5	0.08	0.003	0.04	0.08
x_6	0.04	0.009	0.024	0.64

Table 7: Priority table for J_3

Next, using the priority tables for J_1, J_2 and J_3, we carry out an aggregation procedure. For this, assume that the set of choice parameters of an observer is given by

$$P = \left\{ \begin{array}{l} p_1 = e_1 \wedge e_5 \wedge e_9, \ p_2 = e_2 \wedge e_6 \wedge e_{10}, \ p_3 = e_3 \wedge e_7 \wedge e_{11}, \\ \\ p_4 = e_4 \wedge e_8 \wedge e_{12}, \ p_5 = e_3 \wedge e_5 \wedge e_{12} \end{array} \right\}. \tag{5}$$

In view of the parameter set in (5), we have to take the decision from the available set X. From (5), the resultant fuzzy soft set (R, P) is represented in Table 8.

	p_1	p_2	p_3	p_4	p_5
x_1	0.000352	0.00242	0.000288	0.0000192	0.0000192
x_2	0.0000672	0.000168	0.00118	0.000864	0.0000448
x_3	0.000144	0.0001008	0.000336	0.000907	0.0000448
x_4	0.0000072	0.000269	0.000768	0.000173	0.0000288
x_5	0.000115	0.000529	0.00008	0.000096	0.0000256
x_6	0.0000168	0.00161	0.000144	0.00108	0.000267

Table 8: Resultant fuzzy soft set (R, P)

The comparison-table of the resultant fuzzy soft set (R, P) using Definition 7 is given in Table 9.

	x_1	x_2	x_3	x_4	x_5	x_6
x_1	5	2	2	2	3	3
x_2	3	5	2	4	3	2
x_3	3	4	5	4	5	2
x_4	3	1	1	5	3	1
x_5	2	2	0	2	5	1
x_6	2	3	3	4	4	5

Table 9: Comparison-table

Now, we calculate the row-sum, column-sum and the absolute score of each applicant x_i, using formulae (1), (2) and (4), respectively. This is displayed in Table 10.

	row-sum(r_i)	column-sum(c_i)	Score(S_i)
x_1	17	17	$\left(\frac{0}{1}\right) \times (+1) = 0$
x_2	19	17	$\left(\frac{2}{2}\right) \times (+1) = 1$
x_3	23	13	$\left(\frac{10}{3}\right) \times (+1) = 3.333$
x_4	14	21	$\left(\frac{-7}{4}\right) \times (-1) = 1.75$
x_5	12	23	$\left(\frac{-11}{5}\right) \times (-1) = 2.2$
x_6	21	14	$\left(\frac{17}{6}\right) \times (+1) = 2.833$

Table 10: Decision table

From the Decision table 10, one can see that the applicant x_3 maximizes the absolute score and hence is the best choice for the job according to the opinion of the panel of judges.

3. CONCLUSION

In this study, the application of fuzzy soft sets in decision making problems as first presented in [1, 12, 15] is improved. We agree with Alcantud [1], Feng, et al. [3], and Sooraj [15] when they argue that Roy and Maji's [12] approach has some drawbacks and hence, needs some improvements. Thus, we formulated an algorithm which incorporates the techniques of [1, 12, 15]. Our proposal is generic. It takes multi-source data set and aggregates the inputs into a resultant fuzzy soft set by a more generalized operator noted in [1]. We modified the score formula in [12] by introducing the idea of absolute scores. Further, the idea of standard priority table is proposed. The latter notion is motivated by the concept of W-soft set theory of Lin [8] and membership function for fuzzy soft sets established in [15]. The overall advantage of our algorithm lies in the power of well-determined inference in object recognition problems.

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