Yugoslav Journal of Operations Research
30 (2020), Number 1, 71–89
DOI: https://doi.org/10.2298/YJOR181015025K

A NEW INTERACTIVE APPROACH FOR SOLVING FULLY FUZZY MIXED INTEGER LINEAR PROGRAMMING

Farzaneh KHALILI GOUDARZI
Department of Mathematics, Shiraz university of technology, Shiraz, Iran
f.khalili@sutech.ac.ir

Seyed Hadi NASSERI
Department of Mathematics, University of Mazandaran, Babolsar, Iran
nasseri@umz.ac.ir

Nemat Allah TAGHI-NEZHAD
Department of Mathematics, Faculty of basic sciences, Gonbad Kavous University, Gonbad Kavous, Iran
ntaghinezhad@gonbad.ac.ir

Received: October 2018 / Accepted: September 2019

Abstract: In this paper, a novel method to solve Fully Fuzzy Mixed Integer Linear Programming (FFMILP) problems is presented. Our method is based on the definition of membership function and a fuzzy interactive technique for solving the classical multi-objective programming. It is worthwhile to note that this is the first time that the fully fuzzy mixed integer linear programming problem is discussed and a solving method is presented. To illustrate the steps of the proposed method, some numerical examples are solved and the results are compared with other methods in the literature. Computational results present the application of the method.

Keywords: Fully Fuzzy Mixed Integer Linear Programming, Triangular Fuzzy Number, Fuzzy Interactive Programming, Membership Function.

MSC: 90C05 , 90C70, 68Q10.

1. INTRODUCTION

Linear programming (LP) problem is mostly used in different fields of science and engineering for modeling real world problems [1]. In such cases, using fuzzy set theory the vagueness of data are modeled in mathematical form. That is, some of the LP parameters are represented by fuzzy numbers rather than crisp numbers in many applications. Therefore, developing mathematical models and numerical procedures for the fuzzy LP would be of interest of many researchers. Fuzzy set theory is studied by many researchers in the field of optimization [2, 3, 4, 5]. Also, this concept is adopted for solving fuzzy linear programming problems, but less attention has focused on formulation of Fully Fuzzy Linear Programming (FFLP). Fuzzy linear programming (FLP) problem is first considered by Bellman and Zadeh [6]. A new method to solve FFLP when the constraints are all inequality is proposed by Kumar *e*t al. [7]. A method for solving linear programming problem where all the coefficients are fuzzy numbers is presented by Mariano Jimenez *e*t al. [8]. A new method to solve FFLP problems is proposed by Nasseri *e*t al. [9]. In their method, the definition of membership function and the convenient techniques for solving the classical multi-objective programming is used. After that, FFLP problems is studied by Allahviranloo *e*t al. [10] and a new method based on ranking function is presented. FFLP problems with all parameters and variables as triangular fuzzy numbers is discussed by Lotfi *e*t al. [11]. It is pointed out by Kumar *e*t al. [12] that there is no method in literature to find the exact fuzzy optimal solution of FFLP problems and a new method is proposed to find the fuzzy optimal solution of FFLP problems with equality constraints having non-negative fuzzy variables and unrestricted fuzzy coefficients. To find the exact fuzzy optimal solution of FFLP problems with equality constraints having non-negative fuzzy coefficients and unrestricted fuzzy variables, a method is presented by Kaur and Kumar [13]. Also, their method is used to solve the FFLP problems with equality constraints having non-negative fuzzy variables and unrestricted fuzzy coefficients. Linear programming problems in a fully fuzzy environment is studied by Ullah Khan*e*t al. [14] and a technique is proposed for solving it. An algorithm for solving fuzzy linear programming problems with trapezoidal fuzzy numbers is presented by Stanojevic using a penalty method [15]. Some Multi-choice linear programming (MCLP) problems where the alternative values of the multi-choice parameters are fuzzy numbers is considered by Pradhan and Biswal and a defuzzication method based on incenter point of a triangle is presented to solve it [16]. An algorithm to solve the FFLP problem based on a new lexicographic ordering on triangular fuzzy numbers is suggested by Ezzati *e*t al. [17]. An efficient method to solve FFLP is introduced by Das *e*t al. [18]. Some well-known approaches for solving FLP problems is reviewed by Skandari and Ghaznavi [19] and some of their difficulties is shown by some numerical examples. Also, in this paper, it is shown that some of these methods are not able to solve all the given FLP problems correctly. Thereafter, a new crisp linear programming (CLP) problem is considered and it is presented that its optimal solution is also an optimal solution for Zimmermann and Werner's approaches. Also, another new CLP problem is suggested by them and it is proved that its optimal solutions are efficient. Integer linear programming problem (ILP) is an LP in which some or all of the variables are required to be non-negative integers. ILP is a frequently applied method in optimization [20]. The fuzzy integer linear programming problem is solved by Allahviranloo et al. [21] based on reducing it into a crisp integer linear programming problem. Some models for dealing with fuzzy integer linear programming problems, which have a certain lack of precision of a vague nature in their formulation is studied by Herrera and Verdegay [22] and some methods are presented to solve them with either fuzzy constraints or fuzzy numbers defining the set of constraints. The method is implemented in a number of research studies, as well as in industrial applications. An example of such an application is found in the process of investment portfolio optimization [23]. ILP problem is successfully applied as a supply chain management alternative to traditional ERP systems. Developed mathematical models enable optimization of all supply chain resources, including subcontractors realizing delivery orders in a multi-level structure [23, 24]. High applicability of ILP problem is furthermore confirmed by the results of a study devoted to scheduling in a river transportation system model [25]. Moreover, ILP problem is also used in the development of sequential algorithms carrying out packaging optimization tasks [26]. is more, ILP problem is successfully employed in optimization of Intensity-Modulated Radiotherapy. Since the problem is NP-hard, there exists no simplified method that would enable solving it and therefore an iterative method must be applied [27]. Recall that, in a mixed ILP problem, some variables are required to be integers and others are allowed to be either integers or non-integers. A new method to solve single objective FFMILP problem is presented by Khalili Goodarzi et al. [28] and the method is implemented to open shop scheduling problem. In this paper, a kind of MILP problems whose parameters, coefficients and decision variables are all fuzzy numbers is concentrated. A mixed integer linear programing formulation with a moderate number of variables and constraints is used by Bogdanovi et al. for low discrepancy consecutive k-sums permutation problem [29]. A fuzzy mixed-integer linear programming model is proposed by Ubando et al., for the optimal design of a polygene ration plant with cyclic loads [30]. A fuzzy mixed-integer linear programming model is proposed by Ubando et al for the optimal operational adjustment of an off-grid micro-hydropower-based polygene ration plant seeking to maximize the satisfaction levels of the community utility demands, which are represented as fuzzy constraints [31]. A modeling framework for minimax mixed binary fuzzy linear problems is presented by Arana-Jiménez and Blanco and it is proved that the considered problem can be equivalently formulated as a crisp multiple objective mixed integer programming problem [32]. The modeling and design of a modular energy management system is presented by Luna et al. and its integration to a grid-connected battery-based micro grid [33]. A new method namely, decomposition method is proposed by Pandian and Jayalakshmi for solving integer linear programming problems with fuzzy variables by using classical integer linear programming [34].

The roots of the present paper lie in the following sections: In Section 2, we state some basic notations and definitions of fuzzy sets theory. In Section 3, after introducing the classical FFMILP problem, we present a new algorithm for solving this kind of problems. Section 4 provides a numerical example to illustrate the theory and the solution algorithm. Finally, Section 5 contains the conclusions.

2. PRELIMINARIES

In this section, some basic definitions, arithmetic operations and notations of fuzzy numbers are presented [35, 36].

2.1. Basic definitions

Definition 1. *Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by membership function$\mu_{\tilde{A}} : X \to [0,1]$ which assigns to each element $x \in X$ a real number $\mu_{\tilde{A}}(x)$ in the interval $[0,1]$, where the value of $\mu_{\tilde{A}}(x)$ at* x *represents the grade of membership of x in A. Thus, the nearer the value of $\mu_{\tilde{A}}(x)$ is unity, the higher the grade of membership of x in A.A fuzzy subset A can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written*

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\}.$$

Definition 2. *The support of a fuzzy set $\tilde{A}$ on X, denoted by supp(A), is the set of points x in X at which $\mu_{\tilde{A}}(x) > 0$, i.e.,*

$$supp\,(A) = \{x \in X \mid \mu_{\tilde{A}}(x) > 0\}\,.$$

Definition 3. *The height of a fuzzy set $\tilde{A}$ on X, denoted by hgt ($\tilde{A}$), is the least upper bound of $\mu_{\tilde{A}}(x)$, i.e.,*

$$hgt\ \left(\tilde{A}\ \right) = {}_{x \in X}^{sup}\ \mu_{\tilde{A}}(x)\,.$$

Definition 4. *A fuzzy set $\tilde{A}$ on X is said to be normal if its height is unity, i.e.,if there is $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$. If it is not normal, a fuzzy set is said to be subnormal.*

Definition 5. *The α–cut or α–level set of a fuzzy set is a certain set defined as follow:*

$$\tilde{A}_\alpha = \{\, x \in \mathbb{R}|\, \mu_{\tilde{A}}(x) > \alpha\}\,.$$

Definition 6. *A fuzzy set $\tilde{A}$ of universe set X is convex if and only if for any $x_1, x_2 \in X$ and $\lambda \in [0,1]$, we have:*

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)\, x_2) \geq min\,\{\mu_{\tilde{A}}(x_1)\,, \mu_{\tilde{A}}(x_2)\}\,.$$

Definition 7. *A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}$ whosemembership function is piecewise continuous.*

Definition 8. *A triangular fuzzy number $\tilde{A} = (a,b,c)$ is a fuzzy number on $\mathbb{R}$ with a membership function $\mu_{\tilde{A}}$ defined by:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a,b], \\ \\ \dfrac{x-c}{b-c}, & x \in [b,c], \\ \\ 0, & o.w. \end{cases}$$

We also denote the set of all triangular fuzzy numbers with $\boldsymbol{K}(\mathbb{R})$.
As examples of membership functions for a fuzzy number $\tilde{M}$*, such as approximately* m*, a triangular membership function*

$$\mu_{\tilde{M}}(x) = max(0, 1 - \frac{|x-m|}{a}) \qquad a > 0.$$

Definition 9. *A triangular fuzzy number* (a, b, c) *is said to be non-negative triangular fuzzy number, iff* $a \geq 0$.

2.2. Arithmetic on fuzzy numbers

Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ be two triangular fuzzy numbers. Then, arithmetic operation on these fuzzy numbers can be defined as follows:

1. Addition: $\tilde{A} \oplus \tilde{B} = (a + d, b + e, c + f)$.
2. Symmetry: $-\tilde{A} = (-c, -b, -a)$.
3. Subtraction:$\tilde{A} \ominus \tilde{B} = (a - f, b - e, c - d)$.
4. Equality: $\tilde{A} = \tilde{B}$ iff $a = d, b = e, c = f$.
5. Multiplication: Suppose $\tilde{A}$ be any triangular fuzzy number and $\tilde{B}$ be non-negative triangular fuzzy number, then we define:

$$\tilde{A} \otimes \tilde{B} \simeq \begin{cases} (ad, be, cf), & a \geq 0, \\ (af, be, cf), & a < 0, \; c \geq 0, \\ (af, be, cd), & c < 0. \end{cases}$$

3. FFMILP PROBLEMS AND A NOVEL SOLUTION METHOD

The general form of linear programming problem is as follows:

$$Max \;\; (Min) \sum_{j=1}^{n} c_j x_j$$

$$s.t. \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq b_i, & \forall i = 1, \ldots, m, \\ x_j \geq 0, & j = 1, \ldots, n. \end{cases}$$

In contrast to conventional linear programming problem, fuzzy linear programming problems was first introduced in 1976 by Zimmermann. He considered the LP problems with fuzzy goal and constraints. He proposed to soften the rigid requirements of the decision maker to strictly optimize the objective function and strictly satisfy the constraints. By considering the imprecision or fuzziness of the Decision Makers judgment, he softened the usual linear programming problem into the following fuzzy version:

$$\begin{cases} cx \preccurlyeq z_0, \\ Ax \preccurlyeq b, \\ x \succcurlyeq 0. \end{cases}$$

Where the symbol "$\preccurlyeq$" explains the fuzzy version of the ordinary inequality "$\leq$". These fuzzy inequalities representing in the DM's fuzzy goal and fuzzy constraints mean that "the objective function cx should be essentially smaller than or equal to an aspiration level z_0 of the DM" and "the constraints Ax should be essentially smaller than or equal to b", respectively.
A FFMILP problem with m fuzzy constraints and n variables is formulated as follows:

$$Max\ \ (Min)\ \sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j \qquad \textbf{(3.1)}$$

$$s.t.\ \begin{cases} \sum_{j=1}^{n} \tilde{a}_{ij} \otimes \tilde{x}_j \preccurlyeq, =, \succcurlyeq \tilde{b}_i, & \forall i = 1, \ldots, m, \\ \tilde{x}_j \succcurlyeq 0, & j = 1, \ldots, p, \\ \tilde{x}_j \succcurlyeq 0, \ \ \tilde{x}_j \in \mathbb{Z}, & j = p+1, \ldots, n. \end{cases}$$

Where $\tilde{C}^T = [\tilde{c}_j]_{1\times n}, \tilde{X} = [\tilde{x}_j]_{n\times 1}, \tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_i]_{m\times 1}$and $\tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in \boldsymbol{K}(\mathbb{R})$. According to this definition, the steps of our solution algorithm are as follows:
Initialization Step: Let all $\tilde{c}_j$, $\tilde{x}_j$, $\tilde{a}_{ij}$ and $\tilde{b}_i$ are represented by triangular fuzzy numbers (p_j, q_j, r_j), (a_{ij}, b_{ij}, c_{ij}), (b_i, g_i, h_i) and (x_j, y_j, z_j) respectively. Then, by substituting these values the FFMILP problem, obtained in (3.1), is written as follows:

$$Max\ \ (Min)\ \sum_{j=1}^{n} (p_j, q_j, r_j), \otimes (x_j, y_j, z_j) \qquad \textbf{(3.2)}$$

$$s.t.\ \begin{cases} \sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \preccurlyeq, =, \succcurlyeq (b_i, g_i, h_i), & \forall i = 1, \ldots, m, \\ (x_j, y_j, z_j) \geq 0, & j = 1, \ldots, p, \\ (x_j, x_j, x_j) \geq 0, \ \ x_j \in \mathbb{Z}, & j = p+1, \ldots, n. \end{cases}$$

Step 2: By arithmetic operations defined in subsection 2.2, the fuzzy linear programming problem of Step 1, is converted into the following equivalent problem:

$$Max\,(Min)\ \sum_{j=1}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)=\sum_{j=1}^{n}(\alpha_j,\beta_j,\gamma_j)$$

$$s.t.\begin{cases}\sum_{j=1}^{n} m_{ij}\leq=\geq b_i, & \forall i=1,\dots,m,\\ \sum_{j=1}^{n} n_{ij}\leq=\geq g_i, & \forall i=1,\dots,m,\\ \sum_{j=1}^{n} o_{ij}\leq=\geq h_i, & \forall i=1,\dots,m,\\ (x_j,y_j,z_j)\geq 0, & j=1,\dots,p,\\ x_j\geq 0\ ,\ x_j\in\mathbb{Z}, & j=p+1,\dots,n.\end{cases}$$

Where

$$(a_{ij},b_{ij},c_{ij})\otimes(x_j,y_j,z_j)=(m_{ij},n_{ij},o_{ij}), \qquad \textbf{(3.3)}$$
$$(p_j,q_j,r_j)\otimes(x_j,y_j,z_j)=(\alpha_j,\beta_j,\gamma_j)\,.$$

Step 3: Suppose the problem is in minimizing form, (we can easily expand the problem to the minimizing form), then we convert the objective function into three objectives as follows:

$$Z_1=Max\sum_{j=1}^{n}\beta_j-\alpha_j$$
$$Z_2=Min\sum_{j=1}^{n}\beta_j \qquad \textbf{(3.4)}$$
$$Z_3=Min\sum_{j=1}^{n}\gamma_j-\beta_j$$

$$s.t.\begin{cases}\sum_{j=1}^{n} m_{ij}\leq=\geq b_i, & \forall i=1,\dots,m,\\ \sum_{j=1}^{n} n_{ij}\leq=\geq g_i, & \forall i=1,\dots,m,\\ \sum_{j=1}^{n} o_{ij}\leq=\geq h_i, & \forall i=1,\dots,m,\\ (x_j,y_j,z_j)\geq 0, & j=1,\dots,p,\\ x_j\geq 0\ ,\ x_j\in\mathbb{Z}, & j=p+1,\dots,n.\end{cases}$$

Where (p_j, q_j, r_j), (a_{ij}, b_{ij}, c_{ij}), (b_i, g_i, h_i) and (x_j, y_j, z_j) are triangular fuzzy numbers represent $\tilde{c}_j$, $\tilde{a}_{ij}$, $\tilde{b}_i$ and $\tilde{x}_j$ respectively.

Step 4: Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective function by solving the corresponding model as follows:

$$Z_1{}^{PIS} = Max \sum_{j=1}^{n} \beta_j - \alpha_j \qquad x \in F$$

$$Z_1{}^{NIS} = Min \sum_{j=1}^{n} \beta_j - \alpha_j \qquad x \in F$$

$$Z_2{}^{PIS} = Min \sum_{j=1}^{n} \beta_j \qquad x \in F$$

$$Z_2{}^{NIS} = Max \sum_{j=1}^{n} \beta_j \qquad x \in F$$

$$Z_3{}^{PIS} = Min \sum_{j=1}^{n} \gamma_j - \beta_j \qquad x \in F$$

$$Z_3{}^{NIS} = Max \sum_{j=1}^{n} \gamma_j - \beta_j \qquad x \in F$$

Assumming, $\boldsymbol{F}$ be the set of all constraints, for reducing the computational time, the negative ideal solutions can be estimated as follows. Let v_h^* and $Z_h(v_h^*)$ denote the decision vector associated with the PIS of hth objective function and the corresponding value of hth objective function, respectively.

$$Z_1^{NIS} = Min_{k=1,2,3}\{Z_1(v_k^*)\}.$$

$$Z_h^{NIS} = Max_{k=1,2,3} Z_h(v_k^*), \qquad h = 2, 3.$$

Step 5: Determine a linear membership function for each objective function according to positive and negative ideal points. In practice, $\mu_i(v)\,; i = 1, 2, 3$ presents the satisfaction level of ith objective function for the given solution vector v. The graphs of these membership functions are represented in Figures 1 and 2. See also [37].

Step 6: Convert the auxiliary MILP model into an equivalent single-objective MILP problem by using the following auxiliary crisp formulation:

$$\text{Max } W = \gamma\lambda \; + (1-\gamma) \sum_{i=1}^{3} \theta_i \mu_i(v)$$

$$s.t. \begin{cases} 0 \le \lambda, \gamma \le 1, & \\ \lambda \le \mu_i(v), & i = 1, 2, 3, \\ v \in F(v)\,. & \end{cases}$$

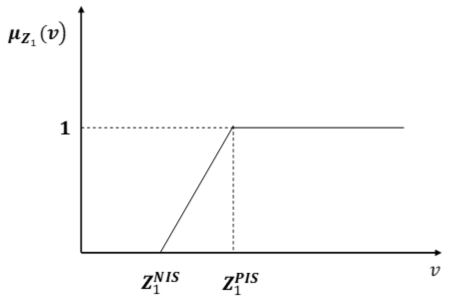

Figure 1: Linear membership function for $\boldsymbol{Z_1}$

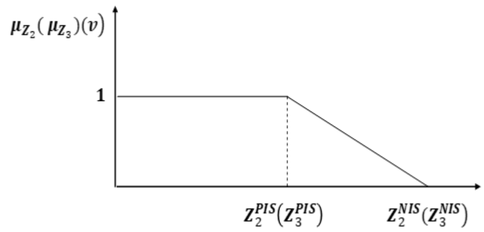

Figure 2: Linear membership function for $\boldsymbol{Z_2(Z_3)}$

where $\mu_i(v); i = 1, 2, 3$ presents the satisfaction level of ith objective function for the given solution vector v and λ denote the minimum satisfaction degree of all objectives. This formulation has a new achievement function defined as a convex combination of the lower bound for satisfaction degree of objectives (λ), and the weighted sum of satisfaction degree of all objectives to ensure yielding an adaptively balanced compromise solution. Moreover, θ_i and γ indicate the relative importance of the ith objective function and the coefficient of compensation, respectively. The selection of θ_i depends to the aims and opinion of decision maker and proportional with the importance of each objective in a proper interval ($\theta_i \in [0, 1]$ and $\sum_i \theta_i = 1$). The main aim in this problem is to find the maximum of minimum satisfaction degree of all objectives in order to find a better solution for the primal FFLP problem.

Step 7: After solving the last problem, the solutions must be put into the objective function of primal FFLP problem in order to find the fuzzy objective value of problem.

4. NUMERICAL EXAMPLES

Here, we give an example to explain the main steps of our solving algorithm. Also, by solving some numerical examples a comparison is presented.

Example 4.1 Consider the following FFMLP problem.

$$Min\ Z = (1,2,3) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 \oplus (0,1,2) \otimes \tilde{x}_3$$

$$s.t. \begin{cases} (0,1,2) \otimes \tilde{x}_1 \oplus (1,2,3) \otimes \tilde{x}_2 \oplus (2,3,4) \otimes \tilde{x}_3 \preccurlyeq (1,10,27), \\ (1,2,3) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 \oplus (0,1,2) \otimes \tilde{x}_3 \succcurlyeq (2,11,28), \\ (3,4,5) \otimes \tilde{x}_1 \oplus (0,1,2) \otimes \tilde{x}_2 \oplus (2,3,4) \otimes \tilde{x}_3 \preccurlyeq (13,17,28), \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \succcurlyeq 0\ ,\ \tilde{x}_3 \in \mathbb{Z}. \end{cases}$$

Based on our proposed method this problem is solved as follows:
Step 1: The problem changes as following:

$$Min\ Z = (1,2,3) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 \oplus (0,1,2) \otimes \tilde{x}_3$$

$$s.t. \begin{cases} (0,1,2) \otimes (x_1,y_1,z_1) \oplus (1,2,\ 3) \otimes (x_2,y_2,z_2) \oplus (2,3,4) \otimes (x_3,y_3,z_3) \preccurlyeq (1,10,27), \\ (1,2,3) \otimes (x_1,y_1,z_1) \oplus (2,3,4) \otimes (x_2,y_2,z_2) \oplus (0,1,2) \otimes (x_3,y_3,z_3) \succcurlyeq (2,11,28), \\ (3,4,5) \otimes (x_1,y_1,z_1) \oplus (0,1,2) \otimes (x_2,y_2,z_2) \oplus (2,3,\ 4) \otimes (x_3,y_3,z_3) \preccurlyeq (13,17,28), \\ (x_1,y_1,z_1)\,,(x_2,y_2,z_2)\,,(x_3,x_3,x_3) \succcurlyeq 0\,(x_3,x_3,x_3) \in \mathbb{Z}. \end{cases}$$

Step 2: Using arithmetic operations, the fuzzy linear programming problem of Step 1, converted into the following equivalent problem:

$$Min\ Z = (x_1 + 2x_2\ ,\ 2y_1 + 3y_2 + y_3, 3z_1 + 4z_2 + 2z_3)$$

$$s.t. \begin{cases} x_2 + 2x_3 \le 1, \\ y_1 + 2y_2 + 3x_3 \le 10, \\ 2z_1 + 3z_2 + 4x_3 \le 27, \\ x_1 + 2x_2 \ge 2, \\ 2y_1 + 3y_2 + x_3 \ge 11, \\ 3z_1 + 4z_2 + 2x_3 \ge 28, \\ 3x_1 + 4z_2x_3 \le 13, \\ 4y_1 + y_2 + 3x_3 \le 17, \\ 5z_1 + 2z_2 + 4x_3 \le 28, \\ (x_1,y_1,z_1),(x_2,y_2,z_2), \\ (x_3,x_3,x_3) \succcurlyeq 0\ ,(x_3,x_3,x_3) \in \mathbb{Z}. \end{cases}$$

Step 3: The objective function is converted into three following objective functions:

$$U_1 = Max(2y_1 + 3y_2 + y_3 - x_1 - 2x_2),$$

$$U_2 = Min(2y_1 + 3y_2 + y_3),$$

$$U_3 = Min(3z_1 + 4z_2 + z_3 - 2y_1 - 3y_{2)}.$$

Table 1: The positive and negative ideal points

i	U_i^{PIS}	U_i^{NIS}
1	14.714	9
2	11	16.714
3	11.286	17.002

Step 4: The positive and negative ideal points for each objective is determined as follows:
Step 5: Based on ZIP and NIS, membership functions for each objective is determined as follows:

$$\mu_1(v) = \begin{cases} 1, & U_1 > 14.714, \\ \dfrac{9 - U_1}{9 - 14.714}, & 9 \le U_1 \le 14.714, \\ 0, & U_1 < 9. \end{cases}$$

$$\mu_2(v) = \begin{cases} 1, & U_2 < 11, \\ \dfrac{16.714 - U_2}{16.714 - 11}, & 11 \le U_2 \le 16.714, \\ 0, & 16.714 < U_2. \end{cases}$$

$$\mu_3(v) = \begin{cases} 1, & U_3 < 11.286, \\ \dfrac{17.002 - U_3}{17.002 - 11.286}, & 11.286 \le U_3 \le 17.002, \\ 0, & U_3 > 17.002. \end{cases}$$

Step 6: Convert the auxiliary LP model into an equivalent single-objective LP problem by using the following auxiliary crisp formulation.

$$Max\ W = \gamma\lambda + (1-\gamma)\sum_{i=1}^{3} \theta_i \mu_i(v)$$

$$s.t, \begin{cases} 0 \le \lambda, \gamma \le 1, \\ \lambda \le \mu_i(v), & i = 1, 2, 3, \\ v \in F(v). \end{cases}$$

The optimal solution for $\gamma = 0,\ \theta_1 = \theta_3 = \frac{1}{6}, \theta_2 = \frac{4}{6}$ is obtained as follows:

$$\lambda = 00.274,\ U_1 = 9,\ U_2 = 11,\ U_3 = 17,$$

$$\tilde{x}_1^* = (x_1^*, y_1^*, z_1^*) = (0,\ 0,\ 4),$$
$$\tilde{x}_2^* = (x_2^*, y_2^*, z_2^*) = (\ 1, 3.666, 4),$$
$$\tilde{x}_3^* = (x_3^*, x_3^*, x_3^*) = (0, 0, 0).$$

By these values, the objective value of the problem is as follows:

$$Z^* = (x_1^* + 2x_2^*\ ,\ 2y_1^* + 3y_2^* + x_3^*, 3z_1^* + 4z_2^* + 2x_3^*) = (2, 11, 28).$$

The optimal solution of our proposed method is $Z^* = (2, 11, 28)$.

Example 4.2 Because there is no example in the literature for FFMILP to evaluate the method, we compare it in partial form of FFLP with other similar one in literature. Consider the following FFLP problem:

$$Min\ Z = (1, 6, 9) \otimes \tilde{x}_1 \oplus (2, 2, 8) \otimes \tilde{x}_2$$
$$s.t. \begin{cases} (0, 1, 1) \otimes \tilde{x}_1 \oplus (2, 2, 3) \otimes \tilde{x}_2 \succcurlyeq (4, 7, 14), \\ (2, 2, 3) \otimes \tilde{x}_1 \oplus (-1, 4, 4) \otimes \tilde{x}_2 \preccurlyeq (-4, 14, 22), \\ (2, 3, 4) \otimes \tilde{x}_1 \ominus (1, 2, 3) \otimes \tilde{x}_2 \preccurlyeq (-12, -3, 6). \end{cases}$$

$\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers.

By considering $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$, the original FFLP problem changes as follows:

$$MinZ = (1, 6, 9) \otimes (x_1, y_1, z_1) \oplus (2, 2, 8) \otimes (x_2, y_2, z_2)$$
$$s.t. \begin{cases} (0, 1, 1) \otimes (x_1, y_1, z_1) \oplus (2, 2, 3) \otimes (x_2, y_2, z_2) \succcurlyeq (4, 7, 14), \\ (2, 2, 3) \otimes (x_1, y_1, z_1) \oplus (-1, 4, 4) \otimes (x_2, y_2, z_2) \preccurlyeq (-4, 14, 22), \\ (2, 3, 4) \otimes (x_1, y_1, z_1) \ominus (1, 2, 3) \otimes (x_2, y_2, z_2) \preccurlyeq (-12, -3, 6). \end{cases}$$

All variables are non-negative triangular fuzzy numbers.

According to our new presented algorithm, this problem is converted to the following equivalent crisp single-objective LP:

$$Max\ W = \lambda\ \gamma + (1-\gamma)\sum_{I=1}^{3}\theta_i\mu_i(v)$$

$$s.t \begin{cases} 5\lambda - Z_1 \leq -3, \\ 5\lambda + Z_2 \leq 12, \\ 17.6667\lambda + Z_3 \leq 43, \\ 2x_2 \geq 4, \\ y_1 + 2y_2 \geq 7, \\ z_1 + 3z_2 \geq 14, \\ 2x_1 - z_2 \leq -4, \\ 2y_1 + 4y_2 \leq 14, \\ 3x_1 + 4z_2 \leq 22, \\ 2x_1 - 3z_2 \leq -12, \\ 3y_1 - 2y_2 \leq -3, \\ 4z_1 - x_2 \leq 6, \\ y_1 - x_1 \geq 0, y_2 - x_2 \geq 0, \\ z_1 - y_1 \geq 0, z_2 - y_2 \geq 0. \end{cases}$$

This problem is a conventional linear programming problem. The optimal solution of this problem for $\gamma = 0.5$, $\theta_1 = 1, \theta_2 = 4$ and $\theta_3 = 1$ is obtained as follows:

$$\tilde{x}_1 = (x_1, y_1, z_1) = (0,0,0)\,,\ \tilde{x}_2 = (x_2, y_2, z_2) = (2, 3.5, 4, 6667).$$

Then, the objective value of the problem is as follows:

$$Z^* = (1,2,3) \otimes (0,0,0) \oplus (2,3,4) \otimes (2,3.5,4,6667) = (4,7,37.333).$$

By solving this problem with the proposed method by [12], the optimal solution is $Z^0 = (4,\ 7, 37.333)$. In comparison with Z^0, our solution for this problem is equal with the solution achieving in [12].

Example 4.3 Consider the problem with non-negative variables as follows.

$$Max\ Z = (2,5,8) \otimes \tilde{x}_1 \oplus (3, \frac{37}{6}, 10) \otimes \tilde{x}_2 \oplus (5, \frac{34}{3}, 15) \otimes \tilde{x}_3$$

$$s.t. \begin{cases} (2,5,8) \otimes \tilde{x}_1 \oplus (3, \frac{41}{6}, 10) \otimes \tilde{x}_2 \oplus (5, \frac{31}{3}, 18) \otimes \tilde{x}_3 \preccurlyeq (6, \frac{50}{3}, 30), \\ (4, \frac{32}{3}, 12) \otimes \tilde{x}_1 \oplus (5, \frac{73}{6}, 20) \otimes \tilde{x}_2 (7, \frac{105}{6}, 30) \otimes \tilde{x}_3 \preccurlyeq (10, 30, 50), \\ (3,5,7) \otimes \tilde{x}_1 \oplus (5,15,20) \otimes \tilde{x}_2 (5,10,15) \otimes \tilde{x}_3 \preccurlyeq (2, \frac{145}{6}, 30). \end{cases}$$

$\tilde{x}_1$, $\tilde{x}_2$ and $\tilde{x}_3$ are non-negative triangular fuzzy numbers.

Based on our new algorithm, for $\gamma = 0.5$, $\theta_1 = \frac{1}{6}, \theta_2 = \frac{4}{6}$ and $\theta_3 = \frac{1}{6}$, the optimal solution of this problem is obtained as follows:

$$\tilde{x}_1 = (x_1, y_1, z_1) = (0, 0, 0.1209677), \tilde{x}_2 = (x_2, y_2, z_2) = (0, 0, 0),$$

$$\tilde{x}_3 = (x_3, y_3, z_3) = (0.4, 1.612903, 1.612903).$$

The objective value of the problem is $Z^* = (2,\ 18.27957,\ 25.16129)$. The optimal solution of the proposed method is $Z^* = (2,\ 18.27957,\ 25.16129)$. By solving this example with proposed method by [12] the optimal solution is $Z^0 = (2,\ 14.21701\ ,30)$. In comparison with Z^0, our solution for this problem is better than the solution achieving in [12].

Example 4.4 Consider the following FILP problem.

$$Max\ \tilde{Z} = 4\tilde{x}_1 + 3\tilde{x}_2$$
$$s.t. \begin{cases} \tilde{x}_1 + 2\tilde{x}_2 \succcurlyeq (4, 8, 12), \\ 2\tilde{x}_1 + \tilde{x}_2 \preccurlyeq (6, 9, 12), \\ \tilde{x}_1, \tilde{x}_2 \succcurlyeq 0 \ and\ integers. \end{cases}$$

The comparison in Table 2 illustrates that our proposed method acheives more effective solution than other previous methods. Accordingly, the obtained optimal solutions of these three methods are depicted in Fig. 3.

Table 2: Comparative results of the example

Methods	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{Z}$
Pandian and Jayalakshmi [34]	$(3, 4, 5)$	$(0, 1, 2)$	$(12, 19, 26)$
Sudhagar and Ganesan [38]	$(4, 4, 4)$	$(0, 1, 3)$	$(16, 19, 25)$
New Method	$(4, 4, 4)$	$(1, 1, 1)$	$(19, 19, 19)$

5. APPLICATION OF THE PROPOSED METHOD IN REAL LIFE PROBLEMS

In this section, in order to illustrate the application of the proposed method, two real case studies from Gandhi Cloth and Stockco Companies are solved. The obtained results demonstrate the validity and efficient performance of the proposed method.

Example 5.1 Gandhi Cloth Company is capable of manufacturing cloth and shirt. For manufacturing each of these products the amounts of string and labor is required which is shown in Table 3. Each week, 150 hours of labor and 160 sq yd of string are available. The selling price for cloth and shirt are shown in Table 4. Formulate an MIP whose solution will maximize Gandhi's weekly profits.

Solution. Let x_1 and x_2 are the amount of cloths and shirts to beproduced. Then, the above problem is formulated as:

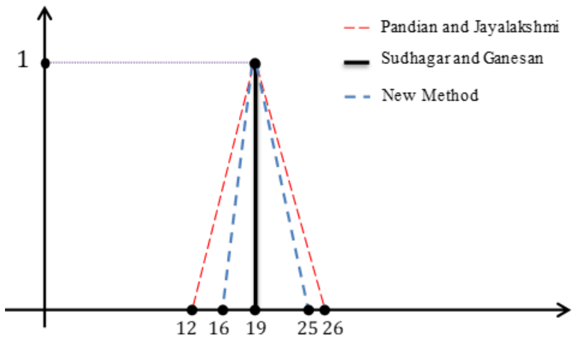

Figure 3: Membership function of the new method vs existing methods

Table 3: Resource Requirements for Gandhi

Products	Labor (Hours)	String(Square Yards)
Shirt	3	4
cloth	2	3

$$Max\ Z=\tilde{5}\otimes\tilde{x}_1\oplus\tilde{4}\otimes\tilde{x}_2$$

$$s.t.\begin{cases}\tilde{3}\otimes\tilde{x}_1\oplus\tilde{2}\otimes\tilde{x}_2\preccurlyeq 1\tilde{5}0,\\ \tilde{4}\otimes\tilde{x}_1\oplus\tilde{3}\otimes\tilde{x}_2\preccurlyeq 1\tilde{6}0,\\ \tilde{x}_1\succcurlyeq 0, \tilde{x}_2\in\mathbb{Z}.\end{cases}$$

Now we take the coefficients of variables as follows respectively:

$$\tilde{5}=(5,6,8),\ \tilde{4}=(4,4,4),\ \tilde{3}=(2,3,5),\ \tilde{2}=(2,2,2),\ 1\tilde{5}0=(140,150,150),$$

$$\tilde{4}=(4,4,7),\ \tilde{3}=(2,3,4),\ 1\tilde{6}0=(155,160,165).$$

Now, the problem can be written as:

$$Max\ Z=(5,6,8)\otimes\tilde{x}_1\oplus(4,4,4)\otimes\tilde{x}_2$$

$$s.t.\begin{cases}(2,3,5)\otimes\tilde{x}_1\oplus(2,2,2)\otimes\tilde{x}_2\preccurlyeq(140,150,150),\\ (4,4,7)\otimes\tilde{x}_1\oplus(2,3,4)\otimes\tilde{x}_2\preccurlyeq(155,160,165),\\ \tilde{x}_1\succcurlyeq 0, \tilde{x}_2\in\mathbb{Z}.\end{cases}$$

$\tilde{x}_1$ and $\tilde{x}_2$ are non-negative triangular fuzzy numbers.

By considering $\tilde{x}_1=(x_1,y_1,z_1)$ and $\tilde{x}_2=(x_2,y_2,z_2)$, the original FFLP problem changes as follows:

Table 4: Cost Information for Gandhi

Products	Sales Price ($)
Shirt	5
cloth	4

$$Max\ Z{=}(5,6,8){\otimes}(x_1,y_1,z_1){\oplus}(4,4,4){\otimes}(x_2,y_2,z_2)$$

$$s.t.\begin{cases}(2,3,5){\otimes}(x_1,y_1,z_1){\oplus}(2,2,2){\otimes}(x_2,y_2,z_2){\preccurlyeq}(140,150,150),\\(4,4,7){\otimes}(x_1,y_1,z_1){\oplus}(2,3,4){\otimes}(x_2,y_2,z_2){\preccurlyeq}(155,160,165),\\ \quad x_1,y_1,z_1{\succcurlyeq}0,x_2,y_2,z_2{\in}\mathbb{Z}.\end{cases}$$

Using arithmetic operations and Steps 2 and 3, the mentioned FFLP problem reduced to the following problem:

$$Z_1{=}Min\ (6y_1{+}4y_2{-}5x_1{-}4x_2)$$

$$Z_2{=}Max\ (6y_1{+}4y_2)$$

$$Z_3{=}Max\ (8z_1+4z_2{-}6y_1{-}4y_2)$$

$$s.t.\begin{cases}2x_1+2x_2{\le}140,\\3y_1+2y_2{\le}150,\\5z_1{+}2z_2{\ge}150,\\4x_1{+}2x_2{\le}150,\\4y_1{+}3y_2{\le}160,\\7z_1{+}2z_2{\le}165,\\y_1{-}x_1{\ge}0,y_2{-}x_2{=}\ 0,\\z_1{-}y_1{\ge}0,z_2{-}y_2{=}\ 0.\end{cases}$$

According Step 4, we have:

$$Z_1^{PIS}{=}\ 0, \qquad Z_1^{NIS}{=}\ 141.426,$$

$$Z_2^{PIS}{=}\ 222\ , \qquad Z_2^{NIS}{=}\ 141.426,$$

$$Z_3^{PIS}{=}\ 330, \qquad Z_3^{NIS}{=}\ -37.712.$$

The optimal solution of this problem for $\gamma = 0,\ \theta_1 = \theta_3 = \frac{1}{6}, \theta_2 = \frac{4}{6}$ is obtained as follows:

$$\tilde{x}_1^*{=}\ (x_1^*,y_1^*,z_1^*) = (0.25, 0.25,\ 8.429),$$

$$\tilde{x}_2^*{=}\ (x_2^*,y_2^*,z_2^*) = (\ 53, 53, 53).$$

By these values, the objective value is as follows:

$$Z^{*}=(5x_1^{*}+4x_2^{*}\ ,\ 6y_1^{*}+4y_2^{*}, 8z_1^{*}+4z_2^{*}) = (213, 213.5, 279.432).$$

The optimal solution of our proposed method is $Z^{*} = (213, 213.5, 279.432)$.

Example 5.2 Stockco is considering four investments. Investment 1 will yield a net present value (NPV) of $16,000; investment 2, an NPV of $22,000; investment 3, an NPV of $12,000; and investment 4, an NPV of $8,000. Each investment requires a certain cash outflow at the present time: investment 1, $5,000; investment 2, $7,000; investment 3, $4,000; and investment 4, $3,000. Currently, $14,000 is available for investment. Formulate an IP whose solution will tell Stockco how to maximize the NPV obtained from investments 1–4.

This real world problem is modeled as the following FFMILP problem:

$$Max\ \mathrm{Z}=(15, 16, 17)\otimes\ \tilde{\mathrm{x}}_1\oplus(20, 22, 23)\otimes\ \widetilde{\mathrm{x}}_2\ \oplus(12, 12, 12)\otimes\ \tilde{\mathrm{x}}_3\oplus(8, 9, 9)\otimes\ \widetilde{\mathrm{x}}_4$$

$$s.t.\ 5\otimes\tilde{\mathrm{x}}_1\oplus7\otimes\tilde{\mathrm{x}}_2\oplus4\otimes\tilde{\mathrm{x}}_3\oplus3\otimes\tilde{\mathrm{x}}_4\preccurlyeq14,$$

$$\tilde{\mathrm{x}}_{\mathrm{i}}\in\{0,1\}\ ,\ \forall i\in\{1,\ldots,4\}\ .$$

The optimal solution of this problem for $\gamma= 0$, $\theta_1{=}\theta_3{=}\frac{1}{6}, \theta_2{=}\frac{4}{6}$ is obtained as follows:

$$\tilde{x}_1^{*} = (x_1^{*}, y_1^{*}, z_1^{*}) = (1, 1, 1),$$

$$\tilde{\mathrm{x}}_2^{*}= (\mathrm{x}_2^{*}, \mathrm{y}_2^{*}, \mathrm{z}_2^{*}) = (1, 1, 1),$$

$$\widetilde{\mathrm{x}}_3^{*}= (\mathrm{x}_3^{*}, \mathrm{y}_3^{*}, \mathrm{z}_3^{*}) = (0, 0, 0),$$

$$\widetilde{\mathrm{x}}_4^{*}= (\mathrm{x}_4^{*}, \mathrm{y}_4^{*}, \mathrm{z}_4^{*}) = (0, 0, 0).$$

By these values, the objective value is $\mathrm{Z}^{*}= (35, 38, 40)$.

6. CONCLUSIONS

In this paper, a new method for solving FFMILP is presented. This method is based on fuzzy interactive method. By using the proposed method, fully fuzzy optimal solution of FFLP, occurring in a real life situation, can be easily obtained. Though, as there is no example in the literature for FFMILP to evaluate the method, we compare it in partial form of FFLP with a similar method in literature. Computational results and illustrative numerical examples show better performance of this method in comparison with other existing methods and also better solutions. In addition, two real-world problems are solved to illustrate the application of the proposed method.

REFERENCES

[1] Taghi-nezhad, N. S., Nasseri, H., Khalili Goudarzi, F., and Taleshian Jelodar, F., "Reactive Scheduling Presentation for an Open Shop problem Focused on jobs' due Dates", *Journal of Production and Operations Management*, 6 (2) (2015) 95–112.

[2] Nasseri, S.H., Taghi-Nezhad, N., and Ebrahimnejad, A., "A Note on Ranking Fuzzy Numbers with an Area Method using Circumcenter of Centroids", *Fuzzy Information and Engineering*, 9(2) (2017) 259–268.

[3] F. Taleshian, J. Fathali and N. A. Taghi-Nezhad, "Fuzzy majority algorithms for the 1-median and 2-median problems on a fuzzy tree", *Fuzzy Information and Engineering*, 10 (2) (2018) 225–248.

[4] Nasseri, S. H., Taghi-Nezhad, N. A., and Ebrahimnejad, A., "A novel method for ranking fuzzy quantities using center of incircle and its application to a petroleum distribution center evaluation problem", *International Journal of Industrial and Systems Engineering*, 27(4) (2017) 457–484.

[5] Taghi-Nezhad, N., "The p-median problem in fuzzy environment: proving fuzzy vertex optimality theorem and its application", *Soft Computing*, 23 (22) (2019) 11399–11407.

[6] Bellman, R., and Zadeh, L., "Decision making in a fuzzy environment", *Management Science*, 17 (1970) 141–164.

[7] Kumar, A., Kaur, J., and Singh, P., "Fuzzy Optimal Solution of Fully Fuzzy Linear Programming Problems with Inequality Constraints", *International Journal of Mathematical and Computer Sciences*, 6 (1) (2010) 37–41.

[8] Jimenez, M., Arenas, M., Bilbao, A., and Victoria Rodrıguez, M., "Linear programming with fuzzy parameters: An interactive method resolution", *European Journal of Operational Research*, 177 (2007) 1599–1609.

[9] Nasseri, S. H., Khalili, F., Taghi-Nezhad, N., and Mortezania, S., "A novel approach for solving fully fuzzy linear programming problems using membership function concepts", *Annals of Fuzzy Mathematics and Informatics*, 7 (3) (2014) 355–368.

[10] Allahviranloo, T., Lotfi, F. H., Kiasary, M., Kiani, N., and Alizadeh, L., "Solving full fuzzy linear programming problem by the ranking function", *Applied Mathematical Scienced*, 2 (2008) 19–32.

[11] Lotfi, F. H., Allahviranloo, T., Jondabeh, M. A., and Alizadeh, L., "Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution", *Applied Mathematical Modeling*, 33 (2009) 3151–3156.

[12] Kumar, A., Kaur, J., and Singh, P., "A new method for solving fully fuzzy linear programming problems", *Applied Mathematical Modeling*, 35 (2011) 817–823.

[13] Kaur, J., and Kumar, A., "Exact fuzzy optimal solution of fully fuzzy linear programming problems with unrestricted fuzzy variables", *Applied Intelligence*, 37 (1) (2012) 145–154.

[14] Khan, I. U., Ahmad, T., and Maan, N., "A simplified novel technique for solving fully fuzzy linear programming problems", *Journal of optimization theory and applications*, 159 (2) (2013) 536–546.

[15] Stanojević, B., "Penalty method for fuzzy linear programming with trapezoidal numbers", *Yugoslav Journal of Operations Research*, 19 (1) (2016) 149–156.

[16] Pradhan, A., and Biswal, M. P., "Linear Programming Problems with some Multi-Choice Fuzzy Parameters", *Yugoslav Journal of Operations Research*, 28 (2) (2018) 249–264.

[17] Ezzati, R., Khorram, E., and Enayati, R., "A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem", *Applied mathematical modelling*, 39 (12) (2015) 3183–3193.

[18] Das, S. K., Mandal, T., and Edalatpanah, S. A., "A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers", *Applied intelligence*, 46 (3) (2017) 509–519.

[19] Skandari, M. N., and Ghaznavi, M., "An Efficient Algorithm for Solving Fuzzy Linear Programming Problems", *Neural Processing Letters*, 46 (3) (2018) 1–20.

[20] Winston, W. L., and Goldberg, B. J., "Operations Research: Applications and Algorithms", *Calif, USA: Thomson/Brooks/Cole, Belmont*, Canada (2004).

[21] Allahviranloo, T., Shamsolkotabi, K., Kiani, N. A., and Alizadeh, L., "Fuzzy integer linear

programming problems", *International Journal of Contemporary Mathematical Sciences*, 2 (4) (2007) 167–181.

[22] Herrera, F., and Verdegay, J. L., "Theory and methodology three models of fuzzy integer linear programming", *European Journal of Operation Research*, 83 (1995) 581–593.

[23] Mansini, R., Ogryczak, W., and Speranza, M., "Twenty years of linear programming based portfolio optimization", *European Journal of Operation Research*, 234 (2) (2014) 518–535.

[24] Martinovic, J., and Scheithauer, G., "Integer linear programming models for the skiving stock problem", *European Journal of Operation Research*, 251 (2) (2016) 356–368.

[25] Mula, J., Lyons, A. C., Hernández, J. E., and Poler, R., "An integer linear programming model to support customer-driven material planning in synchronised, multi-tier supply chains ", *International Journal of Production Research*, 52 (14) (2014) 4267–4278.

[26] Bredström, D., Haugen, K., Olstad, A., and Novotný, J., "A mixed integer linear programming model applied in barge planning for Omya ", *Operations Research Perspectives*, 2 (2015) 150–155.

[27] Fischetti, M., Monaci, M., and Salvagnin, D., "Mixed-integer linear programming heuristics for the prepack optimization problem", *Discrete Optimization*, 22 (2016) 195–205.

[28] Khalili Goodarzi, F., Taghinezhad, N. A., and Nasseri, S. H., "A new fuzzy approach to solve a novel model of open shop scheduling problem", *University Politehnica of Bucharest, Scientific Bulletin-Series A-Applied Mathematics and Physics*, 76 (3) (2014) 199–210.

[29] Bogdanović, M., Maksimović, Z., Simić, A., and Milošević, J., "A mixed integer linear programming formulation for low discrepancy consecutive k-sums permutation problem", *Yugoslav Journal of Operations Research*, 27(1) (2016) 125–132.

[30] Ubando, A. T., et al. "A fuzzy mixed-integer linear programming model for optimal design of polygeneration systems with cyclic loads", *Environmental Progress and Sustainable Energy*, 35 (4) (2016) 1105–1112.

[31] Ubando, A. T., Marfori, I. A. V., Aviso, K. B., and Tan, R. R., "Optimal Operational Adjustment of a Community-Based Off-Grid Polygeneration Plant using a Fuzzy Mixed Integer Linear Programming Model", *Energies*, 12 (4) (2019) 636–653.

[32] Arana-Jiménez, M., and Blanco, V., "On a fully fuzzy framework for minimax mixed integer linear programming", *Computers and Industrial Engineering*, 128 (2019) 170–179.

[33] Luna, A. C., Diaz, N. L., Graells, M., Vasquez, J. C., and Guerrero, J. M., "Mixed-integer-linear-programming-based energy management system for hybrid PV-wind-battery microgrids: Modeling, design, and experimental verification", *IEEE Transactions on Power Electronics*, 32 (4)(2016) 2769–2783.

[34] Pandian, P., and Jayalakshmi, M., "A new method for solving integer linear programming problems with fuzzy variables", *Applied mathematical sciences*, 4 (20)(2010) 997–1004.

[35] Sakawa, M., *Fuzzy sets and interactive multi objective optimization*, Applied Information Technology Springer Science and Business Media, New York, USA, 2013.

[36] Zimmermann, H., "Fuzzy sets, decision making, and expert systems", *Springer Science and Business Media*, 10 (2012).

[37] Torabi, S. T., and Hassini, E., "An interactive possibilistic programming approach for multiple objective supply chain master planning", *Fuzzy Sets and Systems*, 159 (2008) 193–214.

[38] Sudhagar, C., and Ganesan, K., "Fuzzy integer linear programming with fuzzy decision variables", *Applied Mathematical Sciences*, 4 (70) (2010) 3493–3502.